

Quiet Bubbles*

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Abstract

Classic speculative bubbles are loud—price is high and so are price volatility and share turnover. The credit bubble of 2003-2007 is quiet — price is high but price volatility and share turnover are low. We develop a model, based on investor disagreement and short-sales constraints, that explains why credit bubbles are quieter than equity ones. Since debt up-side pay-offs are bounded, debt is less sensitive to disagreement about underlying asset value than equity and hence has a smaller resale option and lower price volatility and turnover. Large debt mispricing requires, in contrast to equity, either greater leverage or investor optimism. An increase in optimism makes debt but not equity bubbles quieter. Even holding fixed average optimism, an increase in disagreement with enough leverage can lead to a large and quiet debt mispricing. Our theory provides a first attempt at a taxonomy of bubbles.

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1. Introduction

Many commentators point to a bubble in credit markets from 2003 to 2007, particularly in the AAA and AA tranches of the sub-prime mortgage collateralized default obligations (CDOs), as the culprit behind the Great Financial Crisis of 2008 to 2009.¹ However, stylized facts, which we gather below, suggest that the credit bubble lacked many of the features that characterize classic episodes. These episodes, such as the Internet bubble, are typically loud—characterized by high price, high price volatility, and high trading volume or share turnover as investors purchase in anticipation of capital gains.² In contrast, the credit bubble is quiet—characterized by high price but low price volatility and low share turnover. In this paper, we make a first attempt at developing a taxonomy of bubbles by showing why credit bubbles are quieter and hence fundamentally different than equity ones.

Our theory builds on the investor disagreement and short-sales constraints framework, which has been used to generate loud equity bubbles. In these models, disagreement and binding short-sales constraints lead to over-pricing in a static setting as pessimists sit out of the market (Miller (1977) and Chen et al. (2002)). In a dynamic setting, investors value the potential to re-sell at a higher price to someone with a higher valuation due to binding short-sales constraints (Harrison and Kreps (1978) and Scheinkman and Xiong (2003)).³ This resale option generates a high price but also high price volatility and high share turnover.

Within this framework, we consider the pricing of a security with an arbitrary concave payoff over an underlying asset or fundamental. Debt emerges as a special case. Investors have disagreement over the underlying asset value. Whereas equity payoffs are linear in the investor beliefs regarding underlying asset value, debt up-side pay-offs are capped at

¹Indeed, there is compelling evidence that these securities were severely mis-priced relative to their risk-adjusted fundamentals (see Coval et al. (2009)).

²See Hong and Stein (2007) for a review of the evidence regarding classic bubbles and Ofek and Richardson (2003) for a focus on the dot-com bubble.

³See Hong and Stein (2007) for a more extensive review of the disagreement approach to the modeling of bubbles. There are other approaches. For the possibility of rational bubbles, see Blanchard and Watson (1983), Tirole (1985), Santos and Woodford (1997), and Allen et al. (1993). For an agency approach, Allen and Gorton (1993) and Allen and Gale (2000). For other behavioral approaches, see Delong et al. (1990) and Abreu and Brunnermeier (2003).

some constant and hence are non-linear (concave) in the investor beliefs about fundamental. We make the standard assumption regarding short-sales constraints. There is compelling evidence that such constraints are even more binding in debt markets than in equity ones. The pricing of debt contracts in this disagreement and short-sales constraints setting is new.

Since debt up-side pay-offs are bounded in contrast to equity, the valuation of debt is less sensitive to disagreement about underlying asset value than equity and hence has a smaller resale option⁴. All else equal, this also means that there is less price volatility and share turnover for a debt claim than for an equity claim. Thus, a debt bubble is quieter than an equity bubble. This result is due simply to the asymmetry in the pay-offs of debt in contrast to equity. However, a debt bubble is also smaller than an equity bubble. Again, the safer is the debt claim, the less sensitive it is to disagreement and therefore the lower the resale option and mispricing. Hence, large mispricings in debt require greater leverage. Since investors are financially constrained, cheaper leverage makes prices more sensitive to disagreements and leads to a larger resale option.

The standard model of heterogenous priors and short-sales constraints has an unbiased distribution of beliefs. The price of an equity claim increases when investors become positively biased over the fundamental. However, both the turnover and the volatility of an equity are independent of investors' average optimism. The linearity of the equity payoff implies that an increase in average optimism does not change the relative disagreement between optimists and pessimists about the asset payoff. An increase in investors average optimism, holding fixed the dispersion in beliefs, makes debt bubbles quieter. This again emanates from the bounded up-side pay-off of debt claims. When investors are very optimistic about the underlying fundamental of the economy, debt payoff becomes more concave (one can think about the debt payoff almost as a risk-free asset) and hence its value becomes very insensitive to beliefs. To the extent there is a bubble on a debt claim, the resale option is

⁴This should be related to the standard argument in Myers and Majluf (1984) that debt claims are less sensitive to private information – our model is the counterpart to this view in a world without private information but with heterogenous beliefs.

limited, and as a consequence, both price volatility and turnover are low. When investors on average become more pessimistic about the economy, investors perceive the debt claim as riskier – closer to an equity claim – and beliefs start having a stronger influence on the asset valuation. The resale option grows and both price volatility and trading volume increase. An increase in average optimism can be viewed as a mechanism that generates for concave claims both large mispricing and low volatility and turnover.

Finally, we show that even in a setting with an unbiased average belief, an increase in the dispersion of priors can make bubbles quieter and larger at the same time, at least provided that leverage is cheap enough. The only ingredients required for this to hold are (1) the concavity of the debt claim and (2) the existence of an interim pay-off (an interest payment for debt or dividend for equity) on which investors currently disagree. Investors with pessimistic priors value the re-sale option more highly than those with optimistic priors it is more likely that the investors with the optimistic priors will be the more optimistic investor tomorrow. As dispersion of priors increases, optimists today value the interim pay-off more highly and buy more shares today. Pessimists have fewer shares with which to re-sell to the optimists tomorrow. This decreases turnover. Price volatility – which essentially reflects the differences in belief between the optimist agents across the future states of nature – also decreases with an increase in the dispersion of priors. When the dispersion becomes sufficiently large, the asset resembles a risk-free asset and there is low disagreement among optimists about the value of the asset regardless of the state of nature — hence low volatility. We also show that mispricing increases with dispersion in priors.

The contribution of our paper is to introduce the asymmetric payoff structure of debt relative to equity into the disagreement and short-sales constraints models of asset price bubbles. We show that the concavity of debt-payoffs and their insensitivity to belief differences generate many interesting new insights on asset price bubbles. Our analysis takes the cost of leverage as exogenously decreasing with the efficiency of the banking sector. We lay out a taxonomy of bubbles distinguishing between debt and equity regardless of what drives

leverage. An earlier literature has examined how to endogenize leverage when investors trade equity claims—so the credit bubble in their setting is a bubble in lending.⁵ In contrast, our paper focuses on bubbles in the trading of actual credits. Another element of our model is the use of heterogeneous priors.⁶ We show that heterogeneous priors lead to novel dynamics in debt prices that do not necessarily occur with equity prices.

Our paper proceeds as follows. We present some stylized facts on the recent credit bubble of 2003-2007 in Section 2. The model and main results are discussed in Section 3. We discuss empirical implications of our model in Section 4. We relate our work to studies on the recent financial crisis in Section 5. We conclude in Section 6.

2. Stylized Facts About the Credit Bubble of 2003-2007

We begin providing some stylized facts regarding the recent credit bubble that motivate our theoretical analysis. By all accounts, sub-prime mortgage CDOs experience little price volatility between 2003 until the onset of the financial crisis in mid-2007. In Figure 1, we plot the prices of the AAA and AA tranches of the sub-prime CDOs. The ABX price index only starts trading in January of 2007, very close to the start of the crisis. Nonetheless, one can see that, in the months between January 2007 until mid-2007, the AAA and AA series are marked by high price and low price volatility. Price volatility only jumps at the beginning of the crisis in mid-2007. This stands in contrast to the behavior of dot-com stock

⁵Geanakoplos (2010) shows that leverage effects in this disagreement setting can persist in general equilibrium though disagreement mutes the amount of lending on the part of pessimists to optimists. Simsek (2010) shows that the amount of lending is further mediated by the nature of the disagreement—disagreement about good states leads to more equilibrium leverage while disagreement about bad states leads to less.

⁶Our use of priors is similar to Morris (1996) who showed that small differences in priors can lead long-run divergences in the Harrison and Kreps (1978) setting that we also consider. We interpret our priors assumption as in Morris (1996) who argues that it applies in settings where there is a financial innovation or a new company and hence there is room for disagreement. The sub-prime mortgage-backed CDOs are new financial instruments and hence the persistence effect that he identifies also holds in our setting. An alternative and more behavioral interpretation of our model is that overconfident investors who overreacted to information as in Odean (1999) and Daniel et al. (1998).

prices—the price volatility of some internet stocks during 1996-2000 (the period *before* the collapse of the internet bubble) exceed 100% per annum, more than three times the typical level of stocks.

Another way to see the quietness is to look at the prices of the credit default swaps for the financial companies that had exposure to these sub-prime mortgage CDOs. This is shown in Figure 2. The price of insurance for the default of these companies as reflected in the spreads of these credit default swaps is extremely low and not very volatile during the years before the crisis. One million dollars of insurance against default cost a buyer only a few thousand dollars of premium each year. This price jumps at the start of the crisis, at about the same time as when price volatility increases for the AAA and AA tranches of the sub-prime mortgage CDOs.

The low price volatility coincided with little share turnover or re-selling of the sub-prime mortgage CDOs before the crisis. Since CDOs are traded over-the-counter, exact numbers on turnover are hard to come by. But anecdotal evidence suggests extremely low trading volume in this market particularly in light of the large amounts of issuance of these securities. Issuance totals around \$100 billion dollars per quarter during the few years before the crisis but most of these credits are held by buyers for the interest that they generate. To try to capture this low trading volume associated with the credit bubble, in Figure 3, we plot the average monthly share turnover for financial stocks. Turnover for finance firms is low and only jumps at the onset of the crisis as does turnover of sub-prime mortgage CDOs according to anecdotal accounts (see, e.g., Michael Lewis (2010)). This stands in contrast to the explosive growth in turnover that coincided with the internet boom in Figure 4—obtained from Hong and Stein (2007). As shown in this figure, the turnover of internet stocks and run-up in valuations dwarfs those of the rest of the market.

3. Model

3.1. Set-up

Our model has three dates $t = 0, 1$, and 2 . There are two assets in the economy. A risk-free asset offers a risk-free rate each period. A risky debt contract with a face value of D has the following pay-off at time 2 given by:

$$m_2 = \min(D, \tilde{G}_2), \quad (1)$$

where

$$\tilde{G}_2 = G + \epsilon_2 \quad (2)$$

and the ϵ_2 is drawn from a standard normal distribution $\Phi(\cdot)$. We think of \tilde{G}_t as the underlying asset value which determines the pay-off of the risky debt. There is an initial supply Q of this risky asset.

Competitive agents hold heterogenous priors relative to the mean of the underlying asset. Agents believe at $t = 0$ that the underlying asset process is actually

$$\tilde{V}_2 = V + \epsilon_2, \quad (3)$$

where V is $F + \sigma$ for a fraction $\frac{1}{2}$ of the agents and $F - \sigma$ for the remaining fraction $\frac{1}{2}$ of the agents. The average prior across the agents is F (i.e. $\mathbb{E}[\tilde{V}] = F$). The variance of the priors distribution is σ^2 (i.e. $\mathbb{E}[(\tilde{V} - F)^2] = \sigma^2$). When $F = G$, the agents' average optimism is equal that of the actual mean of the fundamental G . The larger is F , the greater the average investor optimism.

At $t = 1$, there are two states of nature, which corresponds to the revisions of the agents' beliefs. In state-A, which occurs with probability $\frac{1}{2}$, the σ agents become more optimistic with belief $F + \sigma + \eta$ and the $-\sigma$ agents become more pessimistic with belief $F - \sigma - \eta$. In

state-B, which occurs with probability $\frac{1}{2}$, the σ agents become more pessimistic with belief $F + \sigma - \eta$ and the $-\sigma$ agents become more optimistic with belief $F - \sigma + \eta$. Note also that when we set $\sigma = 0$, we converge to the symmetric priors model. When $\eta > \sigma$, then the initially optimistic σ investors' belief falls below the $-\sigma$ investors' belief in state-B. When $\eta < \sigma$, no such crossing occurs and the initially optimistic σ investor remains more optimistic in both dates. This revision of beliefs is the main shock that determines the price of the asset, its volatility and turnover at $t = 1$. The expected payoff of an agent with belief v regarding m_2 is given by:

$$\pi(v) = E[m_2|v] = \int_{-\infty}^{D-v} (v + \epsilon_2)\phi(\epsilon_2)d\epsilon_2 + D(1 - \Phi(D - v)). \quad (4)$$

This is the payoff of a standard debt claim in the absence of limited liability, which is the usual assumption in these models of disagreement and short-sales constraint. If the value of the fundamental is below the face value of debt (i.e. $v + \epsilon < D$), then the firm defaults on its contract and investors become residual claimant, i.e. they receive $v + \epsilon$. If the realized fundamental value is above the face value of debt, then investors are entitled a fixed payment D . Our analysis below applies more generally to any (weakly) concave expected pay-off function, which would include equity as well standard debt claims. Note also that the unlimited liability assumption is not necessary for most of our results, but it allows us to compare our results with the rest of the literature.

