This PDF is a selection from an out-of-print volume from the National Bureau of Economic Research

Volume Title: Schooling, Experience, and Earnings

Volume Author/Editor: Jacob A. Mincer

Volume Publisher: Columbia University Press

Volume ISBN: 0-870-14265-8

Volume URL: http://www.nber.org/books/minc74-1

Publication Date: 1974

Chapter Title: Distribution Analysis

Chapter Author: Jacob A. Mincer

Chapter URL: http://www.nber.org/chapters/c1764

Chapter pages in book: (p. 24 - 40)

2

Distribution Analysis

2.1 THE SCHOOLING MODEL

The analysis of individual earnings is now adapted to a cross section of workers. I begin with schooling, that is, I restrict human capital investment to schooling alone. In equation (2.1) a subscript i is now attached to the variables Y (earnings) and s (years of schooling) in order to distinguish individual differences in them. For simplicity, I initially disregard individual differences in the (average) rate of return r and in original earning capacity Y_0. The symbol Y_s denotes hypothetical earnings of an individual who does not continue to invest in human capital after the completion of s years of schooling.

$$\ln Y_{si} = \ln Y_0 + rs_i. \tag{2.1}$$

Even at this primitive stage, several important and rather realistic implications follow for the personal distribution of earnings:

1. The positive skewness that is almost always exhibited by distributions of income or earnings may be partly due to the effect of the logarithmic transformation, which converts absolute differences in schooling into percentage differences in earnings. Clearly,

a symmetric distribution of schooling implies a positively skewed distribution of earnings. Indeed, a positive skew of earnings cannot be avoided unless the distribution of schooling is strongly skewed in the negative direction.

2. The larger the dispersion in the distribution of schooling, the larger the relative dispersion and skewness in the distribution of earnings.

3. The higher the rate of return to schooling, the larger the earnings inequality and skewness.

The implications for inequality are shown by taking variances in equation (2.1), assuming both Y_0 and r to be fixed:

$$\sigma^2(\ln Y_s) = r^2\sigma^2(s). \tag{2.2}$$

The implications for skewness are shown most simply in a nonparametric formulation: Let Y_1 be a lower percentile level of earnings corresponding to an s_1 level of schooling; Y_2, symmetric upper percentile corresponding to s_2; Y_m, median earnings; and s_m, median schooling.

Assume first a symmetric distribution of schooling, so that $s_2 - s_m = s_m - s_1 = d$. The absolute (dollar) dispersion in the earnings distribution is $D = Y_2 - Y_1 = (e^{2rd} - 1)Y_1$; so $Y_2 = e^{2rd}Y_1$. The relative dispersion $RD = Y_2/Y_1 = e^{2rd}$. Positive skewness exists when $(Y_1 + Y_2)/2 > Y_m$. A measure of it is

$$\begin{aligned} Sk &= \tfrac{1}{2}(Y_2 - 2Y_m + Y_1) = \tfrac{1}{2}(e^{2rd} - 2e^{rd} + 1)Y_1 \\ &= \tfrac{1}{2}(e^{rd} - 1)^2 Y_1 > 0. \end{aligned}$$

Using Bowley's formula, relative skewness is defined as,

$$RSk = \frac{(Y_2 - Y_m) - (Y_m - Y_1)}{Y_2 - Y_1} = \frac{Sk}{\tfrac{1}{2}(Y_2 - Y_1)} = \frac{(e^{rd} - 1)^2}{e^{2rd} - 1} = \frac{e^{rd} - 1}{e^{rd} + 1} > 0. \tag{2.3}$$

Both measures of skewness are necessarily positive, and all measures of dispersion and skewness are positive functions of the dispersion in the distribution of schooling (d) and of the rate of return on investment in schooling (r).

These conclusions remain unchanged when the distribution of schooling is not symmetric, except that the degree of skewness in the earnings distribution is now additionally affected by the degree of skewness in the distribution of schooling. Let $s_2 - s_m = d_2$; and $s_m - s_1 = d_1$; $d_2 \neq d_1$. Then

$$s_k = Y_2 - 2Y_m + Y_1 = [e^{r(d_1+d_2)} - 2e^{rd_1} + 1]Y_1. \quad (2.4)$$

Let $d_2 = d_1 + \Delta$.

