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# ASYMMETRIC CYCLES

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**ABSTRACT**

I estimate a model in which new technology entails random adjustment costs. Rapid adjustments may cause productivity slowdowns. These slowdowns last longer when retooling is costly. The model explains why growth-rate disasters are more likely than miracles, and why volatility of growth relates negatively to growth over time. I estimate the model, and the estimates have surprising implications. Firms seem to abandon technologies long before they are perfected – current-practice TFP is 17 percent below best-practice.



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# Asymmetric Cycles

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April 11, 2004

**Abstract**

I estimate a model in which new technology entails random adjustment costs. Rapid adjustments may cause productivity slowdowns. These slowdowns last longer when retooling is costly. The model explains why growth-rate disasters are more likely than miracles, and why volatility of growth relates negatively to growth over time. I estimate the model, and the estimates have surprising implications. Firms seem to abandon technologies long before they are perfected – current-practice TFP is 17 percent below best-practice.

## 1 Introduction

Technology shocks play a central role in most business cycle models of the last two decades. We often take such shocks as exogenous and we then study how a model economy responds to them. The present paper starts from the premise that the shocks depend on the technologies we adopt. I study technology adoption in an "$Ak$" growth model with endogenous shocks that can explain a few business-cycle facts. I assume that a technology requires specific skills. The exact nature of these skills is not known before a technology is adopted. Having committed to a technology, firms may face unexpectedly large training costs.

The model generates left-skewed distributions for the growth rates of output, consumption, investment, stock prices, and interest rates. Such skewness is seen in U.S. and other data. The growth process obeys a simple difference equation and I provide estimates of the model's parameters. The model also generates growth-rates that are more volatile in recessions than in booms. This explains the time-series findings of Ramey and Ramey (1991) which I have updated.

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Technological adoption entails free riding incentives analyzed at the industry level by Jovanovic and Lach (1999) and at the aggregate level by Eeckhout and Jovanovic (2002). To keep things tractable here, however, I assume that technological information is of transitory value so that a firm chooses the same technology that all others have chosen. In spite of this, adjustment costs – incurred simultaneously by all firms – play a role similar to diffusion lags and they deliver impulse responses similar to those Lippi and Reichlin (1994) and Forni and Reichlin (1998) report as being induced by diffusion lags.

The parameter estimates imply that firms abandon technologies long before they are perfected. Current-practice TFP is 17 percent below best-practice TFP.

*Plan of paper.*—Section 2 starts with a sketch of the quantitative puzzle and the intuition. Section 3 presents the model and compares it to some evidence. Section 4 discusses the literature and Section 5 concludes.

# 2 Intuitive explanation

The model assumes technological commitment and random adoption costs. The following example shows the intuition behind asymmetric growth rates and sluggish responses to technology. In Figure 1, the vertical axis plots the log of TFP. For technology $A$, log TFP is

$$A - \frac{\lambda}{2}\left(s_A - h\right)^2,$$

where $s_A$ is the ideal skill-mix for technology $A$ and where $h$ is the actual skill mix which we measure on the horizontal axis. Committing to a potential TFP-growth rate of $x$ exposes the adopter to uncertainty about $s_{A+x}$. The law of motion for $s$ is

$$s_{A+x} = s_A + x\varepsilon,$$

and $\varepsilon$ is unknown until *after* the commitment to technology $A + x$ is made. The new log TFP level would then be $A + x - \frac{\lambda}{2}\left(s_A + x\varepsilon - h\right)^2$. Suppose, however, that we start with an initial level of expertise that is ideal for technology $A$. That is, suppose $h = s_A$. Then the new log TFP would be

$$A + x - \frac{\lambda}{2}x^2\varepsilon^2,$$

as illustrated in Figure 1. A large $|\varepsilon|$ produces a growth disaster, whereas a miracle is impossible because the largest possible TFP is $A + x$.