Agents are risk-neutral and are endowed with zero liquid wealth but large illiquid wealth W (which becomes liquid and is perfectly pledgeable at date 2). To be able to trade, these agents need to access a credit market that is imperfectly competitive. The discount rate is 0 but banks charge a positive interest rate, which we call λ^{-1} . In particular, because illiquid assets are large, the loans made by the financial sector are risk-free. Thus, a perfectly competitive credit market is the limiting case where $\lambda \rightarrow \infty$. So λ measures the efficiency

of the financial sector. Let

$$\mu = \frac{1}{1 + \lambda^{-1}} \in [0, 1]. \quad (5)$$

μ is increasing with the efficiency of the credit market. Investors also face quadratic trading costs given by

$$c(\Delta n_t) = \frac{(n_t - n_{t-1})^2}{2\gamma}, \quad (6)$$

where n_t is the shares held by an agent at time t . The parameter γ captures the severity of the trading costs – the higher is γ the lower the trading costs. Note that $n_{-1} = 0$ for all agents, i.e. agents are not endowed with any risky asset. Investors are also short-sales constrained.

Let P_1 be the price of the asset at $t = 1$. Then at $t = 1$, for an investor with belief V_1 and prior V , her optimization problem is given by:

$$J(n_0, V_1) = \max_{n_1} \left\{ n_1 \pi(V_1) - \frac{1}{\mu} \left((n_1 - n_0) P_1 + \frac{(n_1 - n_0)^2}{2\gamma} \right) \right\}, \quad (7)$$

and subject to the short-sales constraint:

$$n_1 \geq 0, \quad (8)$$

and given her inherited position $n_0(V)$ from $t = 0$. If $n_1(V_1) - n_0(V)$ is positive, an agent borrows $(n_1(V_1) - n_0(V)) P_1 + \frac{(n_1(V_1) - n_0(V))^2}{2\gamma}$ to buy additional shares $n_1(V_1) - n_0(V)$. If $n_1(V_1) - n_0(V)$ is negative, an agent gets to lend at the rate λ^{-1} on the sales. The trading cost is symmetric (buying and selling costs are similar) and only affects the number of shares one purchases or sells, and not the entire position (i.e. $n_1 - n_0$ vs. n_1). Let $J(n_0; V_1)$ be the value function of agent- V 's optimization program (see definition in equation 7). $J(n_0; V_1)$ is driven in part by the possibility of the re-sale of the asset bought at $t = 0$ at a higher price to the other agents that get a draw on their belief at $t = 1$.

Let P_0 be the price of the asset at $t = 0$. Then at $t = 0$, agents with prior V have the

following optimization program:

$$\max_{n_0} \left\{ -\frac{1}{\mu} \left(n_0 P_0 + \frac{n_0^2}{2\gamma} \right) + \mathbb{E}[J(n_0; V_1)] \right\}. \quad (9)$$

subject to the constraint at

$$n_0 \geq 0, \quad (10)$$

and where the expectation is taken over the two states of nature A and B. The equilibrium prices in both periods will be determined by the usual market clearing conditions that supply equal demand for the asset.

Our goal here is to solve for the dynamic equilibrium with an eye toward the following three quantities. The first is a mispricing or a bubble measure, which we take to be P_0 and P_1 relative to the price of the asset in the absence of short-sales constraints and no average optimism bias. The second is price volatility between $t = 0$ and $t = 1$. Price volatility is defined simply by

$$\sigma_P = |P_1(\text{state} - A) - P_1(\text{state} - B)|. \quad (11)$$

The reason for this is that our model is essentially a CARA (constant absolute risk aversion) set-up. So share returns, defined as $P_1 - P_0$, and share return volatility $\text{var}(P_1 - P_0)$ are the natural objects of analysis. This reduces to the definition of σ_P . The third object is share turnover. It is simply defined as the expectation of the number of shares exchanged at date 1. Formally:

$$T = \mathbb{E}|n_1^\sigma - n_0^\sigma|, \quad (12)$$

where the expectation is taken over the two states of nature A and B.

3.2. Equilibrium

We now solve for the equilibrium of the model. The following two theorems characterize the equilibrium. In the first theorem, we consider the setting in which $\eta > \sigma$, so that the beliefs

of the σ and $-\sigma$ investors can cross in state-B. In the second theorem, we characterize the setting where $\eta < \sigma$ and no such crossing occurs in state-B.

In either setting, the equilibrium decomposes into three cases depending on the value of $\theta = \frac{Q}{\mu\gamma}$, which is the ratio of the supply of the asset Q to the product of the cheapness of leverage μ and the cost of trading γ . The product of the cheapness of leverage to the cost of trading is analogous to buying power on the part of investors in this short-sales constraints setting. The cheaper is leverage and the lower is the trading cost, the more willing investors are to buy. In this sense, leverage functions similarly to risk aversion. And the equilibrium as such is similar to the one considered in Hong et al. (2006), who consider the pricing of an equity claim with risk averse investors and a finite supply of the security.

More specifically, we now state Theorem 1.

Theorem 1. *Suppose that $\eta > \sigma$, i.e. the initially pessimistic agents become the optimists in state-B. Define $\theta = \frac{Q}{\mu\gamma}$. Then there exists $\theta^1 > \theta^3$ (given in the Proof in the Appendix) such that the equilibrium has the following three cases.*

Case 1. For $\theta \geq \theta^1$, short-sales constraints do not bind and both agents are long at date 0 and 1. The prices are given by

$$P_0 = \frac{1}{4}\mu\pi(\sigma + \eta) + \frac{1}{4}\mu\pi(\sigma - \eta) + \frac{1}{4}\mu\pi(-\sigma + \eta) + \frac{1}{4}\mu\pi(-\sigma - \eta) - \frac{Q}{\gamma} \quad (13)$$

$$P_1(A) = \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(-\sigma - \eta), \quad P_1(B) = \frac{1}{2}\mu\pi(-\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta) \quad (14)$$

Case 2. For $\theta^1 > \theta \geq \theta^3$, the equilibrium has both agents long at date 0. In state-A, only the σ agents are long at date 1. In state-B, both agents are long at date 1.

$$P_0 = \frac{\mu}{3}(\pi(\sigma + \eta) + \pi(\sigma - \eta)) + \frac{\mu}{6}(\pi(-\sigma + \eta) + \pi(\sigma + \eta)) - \frac{4Q}{3\gamma} \quad (15)$$

$$P_1(A) = \mu\pi(\sigma + \eta) + \frac{\mu}{6} (\pi(\sigma - \eta) - \pi(-\sigma + \eta)) - \frac{2Q}{3\gamma}, \quad P_1(B) = \frac{1}{2}\mu\pi(-\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta) \quad (16)$$

Case 3 For $\theta^3 > \theta$, the equilibrium has both agents long at date 0. In state-A, only the σ agents are long at date 1. In state-B, only the $-\sigma$ agents are long at date 1.

$$P_0 = \frac{1}{2} (\mu\pi(\sigma + \eta) + \mu\pi(-\sigma + \eta)) - \frac{2Q}{\gamma} \quad (17)$$

$$P_1(A) = \mu\pi(\sigma + \eta) - \frac{Q}{\gamma}, \quad P_1(B) = \mu\pi(-\sigma + \eta) - \frac{Q}{\gamma} \quad (18)$$

Proof. See Appendix. □

When θ is large, i.e. Case 1, there is little buying power relative to supply, short-sales constraints do not bind at $t = 1$ and both investors will be in the market in both periods. When θ is moderate, i.e. Case 2, short-sales constraint bind in state-A (only the optimistic σ agents are long) but leverage is still sufficiently costly so that short-sales constraints do not bind in state-B (both agents are long). When θ is small, i.e. buying power is large, then the more optimistic agents in each state buy up all the shares, i.e. the σ agents in state-A and the $-\sigma$ agents in state-B (because of the assumption that $\eta > \sigma$). Both types of agents will be long the asset at date 0 since both value the possibility of re-selling at a higher price at date 1.

We now give more details about the economic intuition behind these three cases. The most straightforward case is Case 1: it is the benchmark case – short-sales constraints are never binding. As a consequence, prices at date 1 ($P_1(A)$ and $P_1(B)$) are simply equal-weighted averages of the valuations of the two investors in these two states. There is no resale option in this case. For the purposes of the discussion below, we denote these benchmark prices by \bar{P}_0 , $\bar{P}_1(A)$ and $\bar{P}_1(B)$.

In Case 2, short-sales constraints are binding in state-A, but not in state-B. This is because leverage is cheaper than in Case 1, but not yet large enough to allow the most optimistic agents in state-B to buy all the shares. Notice that disagreement is larger in state-A than in state-B, as $\pi(\sigma + \eta) - \pi(-\sigma - \eta) > \pi(-\sigma + \eta) - \pi(\sigma - \eta)$.

In state-A, only σ agents are long. The effective supply of the asset comes from the $-\sigma$ agents who end up selling all their shares. As a result, the date-1 price depends positively on the σ agents' belief ($\pi(\sigma + \eta)$) and negatively on the effective supply (i.e. the $-\sigma$ agents' date-0 holding of the asset). The date-1 price in state-A given in Theorem 1 is derived from the following expression:

$$P_1(A) = \underbrace{\mu\pi(\sigma + \eta)}_{\text{belief of the } \sigma \text{ agents}} - \underbrace{\frac{n_0^{-\sigma}}{\gamma}}_{\text{supply of from } -\sigma \text{ agents}} .$$

In state-B, short-sales constraints are not binding. As a consequence, the date-0 price is simply an equal-weighted average of the beliefs of the two agents. The price is equal to the benchmark price $\bar{P}_1(B)$.

Importantly, $P_1(A)$ is higher than the benchmark $\bar{P}_1(A)$. As a consequence, $-\sigma$ agents enjoy a resale option – because of binding short-sales constraints in state-A, they can resell the asset for a higher value. The date-0 price can be decomposed in the following way:

$$P_0 = \underbrace{\bar{P}_0}_{\text{benchmark price}} + \underbrace{\frac{1}{2}(P_1(A) - \bar{P}_1(A))}_{\text{resale option}},$$

where \bar{P}_0 is again the date-0 price in the absence of short-sales constraints. The second term of the decomposition reflects that when the state of nature is A (which occurs with probability 1/2), the price becomes larger than what it would be in the absence of a short-sales constraint. The date-0 price in Case 2 thus incorporates this resale option that occurs when the σ agents become so optimist at date-1 and the $-\sigma$ agents are sidelined.

In Case 3, short-sales constraints are binding in both states A and B. The date-0 equi-

librium is now symmetric and agents share the same date-0 holdings, i.e. $n_0^\sigma = n_0^{-\sigma} = Q$. The effective date-1 supply of the asset at date-1 is Q and prices can be written as:

$$P_1(i) = \underbrace{\mu\pi(V_i)}_{\text{belief of optimist agents}} - \underbrace{\frac{Q}{\gamma}}_{\text{supply from pessimists}}$$

The other difference with Case 2 is that the date-0 price now incorporates two resale options – the resale option for agents σ in state-B and the resale option for agents $-\sigma$ in state-A. The date-0 price can be decomposed in the following way:

$$P_0 = \underbrace{\bar{P}_0}_{\text{benchmark price}} + \underbrace{\frac{1}{2}(P_1(A) - \bar{P}_1(A)) + \frac{1}{2}(P_1(B) - \bar{P}_1(B))}_{\text{resale option}}$$

We next describe the equilibrium for the instance where $\eta < \sigma$. The main difference is that in state-B, the σ agent remains more optimistic than the $-\sigma$ agent.

Theorem 2. *Suppose $\eta < \sigma$, i.e. there are no crossing in beliefs between σ and $-\sigma$ agents in state-B. Then there exists $\theta^1 > \theta^2$ (given in the Proof in the Appendix) such that the equilibrium has the following structure.*

Case 1. For $\theta \geq \theta^1$, the equilibrium has no short-sales constraint – both agents are long at date 0 and 1. The prices are given in Case 1 of Theorem 1.

Case 2. For $\theta^1 > \theta \geq \theta^2$, the equilibrium has both agents long at date 0. In state-A, only σ agents long at date 1. In state-B, both agents are long at date 1. The prices are given as in Case 2 of Theorem 1.

Case 3. For $\theta^2 > \theta$, the equilibrium has both agents long at date 0 but only the σ agents long at date 1.

$$P_0 = \left(\frac{1}{2}\mu\pi(\sigma - \eta) + \frac{1}{2}\mu\pi(\sigma + \eta) \right) - \frac{3Q}{2\gamma} \quad (19)$$

$$P_1(A) = \mu\pi(\sigma + \eta) - \frac{Q}{2\gamma}, \quad P_1(B) = \mu\pi(\sigma - \eta) - \frac{Q}{2\gamma} \quad (20)$$

Proof. See Appendix. □

The analysis for Theorem 2 is similar to Theorem 1. Case 1 prices serve as the benchmark in which no resale options exist. There are resale options in both Cases 2 and 3. The only difference (beyond different cut-off values for the definition of the Cases) comes from the nature of the resale option in Case 3. When leverage is sufficiently cheap, the most optimistic agents always end up buying up all the available shares at date 1. However, because we assume now that $\eta < \sigma$, the σ agents always remain the most optimistic agent at date 1, both in states A and B. Thus, in Case 3, only the σ agents get to be long at date 1.