Let relative skewness in the distribution of schooling be defined by $sks = \Delta/d_1$. Then it can be shown (by approximation [1]) that even when sks is negative, the distribution of earnings remains positively skewed when the absolute value of sks does not exceed rd_1 and the latter is less than unity:

$$\frac{d_1 - d_2}{d_1} < rd_1 < 1. \quad (2.5)$$

The empirical usefulness of the schooling model formulated in equation (2.1) may be questioned on two grounds: The initial earnings level Y_0 and the rate of return r cannot be assumed to be the same at all levels of schooling and for all persons. It is merely a convenient simplifying assumption. But if individual values of r are independent of s, and (2.1) is used as a statistical estimating equation, then r must be thought of as an average over all schooling levels and

1. Skewness in the distribution of earnings is positive when

$$e^{2rd_1+r\Delta} - 2e^{rd_1} + 1 > 0$$

or

$$(e^{rd_1} - 1)^2 + e^{2rd_1}(e^{r\Delta} - 1) > 0;\ (e^{rd_1} - 1)^2 > e^{2rd_1}(1 - e^{r\Delta})$$

when $\Delta < 0$, $1 - e^{r\Delta} > 0$.

Taking square roots:

$$e^{rd_1} - 1 > e^{rd_1}\sqrt{1 - e^{r\Delta}};\ e^{rd_1} > \frac{1}{1 - \sqrt{1 - e^{r\Delta}}}.$$

Taking logs: $rd_1 > -\ln(1 - \sqrt{1 - e^{r\Delta}})$. This condition holds when $rd_1 > \sqrt{1 - e^{r\Delta}}$, since for $x < 1$, $x > -\ln(1 - x)$, by the Taylor expansion. Hence $e^{r\Delta} > 1 - r^2d_1^2$ is a sufficient condition for positive skewness in earnings when $\Delta < 0$.

Again, assuming $r^2d_1^2$ sufficiently small, and taking logs, $r\Delta > -r^2d_1^2$, and so $|\Delta/d_1| < rd_1 < 1$ is a sufficient condition for positive skewness in earnings, when $\Delta < 0$.

This condition can also be written as $rd_2 > rd_1(1 - rd_1)$. It is always fulfilled when $rd_2 > 0.25$, so long as $rd_1 < 1$. Skewness was defined with respect to a particular $(s_1 - s_2)$ interval in the distribution of schooling. Therefore, so long as an s_2 can be found such that $r(s_2 - s_m) > 0.25$, where s_m is median schooling, the distribution of earnings must be positively skewed in that interval.

persons, and individual differences in r (and in ln Y_0) are impounded in the statistical residual.

Let rates of return differ by schooling level. Then equation (2.1) becomes

$$\ln Y_s = \ln Y_0 + \sum_{t=1}^{s} r_t = \ln Y_0 + \bar{r}s,$$

where r_t is the marginal rate of return for a particular level of schooling, and $\bar{r}$ is the average. If $\bar{r}$ is not the same for all individuals (i), then

$$\ln Y_{si} = \ln Y_{0i} + \bar{r}_i s_i. \quad (2.6)$$

Now the inequality of earnings in a group is affected not only by the dispersion in schooling and by the average size of the rate of return, as indicated by equation (2.2), but also by the dispersion in the rates of return and by the average level of schooling. This is clearly seen in the case where $\bar{r}_i$ and s_i are independent.[2] Ignoring variation in Y_0:

$$\sigma^2(\ln Y_s) = \bar{r}^2\sigma^2(s) + \bar{s}^2\sigma^2(r) + \sigma^2(s)\sigma^2(r). \quad (2.7)$$

Here $\bar{r}$ is the average of $\bar{r}_i$ across individuals.

Should it not be assumed that $\bar{r}_i$ and s_i are positively related? Presumably, persons who can benefit more (get larger returns) from given amounts of investment will invest more. However, the average rate of return of an individual, $\bar{r}_i$, ceases to be an index of his ability to benefit from schooling investment when individuals with differing amounts of investment are compared, because $\bar{r}_i$ depends, in part, on the level of investment. As spelled out by Becker (1967), the condition for a positive correlation between $\bar{r}_i$ and s_i in a cross section is that the dispersion of "abilities" (levels of demand curves for investment funds) exceed the dispersion of "opportunities" (levels of investment fund supply curves).