*Diffusion lags, learning, and slow adjustment of $h$.*—In the model firms all choose the same technology $A$. This violates the tendency for a technology to spread only gradually among adopters. E.g., Lippi and Reichlin (1994), Jovanovic and Lach (1997) and Forni and Reichlin (1998) argue that such diffusion lags help explain the

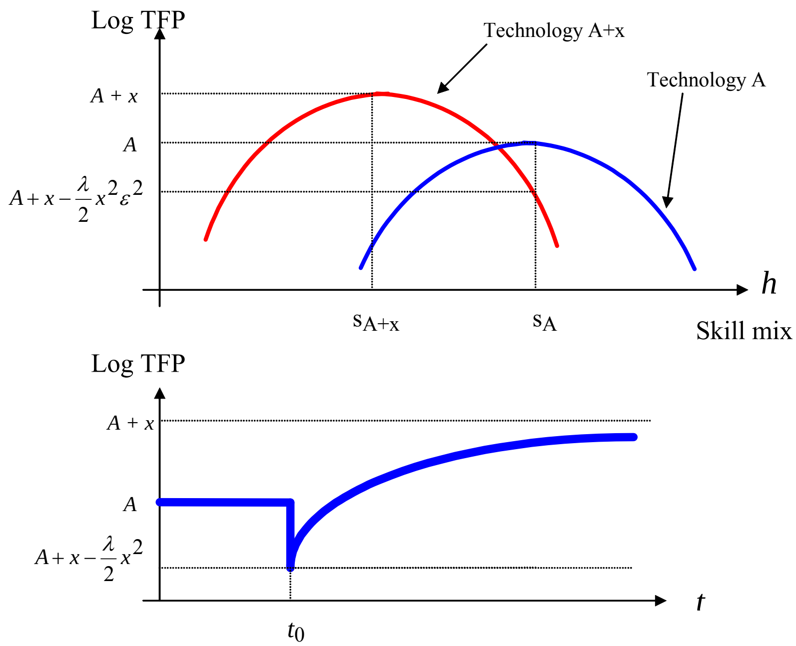



Figure 1: Asymmetric growth and the impulse response of technology shocks

sluggish impulse responses to technology shocks. Yet sluggishness in $h$ within each firm can deliver a similar impulse response. We may think of $h$ as organization capital that the firm owns and that is costly to adjust, as Prescott and Visscher (1980) argue. To see why, suppose a firm starts with expertise $h = s_A$, so that its initial TFP is $A$. Suppose it then permanently switches to technology $A + x$. It turns out that $h_t$ follows a partial adjustment path from $h$ towards $s_{A+x}$:

$$h_{t+1} = \alpha h_t + (1 - \alpha) s_{A+x}, \qquad \text{where } h_0 = s_A$$

where $\alpha$ is estimated to be about 0.6. Since

$$\begin{aligned} h_t &= \alpha^t s_A + \left[1 - \alpha^t\right] s_{A+x} \\ &= s_{A+x} + \alpha^t x\varepsilon \end{aligned}$$

so that

$$\ln(TFP_t) = A + x - \alpha^{2t}\frac{\lambda}{2}x^2\varepsilon^2$$

In other words $\ln TFP$ converges geometrically to its new high. The bottom panel of Figure 1 shows the typical impulse response of $\varepsilon$.

# 3 Model

The model has two types of capital. The first, $k$, is the quantity of capital. The second, $h$, is a non-hierarchical index of expertise and physical-capital type, which I think of as the skill mix.

*Production function.*—With $k$ units of capital, firm has a potential output of

$$y^p = zk.$$

The productivity parameter, $z$, is endogenous and given by

$$z = \exp\left\{A - \frac{\lambda}{2}\left(s_A - h'\right)^2\right\}. \tag{1}$$

Here $A$ is the firm's technology, $h'$ is the firm's skill mix, and $s_A$ is the skill-mix ideal for technology $A$. The cost of technological imbalance is indexed by $\lambda > 0$.

*Adoption of technology.*—Adoption of a better technology is free. A firm can choose a technology level by any amount, $x$, so that starting today at $A$, tomorrow's technology is

$$A' = A\left(1 - \delta\right) + x. \tag{2}$$

where $\delta A$ represents obsolescence. The firm commits to using technology $A$ for at least one period. But $A'$ makes unpredictable demands on the skill mix. Assume that

$$s_{A'} = s_A + x\varepsilon, \tag{3}$$

where $\varepsilon$ is a zero-mean random variable with variance $\sigma^2$. The parameter $\varepsilon$ is time specific. The firm chooses $x$ before seeing $\varepsilon$. Assume $x \geq 0$. I.e., once abandoned, a technology cannot be recalled.