3.3. Comparative Statics

In this section, we consider comparative statics with respect to some of the key parameters of our model: the riskiness of debt D , leverage μ and the average optimism of agents F . These comparative statics yield our first three results.

Proposition 1. *When the equilibrium features short-sales constraint, an increase in D (the riskiness of debt) leads to an increase in (1) mispricing (2) price volatility, and (3) share turnover. Moreover, an increase in D increases the probability that short-sales constraints are binding.*

Proof. See Appendix. □

Our analysis offers a rationale for why debt bubbles are quieter than equity ones or why bubbles in AAA debt are quieter than bubbles in risky or junk debt. Note first that for valuing a debt contract, only the belief on the probability of default of the contract is relevant. This is because conditional on no-default, the payoff is bounded by D and thus insensitive to belief on the distribution of payoffs above D . Thus, when D is low, there is

very little scope for disagreement, and hence less scope for volatility and turnover. When D becomes larger, the impact of beliefs on the expected value of the asset becomes larger – in the extreme, when D grows to infinity, beliefs become relevant for the entire payoff distribution and the scope for disagreement is maximum. Thus, for larger D , both volatility and turnover becomes larger.

The next proposition relates trading behavior and the price of the asset to the cost of leverage in this economy:

Proposition 2. *When the equilibrium features short-sales constraint, for a debt bubble to be as large as an equity bubble requires greater leverage μ . A decrease in the cost of financing (i.e. an increase in μ) leads to an increase in (1) mispricing (2) price volatility, and (3) share turnover and increases the probability that short-sales constraints are binding.*

Proof. See Appendix. □

The intuition for this result comes from the cut-off values for Theorem 1 and 2 involving $\theta = \frac{Q}{\mu\gamma}$. The higher is μ the cheaper is leverage and the greater the buying power of the investors. As a result, disagreements about investors get magnified when μ is greater. Also, the region over which Case 1 exists (i.e. the non-binding short-sales region) shrinks and the resale option rises. With a larger resale option comes greater price volatility and share turnover. So while leverage will make the credit bubble larger, it will also make it louder.

Increasing leverage is not the only way to increase mispricing of the debt claim. Another way is to increase investor average optimism, i.e. F . The next proposition shows the role of the average bias on trading behavior and asset prices.

Proposition 3. *Assume that π is strictly concave. When short-sales constraints are binding, an increase in optimism in the economy (i.e. an increase in the average belief of investors F) leads to (1) higher mispricing (2) lower price volatility, and (3) lower share turnover. Assume that π is linear. When short-sales constraints are binding, an increase in optimism in*

the economy (i.e. an increase in the average belief of investors F) leads to higher mispricing but leaves price volatility and share turnover unaffected.

Proof. See Appendix. □

In our model, provided that the payoff function is strictly concave (or equivalently that $D < \infty$) and that there is a resale option (i.e. once short-sale constraints bind), an increase in the average optimism F will lead to a decrease in share turnover and a decrease in price volatility while the mispricing will clearly increase. In other words, an increase in average optimism makes the bubble bigger and quieter at the same time. The intuition is that as F increases, the date-1 differences in opinion about the concave payoff are attenuated: this is because as F increases, the debt claim appears safer and safer and thus fixed differences in belief about the fundamental leads to lower and lower disagreement about the payoff of the debt claim. In this sense, an increase in average optimism is similar to a decrease in the ex-post dispersion of belief and thus leads to a lower resale option. As a result, price volatility and share turnover decrease.

Interestingly, if the asset was a straight equity claim, both volatility and turnover would be left unaffected by variations in the average optimism – even in the case of binding short-sales constraint. This is because differences in opinion about an asset with a linear payoff are invariant to a translation in initial beliefs. Thus while an increase in optimism would obviously inflate the price of an equity, it would not change its price volatility nor its turnover.

3.4. Extension: Interim Payoffs and Dispersed Priors

We showed in the previous section how to make credit bubbles quieter than equity bubbles while keeping the amount of mispricing constant. We did this by allowing for aggregate optimism – i.e. for the possibility that the average belief on the fundamental is higher than the true expected value. In this section, we highlight another mechanism to make credit bubbles quieter and larger while keeping the distribution belief unbiased.

In order to do so, we introduce an additional ingredient to our model. Agents now receive an interim payoff $\pi(G + \epsilon_1)$ at date-1 from holding the asset at date 0. One can think of this pay-off as an interest payment in the context of debt and a dividend payment in the context of equity. Investors now hold the asset both for the utility of the interim pay-off and for ability to resell it to more optimistic agents in the future. This $t = 1$ cash-flow occurs before the states of nature A and B are realized. We assume that the proceeds from this interim cash flow, as well as the payment of the date-0 and date-1 transaction costs all occur on the terminal date. This assumption is made purely for tractability reason so we do not have to keep track of the interim wealth of the investors.

Before we state the formal proposition, we explain the intuition for how this interim payoff generates quietness and a large mispricing at the same time. The σ agents are more optimistic about the interim payoff than the $-\sigma$ agents. This interim payoff motive leads them to buy more shares of the asset than the pessimistic agents at date 0. This occurs despite the fact that both types of agents share the same valuation for the date-1 resale option, described in the previous section. This effect is entirely driven by the interim payoff: in the previous pure resale option setting, all agents ended up long at date 0 as there was no disagreement about the value of the resale option.

Consider now an equilibrium where in state-B the σ agents end up buying all the shares. This corresponds to the setting in Theorem 2 where the interim belief shock is not too large, i.e. $\eta < \sigma$. As dispersion (i.e. σ) increases, so does the disagreement about the interim payoff. As a consequence, the σ agents hold more and more shares. This decreases turnover as the $-\sigma$ agents supply of shares with which they can resell to the optimistic σ agents at date 1 decreases.

Price volatility reflects the differences in belief between the optimistic agents across the two states of nature decreases with an increase in the dispersion of priors. This result is driven by the concavity of the asset payoff. When dispersion increases, the asset appears safer and safer for the σ agents. As a result, a marginal increase in the belief about the

fundamental becomes less and less significant on the asset valuation. When σ becomes sufficiently large, the asset resembles a risk-free asset and volatility goes down to 0. This last mechanism would not arise with an equity claim.

Finally, mispricing (i.e. the spread between the date-0 price and the no-short-sales constraint price – will increase with an increase in dispersion. This is because as dispersion increases, the valuation for the interim pay-off and the marginal buyer’s valuation at $t = 1$ both increase, leading to a higher date-0 price.

The following proposition formalizes this intuition.

Proposition 4. *Assume that leverage is sufficiently cheap and dispersion sufficiently large so that:*

$$\frac{\pi(\sigma) - \pi(-\sigma)}{2} + \pi(\sigma - \eta) - \pi(-\sigma + \eta) > \frac{Q}{\mu\gamma} \quad \text{and} \quad \eta < \sigma.$$

Then an increase in dispersion σ leads to (1) a strict increase in date-0 mispricing (2) a weak decrease in turnover and (3) a strict decrease in volatility.

In other words, provided that leverage is cheap and initial beliefs are sufficiently dispersed, an increase in dispersion leads to a larger and quieter bubble.

It is important to emphasize that our effect exists irrespective of whether or not beliefs cross at date 1 or the assumption that $\sigma > \eta$. However, when $\sigma < \eta$, other countervailing effects also exist that make the overall impact of dispersion on equilibrium turnover and volatility ambiguous. For instance, when $\sigma < \eta$ and leverage is sufficiently cheap, the equilibrium will feature the σ agents only being long in state-A and the $-\sigma$ agents only being long in state-B, while both agents will be long at date 0. In such an equilibrium, an increase in dispersion will reduce the date-0 holding of the $-\sigma$ agents and thus reduce date-1 turnover in state-A. However, because $\eta > \sigma$, the $-\sigma$ agents end up buying all the asset in state-B. An increase in dispersion leads to a decrease in the date-0 holding of $-\sigma$ agents and will thus lead to an increase in turnover in state-B ($-\sigma$ agents have to buy more at equilibrium). These two effects exactly cancel each other and turnover ends up being

independent of dispersion.

4. Empirical Implications

In this section, we relate the predictions of our model to the stylized facts regarding the recent credit bubble presented above. In particular, disagreement and short-sales constraints leads to a concentration of an asset in the hands of the optimists. To this point, we provide evidence regarding the concentration of risk-taking in the hands of a small fraction of financial institutions.

Propositions 1-3 explain why the recent credit bubble is quieter than the Internet bubble. Internet bubbles can be fueled by resale and trading since equity pay-offs are linear in asset value. In contrast, the capped upside of credits such as the subprime mortgage CDOs means that more optimism or leverage is required for there to be a big mispricing in debt than in equity. Optimism makes the credit bubble quiet. Interestingly, when investors become more pessimistic at the onset of the crisis, disagreement becomes much more important in the valuation of debt contracts. This in turn triggers an increase in both price volatility and turnover and the bubble stops being “quiet”. This time-series behavior can be seen in the figures above as price volatility increases in the year preceding the crisis and the collapse of these markets. The collapse of the market is outside the model and is likely due to the impact of the high leverage, defaults and fire-sales.

A distinct prediction that emanates from our model and indeed all disagreement and short-sales constraints models is that disagreement leads to concentrated positions in the hands of the optimists. This prediction can be seen in accounts of the sub-prime mortgage CDO bubble during the years of 2003-2008 as being due to the concentrated positions of key institutional players such as AIG-FP which we discuss below (see Michael Lewis (2010)). Big banks’ exposures to these structured credits are responsible for their demise. Michael Lewis (2010) provides a detailed account of the history of this market. More specifically,

we make the case based on his account and other sources that an example of the extreme optimists with deep pockets in our model is AIG-FG – the financial markets group of AIG. AIG-FG insured 50 billion dollars worth of sub-prime AAA tranches between 2002 and 2005 at extremely low prices, years during which the issuance in the market was still relatively low, on the order of 100 billion dollars a year. Hence the bubble in CDOs was quiet because AIG-FP ended up being the optimist buyer of a sizable fraction of the supply of these securities. Michael Lewis (2010) provides evidence that AIG-FP, the trading division of AIG responsible for insuring sub-prime CDOs, and banks like it maybe the optimistic σ -agents in our model and as a result might have influenced prices and issuance in this market.

We begin with a summary of Michael Lewis (2010)’s extremely detailed timeline of the events surrounding the sub-prime mortgage CDO market’s rise between 2003 to 2007 and its implosion in 2008.

The Timeline of Sub-prime Mortgage CDO Bubble

- Before 2005, AIG-FP was de-facto the main optimistic buyer of sub-prime mortgages because it offered extremely cheap insurance for buyers of AAA tranches of these CDOs.
- In early years before 2005, AIG-FP ended up with a \$50 billion position in the CDO market, which was one-fourth of the total annual issuance then.
- AIG FP stopped insuring new mortgages after 2005, though they maintained insurance on old ones.
- After 2005, other banks including German Banks and UBS that were previously pessimistic started buying.
- After 2006, shorting becomes possible in the market through synthetic shorts on so-called mezzanine tranches (the worst quality sub-prime names) and the introduction of ABX indices.

- Market begins to collapse in 2007.

Michael Lewis (2010) makes the case, and convincingly from our perspective, that the market for sub-prime mortgage CDOs might not have taken off between 2003 and 2005 without the extremely cheap insurance offered by AIG-FP, which was then the largest insurance company in the world and one of the few companies with a AAA-rating and perceived invulnerable balance sheet. AIG-FP's extremely low insurance to companies like Goldman who underwrote the mortgages effectively makes them the optimistically skewed x -agent in our model since AIG naturally had access to extreme leverage by virtue of them being able to write insurance contracts.

For instance, Michael Lewis (2010) writes, "Stage Two, beginning at the end of 2004, was to replace the student loans and the auto loans and the rest with bigger piles consisting of nothing but U.S. sub-prime mortgage loans...The consumer loan piles that Wall Street firms, led by Goldman Sachs, asked AIG-FP to insure went from being 2 percent sub-prime mortgages to being 95 percent sub-prime mortgages. In a matter of months, AIG-FP in effect bought \$50 billion in triple B rated sub-prime mortgage by insuring them against default."

Why did AIG-FP take on such a large position? They did not think home prices could fall—i.e. our skewed priors assumption. Michael Lewis (2010) writes, "Confronted with the new fact that his company was effectively long \$50 billion in triple-B rated sub-prime mortgage bonds, masquerading as triple A-rated diversified pools of consumer loans—Cassano at first sought to rationalize it. He clearly thought that any money he received for selling default insurance on highly rated bonds was free money. For the bonds to default, he now said, U.S. home prices had to fall and Joe Cassano didn't believe house prices could ever fall everywhere in the country at once. After all, Moody's and S&P had rated this stuff triple-A!" Indeed, AIG FP continued to keep their insurance contracts even after 2005. In sum, anecdotal evidence very directly points to the central role of optimistic priors and leverage in influencing the sub-prime mortgage CDO bubble.