There are no a priori reasons for specifying which dispersion is greater, and the empirical evidence[3] suggests there is little if any correlation between rates of return and quantities invested across

2. By a theorem of L. Goodman (1960).

3. See Tables 3.3 and 4.4 in Part II. In bodies of data in which Y_{0i}, $\bar{r}_i$, and s_i are correlated, empirical estimates of the coefficient of $\bar{r}$ will be biased. In that case the expression for the inequality of earnings (2.7) will contain additional variance and covariance terms.

individuals. Hereafter, the symbol $\bar{r}$ will not be used. Instead, unless it is otherwise stated, r will denote the average rate of return.

By definition, the schooling model described in (2.1) applies to the earnings of individuals who do not make post-school investments in human capital. Because such individuals are rare, these earnings cannot be directly observed. They can be rather crudely estimated, as explained in Chapter 1, by earnings at the overtaking stage of the life cycle (see Figure 1.2). In the following section, the earnings model is expanded beyond schooling to take account of variation in earnings due to life-cycle and individual differences in the distribution of post-school investments.

2.2 COMPARATIVE ANALYSIS OF EARNINGS PROFILES

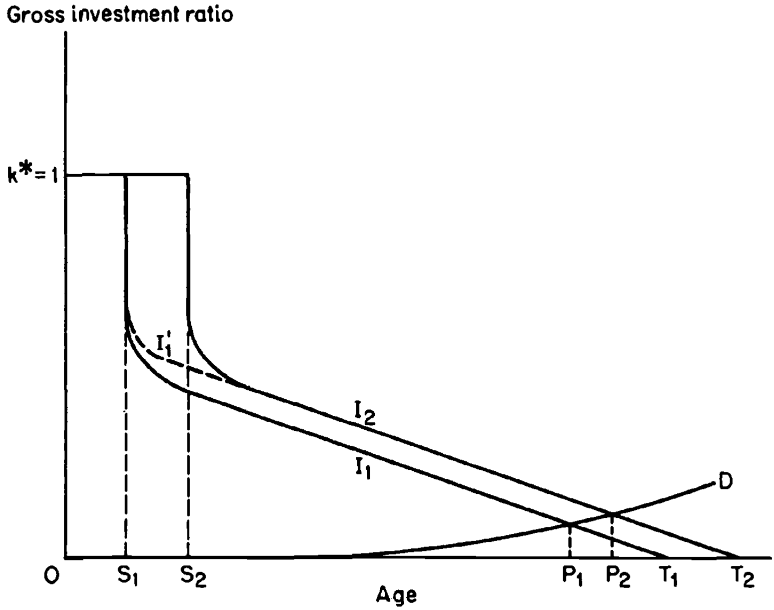
Figure 2.1 portrays investment profiles of three individuals whose gross investment at each age is measured in "time-equivalent" units (k^*), that is, as a ratio to earning capacity. The three investment profiles $I_i = k^*_{ij}$ and a common depreciation curve $D = (\delta/r)_j$ are drawn schematically. Here i denotes the individual, j his age, δ the depreciation rate.

Individuals who invest more than others have their investment profiles shifted upward. I'_1 describes investment behavior of an individual with the same level of schooling (s_1) as I_1, but larger post-school investments while I_2 describes the investment profile of a person with more schooling (s_2) than I_1 but the same level of post-school investment as I'_1. I_1 and I_2 need not be parallel, but they are plausibly near-parallel, given that the expected period of gross investment T extends over most of a lifetime.

Consider now the two comparisons, and define experience as chronological time (j) since the start of post-school investments. Note that the net investment ratio k_j is given by the vertical difference between I and D, and recall that the growth rate of earning capacity in period j is given by rk_j.

If the increase in investment (from I_1 to I'_1) is restricted to post-school investment, meaning that *schooling* (s_1) is the same in both cases, then net investments (k_j) are larger for each additional year of *experience and of age,* and peak earning time (P_1) is shifted to a later age (P_2) and to a later year of experience. Earning capacity rises more rapidly at each age and for a longer period, reaching a higher level at P_2. Even if the increase in investment includes also an in-

