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ABSTRACT

While empirical evidence finds only a weak relationship between nominal exchange rates and macroeconomic fundamentals, forex markets participants often attribute exchange rate movements to a macroeconomic variable. The variables that matter, however, appear to change over time and some variable is typically taken as a scapegoat. For example, the current dollar weakness appears to be caused almost exclusively by the large current account deficit, while its previous strength was explained mainly by growth differentials. In this paper, we propose an explanation of this phenomenon in a simple monetary model of the exchange rate with noisy rational expectations, where investors have heterogeneous information on some structural parameter of the economy. In this context, there may be rational confusion about the true source of exchange rate fluctuations, so that if an unobservable variable affects the exchange rate, investors may attribute this movement to some current macroeconomic fundamental. We show that this effect applies only to variables with large imbalances. The model thus implies that the impact of macroeconomic variables on the exchange rate changes over time.

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1 Introduction

There is a peculiar mismatch between explanations given by market analysts for observed exchange rate fluctuations and the academic consensus about exchange rates. The academic consensus, based on the seminal work of Richard A. Meese and Kenneth Rogoff (1983) and subsequent literature, is that macroeconomic variables have little explanatory power for exchange rates in the short to medium run. On the other hand, market analysts often point to particular macro developments in accounting for exchange rates. For example, the large depreciation of the euro relative to the dollar subsequent to its inauguration in January 1999 was blamed on the strong growth performance of the US economy relative to the European economy. More recently the appreciation of the euro relative to the dollar has been blamed on the large U.S. current account deficit.[1] That practitioners regularly change the weight they attach to different macro indicators is widely reported in the financial press. It has also been confirmed by Yin-Wong Cheung and Menzie Chinn (2001), who surveyed US foreign exchange traders.

The varying weight that traders give to different macro indicators may explain why formal models of exchange rates have found so little explanatory power of macro variables. In contrast to existing models, the relationship between macro variables and

[1]For example, in the Financial Times of December 1, 2003, one can read: "The dollar's latest stumble ... came despite optimistic economic data from the US. But analysts said the movement of the US currency was no longer driven by growth fundamentals. All the focus is on the deficit now..."

the exchange rate appears to be highly unstable. Cheung et. al. (2002) find that some models, with certain macro variables, do well in some periods but not in others.

One explanation for this parameter instability is a scapegoat story: some variable is given an 'excessive' weight during some period. The exchange rate may change for reasons that have nothing to do with observed macro fundamentals, for example due to unobserved liquidity trades. As the market rationally searches for an explanation for the observed exchange rate change, it may attribute it to some observed macro indicator. This macro indicator then becomes a natural scapegoat and influences trading strategies. Over time different observed variables can be taken as scapegoats, so that the weights attributed to macro variables change.

In this paper we formalize this scapegoat story in the context of a simple rational expectations model. The model illustrates how a variable can become a scapegoat and illuminates the implications for the exchange rate. The basic mechanism behind this scapegoat story is that there is "confusion" in the market about the true source of exchange rate fluctuations. This happens because investors have different views about the importance of various observed macro variables. We model this heterogeneity with investors receiving different private signals about some structural parameters. Investors therefore do not know whether an exchange rate fluctuation can be explained by unobserved fundamentals, such as liquidity trades, or by a larger than expected weight to certain observed macro fundamentals. In such an environment it is natural to blame the variables you can observe, i.e., the macro fundamentals.

Although models with investor heterogeneity are common in the finance literature, they have not often been used to analyze the foreign exchange market. In related work, Philippe Bacchetta and Eric van Wincoop (2003), we develop a fully dynamic macro model of exchange rate determination where investors have different information about future macro variables. We show that such a framework can lead to a disconnect between observed macro fundamentals and exchange rates in the short to medium run, but a closer relationship in the long-run. In that paper it is assumed that all investors know the model and its parameters. In contrast, here we assume that investors are incompletely, and heterogeneously, informed about some parameters and therefore about the importance of various macro indicators.

In the next section we present a simple monetary model of the exchange rate, where investors have different views about the growth rate of fundamental variables. In Section 2, we show how the model is solved and how a scapegoat can emerge. A crucial element is that investors use the exchange rate as a source of information on imperfectly known parameters. In Section 3, we examine the implications for the exchange rate and provide concluding comments in Section 4.