In Figure 5, we plot a figure shown from Stein (2010), who argues that the huge growth in

issuance in the non-traditional CDO market was an important sign that a bubble might have taken hold here compared to traditional structured products. Indeed, this plot of issuance activity between 2000 and 2009 shows that activity really jumped in 2004 when AIG begins to insure significant amounts of the the sub-prime names as described in Michael Lewis (2010).

Finally, we provide evidence that short-sales constraints were tightly binding until around 2006 when synthetic mezzanine ABS CDOs allowed hedge funds to short sub-prime CDOs, thereby leading to the implosion of the credit bubble. In Figure 6, we plot issuance of synthetic mezzanine ABS CDOs which is how hedge funds such as John Paulson's finally were able to short the sub-prime mortgage CDOs. Notice that there was very little shorting in this market until the end as issuances of this type of CDO are not sizable until 2006. In other words, short-sales constraints were binding tightly until around 2007, consistent with the premise of our model. The collapse coincided with a large supply of these securities in 2007, similar to what happened during the dot-com period. This collapse effect is already modeled in Hong et al. (2006).

5. Narratives of the Financial Crisis

Our theory complements recent work on the financial crisis. An important narrative is that a wave of money from China needed a safe place to park and in light of the lack of sovereign debt, Wall Street created AAA securities as a new parking place for this money. This search for safe assets led to inflated prices through a variety of channels. Caballero and Krishnamurthy (2009) provides a theory of how search for safe assets led to a shrinking of the risk premium. Gennaioli et al. (2010) focuses on the neglected risks from this demand for safety. Another narrative is that of agency and risk-shifting as in Allen and Gale (2000), which can also deliver asset price bubbles and would seem vindicated by the bailouts of the big banks. Gorton et al. (2010) provide a theory of why these securitized debt products are a

good substitute for Treasuries since they are information insensitive but when the economy is hit by a negative shock, these CDOs become more sensitive to private information and results in a loss of liquidity and trade (Holmstrom (2008)).

Our mechanism shares the most with the last approach in terms of our emphasis on the insensitivity of debt to underlying disagreement about asset values. By focusing on disagreement instead of asymmetric information leads to an opposite prediction compared to theirs, in which there is little trading in good times and more trading in bad times in CDOs. Our prediction is borne out in anecdotal accounts of the rise in speculative trade of CDOs after 2006-2007 but before the financial crisis in 2008. Our approach cannot capture the financial crisis and the ensuing freeze in trade.

Our theory also has a number of implications that are distinct from this earlier work. First, it offers a unified approach to explain both “classic” bubbles such as the dot-com and so called the recent so-called credit bubble.⁷ Second, it offers a story for how a financial crisis of this size could emerge despite our recent experience with bubbles. Anecdotal accounts of the crisis invariably point to how the crisis and the credit bubble caught everyone by surprise. Indeed, this is an especially large conundrum when one considers that sophisticated finance companies such as Goldman Sachs were able to survive and indeed thrive through many prior speculative episodes, including the the dot-com boom and bust. So why did companies like Goldman Sachs get caught this time?

Our unified theory offers a rationale for why smart investment banks and regulators failed to see the bubble on time. The low price volatility made these instruments appear safer in contrast to the high price volatility of dot-com stocks which made traders and regulator more aware of their dangers. Interestingly, our theory also predicts that riskier tranches of CDOs trade more like equity and hence have more price volatility and turnover, which is

⁷Our paper focuses on mispricing of the mortgages as opposed to the homes since evidence from Mian and Sufi (2009) indicate that the run-up in home prices was due to the cheap subprime home loans and that households took out refinancing from the equity of their homes for consumption as opposed to the purchase of additional homes, suggesting that there was limited speculation among households in the physical homes themselves. Khandani et al. (2010) make a similar point that the financial crisis was due to the refinancing of home loans as opposed to the speculative aspects described in the media.

also consistent with the evidence. These securities were also less responsible for the failure of the investment banks which were mostly caught with the higher rated AAA tranches of the sub-prime mortgage CDOs. Other theories for the crisis based on agency or financial innovation cannot explain why banks did not get caught during the dot-com bubble since they had the same agency problems and innovative products to price.

Third, our analysis also has implications for thinking about the Dodd-Frank financial reform package. This legislation establishes, among many things, an Office of Financial Research and a Financial Stability Oversight Committee that are meant to detect the next bubble. The premise of the Office of Financial Research is that by forcing finance firms to disclose their positions, regulators will be in a better position to detect the next speculative episode. Our model suggests that the quietness of the credit bubble would have made it difficult to detect regardless. Hence, regulators should distinguish between loud versus quiet bubbles as the signs are very different.

Fourth, the role of leverage in amplifying shocks has been understood and is clearly critical in melt-down or fire-sale part of the crisis (see Kiyotaki and Moore (1997) and Shleifer and Vishny (1997)). Our analysis shows that leverage can lower price volatility when there is an asset price bubble. Our analysis also echoes Hyman Minsky's warnings of bubbles fueled by credit. All in all, this paper can be thought of as offering an explanation for why bubbles fueled by credit are more dangerous – because they are quiet.

Finally, our theory is also consistent with the literature analyzing the destabilizing role of institutions in financial markets. This literature has pointed mostly toward constraints as the culprit for demand shocks (see, e.g., Vayanos and Gromb (2010)). Our theory offers a novel approach – namely that that outlier beliefs in institutional settings are more likely to be amplified with access to leverage.

6. Conclusion

With the onset of the financial crisis, the term "bubble" is being used, from our perspective too liberally, to describe any type of potential mispricings in the market ranging from equities to housing and credit. The term speculative bubble has traditionally been used to connote the idea that investors purchase an asset knowing that the price is high because they anticipate capital gains. The classic speculative episodes such as the recent dot-com bubble fit this definition and usually come with high price, high price volatility and high turnover. We argue that the credit bubble in sub-prime mortgage CDOs is fundamentally different as it was quieter —price is high but price volatility and turnover are low.

We offer first attempt at a taxonomy of bubbles that distinguishes between loud equity bubbles and quiet credit bubbles. This theory builds on the platform of disagreement and short-sales constraints that are key to getting loud bubbles. We show that credit bubbles are quieter than equity ones. The up-side concavity of debt pay-offs means debt instruments (especially higher rated ones) are less disagreement-sensitive than lower rated credit or equity. As a consequence, bubbles on debt claims are essentially characterized by lower price volatility, lower turnover and lower mispricing. To obtain large debt mispricing, either high leverage, average investor optimism or large dispersion of beliefs are required. Interestingly, average optimism does not make equity bubbles quieter. Future work elaborating on this taxonomy and providing other historical evidence would be very valuable.

A. Appendix

Proof of Theorem 1 and 2.

At date 1, the demand of an agent with belief V_1 and with prior V facing a price P_1 is easily defined as the first-order condition to problem 7:

$$n_1(V_1) = \max \{0, n_0(V) + \gamma (\mu\pi(V_1) - P_1)\}$$

In state-A, the date-1 equilibrium structure can be decomposed the following way:

1. Both agents are long at date 1. Their demand is given by:

$$\begin{cases} n_1^\sigma(A) = n_0^\sigma + \gamma (\mu\pi(\sigma + \eta) - P_1(A)) \\ n_1^{-\sigma}(A) = n_0^{-\sigma} + \gamma (\mu\pi(-\sigma - \eta) - P_1(A)) \end{cases}$$

Because $n_0^\sigma + n_0^{-\sigma} = 2Q$, the date-1 market clearing condition ($\frac{n_1^{-\sigma}}{2} + \frac{n_1^\sigma}{2} = Q$) imposes that the date-1 price following shock 1 is:

$$P_1(A) = \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(-\sigma - \eta)$$

This can be the date-1 equilibrium if and only if the $-\sigma$ agents want to be long, i.e. $n_1^{-\sigma} > 0$:

$$n_0^{-\sigma} > \frac{\gamma\mu}{2} (\pi(\sigma + \eta) - \pi(-\sigma - \eta))$$

2. Only agent σ is long. The price in this case is simply given by the date-1 market clearing condition ($\frac{n_1^\sigma}{2} = Q$) and using the fact that $Q - \frac{n_0^\sigma}{2} = -\frac{n_0^{-\sigma}}{2}$:

$$P_1(A) = \mu\pi(\sigma + \eta) - \frac{n_0^{-\sigma}}{\gamma}$$

$-\sigma$ agents will be sidelined if and only if $n_0^{-\sigma} + \gamma (\mu\pi(-\sigma - \eta) - P_1(A)) < 0$, i.e.:

$$n_0^{-\sigma} < \frac{\gamma\mu}{2} (\pi(\sigma + \eta) - \pi(-\sigma - \eta))$$

Consider now state B. Then three situations are possible:

1. Both agents are long at date 1. This case is similar to the first case considered in state A. The price

is thus again simply the average of the two agents beliefs, but these beliefs are now different:

$$P_1(B) = \frac{1}{2}\mu\pi(-\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta)$$

This can be the equilibrium price if and only if both $-\sigma$ and the σ agents want to be long, i.e.:

$$\begin{cases} n_1^\sigma > 0 \Leftrightarrow n_0^{-\sigma} > \frac{\gamma\mu}{2} (\pi(\sigma - \eta) - \pi(-\sigma + \eta)) \\ n_1^{-\sigma} > 0 \Leftrightarrow n_0^\sigma > \frac{\gamma\mu}{2} (\pi(-\sigma + \eta) - \pi(\sigma - \eta)) \end{cases}$$

2. Only agent σ is long. This case is similar to case 2 in state A, except that the σ agents belief is now $\sigma - \eta$:

$$P_1(B) = \mu\pi(\sigma - \eta) - \frac{n_0^{-\sigma}}{\gamma}$$

$-\sigma$ agents will be sidelined when $n_0^{-\sigma} + \gamma(\mu\pi(-\sigma + \eta) - P_1(B)) < 0$ i.e.:

$$n_0^{-\sigma} < \frac{\gamma\mu}{2} (\pi(\sigma - \eta) - \pi(-\sigma + \eta))$$

3. Only agent $-\sigma$ is long. The price in this case is simply given by the date-1 market clearing condition ($\frac{n_1^{-\sigma}}{2} = Q$) and using the fact that $Q - \frac{n_0^{-\sigma}}{2} = -\frac{n_0^\sigma}{2}$:

$$P_1(B) = \mu\pi(-\sigma + \eta) - \frac{n_0^\sigma}{\gamma}$$

σ agents will be sidelined when $n_0^\sigma + \gamma(\mu\pi(\sigma - \eta) - P_1(B)) < 0$ i.e.:

$$n_0^\sigma < \frac{\gamma\mu}{2} (\pi(-\sigma + \eta) - \pi(\sigma - \eta))$$

This ends up the description of the date-1 equilibrium structure. We now solve for the date-0 equilibrium structure. We will show that the equilibrium can only feature both agents long at date-0. Intuitively, if $-\sigma$ agents were sidelined at date-0, by buying a small quantity of the asset at date-0 and reselling it to the σ agents, she would enjoy the same marginal benefit but would have a zero marginal cost.

More precisely, we show that only the following four equilibria are sustainable.