FIGURE 2.1
AGE PROFILES OF INVESTMENT RATIOS



crease in schooling (shift from I_1 to I_2), the conclusion for the *age-earnings* profiles remains the same. This is not necessarily true, however, for the description of experience-earnings profiles, since the same age no longer represents the same year of experience as it did before. For example, if the shift from I_1 to I_2 is, indeed, parallel, as in Figure 2.1, meaning that *post-school gross investment* ratios remain unchanged, net investment ratios (k_j) will not be greater for each year of experience. In fact, they will actually be somewhat smaller if D has an upward slope; and peak earnings will be reached at an earlier year of experience. With D relatively flat, the (log) experience-earnings profiles are nearly parallel, though the age-earnings profiles diverge.[4]

4. This is exact when D is horizontal and the same in all schooling groups. Then the parallel shift of gross investment I_j implies the same parallel shift in net investments k_j. In that case, the logarithmic experience-earnings profiles would be exactly alike in the two cases, except for a difference in levels. At a given year of experience the ratio of earnings Y_{s_2+j}/Y_{s_1+j} would be equal to Y_{s_2}/Y_{s_1}. Thus, relative differentials in earnings between the two schooling groups would be the same at any level of experience, with or without post-school investments.

This analysis underscores the importance of distinguishing between age profiles and experience profiles of earnings. In the special case just discussed, shapes of experience profiles of (log) earnings are the same in all schooling groups, though shapes of age profiles are not. While relative intergroup differentials in earnings do not change with experience, they grow with age! This is because at given ages, earnings rise less rapidly (and decline more rapidly at advanced ages) for the lower profiles. For groups with more schooling earnings peak at the same or earlier years of experience, but at later ages.

If an increase in investment ratios results from both longer schooling and more "time" spent in post-school investments, that is, $s_2 > s_1$ and $k_{2j} > k_{1j}$ for each j, then log earnings profiles for both age and experience will be steeper and peak later than if either schooling or post-school investment are the same, though this behavior will be less evident in the latter profiles than in the former.

Note also that the steeper the upward slope of D in the neighborhood of its intersection with I, the less the difference in age at which earnings begin to decline in all schooling groups. If retirement age is related to the time of onset of declining earning power, this analysis might well explain why persons with more schooling retire later in life, and yet have a somewhat shorter earnings span.

So long as gross investment extends over the working life and retirement age is not earlier for the more educated, I_2 is likely to exceed I_1 at each age. This is the simplest interpretation of the universally observed divergence ("fanning out") of *age profiles* of earnings. Note that if I_2 declines more steeply than I_1 (without intersecting), *logarithmic age profiles* will still *diverge* ("fan out"), but *log experience* profiles will *converge:* earnings of higher schooling groups will grow at a somewhat slower rate. *Dollar age profiles* will fan out, a fortiori, and so may[5] dollar experience profiles, even though log experience profiles converge.

A positive correlation between dollar investments in schooling and at work does not constitute evidence against the possibilities for

5. A convergence of log experience profiles would mean that the more schooled persons spend less "time" in post-school investment. However, they clearly spend more in dollar terms, if $I_2 > I_1$ at each age. The sufficient condition for a positive correlation between schooling and post-school investments in dollars is even weaker: $C_2 > C_1$ in each year of experience.

substitution between the two forms of investment in human capital. Rather, it reflects the dominance of individual differences in factors determining the scale of total human capital accumulation. Individuals who invest more in human capital, invest more in both forms of it. Evidently, abilities to learn on the job are positively, though far from perfectly, related to abilities to learn at school, and so are financial opportunities to incur such investments. Indeed, given imperfect markets for human capital, it would not be surprising to find that just as schooling investments are positively related to personal (family) income, so are post-school investments to personal earning capacity, that is, to the preceding schooling investments.

As already noted, a positive correlation between dollar schooling and post-school investments does not imply a positive correlation between these investments in "time" units. If individuals with differing amounts of schooling all have the same post-school investment ratios, as indicated by the parallel investment profiles in Figure 2.1, then the correlation between "time" spent in school and in post-school investments is zero. However, *dollar* post-school investments are larger in proportion to the larger earning capacity (initial gross earnings) of the more schooled. This case can be described as one of unitary elasticity of post-school investments with respect to earning capacity. The positive elasticity is less than 1 when dollar post-school investments are larger at higher schooling levels, but less than in proportion to the higher earning capacity.