*Adjustment of $h$.*—The firm starts with skill mix $h$. Before producing, it can adjust its skill mix from $h$ to $h'$ at a cost of

$$C\left(y^p, h, h'\right) \equiv \left[1 - \exp\left\{-\frac{\theta}{2}\left(h - h'\right)^2\right\}\right] y^p$$

The cost of redressing technological imbalance is indexed by $\theta > 0$. I refer to this loosely as a retooling cost.

*The firm's decision problem.*—Firms will choose their $x$ and $h'$ so as to maximize the productivity of the capital that they raised in the previous period. A firm produces for one period and then liquidates. In the *pre-pre-production* period it

1. raises capital $k$ from shareholders,

2. chooses $x$ which commits it to using technology $A'$ as given by (2),

3. freely inherits the prevailing skill mix $h$.

In the *production period* the firm does the following in sequence: It

1. observes $s_{A'}$ as given by (3),

2. chooses $h'$,

3. produces and pays a dividend

$$y = y^p - C\left(y^p, h, h'\right)$$

4. liquidates; the salvage value of its $k$ and $h'$ is zero.

*Choice of* $h'$.—Suppose that (having committed to $A'$ in the previous period) at the start of the production period the firm has observed that $s_{A'} = s'$. The firm then chooses $h'$ to solve

$$\max_{h'}\left\{y^p - C\left(y, h, h'\right)\right\} = k \max_{h'} \exp\left\{A' - \frac{\lambda}{2}\left(s' - h'\right)^2 - \frac{\theta}{2}\left(h' - h\right)^2\right\} \tag{4}$$

The first-order condition is $\lambda\left(s' - h'\right) - \theta\left(h' - h\right) = 0$ and at its solution, the second-order derivative w.r.t. $h'$ is negative. The optimal $h'$ is a convex combination of starting skill mix $h$, and ideal skill mix $s'$:

$$h' = \alpha h + (1 - \alpha)\, s' \tag{5}$$

where

$$\alpha = \frac{\theta}{\lambda + \theta}. \tag{6}$$

Substituting into (4), its maximized value is the firm's output:

$$y = Z\left(A', s' - h\right) k$$

where

$$Z\left(A', s' - h\right) \equiv \exp\left(A' - \frac{\alpha\lambda}{2}\left(s' - h\right)^2\right) \tag{7}$$

is the average product of capital, or maximized TFP, which depends only on $s' - h$, the "skill-mix gap" that exists at the start of the production period, after $s'$ has been drawn, but before the firm has adjusted $h$.

*The choice of* $x$.—The firm chooses its technology in the pre-production period, before knowing $s'$. The state-of-the-art technology is summarized by the pair $(A, s)$, and the skill mix is $h$. All firms face the same shock $\varepsilon$ and so tomorrow's aggregate output and consumption will depend on $\varepsilon$. All firms will choose the same $(x, h')$ pair. This means that the firm's dividend will be correlated with tomorrow's aggregate consumption. Let $p(A, \varepsilon)$ be today's price of a unit of consumption tomorrow if the aggregate shock is $\varepsilon$. In (16) we shall see that if all other firms choose the value $x^*$,

$$p(A, \varepsilon, x^*) = \frac{1}{Z([1-\delta]A + x^*, s + x^*\varepsilon - h)} \tag{8}$$

The optimal $x$ maximizes the pre-production value of the firm per unit of $k$ raised. This value, $v$, depends on the firm's pre-production state $(A, s-h)$ as follows:

$$\begin{aligned} v(A, s-h, x^*) &\equiv \max_x \int p(A, \varepsilon) Z([1-\delta]A + x, s + x\varepsilon - h) \, dF(\varepsilon) \\ &= 1. \end{aligned} \tag{9}$$

The amount the market is willing to pay for a claim to the firm's dividend in the next period is $v(A, s-h)$, which must equal unity because cost of capital is 1. At this price and value, a firm breaks even on each unit of $k$ that it raises.