2 A Model with Heterogeneous Beliefs

Our starting point is the standard monetary model of exchange rate determination. It contains three equations. The first is a purchasing power parity equation: $p_t = p_t^* + s_t$, where s_t is the log of the nominal exchange rate. The second is a money demand

equation: $m_t - p_t = y_t - \alpha i_t$ (and foreign analogue). The third is an interest arbitrage equation:

$$\overline{E}_t(s_{t+1}) - s_t = i_t - i_t^* + \gamma b_t \sigma_t^2 \tag{1}$$

Here $\overline{E}_t$ denotes the average expectation of individual investors and σ_t^2 is the conditional variance of next period's exchange rate. b_t is the unobserved net supply of foreign currency based on non-speculative trade (such as liquidity trades) and has a normal distribution $N(0, \sigma_b^2)$. This interest parity equation can be derived from a standard portfolio choice model with constant absolute risk-aversion γ. We refer the reader to Olivier Jeanne and Andrew K. Rose (2002) and Bacchetta and van Wincoop (2003) for formal derivations.

As usual, (1) is solved forward after substituting the purchasing power parity and money demand equations, leading to an expression equating the current exchange rate to the present value of expected future fundamentals. In our context, however, we are dealing with average instead of single expectations. In Bacchetta and van Wincoop (2003) we show that this implies that the law of iterated expectations may not hold and that the exchange rate may depend on higher order expectations as in John Maynard Keynes' beauty contest paradigm (average expectations of average expectations, and so on). While the presence of higher order expectations has interesting implications for asset price behavior in general (see Franklin Allen, Stephen Morris, and Hyun Song Shin (2003), and Bacchetta and van Wincoop (2004)), we abstract from them in this short paper by assuming that information heterogeneity lasts only one period.

We assume that starting at date 2 investors have common information about future output levels and money supplies. To keep things simple we assume that $\overline{E}_2(m_t) = \overline{E}_2(y_t) = 0$ for $t > 2$ and that the foreign money supply and output level are zero at all times. Since $\overline{E}_1(b_t) = 0$ for $t > 2$, we have $\overline{E}_1(s_t) = 0$ for $t > 2$ (ruling out bubbles). Then,

$$s_1 = \frac{1}{1+\alpha}(m_1 - y_1) + \frac{\alpha}{(1+\alpha)^2}\overline{E}_1(m_2 - y_2) - \frac{\alpha}{1+\alpha}\gamma\sigma_1^2 b_1 \tag{2}$$

The exchange rate depends on current and expected future macro fundamentals minus a risk-premium term that depends on liquidity trade. Investors need to forecast money and output at time 2. We assume the following autoregressive structure (applying only at time 2):

$$\begin{aligned} m_2 - \overline{m} &= \rho_m(m_1 - \overline{m}) + \varepsilon_2^m \\ y_2 - \overline{y} &= \rho_y(y_1 - \overline{y}) + \varepsilon_2^y \end{aligned} \tag{3}$$

with $0 < \rho_j < 0$ and $\varepsilon_2^j \sim N(0, \sigma_j^2)$, $j = m, y$. The persistence coefficients ρ_m and ρ_y are a key element of the model. The larger ρ_m, the bigger the impact of the current money supply on the exchange rate, and similarly for output. However, investors do not know the persistence coefficients and only receive private signals about them:

$$\begin{aligned} v_m^i &= \rho_m + \varepsilon_m^i \qquad \varepsilon_m^i \sim N(0, \sigma_v^2) \\ v_y^i &= \rho_y + \varepsilon_y^i \qquad \varepsilon_y^i \sim N(0, \sigma_v^2) \end{aligned}$$

We assume that errors in the private signals average to zero across all investors. We

can plug the expectation derived from (3) into (2) to get:

$$s_1 = q_1 + \tilde{\alpha}(m_1 - \overline{m})\overline{E}_1(\rho_m) - \tilde{\alpha}(y_1 - \overline{y})\overline{E}_1(\rho_y) - \frac{\alpha}{1+\alpha}\gamma\sigma_1^2 b_1 \tag{4}$$

where $\tilde{\alpha} \equiv \frac{\alpha}{(1+\alpha)^2}$ and $q_1 \equiv \frac{1}{1+\alpha}(m_1 - y_1) + \tilde{\alpha}(\overline{m} - \overline{y})$. Equation (4) shows that the exchange rate depends in a straightforward manner on the observable variables m_1 and y_1 as well as the unobservable b_1. Moreover, it depends on the average expectations of the persistence coefficients, $\overline{E}_1(\rho_m)$ and $\overline{E}_1(\rho_y)$. With heterogeneous information, individual investors do not know these average expectations and cannot disentangle shocks to b_1 from shocks to $\overline{E}_1(\rho_m)$ and $\overline{E}_1(\rho_y)$. This difficulty in the inference process can lead to rational confusion, which in turn can lead to attributing the wrong weight to fundamental variables.