1. **Equilibrium 1:** Consider first the equilibrium where all agents are long at date 1, for state A and B and both agents are long at date 0. This corresponds to Case 1 in Theorem 1 and 2. If this is the equilibrium, then the σ agents program at date 0 writes:

$$\max_{n_0} \frac{1}{2} J(n_0, \sigma + \eta) + \frac{1}{2} J(n_0, \sigma - \eta) - \frac{1}{\mu} \left(n_0 P_0 + \frac{n_0^2}{2\gamma} \right)$$

Which yields the following first order condition:

$$n_0 = \gamma \left(\frac{\mu}{2} \frac{\partial J}{\partial n_0}(n_0, \sigma + \eta) + \frac{\mu}{2} \frac{\partial J}{\partial n_0}(n_0, -\sigma + \eta) - P_0 \right)$$

Remember that: $J(n_0, V_1) = \max_{n_1 \geq 0} \left\{ n_1 \pi(V_1) - \frac{1}{\mu} \left((n_1 - n_0) P_1 + \frac{(n_1 - n_0)^2}{2\gamma} \right) \right\}$

When agents are not short-sales constrained, J is defined by an interior solution and we can apply the envelop theorem as well as the first order condition, so that:

$$\frac{\partial J}{\partial n_0}(n_0, V_1) = \frac{P_1}{\mu} + \frac{n_1 - n_0}{\gamma \mu} = \frac{P_1}{\mu} + \pi(V_1) - \frac{P_1}{\mu} = \pi(V_1)$$

In the equilibrium we consider, σ agents are not short-sales constrained, thus, the first order condition of their portfolio decision can be re-written:

$$n_0 = \gamma \left(\frac{1}{2} \mu \pi(\sigma + \eta) + \frac{1}{2} \mu \pi(\sigma - \eta) - P_0 \right)$$

Similarly, the $-\sigma$ agents are not short-sales constrained at date 1, so that eventually date-0 demands simply write:

$$\begin{cases} n_0^\sigma = \gamma \left(\frac{1}{2} \mu \pi(\sigma + \eta) + \frac{1}{2} \mu \pi(\sigma - \eta) - P_0 \right) \\ n_0^{-\sigma} = \gamma \left(\frac{1}{2} \mu \pi(-\sigma + \eta) + \frac{1}{2} \mu \pi(-\sigma - \eta) - P_0 \right) \end{cases}$$

The date-0 equilibrium price is a standard market clearing condition:

$$\frac{n_0^\sigma}{2} + \frac{n_0^{-\sigma}}{2} = Q \Leftrightarrow P_0 = \frac{1}{4} \mu \pi(\sigma + \eta) + \frac{1}{4} \mu \pi(\sigma - \eta) + \frac{1}{4} \mu \pi(-\sigma + \eta) + \frac{1}{4} \mu \pi(-\sigma - \eta) - \frac{Q}{\gamma}$$

For this to be an equilibrium, it needs to be first that $n_0^\sigma > 0$ and $n_0^{-\sigma} > 0$. It is always true that: $n_0^\sigma > 0$. Moreover:

$$n_0^{-\sigma} > 0 \Leftrightarrow \frac{Q}{\mu \gamma} > \frac{1}{4} \pi(\sigma + \eta) + \frac{1}{4} \pi(\sigma - \eta) - \left(\frac{1}{4} \pi(-\sigma + \eta) + \frac{1}{4} \pi(-\sigma - \eta) \right) \quad (21)$$

This equilibrium is also sustainable if and only if the date-0 demands allow the date 1 equilib-

rium in state A and B to hold. This implies that $n_0^{-\sigma} > \frac{\gamma\mu}{2} (\pi(\sigma + \eta) - \pi(-\sigma - \eta))$ and $n_0^\sigma > \frac{\gamma\mu}{2} (\pi(-\sigma + \eta) - \pi(\sigma - \eta))$, i.e.:

$$\begin{cases} \frac{Q}{\mu\gamma} > \frac{3}{4} (\pi(\sigma + \eta) - \pi(-\sigma - \eta)) + \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{4} \\ \frac{Q}{\mu\gamma} > \frac{3}{4} (\pi(-\sigma + \eta) - \pi(\sigma - \eta)) + \frac{\pi(-\sigma - \eta) - \pi(\sigma + \eta)}{4} \end{cases} \quad (22)$$

Note that the right hand side of the first inequality in (24) is strictly greater than the right hand side of the second inequality in (24). Note also that the right hand side of the first inequality in (24) is greater than the right hand side of inequality (23). As a consequence, the condition defining this equilibrium is simply:

$$\frac{Q}{\mu\gamma} > \frac{3}{4} (\pi(\sigma + \eta) - \pi(-\sigma - \eta)) + \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{4}$$

Expected turnover in this equilibrium is:

$$\mathbb{T} = \underbrace{\frac{1}{2}}_{\text{prob. of A}} \times \underbrace{\frac{1}{2}}_{\sigma \text{ weight}} \times \underbrace{\mu\gamma \left(\frac{\pi(\sigma + \eta) - \pi(-\sigma - \eta)}{2} \right)}_{=|n_1^\sigma(A) - n_0^\sigma|} + \underbrace{\frac{1}{2}}_{\text{prob. of B}} \times \underbrace{\frac{1}{2}}_{-\sigma \text{ weight}} \times \underbrace{\mu\gamma |\pi(-\sigma + \eta) - \pi(\sigma - \eta)|}_{=|n_1^\sigma(B) - n_0^\sigma|}$$

So that:

$$\mathbb{T} = \frac{\mu\gamma}{8} (\pi(\sigma + \eta) - \pi(-\sigma - \eta) + |\pi(-\sigma + \eta) - \pi(\sigma - \eta)|)$$

Price volatility is:

$$\sigma_P = |P_1(B) - P_1(A)| = \frac{1}{2}\mu (\pi(\sigma - \eta) - \pi(\sigma + \eta) + \pi(-\sigma + \eta) - \pi(-\sigma - \eta))$$

There is no resale option in this equilibrium, as there are no binding short-sales constraint in this equilibrium.

2. **Equilibrium 2:** Consider now the case where (1) both types of agents are long at date 0 (2) following state-A only the σ agents are long and (2) following state-B both types of agents are long. This corresponds to Case 2 in Theorem 1 and 2. In this equilibrium σ agents are never short-sales constrained. Thus, their demand is similar to equilibrium 1:

$$n_0^\sigma = \gamma \left(\frac{1}{2} \mu \pi(\sigma + \eta) + \frac{1}{2} \mu \pi(\sigma - \eta) - P_0 \right)$$

However, $-\sigma$ agents are now short-sales constrained at date 1. As a consequence, their value function in this state is:

$$J(n_0, V) = \frac{1}{\mu} \left(n_0 P_1 - \frac{n_0^2}{2\gamma} \right)$$

Therefore, the first-order condition of $-\sigma$ agents can be rewritten as:

$$\begin{aligned} n_0 &= \gamma \left(\frac{\mu}{2} \frac{\partial J}{\partial n_0}(n_0, -\sigma - \eta) + \frac{\mu}{2} \frac{\partial J}{\partial n_0}(n_0, -\sigma + \eta) - P_0 \right) \\ &= \gamma \left(\frac{\mu}{2} \times \frac{1}{\mu} \left(P_1(A) - \frac{n_0}{\gamma} \right) + \frac{\mu}{2} \pi(-\sigma + \eta) - P_0 \right) \end{aligned}$$

We know that in this equilibrium $P_1(A) = \mu \pi(\sigma + \eta) - \frac{n_0^{-\sigma}}{\gamma}$. Thus:

$$n_0^{-\sigma} = \gamma \left(\frac{1}{2} \mu \pi(\sigma + \eta) + \frac{1}{2} \mu \pi(-\sigma + \eta) - P_0 \right) - n_0^\sigma$$

Eventually, the date-0 demands are given by:

$$\begin{cases} n_0^\sigma = \gamma \left(\frac{1}{2} \mu \pi(\sigma + \eta) + \frac{1}{2} \mu \pi(\sigma - \eta) - P_0 \right) \\ n_0^{-\sigma} = \frac{1}{2} \gamma \left(\frac{1}{2} \mu \pi(-\sigma + \eta) + \frac{1}{2} \mu \pi(\sigma + \eta) - P_0 \right) \end{cases}$$

The date-0 price is simply computed with the date-0 market clearing condition ($\frac{n_0^\sigma + n_0^{-\sigma}}{2} = Q$):

$$P_0 = \frac{\mu}{2} \pi(\sigma + \eta) + \frac{\mu}{3} \pi(\sigma - \eta) + \frac{\mu}{6} \pi(-\sigma + \eta) - \frac{4}{3} \frac{Q}{\gamma}$$

We can compute the date-0 equilibrium demand, including the endogenous date-0 equilibrium price:

$$\begin{cases} n_0^\sigma = \frac{4}{3} Q + \frac{\gamma \mu}{6} (\pi(\sigma - \eta) - \pi(-\sigma + \eta)) \\ n_0^{-\sigma} = \frac{2}{3} Q - \frac{\gamma \mu}{6} (\pi(\sigma - \eta) - \pi(-\sigma + \eta)) \end{cases}$$

The equilibrium implies that the $-\sigma$ agents are long at date 0. This is the case if:

$$\frac{Q}{\mu \gamma} > \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{4}$$

The other conditions for this equilibrium to exist is that at date 1, the date-0 demands are such that (1) the $-\sigma$ agents are sidelined in state A and (2) both types of agents are long in state B. There are two possible cases:

- (a) If $\eta < \sigma$, then the binding condition for the date-1 equilibrium to exist is that the $-\sigma$ agents are sidelined in state A and the $-\sigma$ agents are long in state B (this is intuitive as under the assumption that $\eta < \sigma$, the $-\sigma$ agents are the pessimists in state B, and thus are the more likely to be sidelined). Using the equilibrium conditions derived for the date-1 equilibrium, these two conditions can simply be written as:

$$\begin{cases} \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{4} + \frac{3}{4}(\pi(\sigma + \eta) - \pi(-\sigma - \eta)) > \frac{Q}{\mu\gamma} \\ \frac{Q}{\mu\gamma} > \pi(\sigma - \eta) - \pi(-\sigma + \eta) \end{cases}$$

- (b) If $\eta > \sigma$, then the binding condition for the date-1 equilibrium to exist is that the $-\sigma$ agents are sidelined in state A and the σ agents are long in state B (this is intuitive as under the assumption that $\eta > \sigma$, the σ agents are the pessimists in state B, and thus are the more likely to be sidelined). Using the equilibrium conditions derived for the date-1 equilibrium, these two conditions can simply be written as:

$$\begin{cases} \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{4} + \frac{3}{4}(\pi(\sigma + \eta) - \pi(-\sigma - \eta)) > \frac{Q}{\mu\gamma} \\ \frac{Q}{\mu\gamma} > \frac{1}{2}(\pi(-\sigma + \eta) - \pi(\sigma - \eta)) \end{cases}$$

Note in particular that the condition that the $-\sigma$ agents are long at date 0 is never binding:

We can now compute turnover in a fashion similar to equilibrium 1:

$$\begin{aligned} \mathbb{T} &= \frac{1}{2} \times \frac{1}{2} \times n_0^{-\sigma} + \frac{1}{2} \times \frac{1}{2} |n_1^\sigma(B) - n_0^\sigma(B)| \\ &= \frac{Q}{6} - \frac{\gamma\mu}{24} (\pi(\sigma - \eta) - \pi(-\sigma + \eta)) + \frac{1}{8}\gamma\mu |\pi(\sigma - \eta) - \pi(-\sigma + \eta)| \end{aligned}$$

Volatility is:

$$\sigma_P = |P_1(B) - P_1(A)| = \mu\pi(\sigma + \eta) - \frac{1}{3}\mu\pi(\sigma - \eta) - \frac{2}{3}\mu\pi(-\sigma + \eta) - \frac{2Q}{3}$$

The resale option – defined as the difference between P_0 and the price in the absence of short-sales constraints is:

$$\text{resale} = \frac{1}{4} (\pi(\sigma + \eta) - \pi(-\sigma - \eta)) + \frac{1}{12} (\pi(\sigma - \eta) - \pi(-\sigma + \eta)) - \frac{Q}{3\gamma}$$

Denote by $\bar{P}_1(A)$ the price in state A in equilibrium 1. Then one easily sees that the resale option is:

$$\text{resale} = \underbrace{\frac{1}{2}}_{\text{prob. of state A}} (P_1(A) - \bar{P}_1(A))$$

3. **Equilibrium 3:** Consider now the equilibrium where following state-A, only σ agents are long and following state-B, only $-\sigma$ agents are long. Both agents are long at date 0. This corresponds to Case 3 in Theorem 1. Indeed, this equilibrium will only occur if $\eta > \sigma$, i.e. if agents $-\sigma$ become the optimists in state B.

In this equilibrium, σ agents are short-sales constrained in state B and $-\sigma$ agents are short-sales constrained in state A. As a consequence, the first order condition of both types of agents is similar to that of $-\sigma$ agents in equilibrium 2, i.e.:

$$\begin{cases} n_0^\sigma = \gamma \left(\frac{1}{2} \mu \pi(\sigma + \eta) + \frac{1}{2} \mu \pi(-\sigma + \eta) - P_0 \right) - n_0^\sigma \\ n_0^{-\sigma} = \gamma \left(\frac{1}{2} \mu \pi(\sigma + \eta) + \frac{1}{2} \mu \pi(-\sigma + \eta) - P_0 \right) - n_0^{-\sigma} \end{cases}$$

Date-0 demands are thus easily found:

$$\begin{cases} n_0^\sigma = \frac{1}{2} \gamma \left(\frac{1}{2} \mu \pi(\sigma + \eta) + \frac{1}{2} \mu \pi(-\sigma + \eta) - P_0 \right) \\ n_0^{-\sigma} = \frac{1}{2} \gamma \left(\frac{1}{2} \mu \pi(\sigma + \eta) + \frac{1}{2} \mu \pi(-\sigma + \eta) - P_0 \right) \end{cases}$$

In particular, we notice that this equilibrium has symmetric date-0 demand. The date-0 price is again given by the standard date-0 market clearing condition:

$$P_0 = \frac{1}{2} (\mu \pi(\sigma + \eta) + \mu \pi(-\sigma + \eta)) - \frac{2Q}{\gamma}$$

Because the equilibrium is symmetric, the date-0 demand are trivially given by $n_0^\sigma = n_0^{-\sigma} = Q$.

The relevant condition for this equilibrium to exist is that the σ agents are sidelined in state B. This condition is:

$$\frac{1}{2} (\pi(-\sigma + \eta) - \pi(\sigma - \eta)) > \frac{Q}{\mu\gamma}$$

In particular, notice that this last condition can be satisfied only if $\sigma < \eta$.

Turnover in this equilibrium is $\mathbb{T} = \frac{1}{2} \times \frac{1}{2} \times Q + \frac{1}{2} \times \frac{1}{2} \times Q = \frac{Q}{2}$.