As (9) shows, firms' choices of $x$ are interdependent. To find the equilibrium choice of $x$ we now differentiate the RHS of (9) w.r.t. $x$ in and substituting from (8) into the resulting expression. We then evaluate the FOC at the symmetric equilibrium $x = x^*$, and obtain

$$\int [1 - \lambda\alpha\varepsilon(s + x\varepsilon - h)] \, dF(\varepsilon) = 0.$$

Since $\varepsilon$ has mean zero and variance $\sigma^2$, and since $(x, s, h)$ are predetermined, this reads $1 - \lambda\alpha x\sigma^2 = 0$, so that

$$x = \frac{1}{\lambda\alpha\sigma^2} = \frac{1}{\sigma^2}\left(\frac{1}{\lambda} + \frac{1}{\theta}\right). \tag{10}$$

where the second equality follows from the (6). Now we see clearly what the barriers to technological adoption are. If $\lambda$ or $\theta$ or $\sigma^2$ were zero, $x$ would be infinite.

*Preferences.*—Households are infinitely lived with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t. \tag{11}$$

*Asset markets.*—The number of households and the number of firms are both normalized to unity. This double normalization is fine because firm-size is indeterminate. Then $y$ and $k$ are output and capital per consumer. A household owns one-period shares of firms, and dividends are its only income. Because the representative firm grows over time, let us define shares in terms of pieces of capital rather than of firms. That is, let $n$ be the number of units of capital that the household owns. From (9), the price of a share is unity.

*The behavior of the aggregate state.*—The pricing of assets will not depend on the capital stock so that for the consumer's savings problem, at least, the aggregate state will be $(A, s, h_{-1})$. Let $u = s - h_{-1}$. From (7) it follows that $(s, h_{-1})$ matters for aggregate output only through $u$. We shall show that $u$ follows the Markov process

$$u' = \alpha u + x\varepsilon,$$

so that its transition function is

$$\Phi\left(u', u\right) = F\left(\frac{u' - \alpha u}{x}\right),$$

where $F$ is the C.D.F. of $\varepsilon$.

*The savings decision.*—If it owns $n$ shares, a household's wealth is $Z\left(A, u\right) n$. Its budget constraint therefore is

$$n' + c = Z\left(A, u\right) n. \tag{12}$$

The consumer's state is $\left(n, u\right)$, and the Bellman equation is

$$w\left(A, n, u\right) = \max_{n'} \left\{ \ln\left(Z\left(A, u\right) n - n'\right) + \beta \int w\left(\left[1 - \delta\right] A + x, n', u'\right) d\Phi\left(u', u\right) \right\}. \tag{13}$$

The first appendix shows that optimal consumption is

$$c = \left(1 - \beta\right) Zn. \tag{14}$$

and saving is

$$n' = \beta Zn$$

At equilibrium,

$$n = k. \tag{15}$$

so that $k' = \beta Z\left(A, u\right) k$ and

$$\begin{aligned} \frac{U'\left(c'\right)}{U'\left(c\right)} &= \frac{c}{c'} = \frac{\left(1 - \beta\right) Zk}{\left(1 - \beta\right) Z'\left(\beta Zk\right)} = \frac{1}{\beta Z\left(A', u'\right)} \\ &= \frac{1}{\beta Z\left(\left[1 - \delta\right] A + x^{*}, s + x^{*}\varepsilon - h\right)}, \end{aligned} \tag{16}$$

which proves (8).