We may now summarize our conclusions concerning comparative earnings profiles for different schooling groups, and the implications of these comparisons for earnings differentials by schooling, age, and experience. So long as the elasticity (or "marginal propensity to invest") is positive with respect to earning capacity (correlation between dollar schooling and post-school investments is positive), dollar earnings grow faster in upper schooling groups, at given years of experience and—a fortiori—of age. Logarithmic profiles fan out with age, so long as $l_2 > l$, but not necessarily with experience. They converge with experience if the "income elasticity of investment" is less than 1, that is, when the correlation between "time" in schooling and in post-school investment is negative.

Hence "skill differentials" in dollar earnings which are attributable to schooling differences can be expected to grow with age and experience, and relative (percentage) differentials to grow with age.

The latter also grow with experience, but only if the elasticity of post-school investments with respect to earnings capacity exceeds 1. They decline with experience if the elasticity is less than 1, and remain fixed at all stages of experience when the elasticity is 1.

2.3 DISTRIBUTION OF EARNINGS

Thus far I dealt with intergroup differences in earnings of persons differing in schooling and age.

However, within groups of workers of the same schooling and age, earnings inequality is far from negligible. There are several reasons for this: (1) differences in accumulated human capital, despite the same *length* of schooling, because of differences in schooling quality or rates of return to schooling; (2) differences in post-school investment behavior;[6] and (3) differences in rates of return to post-school investments.

2.3.1 VARIANCES

Assume first that individuals who complete a given level of schooling have the same gross earnings (earning capacity) Y_s and rates of return r_j, but differ in their post-school investment behavior.

Individual differences in post-school investments were illustrated in Figure 2.1. The conclusion was that earnings of individuals who invest more in each year j rise more rapidly with experience and for a longer period. This means that relative (as well as absolute) dispersion of *gross earnings* within a schooling cohort rises with experience until peak earnings are reached by the largest investors.

Note, however, that the change in dispersion of *net earnings* with age is not monotonic: Assuming, as I have thus far, that Y_s and r are fixed within schooling groups, earnings of investors are initially smaller than those of noninvestors. Only after the overtaking year of experience ($\hat{j}$) do their earnings surpass those of noninvestors. In this case, earnings profiles of individuals with the same schooling but differing in post-school investments will *cross* in the

6. Such as job training, job search, or investment in health. Effects of differences in job search behavior have been analyzed by Stigler in his pioneering work on information in labor markets (1962).

neighborhood of $\hat{j}$, reaching the smallest dispersion in that neighborhood. More generally, $\hat{j}$ is not the same for each investor, but it has a strong central tendency, if, in the period preceding $\hat{j}$, the rate of decline of investments, is similar even though its volume differs among investors.

In the special case, where Y_s and $\hat{j}$ are the same for all, dispersion first diminishes, reaching zero at time $\hat{j}$, and increases thereafter. If $\hat{j}$ differs, a minimum but nonzero dispersion is reached at some average $\hat{j}$.

The assumption that initial post-school earning capacities Y_s are the same among persons with the same schooling is not tenable. For the moment, let us keep r the same for these individuals and for all their investments. Let i indicate individual differences in the earnings function:

$$E_{ij} = Y_{si} + r \sum_{j} C_{ij}.$$

Then,

$$\sigma^2(E_{ij}) = \sigma^2(Y_{si}) + r^2\sigma^2\left(\sum_{j} C_{ij}\right) + 2\rho r\sigma(Y_{si})\sigma\left(\sum_{j} C_{ij}\right), \qquad (2.8)$$

where ρ is the correlation between dollar post-school investments and (dollar) earning capacity. If this correlation is nonnegative, the dollar variance of gross earnings must rise with experience (j), since $\sigma^2(\Sigma_j C_j)$ increases with j. This is because the variance of a sum must increase when the sum is generated by positively correlated increments.

If the positive correlation ρ is not too weak, the monotonic growth in dollar variances will also be observed in *net* earnings,[7] since $\sigma^2(Y_0) = \sigma^2(Y_s - C_0)$, and $\sigma^2(Y_0) < \sigma^2(Y_s)$, so long as $\rho(C_0, Y_s) > \sigma(C_0)/\sigma(Y_s)$. That is, the initial (first-year) variance in net earnings will be smaller than the variance at overtaking, which will, in turn, be smaller than subsequent variances, according to (2.8). The size order of the variances is changed if ρ is small. By the same token, if ρ is negative and sufficiently large, a monotonic decline occurs.