3 Finding a Scapegoat

The equilibrium exchange rate can be solved with a simple signal extraction procedure. Based on (4), we first conjecture that the exchange rate takes the form

$$s_1 = q_1 + \tilde{\alpha}\rho_m(m_1 - \overline{m}) - \tilde{\alpha}\rho_y(y_1 - \overline{y}) - \lambda b_1 \tag{5}$$

for some positive λ. The exchange rate depends linearly on the unknown persistence coefficients ρ_m and ρ_y, and therefore provides a public signal of these parameters. It is therefore optimal for individual investors to use both their private signals and the exchange rate as basis for estimating ρ_m and ρ_y. We now describe the inference process and the solution for the exchange rate in the case where $y_1 = \overline{y}$, so that investors are

only interested in estimating ρ_m. The more general case can be found in a Technical Appendix available upon request.

From (5), investors can use an adjusted exchange rate signal that is normally distributed: $\frac{s_1 - q_1}{\tilde{\alpha}(m_1 - \overline{m})}$. Since the private signal is also normal, the optimal inference of ρ_m is a linear combination of the two signals. Aggregating the expectations of ρ_m over individuals, we get:[2]

$$\overline{E}(\rho_m) = \delta_v \rho_m + (1 - \delta_v) \frac{s_1 - q_1}{\tilde{\alpha}(m_1 - \overline{m})} \tag{6}$$

where $0 < \delta_v < 1$ depends on the (endogenous) relative precision of private and public signals ($\delta_v = \frac{1/\sigma_v^2}{(1/\sigma_v^2) + ((m_1 - \overline{m})^2 / \lambda^2 \sigma_b^2)}$). A crucial element in the analysis is that the expectation of ρ_m depends on the value of the exchange rate. Using (5), one can substitute s_1 to obtain:

$$\overline{E}(\rho_m) = \rho_m - k(m_1 - \overline{m}) b_1 \tag{7}$$

where $k = \frac{(1 - \delta_v)\lambda}{\tilde{\alpha}(m_1 - \overline{m})^2} > 0$. It is easily verified that the conjectured equation (5) is confirmed if one substitutes (7) into (4), with $\lambda = \frac{\alpha}{1+\alpha} \gamma \sigma_1^2 / \delta_v$.

Equation (7) illustrates how an observed macro variable can become a scapegoat. Investors know that the equilibrium exchange rate takes the form (5). While they know the functional form, they do not know the persistence ρ_m and liquidity trades b_1. When an investor sees that the exchange rate is higher than expected based on the private signal of persistence, there can be only two explanations: either unobserved liquidity sales have reduced b_1 or the persistence coefficient differs from the private

[2]Here we use the fact that summing over the signals v_m^i gives ρ_m.

signal. In the case where money supply is large, so that $m_1 - \bar{m}$ is large, a high exchange rate can then be explained by a large persistence coefficient. The reason is that more persistence leads to a bigger expected second period money supply when the money supply is above its mean in period 1.

Now assume that the high level of the exchange rate is actually caused by liquidity trades. It then becomes rational to make money the scapegoat when the money supply is unusually large. Even if investors do not believe that the high money supply will be so persistent based on their private information, they will each believe that others have private signals indicating that money supply is persistent. The scapegoat is captured by (7), which shows that the expected persistence rises when $b_1 < 0$ and $m_1 - \bar{m} > 0$.

The rational confusion that leads to making money the scapegoat is market-wide. Based on private information alone the average expectation of ρ_m is equal to ρ_m. However, when $b_1 < 0$ and $m_1 - \overline{m} > 0$ investors systematically, and incorrectly, believe that ρ_m is larger than it actually is. They all believe that others must have information indicating that ρ_m is very large, even if no investor actually has such information.

Output can similarly become a scapegoat. The Technical Appendix shows that if $y_1 \neq \overline{y}$ we have $\bar{E}(\rho_y) = \rho_y + \widetilde{k}(y_1 - \bar{y})b_1$. If the exchange rate is high due to liquidity trades and output is below its mean ($y_1 - \bar{y} < 0$), investors revise upward their expectation of ρ_y and output becomes a scapegoat. The larger the deviations from the mean, the more likely it is to blame one of the macro variables.