Price volatility is:

$$\sigma_P = |P_1(A) - P_1(B)| = \mu (\pi(\sigma + \eta) - \pi(-\sigma + \eta))$$

The resale option is computed as:

$$\text{resale} = \frac{\mu}{4} (\pi(\sigma + \eta) - \pi(\sigma - \eta) + \pi(-\sigma + \eta) - \pi(-\sigma - \eta)) - \frac{Q}{\gamma}$$

Again, it is easily checked that the resale option is the difference between the date-1 prices in this equilibrium and the date-1 prices in the no-short-sales constraint equilibrium, i.e.:

$$\text{resale} = \frac{1}{2}(P_1(A) - \bar{P}_1(A)) + \frac{1}{2}(P_1(B) - \bar{P}_1(B))$$

4. **Equilibrium 4:** Finally, consider the equilibrium where following state-A only σ agents are long and following state-B, only σ agents are long. Both agents are long at date 0. This corresponds to Case 3 in Theorem 2. This equilibrium happens when (1) the σ agents are always the optimists ($\sigma > \eta$) (2) leverage is cheap (so that at date 1, only the most optimistic agents are long the asset).

In this equilibrium, σ agents are never short-sales constrained, so their demand is similar to equilibrium 1, i.e.:

$$n_0^\sigma = \gamma \left(\frac{1}{2} \mu \pi(\sigma + \eta) + \frac{1}{2} \mu \pi(\sigma - \eta) - P_0 \right)$$

$-\sigma$ agents, however, are short sales constrained in both states. Their program can thus be written as:

$$\max_{n_0} \left\{ \frac{1}{2\mu} \left(P_1(A)n_0 - \frac{n_0^2}{2\gamma} \right) + \frac{1}{2\mu} \left(P_1(B)n_0 - \frac{n_0^2}{2\gamma} \right) - \frac{1}{\mu} \left(n_0 P_0 - \frac{n_0^2}{2\gamma} \right) \right\}$$

The first order condition of this program yields:

$$2n_0^{-\sigma} = \gamma \left(\frac{1}{2} (P_1(A) + P_1(B)) - P_0 \right)$$

But we know that in this equilibrium: $P_1(A) = \mu\pi(\sigma + \eta) - \frac{n_0^{-\sigma}}{\gamma}$ and $P_1(B) = \mu\pi(\sigma - \eta) - \frac{n_0^{-\sigma}}{\gamma}$

Thus, the first-order condition can be rewritten:

$$3n_0^{-\sigma} = \gamma \left(\frac{1}{2} \mu \pi(\sigma - \eta) + \frac{1}{2} \mu \pi(\sigma + \eta) - P_0 \right)$$

Date-0 demands are thus given by:

$$\begin{cases} n_0^\sigma = \gamma \left(\frac{1}{2} \mu \pi(\sigma + \eta) + \frac{1}{2} \mu \pi(\sigma - \eta) - P_0 \right) \\ n_0^{-\sigma} = \frac{1}{3} \gamma \left(\frac{1}{2} \mu \pi(\sigma - \eta) + \frac{1}{2} \mu \pi(\sigma + \eta) - P_0 \right) \end{cases}$$

And the date-0 price is derived from the date-0 market clearing condition:

$$P_0 = \left(\frac{1}{2} \mu \pi(\sigma - \eta) + \frac{1}{2} \mu \pi(\sigma + \eta) \right) - \frac{3Q}{2\gamma}$$

Equilibrium date-0 demands can be computed using the date-0 equilibrium price:

$$\begin{cases} n_0^\sigma = \frac{3}{2} Q > 0 \\ n_0^{-\sigma} = \frac{Q}{2} > 0 \end{cases}$$

Note that this equilibrium is asymmetric: σ agents hold more share than $-\sigma$ agents. This is because they are never short-sales constrained.

The relevant condition that defines this equilibrium is that the $-\sigma$ agents are sidelined in state B. This condition is simply:

$$\pi(\sigma - \eta) - \pi(-\sigma + \eta) > \frac{Q}{\mu\gamma}$$

Turnover in this equilibrium comes from the deterministic resale of $-\sigma$ agents holdings, i.e.:

$$\mathbb{T} = \frac{1}{2} \times n_0^{-\sigma} = \frac{Q}{2}$$

And volatility is:

$$\sigma_P = |P_1(A) - P_1(B)| = \mu (\pi(\sigma + \eta) - \pi(\sigma - \eta))$$

The resale option:

$$\text{resale} = \frac{\mu}{4} (\pi(\sigma + \eta) + \pi(\sigma - \eta) - \pi(-\sigma + \eta) - \pi(-\sigma - \eta)) - \frac{Q}{\gamma}$$

We now turn to the equilibrium structure. Two cases arise: Formally:

1. Assume first that $\eta < \sigma$. Call $\theta = \frac{Q}{\mu\gamma}$. Then for $\theta > \frac{3}{4}(\pi(\sigma + \eta) - \pi(-\sigma - \eta)) + \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{4} = \theta^1$ (i.e. low leverage), equilibrium 1 exists (no binding short-sales constraint – the benchmark case). For $\theta^1 > \theta > \frac{1}{2}(\pi(\sigma - \eta) - \pi(-\sigma + \eta)) = \theta^2$ (i.e. intermediate leverage), equilibrium 2 exists (binding short-sales constraint for $-\sigma$ agents in state A, no short-sales constraint in state B). For $\theta^2 > \theta$ (high leverage), then we are in equilibrium 4 (binding short-sales constraint for the pessimist agents in both state A and B, i.e. $-\sigma$ agents).
2. Assume now that $\eta > \sigma$. For $\theta > \theta^1$ (low leverage), we are in equilibrium 1 (no binding short-sales constraint). For $\theta^1 > \theta > \frac{1}{2}(\pi(-\sigma + \eta) - \pi(\sigma - \eta)) = \theta^3$ (intermediate leverage), we are in equilibrium 2 (binding short-sales constraint for $-\sigma$ agents in state A, no short-sales constraint in state B). For $\theta^3 > \theta$ (high leverage), we are in equilibrium 3 (binding short-sales constraint for the pessimist agents in both state A and B, i.e. $-\sigma$ agents in state A and σ agents in state B).

Proof of Proposition 1, 2 and 3

Finally, we turn to the comparative statics in D , μ and F . Note that if $x > y$, $\frac{\partial(\pi(x) - \pi(y))}{\partial D} = \Phi(D - F - y) - \Phi(D - F - x) > 0$ as Φ is increasing. This result proves that for equilibrium 2, 3 and 4, the resale option, turnover and volatility are increasing with D .

Note also that for $x > y$, provided that $D < \infty$, $\frac{\partial(\pi(x) - \pi(y))}{\partial F} = \Phi(D - F - x) - \Phi(D - F - y) < 0$ as Φ is increasing. This result proves that for equilibrium 2, 3 and 4, the resale option, turnover and volatility are decreasing with F , the average bias among investors. Note that when $D = \infty$, i.e. the contract is an equity, the resale option, turnover and volatility are independent of F .

Finally, it is trivial to see that turnover, resale volatility and the resale option increase with μ , the cheapness of leverage.

Proof of Proposition 4. We start by describing generally the date-1 equilibrium structure. Consider first the date-1 equilibrium in state A. It can be decomposed in two situations:

1. Both agents are long at date 1. This makes the price equal to $P_1(A) = \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(-\sigma - \eta)$. This can be the equilibrium price if and only if the $-\sigma$ agents want to be long, i.e.:

$$n_0^{-\sigma} > \frac{\gamma\mu}{2}(\pi(\sigma + \eta) - \pi(-\sigma - \eta))$$

2. Only agent σ is long. The price in this case is simply:

$$P_1(A) = \mu\pi(\sigma + \eta) - \frac{n_0^{-\sigma}}{\gamma}$$

Of course, $-\sigma$ agents will be sidelined if and only if:

$$n_0^{-\sigma} < \frac{\gamma\mu}{2} (\pi(\sigma + \eta) - \pi(-\sigma - \eta))$$

Consider now state B. Then two cases may arise:

1. Both agents are long at date 1. This makes the price equal to $P_1(B) = \frac{1}{2}\mu\pi(-\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta)$.

This can be the equilibrium price if and only if the $-\sigma$ and the σ agents want to be long, i.e.:

$$\begin{cases} n_0^{-\sigma} > \frac{\gamma\mu}{2} (\pi(\sigma - \eta) - \pi(-\sigma + \eta)) \\ n_0^{\sigma} > \frac{\gamma\mu}{2} (\pi(-\sigma + \eta) - \pi(\sigma - \eta)) \end{cases}$$

2. Only agent σ is long. The price in this case is simply:

$$P_1(B) = \mu\pi(\sigma - \eta) - \frac{n_0^{-\sigma}}{\gamma}$$

Of course, $-\sigma$ agents will be sidelined if and only if:

$$n_0^{-\sigma} < \frac{\gamma\mu}{2} (\pi(\sigma - \eta) - \pi(-\sigma + \eta))$$

This ends up the description of the date-1 equilibrium structure. For completeness, we expose here the entire equilibrium. The equilibria mentioned in the proposition correspond to equilibrium 4 and 5 in the following analysis:

1. **Equilibrium 1:** Consider first the equilibrium where all agents are long at date 1, in state A and B, and both agents are long at date 0. Then the date-0 demands are simply given by:

$$\begin{cases} n_0^{\sigma} = \gamma \left(\mu\pi(\sigma) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta) - P_0 \right) \\ n_0^{-\sigma} = \gamma \left(\mu\pi(-\sigma) + \frac{1}{2}\mu\pi(-\sigma + \eta) + \frac{1}{2}\mu\pi(-\sigma - \eta) - P_0 \right) \end{cases}$$

And the date-0 equilibrium price is:

$$P_0 = \frac{1}{2}\mu\pi(\sigma) + \frac{1}{2}\mu\pi(-\sigma) + \frac{1}{4}\mu\pi(\sigma + \eta) + \frac{1}{4}\mu\pi(\sigma - \eta) + \frac{1}{4}\mu\pi(-\sigma + \eta) + \frac{1}{4}\mu\pi(-\sigma - \eta) - \frac{Q}{\gamma}$$

For this to be an equilibrium, it needs to be first that $n_0^{\sigma} > 0$ and $n_0^{-\sigma} > 0$. It is always true that: $n_0^{\sigma} > 0$. And:

$$n_0^{-\sigma} > 0 \Leftrightarrow \frac{Q}{\mu\gamma} > \frac{1}{2}\pi(\sigma) + \frac{1}{4}\pi(\sigma + \eta) + \frac{1}{4}\pi(\sigma - \eta) - \left(\frac{1}{2}\pi(-\sigma) + \frac{1}{4}\pi(-\sigma + \eta) + \frac{1}{4}\pi(-\sigma - \eta) \right) \quad (23)$$

It also needs to be the case that the equilibrium 1 is sustainable. This implies:

$$\begin{cases} \frac{Q}{\mu\gamma} > \frac{\pi(\sigma) - \pi(-\sigma)}{2} + \frac{3}{4}(\pi(\sigma + \eta) - \pi(-\sigma - \eta)) + \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{4} \\ \frac{Q}{\mu\gamma} > \frac{\pi(-\sigma) - \pi(\sigma)}{2} + \frac{3}{4}(\pi(-\sigma + \eta) - \pi(\sigma - \eta)) + \frac{\pi(-\sigma - \eta) - \pi(\sigma + \eta)}{4} \end{cases} \quad (24)$$

Note that the right hand side of the first inequality in 24 is strictly greater than the right hand side of the second inequality in 24. Note also that the right hand side of the first inequality in 24 is greater than the right hand side of inequality 23. As a consequence, the condition defining this equilibrium is simply:

$$\frac{Q}{\mu\gamma} > \frac{\pi(\sigma) - \pi(-\sigma)}{2} + \frac{3}{4}(\pi(\sigma + \eta) - \pi(-\sigma - \eta)) + \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{4}$$

Expected turnover in this equilibrium is:

$$\mathbb{T} = \frac{1}{8}\mu\gamma(\pi(\sigma + \eta) - \pi(-\sigma - \eta)) + \frac{1}{8}\mu\gamma|\pi(-\sigma + \eta) - \pi(\sigma - \eta)|$$

Price volatility is:

$$\sigma_P = \frac{1}{2}\mu(\pi(-\sigma + \eta) + \pi(\sigma - \eta) - \pi(\sigma + \eta) - \pi(-\sigma - \eta))$$

There is no resale option in this equilibrium.