7. An example of the effects of such a positive correlation is the growth in the dispersion of earnings due to better recognition of differences in productivity of workers whose initial wages were similar. This may be viewed as worker investment in employer information about their quality. Cf. Stigler (1962).

Exactly the same arithmetic applies to variances of logs. Their profiles depend on the correlation between initial post-school earning capacity ($\ln Y_{si}$) and investment ratios k_i. A strong positive correlation leads to monotonic growth of relative (log) variances with experience, a strong negative correlation produces a monotonic decline, while a weak correlation creates U-shaped experience profiles of log variances. The bottom of the U-shaped profile is found at the overtaking period only when the correlation is zero. Negative correlations shift it to later periods; positive correlations, to earlier periods.

If, as is suggested by Figure 2.1, positive and near-unitary elasticities hold, we would expect to observe dollar variances monotonically increasing with experience but U-shaped profiles of relative variances.

According to the same kind of analysis, the dollar variance of earnings within a schooling group at the "overtaking" stage of experience is larger the higher the schooling level. Since

$$Y_{si} = Y_0 + r \sum_s C_{si},$$

therefore,

$$\sigma^2(Y_{si}) = \sigma^2(Y_0) + r^2\sigma^2\left(\sum_s C_{si}\right), \tag{2.9}$$

and $\sigma^2(\Sigma_s C_{si})$ grows with increments of schooling. Other things equal, particularly r and the correlation parameter ρ, expression (2.9) implies that dollar variances of earnings increase with level of schooling at each stage of experience.

The relation between relative (log) variances and level of schooling would be the same if similar assumptions could be made about correlations between time-equivalents of investment components. This is not the case, however, as the empirical analysis in Part II indicates.

Thus far I have neglected individual differences in rates of return. Once differences in r_i are assumed, age changes in dispersion can be generated, provided post-school investment is assumed as well, since variations in rates of return alone are not sufficient to generate age changes in the dispersion of earnings. However, it is not necessary in this case to assume that post-school investment differs among persons.

For simplicity, look at gross earnings:

$$E_{ji} = Y_{si} + r_i \sum_j C_{ji}.$$

Assume $\sigma^2(r_i) > 0$, and $C_{ji} = C_j$ for all i. If $C_j = 0$, $\sigma^2(E_{ji})$ remains fixed throughout earning life. But if $C_j > 0$, $\sigma^2(E_{ji})$ increases with j, assuming that r_i and Y_{si} are not negatively correlated:

$$\sigma^2(E_{ji}) = \sigma^2(Y_{si}) + (\Sigma C_j)^2\sigma^2(r) + 2\rho(\Sigma C_j)\sigma(Y_s)\sigma(r). \qquad (2.10)$$

Note that variances of net earnings are the same as variances of gross earnings when investments are the same for all. A similar monotonic growth of relative variances can be derived from the logarithmic formulation. If reversals or declines in profiles of variances are observed, the hypothesis that post-school investments do not vary among individuals must be rejected. In the logarithmic case the implication is that $\sigma^2(k_i) > 0$. This test is of some importance, because the dispersion in earnings of persons with the same schooling represents an exaggerated index of risk if it is attributed solely to variation in rates of return. A general approach is to assume both $\sigma^2(C_{ij}) > 0$ and $\sigma^2(r_i) > 0$. The empirical implications remain qualitatively the same as when only $\sigma^2(C_{ij}) > 0$.

I conclude that the fanning out of dollar variances and the possible reversals or declines in profiles of relative variances of earnings within schooling groups reflect systematic age increments and individual differences in the scale of human capital investments, rather than random increments ("shocks") in earnings, as the exclusively stochastic theories of income distribution would have it.[8]

Finally, the conclusions about the determinants of earnings dispersion that were expressed for the schooling model by (2.7) [9] can be directly generalized by earnings function (1.4). The logarithmic version is required for studying relative inequality, and a simplified formulation parallel to (2.7), in which correlations among terms are ignored, is derived as follows:

$$\ln E_{ji} = \ln Y_{si} + r_{ji}K_{ji},$$

where

$$K_j = \sum_{t=0}^{j-1} k_t.$$

8. See Part II, Table 7.2, for empirical evidence against random shock models.

9. Section A, above.

Then

$$\sigma^2(\ln E_j) = \sigma^2(\ln Y_s) + \bar{r}_j^2\sigma^2(K_j) + \bar{K}_j^2\sigma^2(r_j) + \sigma^2(K_j)\sigma^2(r_j). \quad (2.11)$$