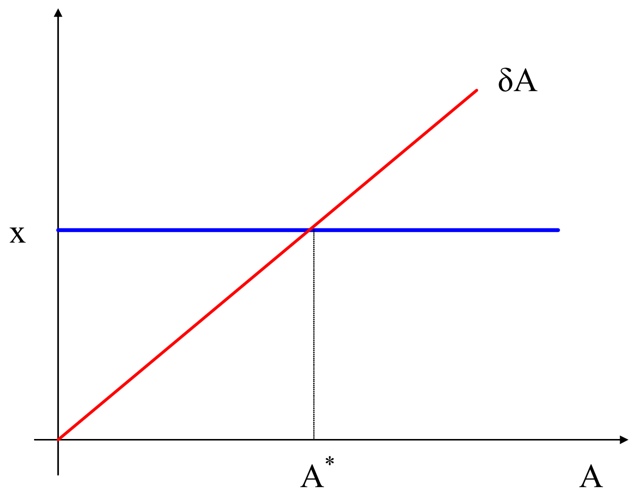



Figure 2: DETERMINATION OF $A^*$

### 3.1 The growth process

Since $x$ is a constant, $A_t$ converges monotonically to $A^*$ that uniquely solves

$$x = \delta A, \tag{17}$$

as shown in Figure (2).In RBC analysis we often treat the technology parameter as stationary, and I shall do the same and assume that $A$ is at $A^*$.

I will also assume that the costs of adjusting $h$ consist entirely of foregone output. Measured output then is

$$y = Z\left(A^*, u\right)k.$$

Then (4) implies

$$\begin{aligned} \ln y &= \ln k + A^* - \frac{\lambda}{2}\left(s' - h'\right)^2 - \frac{\theta}{2}\left(h - h'\right)^2 \\ &= \ln \beta + A^* + \ln y_{-1} - \frac{\lambda}{2}\left(\alpha u\right)^2 - \frac{\theta}{2}\left(\left[1-\alpha\right]u\right)^2 \\ &= \psi_0 + \ln y_{-1} - \psi u^2 \end{aligned} \tag{18}$$

where

$$\psi_0 = \ln \beta + A^* \quad \text{and}^1 \quad \psi = \frac{1}{2}\lambda\alpha. \tag{16}$$

The second line of (18) follows because (14) and (15) imply that $k = \beta y_{-1}$, and upon applying (5) and the definition $u \equiv s - h_{-1}$. Thus letting $\Delta \ln y_t \equiv \ln y_{t+1} - \ln y_t$,

$$\Delta \ln y_t = \psi_0 - \psi u_{t+1}^2. \tag{17}$$

*Long-run growth.*—The long-run-average growth rate of output is gotten by taking the unconditional expectation in (17) which leads to the following result (proved in the appendix):

**Proposition 1** *The long-run growth rate has a mean of*

$$E\left(\Delta \ln y\right) = \ln \beta + \frac{1}{\lambda \alpha \sigma^2}\left(\frac{1}{\delta} - \frac{1}{2\left(1-\alpha^2\right)}\right), \tag{18}$$

*and variance*

$$Var\left(\Delta \ln y\right) = \left(\frac{1}{\lambda \alpha \sigma^2}\right)^2 \frac{1}{1-\alpha^4}\left(\frac{1}{2} + \frac{\alpha^2}{1-\alpha^2}\right). \tag{19}$$

Long-run growth increases with $\beta$, and decreases with $\alpha, \lambda$, and $\sigma^2$.

*The process for u.*—From (5), $h' = \alpha h + (1-\alpha) s'$, so that

$$u' = s' - h = s + x\varepsilon - \alpha h_{-1} - (1-\alpha) s = \alpha \left(s - h_{-1}\right) + x\varepsilon.$$

Since $\varepsilon$ is independent of $u$ we adopt the convention of dating it at $t+1$ and we therefore have the time-series process

$$u_{t+1} = \alpha u_t + x\varepsilon_{t+1}. \tag{20}$$

*The case where $\varepsilon$ is normally distributed.*—If $\varepsilon_t$ is normally distributed, the stationary distribution of $u_t$ is also normal with mean zero an variance

$$\sigma_u^2 = \frac{x^2\sigma^2}{1-\alpha^2}. \tag{21}$$

Now, the stationary distribution of the square of a standard normal variate, is $\chi^2_{(1)}$. Denote by $v$ the square of such a variable, i.e.,

$$v = \left(\frac{\sqrt{1-\alpha^2}}{x\sigma} u\right)^2 \tag{22}$$

Then $v$ has a Chi-squared distribution with 1 degree of freedom:

$$v^{-\frac{1}{2}} \frac{1}{2\pi} \exp\left(-\frac{1}{2} v^{\frac{1}{2}}\right) \equiv g\left(v\right), \tag{23}$$

for $v \geq 0$.