2. **Equilibrium 2:** Consider now the case where only σ agents are long in state A and both agents are long following state B. Both agents are long at date 0. Date-0 demands are:

$$\begin{cases} n_0^\sigma = \gamma \left(\mu\pi(\sigma) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta) - P_0 \right) \\ n_0^{-\sigma} = \frac{1}{2}\gamma \left(\mu\pi(-\sigma) + \frac{1}{2}\mu\pi(-\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma + \eta) - P_0 \right) \end{cases}$$

And the date-0 price:

$$P_0 = \frac{2}{3} \left(\mu\pi(\sigma) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta) \right) + \frac{1}{3} \left(\mu\pi(-\sigma) + \frac{1}{2}\mu\pi(-\sigma + \eta) + \frac{1}{2}\mu\pi(-\sigma - \eta) \right) - \frac{4}{3} \frac{Q}{\gamma}$$

We can compute the date-0 equilibrium demand:

$$\begin{cases} n_0^\sigma = \frac{4}{3}Q + \frac{\gamma\mu}{3} \left(\pi(\sigma) - \pi(-\sigma) + \frac{1}{2}(\pi(\sigma - \eta) - \pi(-\sigma + \eta)) \right) \\ n_0^{-\sigma} = \frac{2}{3}Q - \frac{\gamma\mu}{3} \left(\pi(\sigma) - \pi(-\sigma) + \frac{1}{2}(\pi(\sigma - \eta) - \pi(-\sigma + \eta)) \right) \end{cases}$$

Thus, the $-\sigma$ agents are long at date 0 if and only if:

$$\frac{Q}{\mu\gamma} > \frac{\pi(\sigma) - \pi(-\sigma)}{2} + \frac{\pi(\sigma + \eta) - \pi(-\sigma + \eta)}{4}$$

Eventually, we have to consider three cases:

(a) If $\eta < \sigma$, then the equilibrium exists provided that:

$$\begin{cases} \frac{\pi(\sigma) - \pi(-\sigma)}{2} + \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{4} + \frac{3}{4}(\pi(\sigma + \eta) - \pi(-\sigma - \eta)) > \frac{Q}{\mu\gamma} \\ \frac{Q}{\mu\gamma} > \frac{\pi(\sigma) - \pi(-\sigma)}{2} + (\pi(\sigma - \eta) - \pi(-\sigma + \eta)) \end{cases}$$

(b) For $\eta \in [\sigma, 2\sigma]$, the equilibrium exists if and only if:

$$\begin{cases} \frac{\pi(\sigma) - \pi(-\sigma)}{2} + \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{4} + \frac{3}{4}(\pi(\sigma + \eta) - \pi(-\sigma - \eta)) > \frac{Q}{\mu\gamma} \\ \frac{Q}{\mu\gamma} > \frac{\pi(\sigma) - \pi(-\sigma)}{2} + \frac{1}{4}(\pi(\sigma - \eta) - \pi(-\sigma + \eta)) \end{cases}$$

(c) For $\eta > 2\sigma$, the equilibrium exists if and only if:

$$\begin{cases} \frac{\pi(\sigma) - \pi(-\sigma)}{2} + \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{4} + \frac{3}{4}(\pi(\sigma + \eta) - \pi(-\sigma - \eta)) > \frac{Q}{\mu\gamma} \\ \frac{Q}{\mu\gamma} > \frac{\pi(-\sigma) - \pi(\sigma)}{4} + \frac{1}{2}(\pi(-\sigma + \eta) - \pi(\sigma - \eta)) \end{cases}$$

Turnover is:

$$\begin{aligned} \mathbb{T} &= \frac{1}{4}n_0^{-\sigma} + \frac{1}{8}\gamma\mu |\pi(\sigma - \eta) - \pi(-\sigma + \eta)| \\ &= \frac{Q}{6} - \frac{\gamma\mu}{12} \left(\pi(\sigma) - \pi(-\sigma) + \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{2} \right) + \frac{1}{8}\gamma\mu |\pi(\sigma - \eta) - \pi(-\sigma + \eta)| \end{aligned}$$

Consider first the case where $\sigma > \eta$, then

$$\mathbb{T} = \frac{Q}{6} - \frac{\gamma\mu}{12} (\pi(\sigma) - \pi(-\sigma) - (\pi(\sigma - \eta) - \pi(-\sigma + \eta)))$$

Then, turnover is decreasing with μ and γ and decreasing with D .

Assume now that $\sigma < \eta$. Then:

$$\mathbb{T} = \frac{Q}{6} + \frac{\gamma\mu}{6} \left(\pi(-\sigma + \eta) - \pi(\sigma - \eta) - \frac{\pi(\sigma) - \pi(-\sigma)}{2} \right)$$

Volatility is:

$$\sigma_P = -\frac{2Q}{3} + \frac{\mu}{3} (\pi(\sigma) - \pi(-\sigma)) + \mu\pi(\sigma + \eta) - \frac{1}{3}\mu\pi(\sigma - \eta) - \frac{2}{3}\mu\pi(-\sigma + \eta)$$

The mispricing in this equilibrium is:

$$\text{mispricing} = -\frac{Q}{3\gamma} + \frac{\mu}{6} \left(\pi(\sigma) - \pi(-\sigma) + \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{2} + \frac{3}{2} (\pi(\sigma + \eta) - \pi(-\sigma - \eta)) \right)$$

Note that here again, if $\bar{P}_1(i)$ denotes the price in state i in equilibrium 1, i.e. in the absence of short-sales constraint, then the mispricing here can be decomposed in the following way:

$$\text{mispricing} = \frac{1}{2} (P_1(A) - \bar{P}_1(A))$$

Mispricing here comes purely from the resale option agents $-\sigma$ have at date 1 in state A.

3. **Equilibrium 3:** Consider now the equilibrium where only σ agents are long in state A and only $-\sigma$ agents are long in state B, and both agents are long at date 0.

Date-0 demands are:

$$\begin{cases} n_0^\sigma = \frac{1}{2}\gamma \left(\mu\pi(\sigma) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(-\sigma + \eta) - P_0 \right) \\ n_0^{-\sigma} = \frac{1}{2}\gamma \left(\mu\pi(-\sigma) + \frac{1}{2}\mu\pi(-\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma + \eta) - P_0 \right) \end{cases}$$

The date-0 price follows from the date-0 market clearing condition:

$$P_0 = \frac{1}{2} (\mu\pi(\sigma) + \mu\pi(-\sigma)) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(-\sigma + \eta) - \frac{2Q}{\gamma}$$

We can compute the date-0 equilibrium demand:

$$\begin{cases} n_0^\sigma = Q + \frac{1}{4}\mu\gamma(\pi(\sigma) - \pi(-\sigma)) > 0 \\ n_0^{-\sigma} = Q - \frac{1}{4}\mu\gamma(\pi(\sigma) - \pi(-\sigma)) \end{cases}$$

Note that contrary to equilibrium 3 in the previous model, the equilibrium demands are no longer symmetric. This is because of the different valuations for the interim payoff (the ‘‘Miller’’ piece). As a consequence, σ agents have larger date-0 holdings than agents $-\sigma$.

The $-\sigma$ agents are long at date 0 if and only if:

$$\frac{Q}{\mu\gamma} > \frac{1}{4}(\pi(\sigma) - \pi(-\sigma))$$

Eventually, the equilibrium exists if and only if:

$$\frac{1}{2}(\pi(-\sigma + \eta) - \pi(\sigma - \eta)) - \frac{1}{4}(\pi(\sigma) - \pi(-\sigma)) > \frac{Q}{\mu\gamma} > \frac{1}{4}(\pi(\sigma) - \pi(-\sigma))$$

Turnover in this equilibrium is simply given by σ agents reselling everything in state B and $-\sigma$ agents reselling everything in state A. Because the sum of σ and $-\sigma$ agents date-0 holding has to be equal to $2Q$, this implies that turnover is:

$$\mathbb{T} = \frac{Q}{2}$$

Price volatility is:

$$\sigma_P = \frac{\mu\gamma}{2}(\pi(\sigma) - \pi(-\sigma)) + \mu(\pi(\sigma + \eta) - \pi(-\sigma + \eta))$$

The mispricing is:

$$\text{mispricing} = -\frac{Q}{\gamma} + \frac{\mu}{4}(\pi(-\sigma + \eta) - \pi(\sigma - \eta) + \pi(\sigma + \eta) - \pi(-\sigma - \eta))$$

Again, mispricing can simply be decomposed as:

$$\text{mispricing} = \frac{1}{2}(P_1(A) - \bar{P}_1(A)) + \frac{1}{2}(P_1(B) - \bar{P}_1(B))$$

Thus, mispricing comes solely from the date-1 resale option of both agents.

4. **Equilibrium 4:** Consider the equilibrium where only the σ agents are long in state A and B, and both agents are long at date 0.

Date-0 demands are:

$$\begin{cases} n_0^\sigma = \gamma \left(\mu\pi(\sigma) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta) - P_0 \right) \\ n_0^{-\sigma} = \frac{1}{3}\gamma \left(\mu\pi(-\sigma) + \frac{1}{2}\mu\pi(\sigma - \eta) + \frac{1}{2}\mu\pi(\sigma + \eta) - P_0 \right) \end{cases}$$

And the date-0 price comes from the date-0 market clearing condition:

$$P_0 = \frac{3}{4} \left(\mu\pi(\sigma) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta) \right) + \frac{1}{4} \left(\mu\pi(-\sigma) + \frac{1}{2}\mu\pi(\sigma - \eta) + \frac{1}{2}\mu\pi(\sigma + \eta) \right) - \frac{3Q}{2\gamma}$$

Equilibrium date-0 demands can be computed using the date-0 equilibrium price:

$$\begin{cases} n_0^\sigma = \frac{3}{2}Q + \frac{1}{4}\mu\gamma(\pi(\sigma) - \pi(-\sigma)) > 0 \\ n_0^{-\sigma} = \frac{Q}{2} - \frac{1}{4}\mu\gamma(\pi(\sigma) - \pi(-\sigma)) \end{cases}$$

This equilibrium is sustainable if and only if:

$$\frac{\pi(\sigma) - \pi(-\sigma)}{2} + \pi(\sigma - \eta) - \pi(-\sigma + \eta) > \frac{Q}{\mu\gamma} > \frac{1}{2}(\pi(\sigma) - \pi(-\sigma))$$

Turnover comes from the deterministic resale of $-\sigma$ agents date-0 holdings:

$$\mathbb{T} = \frac{1}{2}n_0^{-\sigma} = \frac{Q}{4} - \frac{1}{8}\mu\gamma(\pi(\sigma) - \pi(-\sigma))$$

And volatility is:

$$\sigma_P = |P_1(A) - P_1(B)| = \mu(\pi(\sigma + \eta) - \pi(\sigma - \eta))$$

The mispricing in this equilibrium is simply:

$$\text{mispricing} = \frac{1}{4}\mu(\pi(\sigma) - \pi(-\sigma) + \pi(\sigma + \eta) + \pi(\sigma - \eta) - \pi(-\sigma - \eta) - \pi(-\sigma + \eta)) - \frac{Q}{\gamma}$$

We obtain again the same decomposition of the resale option, i.e.:

$$\text{mispricing} = \frac{1}{2}(P_1(A) - \bar{P}_1(A)) + \frac{1}{2}(P_1(B) - \bar{P}_1(B))$$

Again, the mispricing here comes solely from the resale option of $-\sigma$ agents.

5. **Equilibrium 5:** Consider the equilibrium where only σ agents are long in state A and B and only σ agents are long at date 0.

Date-0 demands are:

$$\begin{cases} n_0^\sigma = \gamma \left(\mu\pi(\sigma) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta) - P_0 \right) \\ n_0^{-\sigma} = \max \left(0, \frac{1}{3}\gamma \left(\mu\pi(-\sigma) + \frac{1}{2}\mu\pi(\sigma - \eta) + \frac{1}{2}\mu\pi(\sigma + \eta) - P_0 \right) \right) \end{cases}$$

And the date-0 price:

$$P_0 = \mu\pi(\sigma) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta) - \frac{2Q}{\gamma}$$

The date-0 demands are: $n_0^\sigma = 2Q$ and $n_0^{-\sigma} = 0$. This is the equilibrium provided that $\eta < \sigma$ and:

$$\frac{Q}{\mu\gamma} < \frac{\pi(\sigma) - \pi(-\sigma)}{2}$$

In this equilibrium, turnover is 0 $\mathbb{T} = 0$ and volatility is:

$$\sigma_P = \mu(\pi(\sigma + \eta) - \pi(\sigma - \eta))$$

Mispricing is:

$$\text{mispricing} = -\frac{Q}{\gamma} + \frac{1}{2}\mu \left(\pi(\sigma) + \frac{1}{2}\pi(\sigma + \eta) + \frac{1}{2}\pi(\sigma - \eta) - \pi(-\sigma) - \frac{1}{2}\pi(-\sigma - \eta) - \frac{1}{2}\pi(-\sigma + \eta) \right)$$

Mispricing can now be decomposed in the following way:

$$\text{mispricing} = \frac{1}{2}\mu(\pi(\sigma) - \pi(-\sigma)) - \frac{Q}{\gamma} + \frac{1}{2}(P_1(A) - \bar{P}_1(A)) + \frac{1}{2}(P_1(B) - \bar{P}_1(B))$$

The first term in the previous expression corresponds to the ‘‘Miller’’ piece. Because of the interim payoff and high leverage, $-\sigma$ agents are sidelined *at date 0*. As a consequence, their negative view about the interim payoff is not impounded in the price, which leads to a positive mispricing. More precisely, if one considers the static model where only the interim payoff is valued by investors, then the price with no short-sales constraint would trivially be $\frac{\mu}{2}(\pi(\sigma) + \pi(-\sigma)) - \frac{Q}{\gamma}$. With short-sales constraint, the price would become: $\pi(\sigma) - \frac{2Q}{\gamma}$, as only the σ agents, in proportion 1/2 would be long

the asset. The first term in the previous decomposition corresponds to the difference between these two prices, i.e. the mispricing from the static interim payoff.