The positive determinants of $\sigma^2(\ln Y_s)$ in (2.7) were initial capacity levels and dispersions in schooling investments s and in rates of return r_s. Now (2.11) adds the correspondingparameters of the distribution of post-school investments as parameters of inequality of gross earnings in an experience group j. Incidentally, the inequality determined in the schooling model, $\sigma^2(\ln Y_{si})$, can be seen, under simplified assumptions, as the inequality in a particular experience group, when $j = \hat{j}$. The overall inequality, however, is of a distribution of earnings of workers who are at different levels of experience in their working life.

2.3.2 Aggregation of Variances

The aggregation of variances of overall years of experience in a schooling group is visualized by the well-known aggregation formula for variances:

$$\sigma_T^2 = \sum \frac{n_j}{n} (\sigma_j^2 + d_j^2), \quad (2.12)$$

where T is an aggregate of several j groups; σ_j^2, the within-group variances; $d_j = \mu_j - \mu_T$, the differences between the means of group j and the overall mean; n_j, the number of observations in j; and n, the total number of observations.

The size of d_j^2 is clearly a positive function of the rate of growth of mean earnings with experience. In dollar terms, therefore, we should expect variances of earnings to increase with length of schooling, if relative frequencies of numbers of workers are similar by years of experience. However, because of upward secular trends in schooling, these frequencies are not similar: there are relatively fewer older workers in the upper schooling groups. Consequently, the increase in dollar variances of earnings with schooling is somewhat attenuated. The conclusions about relative variances of earnings classified by schooling cannot be determined a priori. A discussion of findings based on empirical data is deferred to Part II.

Formula (2.12) is equally applicable to an aggregation of variances over all years of schooling in a given experience group. Because σ_j and d_j in dollar terms increase with experience, increases in the dispersion of earnings by experience and (a fortiori) by age are predictable. Relative variances are not expected to grow monotonically with experience, because reversals are likely to arise some time during the working life. In a classification by age, the growth of relative variances with age is likely to dominate, but reversals may still occur. The attenuation of inequality which is due to secular trends in schooling holds for schooling, experience, and age aggregations, as well as for total inequality observed in the cross section.[10]

2.3.3 SKEWNESS

Positive skewness is a well-known feature of income distributions. Human capital models can explain skewness in several different, not mutually exclusively ways:

a. It will be recalled that the distribution of investment time-equivalents, $h_i = S_i + K_i$, tends to impart positive skewness to the distribution of earnings, even when investments are symmetric. Suppose, therefore, that without investments, the distribution of earnings Y_0 would be symmetric. In that case, the distribution of ln Y_0 would be negatively skewed and so would the distribution of ln Y_i, given a symmetric distribution of investments. Thus, unless the distribution of investments has a strong positive skew, the *logarithmic* distribution of earnings will be negatively skewed. At the same time, unless the distribution of investments has a substantial negative skew, the distribution of *dollar* earnings will be positively skewed. *A normal distribution of "raw abilities" is, therefore, likely to produce a positively skewed distribution of earnings with a shape intermediate between normal and log normal.* The larger the investment component $r(S + K)$ in earnings, the better the log normal rather than normal approximation.

b. Assume that the distribution of r_i is symmetric, and ignore variation in Y_{0t}. In that case, even for fixed h, the distribution of earnings would be positively skewed. As before, positive skewness would be accentuated at higher levels of investment h.

10. To state that inequality in the cross section is, in part, affected by the rate of change of secular trends in schooling is to ignore possible feedbacks of such trends on rates of return. Such effects depend on the nature of the trends, a subject outside the scope of this study.

c. An important conclusion emerges from the analysis of within-group variances: systematic allocations of individuals' investments in human capital over their life cycle and systematic differences among individuals in the scale of their investments show up as positive correlations among instalments of investment within and between the schooling and post-school stage. Consequently, dollar variances of earnings are positively related to experience (age within schooling groups) and to schooling (at given years of experience). Since average earnings must grow with schooling and experience, the allocation of human capital investments produces a positive correlation between means and variances across subgroups of workers defined by schooling and experience. The positive correlation between means and variances of subsets of the distribution leads to positive skewness in the aggregate. This explanation of skewness is additional and independent of assumptions about the shape of the distribution of schooling which were emphasized in the schooling model.[11] It is the only explanation inherent in the human capital model of individual behavior.