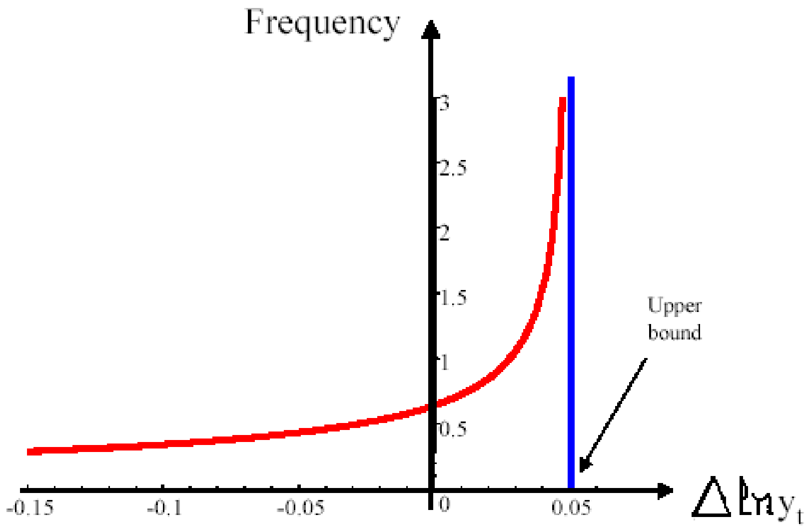



Figure 3: $\chi^2$ DISTRIBUTION OF $\Delta \ln y$ WHEN $\varepsilon$ is NORMAL.

Figure 3 shows the long-run distribution of output growth (given in [17]). It is also distributed $\chi_1^2$, except that the tail is on the left. Output growth is negative if $u_t^2 > \psi_0/\psi$. And why is $u_t$ missing in (17)? It is because savings exactly offset the influence of $u_t$: Savings are proportional to $y_t$ so that fluctuations in $y_t$ do not affect the growth rate – a drop in $y_t$ simply translates into an equal percentage drop in $k_{t+1}$. On the other hand, fluctuations in $y_{t+1}$ do get into the growth rate between $t$ and $t+1$, and the distribution of the level $y_{t+1}$ is skewed to the left. Hence the asymmetry in the growth rate of $y$. This asymmetry should also show up in consumption and investment growth.

*The distribution of growth rates in U.S. GDP per capita.*—The top panels of Figure 4 show the frequency distribution of growth rates of per-capita output at a five-year frequency. The labeling refers to the last year of a five-year interval so, for example, the growth rate for 1940 means $\ln y_{1940} - \ln y_{1935}$. With the three observations the three wars (Civil 1860-65, WW1 1915-20, WW2 1940-45) taken out, the numbers are decidedly skewed to the left. Omitted were those 5-year intervals that most naturally contain the most intense war-time years). The two histograms look a little different because the number of bins in both histograms is the same – 25 bins. As a result, bin size is slightly different and, hence, the 2 left-most observations are paired in the right histograms and not paired in the left one. The kernel density estimates are also reported in the bottom panels.

*The distribution of conditional TFP levels in U.S. plants.*—TFP levels are non-stationary, but their distribution conditional on lagged values should also be skewed to the left. Now, in the model firms are identical, and each uses the same technology.