4 The Scapegoat Effect on Exchange Rates

When a macro variable becomes a scapegoat, it has a much larger impact on the exchange rate than otherwise due to confusion with liquidity trade. In this regard an important role is played by the parameter $\lambda = \frac{\alpha}{1+\alpha}\gamma\sigma_1^2/\delta_v$ that multiplies liquidity trade in the equilibrium exchange rate. This coefficient is larger than in the risk-premium term in (2) or (4) because $\delta_v < 1$. This implies that the impact of b_1 is amplified with heterogeneous information.[3] More important in our context, it is easy to check that λ depends positively on $(m_1 - \bar{m})^2$. In other words, $\partial\lambda/\partial(m_1 - \overline{m})^2 > 0$. If the deviation of money from its mean is large, there is more rational confusion that leads to a bigger amplification of the impact of liquidity trade on the exchange rate.

The impact of current money supply on the exchange rate is found by differentiating (5):

$$\frac{\partial s_1}{\partial m_1} = \frac{1}{1+\alpha} + \tilde{\alpha}\rho_m - \nu(m_1 - \overline{m})b_1 \tag{8}$$

where $\nu = 2\partial\lambda/\partial(m_1 - \overline{m})^2 > 0$. The impact of m_1 therefore depends on b_1. For example, if $m_1 > \overline{m}$ a negative b_1 increases the impact of m_1.

With no liquidity shocks the derivative of the exchange rate with respect to money would be only $\frac{1}{1+\alpha} + \tilde{\alpha}\rho_m$, the same as it would be under perfect foresight. A similar impact obtains if money supply is close to its normal level $\overline{m}$. It is the interplay between the unobserved liquidity trades and the unusual size of the money supply that

[3] This amplification of shocks is similar to the one discussed in detail in Bacchetta and van Wincoop (2003).

delivers the scapegoat effect and its increased weight in the equilibrium exchange rate.

The derivative in (8) is not known to market participants since ρ_m is unknown. One can compute the average expectation of (8), by using (5) to (7) (see the Technical Appendix for details). This gives

$$\overline{E}\frac{\partial s_1}{\partial m_1} = \frac{1}{1+\alpha} + \tilde{\alpha}\rho_m - (\nu\delta_v + \tilde{\alpha}k)(m_1 - \overline{m})b_1 \tag{9}$$

Clearly, the same reasoning applies to the perceived impact of m_1 on s_1 as to the actual impact. The reason is that ρ_m is expected to be larger than it actually is when m_1 is large and b_1 is negative.

We conclude this section with two remarks. First, it is worth noting that when the impact of money on the exchange rate is large as a result of the scapegoat effect, the impact of liquidity trade is also magnified. The rational confusion raises the impact of both money shocks and liquidity trades. Second, we have focused on the case where macro variables have increased weight. The opposite can also occur, for example, when the exchange rate is high due to a negative b_1 and the money supply is below normal. On average, macro variables have the correct weight because on average $b_1 = 0$. However, this average weight may be small, so that observed macro variables do not consistently contribute much to observed exchange rate volatility.

5 Conclusion

In this short paper we have developed a simple model to illuminate some implications of information dispersion for the importance of macro variables in the equilibrium

exchange rate. We have shown that when unobserved speculative trades are responsible for an exchange rate depreciation, an unusually high money supply can easily be made the scapegoat. We introduced only two macro fundamentals, money supply and output, but in reality one can have a large number of such macro indicators. When a macro fundamental becomes a scapegoat, its impact on the exchange rate can be much larger than otherwise.

While the model we adopted here is close to static in nature in order to keep things simple, future work should naturally focus on a more dynamic model with information heterogeneity of the sort described here. This can account for phenomena such as parameter instability and changing weight given by investors to different macro indicators.

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6 Technical Appendix

6.1 Solving the model with inference on two parameters

In this Appendix we solve the case where $m_1 \neq \overline{m}$ and $y_1 \neq \overline{y}$ so that investors make inference about both ρ_m and ρ_y. The reasoning with two variables can easily be extended to a model with N variables and N signals. We start from equation (4) and conjecture equation (5). Then $s_1 - q_1$ is normally distributed with variance $\lambda^2\sigma_b^2$ and gives information on both ρ_m and ρ_y. This is combined with private signals v_m^i and v_y^i. Let the vectors of signals be $Y' = (s_1 - q_1, v_m^i, v_y^i)$ and define $\boldsymbol{\beta}' = (\rho_m,\ \rho_y)$. We can write:

$$\begin{bmatrix} s_1 - q_1 \\ v_m^i \\ v_y^i \end{bmatrix} = \begin{bmatrix} \widetilde{\alpha}(m_1 - \overline{m}) & -\widetilde{\alpha}(y_1 - \overline{y}) \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \rho_m \\ \rho_y \end{bmatrix} + \begin{bmatrix} -\lambda b_1 \\ \varepsilon_m^i \\ \varepsilon_y^i \end{bmatrix} \tag{10}$$

or

$$Y = X\boldsymbol{\beta} + \boldsymbol{\varepsilon}^i \tag{11}$$

Moreover, define the variance-covariance matrix:

$$\Sigma = \begin{bmatrix} \lambda^2\sigma_b^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_v^2 \end{bmatrix}$$

Since the signals are normal, the best estimate of $\boldsymbol{\beta}$ is a linear regression:

$$E_1^i\boldsymbol{\beta} = \boldsymbol{\beta} + \left(X'\Sigma^{-1}X\right)^{-1} X'\Sigma^{-1}\boldsymbol{\varepsilon}^i \tag{12}$$

The average expectation is then:

$$\overline{E}_1\boldsymbol{\beta} = \boldsymbol{\beta} + \left(X'\Sigma^{-1}X\right)^{-1} X'\Sigma^{-1}\boldsymbol{\varepsilon} \tag{13}$$

where $\boldsymbol{\varepsilon}' = (-\lambda b_1, 0, 0)$ since the aggregate signal errors are zero. Writing out (13) gives:

$$\overline{E}_1 \begin{bmatrix} \rho_m \\ \\ \rho_y \end{bmatrix} = \begin{bmatrix} \rho_m \\ \\ \rho_y \end{bmatrix} + \frac{1}{\sigma_v^2}\frac{1}{D} \begin{bmatrix} \frac{\widetilde{\alpha}^2(y_1-\overline{y})^2}{\lambda^2\sigma_b^2} + \frac{1}{\sigma_v^2} & \frac{\widetilde{\alpha}^2(y_1-\overline{y})(m_1-\overline{m})}{\lambda^2\sigma_b^2} \\ \frac{\widetilde{\alpha}^2(y_1-\overline{y})(m_1-\overline{m})}{\lambda^2\sigma_b^2} & \frac{\widetilde{\alpha}^2(m_1-\overline{m})^2}{\lambda^2\sigma_b^2} + \frac{1}{\sigma_v^2} \end{bmatrix} \begin{bmatrix} -\frac{\widetilde{\alpha}(m_1-\overline{m})}{\lambda^2\sigma_b^2}\lambda b_1 \\ \\ \frac{\widetilde{\alpha}(y_1-\overline{y})}{\lambda^2\sigma_b^2}\lambda b_1 \end{bmatrix}$$

where

$$D \equiv \frac{\widetilde{\alpha}^2}{\lambda^2\sigma_b^2}Z + \frac{1}{\sigma_v^2}$$

and

$$Z \equiv (m_1 - \overline{m})^2 + (y_1 - \overline{y})^2$$

We can rewrite:

$$\overline{E}_1 \begin{bmatrix} \rho_m \\ \\ \rho_y \end{bmatrix} = \begin{bmatrix} \rho_m \\ \\ \rho_y \end{bmatrix} + \frac{\lambda b_1}{D}\frac{\widetilde{\alpha}}{\lambda^2\sigma_b^2} \begin{bmatrix} -(m_1 - \overline{m}) \\ \\ (y_1 - \overline{y}) \end{bmatrix} \tag{14}$$

We can then use (14) to get equation (7), with its analogue for ρ_y, where $k = \frac{1}{D}\frac{\widetilde{\alpha}}{\lambda\sigma_b^2}$. Substituting (14) into (4) gives:

$$\lambda = \frac{\alpha}{1+\alpha}\gamma\sigma_1^2 + \widetilde{\alpha}kZ \tag{15}$$

Solving for k and D gives:

$$\lambda = \frac{\frac{\alpha}{1+\alpha}\gamma\sigma_1^2\left(\sigma_v^2\widetilde{\alpha}^2Z + \lambda^2\sigma_b^2\right)}{\lambda^2\sigma_b^2} \tag{16}$$

The next step is to compute σ_1^2. First, we assume that the conditional variance of the exchange is a constant for $t > 1$, i.e. $var_t(s_{t+1}) = \sigma^2$ for $t > 1$.[4] It is easy to see that:

$$s_2 = \frac{1}{1+\alpha}(m_2 - y_2) - \frac{\alpha}{1+\alpha}\gamma\sigma^2 b_2$$