6. **Equilibrium 6:** Consider now the equilibrium where only σ agents are long in state A and only $-\sigma$ agents are long in state B, and only agents σ are long at date 0.

Date-0 demands are:

$$\begin{cases} n_0^\sigma = \frac{1}{2}\gamma \left(\mu\pi(\sigma) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(-\sigma + \eta) - P_0 \right) \\ n_0^{-\sigma} = \max \left(0, \frac{1}{2}\gamma \left(\mu\pi(-\sigma) + \frac{1}{2}\mu\pi(-\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma + \eta) - P_0 \right) \right) \end{cases}$$

And the date-0 price:

$$P_0 = \mu\pi(\sigma) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(-\sigma + \eta) - \frac{4Q}{\gamma}$$

This is an equilibrium if and only if:

$$\begin{cases} \frac{Q}{\mu\gamma} < \frac{\pi(\sigma) - \pi(-\sigma)}{4} \\ \frac{Q}{\mu\gamma} < \frac{\pi(-\sigma + \eta) - \pi(\sigma - \eta)}{4} \end{cases}$$

In this equilibrium turnover comes from σ agents reselling their date-0 holding in state B:

$$\mathbb{T} = \frac{1}{2} \times \frac{n_0^\sigma}{2} = \frac{Q}{2}$$

Price volatility is simply derived as:

$$\sigma_P = |P_1(A) - P_1(B)| = \frac{2Q}{\gamma} + \mu(\pi(\sigma + \eta) - \pi(-\sigma + \eta))$$

Mispricing is easily computed as:

$$\text{mispricing} = \frac{\mu}{2} \left(\pi(\sigma) - \pi(-\sigma) + \frac{\pi(\sigma + \eta) - \pi(-\sigma - \eta)}{2} + \frac{\pi(-\sigma + \eta) - \pi(\sigma - \eta)}{2} \right) - \frac{3Q}{\gamma}$$

Using the usual decomposition, we can rewrite the mispricing as:

$$\text{mispricing} = \underbrace{\frac{1}{2}\mu(\pi(\sigma) - \pi(-\sigma)) - \frac{Q}{\gamma}}_{\text{Miller mispricing}} + \underbrace{\frac{1}{2}(P_1(A) - \bar{P}_1(A)) + \frac{1}{2}(P_1(B) - \bar{P}_1(B))}_{\text{resale option}}$$

7. **Equilibrium 7:** Consider now the case where only σ agents are long in state A and both agents are long in state B. Only σ agents are long at date 0. Date-0 demands are:

$$\begin{cases} n_0^\sigma = \gamma \left(\mu\pi(\sigma) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta) - P_0 \right) \\ n_0^{-\sigma} = \max \left(0, \frac{1}{2}\gamma \left(\mu\pi(-\sigma) + \frac{1}{2}\mu\pi(-\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma + \eta) - P_0 \right) \right) \end{cases}$$

And the date-0 price:

$$P_0 = \mu\pi(\sigma) + \frac{1}{2}\mu\pi(\sigma + \eta) + \frac{1}{2}\mu\pi(\sigma - \eta) - \frac{2Q}{\gamma}$$

This is an equilibrium if and only if $\eta > \sigma$ and:

$$\frac{\pi(\sigma) - \pi(-\sigma)}{2} + \frac{\pi(\sigma - \eta) - \pi(-\sigma + \eta)}{4} > \frac{Q}{\mu\gamma} > \frac{\pi(-\sigma + \eta) - \pi(\sigma - \eta)}{4}$$

In this equilibrium, turnover comes from the $-\sigma$ agents purchasing some shares in state B:

$$T = \frac{1}{8}\gamma\mu(\pi(-\sigma + \eta) - \pi(\sigma - \eta))$$

And volatility:

$$\sigma_P = \mu\pi(\sigma + \eta) - \frac{1}{2}(\pi(-\sigma + \eta) + \pi(\sigma - \eta))$$

Mispricing in this equilibrium is similar to that of equilibrium 5.

We now turn more precisely to proposition 4. Assume that $\eta < \sigma$ and $\frac{Q}{\mu\gamma} < \frac{\pi(\sigma) - \pi(-\sigma)}{2} + \pi(\sigma - \eta) - \pi(-\sigma + \eta)$. When $\frac{Q}{\mu\gamma} > \frac{1}{2}(\pi(\sigma) - \pi(-\sigma))$, then equilibrium 4 exists. Because π is increasing with σ , turnover is clearly strictly decreasing with σ . Because π is concave, volatility is strictly decreasing with σ . Finally, because π is increasing with σ , mispricing is strictly increasing with σ .

When $\frac{Q}{\mu\gamma} < \frac{1}{2}(\pi(\sigma) - \pi(-\sigma))$, then equilibrium 5 exists. In this equilibrium, turnover is 0. Volatility is strictly decreasing with σ as π is strictly concave as long as $D < \infty$. Finally, mispricing is strictly increasing with σ as π is strictly increasing.

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Figure 1: ABX Prices

The figure plots the ABX 7-1 Prices for various credit tranches including AAA, AA, A, BBB, and BBB-.

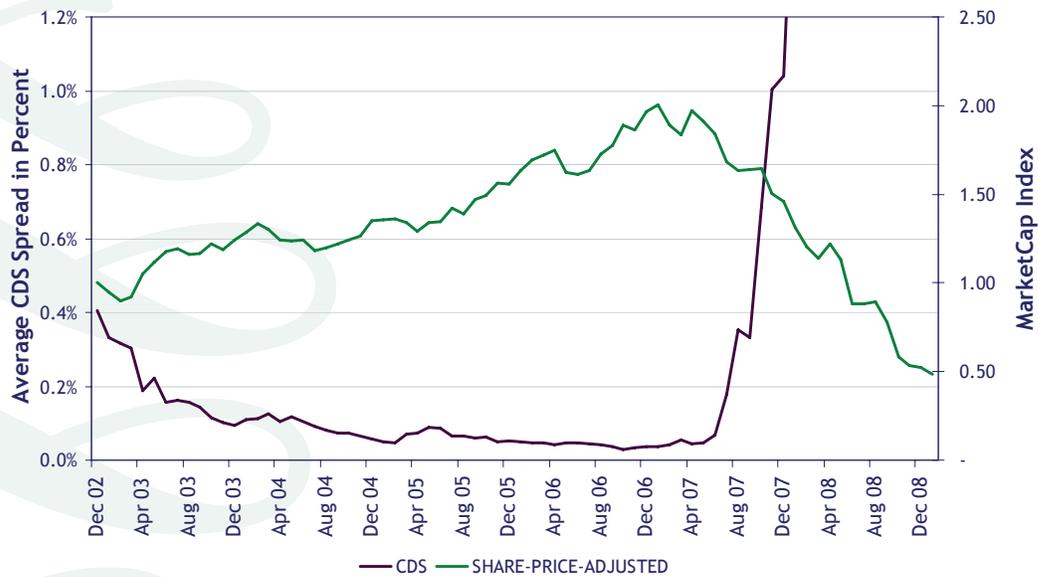


Figure 2: CDS Prices of Basket of Finance Companies

The figure plots the average CDS prices for a basket of large finance companies between December 2002 and December 2008.



Financial firms' CDS and share prices



Source: Moody's KMV, FSA Calculations

Firms included: Ambac, Aviva, Banco Santander, Barclays, Berkshire Hathaway, Bradford & Bingley, Citigroup, Deutsche Bank, Fortis, HBOS, Lehman Brothers, Merrill Lynch, Morgan Stanley, National Australia Bank, Royal Bank of Scotland and UBS.
CDS series peaks at 6.54% in September 2008.

Figure 3: Monthly Share Turnover of Financial Stocks

The figure plots the average monthly share turnover of financial stocks.

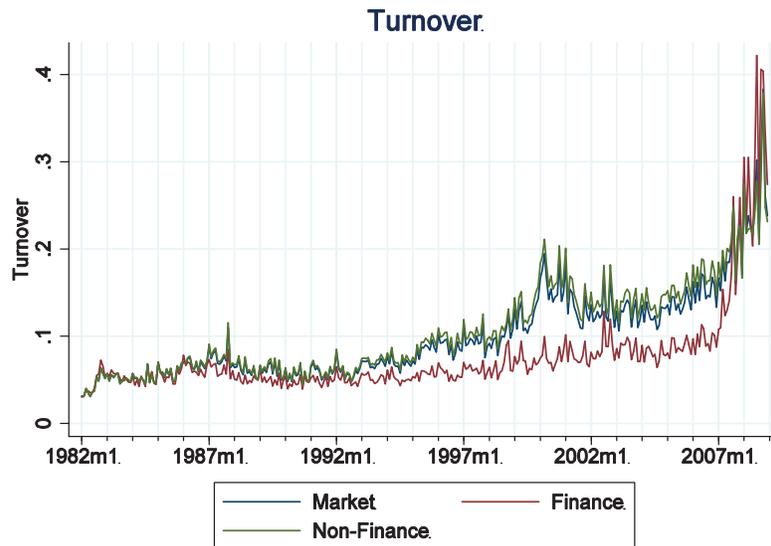


Figure 4: Monthly Share Turnover of Internet Stocks

The figure plots the average monthly share turnover of internet stocks compared to the rest of the market.

Prices and Turnover for Internet and Non-Internet Stocks, 1997-2002

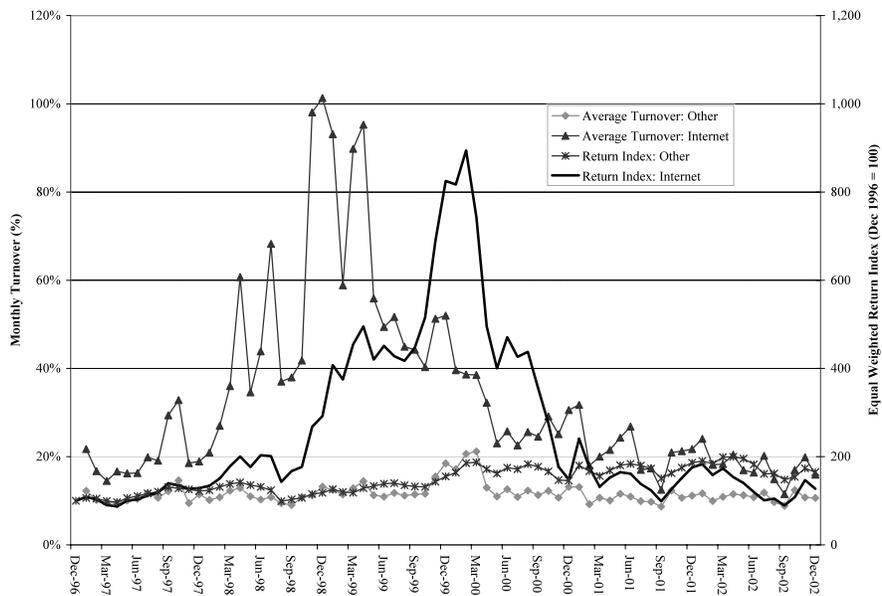


Figure 5: Traditional and Non-Traditional Issuance of Asset-Backed Securities (Quarterly)

The figure plots the issuance of traditional and non-traditional asset-backed securities by quarter.

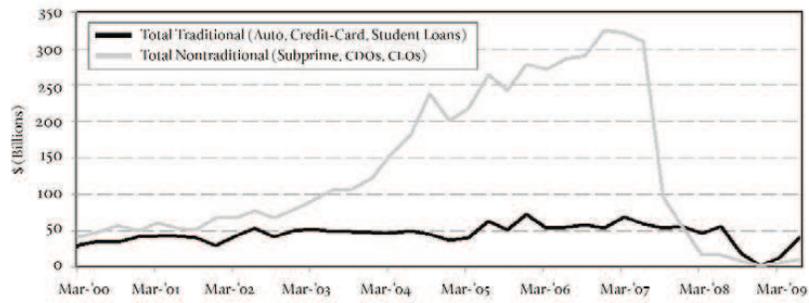
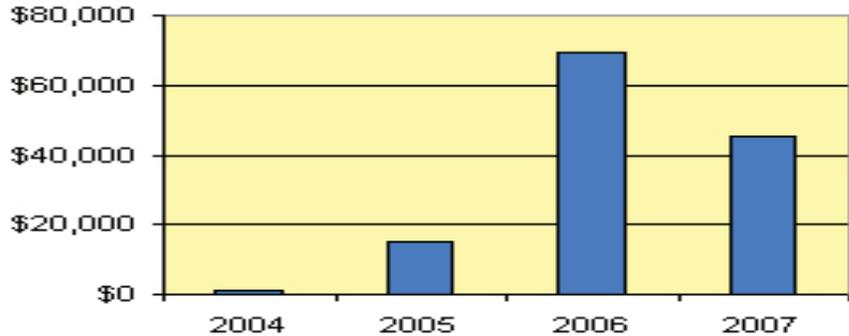


Figure 6: Synthetic Mezzanine ABS CDO Issuance

The figure plots the issuance of synthetic mezzanine ABS CDOs by year.

The Big Bet

Volume of synthetic mezzanine ABS CDOs created by total value of bonds on which bets were made, in millions of dollars



Source: Citigroup