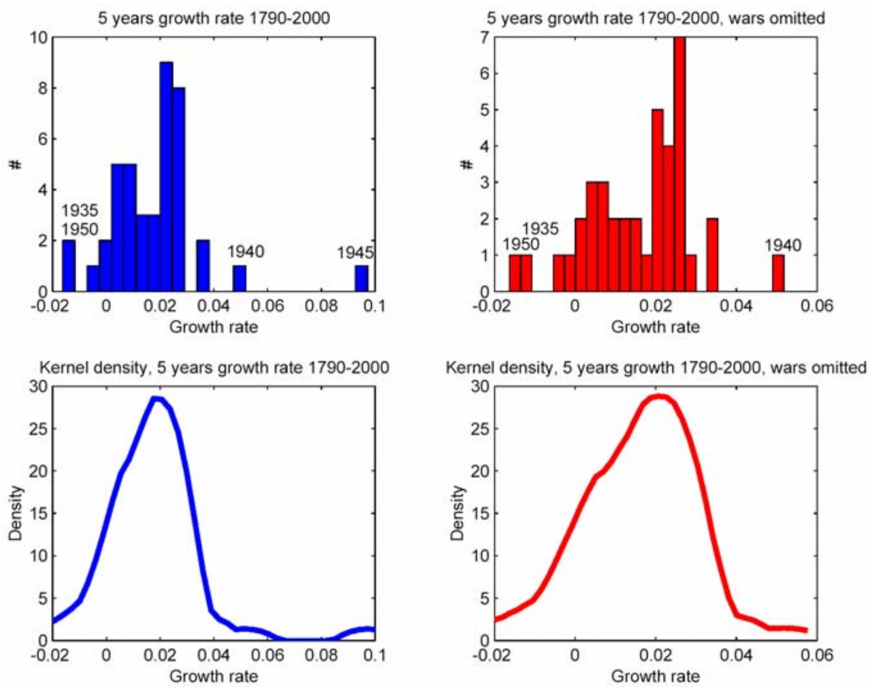



Figure 4: Distributions of five-year growth rates with and without wars

In fact, however, while technology has an aggregate component, there also are firm-specific technological differences. Stretching the model somewhat, we may thus look for a left skew in the distribution of firm-level TFP. Such asymmetries have been found in the frontier-production-function literature (Caves and Barton, 1990). Figure reproduces the results reported in Figure 2 of Baily Hulten and Campbell (1993). The six histograms pertain to plants' TFP levels in 1987 conditional on their values in 1982. Each histogram pertains to a separate productivity range in 1982. In other words there are six conditioning sets in 1982, and the sets are monotonically increasing as we move down and to the right in the panel. The means rise monotonically, which implies that TFP levels are positively autocorrelated

**Table 1**: *Plant TFP moments*

| Mean | Variance | Skewness |
|---|---|---|
| $-.29$ | .19 | $-.016$ |
| $-.08$ | .12 | $-.009$ |
| $-.04$ | .14 | .016 |
| .08 | .09 | $-.023$ |
| .12 | .16 | $-.071$ |
| .28 | .12 | $-.023$ |

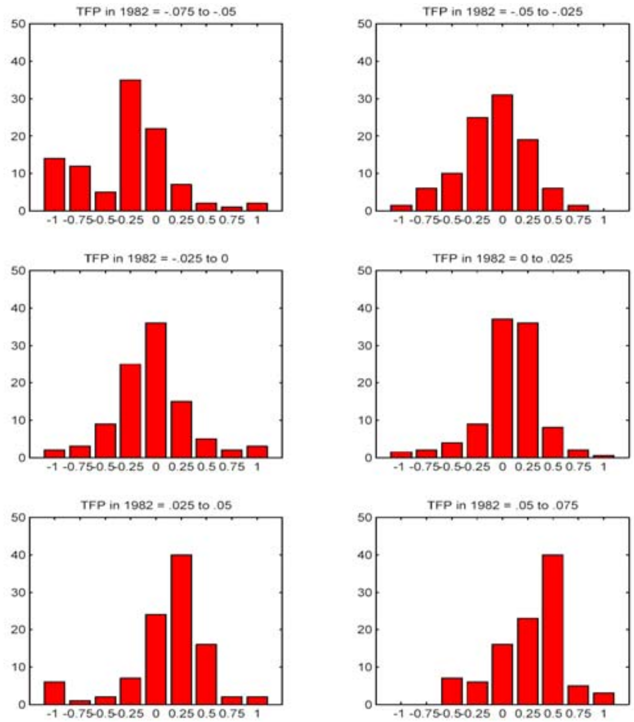



Figure 5: Conditional distributions of plant TFP

## 3.2 Growth and retooling

Recessions are retooling episodes here in the sense that $h$ adjusts most when output is low. Let $r$ denote the retooling cost relative to potential output:

$$r \equiv \frac{1}{y^p} C\left(y^p, h, h'\right).$$

Then we have

**Proposition 2**

$$r = 1 - \exp\left\{(1-\alpha)\left(\Delta \ln y_t - \psi_0\right)\right\} \tag{24}$$