Thus, using (3) :

$$\sigma_1^2 = var(s_2) = \frac{1}{(1+\alpha)^2} var\left[(m_1 - \overline{m})\rho_m - (y_1 - \overline{y})\rho_y + \varepsilon_m - \varepsilon_y\right] + \frac{\alpha^2}{(1+\alpha)^2}\gamma^2\sigma^4\sigma_b^2$$

This can be written as:

$$\sigma_1^2 = A + n'\left(X'\Sigma^{-1}X\right)^{-1} n \tag{17}$$

where A is a constant, $n' = \left[\frac{1}{1+\alpha}(m_1 - \overline{m}), -\frac{1}{1+\alpha}(y_1 - \overline{y})\right]$ and we use the fact that $var(\beta) = (X'\Sigma^{-1}X)^{-1}$. From (17), we get:

$$\sigma_1^2 = A + \frac{1}{(1+\alpha)^2}\frac{Z}{D}$$

Given the definition of D, it is easy to see that:

$$\frac{\partial\sigma_1^2}{\partial Z} = \frac{1}{(1+\alpha)^2}\frac{1}{D^2}\left[D - \frac{\tilde{\alpha}^2 Z}{\lambda^2\sigma_b^2}\right] = \frac{1}{(1+\alpha)^2}\frac{1}{D^2}\frac{1}{\sigma_v^2} > 0$$

Finally, by differentiating (16) with respect to λ and to Z, we get:

$$\frac{\partial\lambda}{\partial Z} = \frac{\frac{\alpha}{1+\alpha}\gamma\sigma_1^2\sigma_v^2\tilde{\alpha}^2 + \frac{\lambda^3\sigma_b^2}{\sigma_1^2}\frac{\partial\sigma_1^2}{\partial Z}}{3\lambda^2\sigma_b^2 - \frac{2\alpha}{1+\alpha}\gamma\sigma_1^2\lambda\sigma_b^2}$$

[4]This is justified by assuming that we are at a steady state, except for periods 1 and 2. Since the model implies that $s_t = \frac{1}{1+\alpha}(m_t - y_t) - \frac{\alpha}{1+\alpha}\gamma\sigma_t^2 b_t$, we have $\sigma_t^2 = \frac{1}{(1+\alpha)^2}(var(m_t - y_t)) + \left(\frac{\alpha}{1+\alpha}\right)^2\gamma^2\sigma_b^2\sigma_{t+1}^2$. We assume that $var(m_t - y_t)$ is constant. This is a difference equation in σ_t^2 that in general leads to two steady-state values. It can be shown, however, that only the smallest value is stable. We assume that σ^2 is equal to that value.

we can simplify the denominator by using the definitions of λ and of k:

$$\frac{\partial \lambda}{\partial Z} = \frac{\frac{\alpha}{1+\alpha}\gamma\sigma_1^2\sigma_v^2\widetilde{\alpha}^2 + \frac{\lambda^3\sigma_b^2}{\sigma_1^2}\frac{\partial\sigma_1^2}{\partial Z}}{\lambda^2\sigma_b^2 + 2\widetilde{\alpha}^2\frac{Z}{D}}$$

Given that $\frac{\partial\sigma_1^2}{\partial Z} > 0$, $\frac{\partial\lambda}{\partial Z} > 0$.

6.2 Deriving Equations (8) and (9)

Here we specialize to one parameter, ρ_m, so that $Z = (m_1 - \overline{m})^2$. Notice also that in this case $\delta_v = \frac{1}{\sigma_v^2}\frac{1}{D}$. To get equation (8), using (5) gives:

$$\frac{\partial s_1}{\partial m_1} = \frac{1}{1+\alpha} + \tilde{\alpha}\rho_m - \frac{\partial\lambda}{\partial m_1}b_1 \tag{18}$$

We then get (8) by noting that $\frac{\partial\lambda}{\partial m_1} = 2\frac{\partial\lambda}{\partial Z}(m_1 - \overline{m}) = \nu(m_1 - \overline{m})$.

To get equation (9), using (5) gives:

$$\overline{E}\frac{\partial s_1}{\partial m_1} = \frac{1}{1+\alpha} + \tilde{\alpha}\overline{E}\rho_m - \frac{\partial\lambda}{\partial m_1}\overline{E}b_1 \tag{19}$$

Taking expectations of (5) and using (6), we find $\overline{E}b_1 = \delta_v b_1$. We substitute this and $\overline{E}\rho_m$ from (7) to find (9).