The conclusions about positive skewness in the aggregate do not apply to the logarithms of earnings, because, as the previous discussion suggested, log earnings are likely to be negatively skewed within groups, and an a-priori case for a positive correlation between logarithmic means and variances in subgroups is not clear.[12]

The effects of secular trends in schooling on the distribution of earnings is an important example of the distinction between observations in cohorts and in cross sections. Though the theoretical analysis is carried out in longitudinal (cohort) terms, empirical analysis and interest focus on the distribution of earnings in a cross section in a given period of time. The possible considerations impinging on this distinction are too numerous for a useful a-priori analysis, given the limited information available. However, the distinction between cohorts and cross sections receives attention, where possible and appropriate, in the empirical analysis of Part II, below.

---

11. See the mathematical note at the end of this chapter.

12. Both in dollars and in logs, aggregate skewness in the cross section is also affected by secular trends in schooling in a manner analogous to the effects on variances, as discussed above.

2.4 MATHEMATICAL NOTE ON SKEWNESS

Let

N = population of an aggregate; M, its mean; σ, its standard deviation; and α, its third moment (skewness).
n_i = population of component i; M_i, its mean; σ_i, its standard deviation; and α_i, its skewness.
$d_i = M_i - M$.

Then [13]

$$\alpha = \frac{\Sigma_i n_i \sigma_i^3 \alpha_i + \Sigma_i n_i d_i (3\sigma_i^2 + d_i^2)}{N\sigma^3}. \qquad (2.13)$$

With the help of this relation we can (1) investigate the conditions under which a combination of symmetric distributions of the components results in a positively skewed aggregate, (2) show that the theoretical model ensures such a result.

Let $\alpha_i = 0$, hence

$$\alpha = \frac{\Sigma_i n_i d_i (3\sigma_i^2 + d_i^2)}{N\sigma^3}$$

$$= \frac{\Sigma_i n_i d_i^3 + \Sigma_i 3 n_i d_i \sigma_i^2}{N\sigma^3}. \qquad (2.14)$$

Since the denominator is positive, aggregative skewness will be positive ($\alpha > 0$), if and only if:

$$\Sigma_i n_i d_i^3 + \Sigma_i 3 n_i d_i \sigma_i^2 > 0.$$

A. If no intragroup dispersion exists ($\sigma_i = 0$), or if all component dispersions are the same ($\sigma_i = C$), the second term vanishes:

$$\Sigma_i 3 n_i d_i \sigma_i^2 = 3C\Sigma_i n_i (M_i - M) = 3C(NM - NM) = 0.$$

In this case aggregate skewness is positive, if and only if

$$\Sigma_i n_i d_i^3 > 0. \qquad (2.15)$$

This expression is, in fact, the third moment in the distribution of component means around the aggregate mean. When the i's are interpreted as schooling groups, expression (2.15) measures skew-

13. For derivation, see Bates (1935, pp. 95–98).

ness introduced by the distribution of schooling alone. In the schooling model this is positive, provided the skewness of the schooling distribution is not excessively negative.

B. If intragroup dispersion does exist ($\sigma_i > 0$), and differs from group to group, the factor $3\Sigma_i n_i d_i \sigma_i^2$ can be interpreted as the contribution of intragroup differentials to aggregate skewness. The condition for

$$\Sigma_i 3 n_i d_i \sigma_i^2 > 0 \tag{2.16}$$

is

$$\Sigma_i n_i M_i \sigma_i^2 > \Sigma_i n_i M \sigma_i^2.$$

Dividing both sides of the inequality by $\Sigma_i n_i M_i = NM$ we get:

$$\frac{\Sigma_i n_i M_i \sigma_i^2}{\Sigma_i n_i M_i} > \frac{\Sigma_i n_i \sigma_i^2}{\Sigma_i n_i}. \tag{2.17}$$

In other words, in order for (2.16) to hold, the weighted average of the intragroup variances weighted by $n_i M_i$ must exceed the average of these variances weighted by n_i. Clearly, this occurs when the σ_i^2's are positively correlated with the M_i's. This condition holds in the complete model, in which intragroup dispersion is expected to increase with the average accumulated investment $(S + K)$, hence with average earnings.