**Proof.** From the definition of $C()$,

$$r = 1 - \exp\left\{-\frac{\theta}{2}\left(h-h'\right)^2\right\} = 1 - \exp\left\{-\frac{\theta}{2}\left(1-\alpha\right)^2 u^2\right\}$$

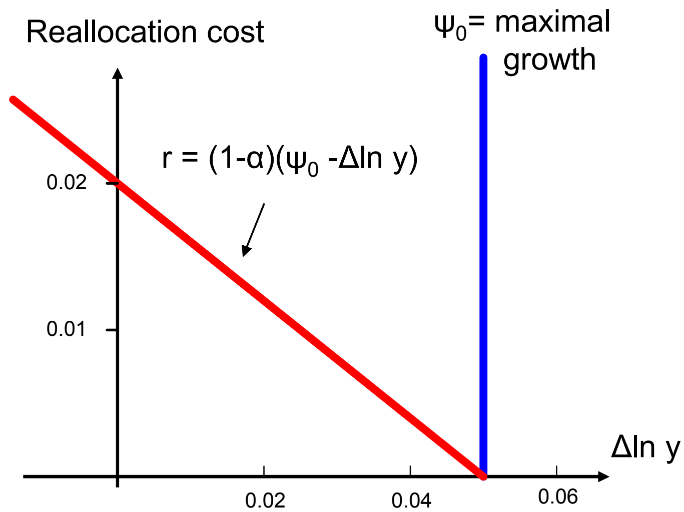
Then (17) implies

$$r = 1 - \exp\left\{\frac{\theta}{2}\left(1-\alpha\right)^2\left[\frac{\Delta \ln y_t - \psi_0}{\psi}\right]\right\}.$$

But

$$\frac{\theta}{2\psi}(1-\alpha)^2 = \frac{\theta(1-\alpha)^2}{\lambda\alpha^2+\theta(1-\alpha)^2} = \frac{\theta\lambda^2}{\lambda\theta^2+\theta\lambda^2} = \frac{\lambda}{\lambda+\theta} = 1-\alpha.$$

■




Costs of adjusting $h$ as a percentage of output

The costs of growth are in the form of lower output due to the adjustment of $h$. These costs depend mainly on $\alpha$. The above Figure plots the relation in (24) evaluated at the estimated parameter values (5 yrs no war) in column 4 of the Table of estimates below: $\alpha = 0.6$ and $\psi_0 = 0.05$ (the annualized value of $\hat{\psi}_0$).

### 3.3 Growth vs. volatility of $\Delta y_t$ over time

When $\theta > 0$, and hence when $\alpha > 0$, the model predicts a negative correlation between output growth and output-growth variability over time. This is seen intuitively in Figure 3. The conditional variance is higher if we know that $\Delta \ln y$ is likely to be low. The latter, in turn, follows because $u$ us autocorrelated — when $u$ strays far from the origin, it will probably remain far from the origin in the next period as well. Conditional on $u$, this implies lower expected growth but, because $u^2$ is an increasing and convex function of $|u|$, it also implies a higher variance of growth. Formally,

**Proposition 3** *The time-series relation between growth and its variability is negative.*

**Proof.** In (17) we condition the mean and variance of $\Delta \ln y_t$ on the lagged value of $u$, i.e., on $u_t$. As $u_t$ varies over time, the conditional mean and variance of $\Delta \ln y_t$ shift. Showing that the two move in opposite ways when $u_t$ shifts is equivalent to showing that the conditional mean and conditional variance of $u_{t+1}^2$ move in the same direction as $u_t$ changes. Note that

$$\begin{aligned} E\left(u_{t+1}^2 \mid u\right) &= \left[E\left(u_{t+1} \mid u\right)\right]^2 + Var\left(u_{t+1} \mid u\right) \\ &= \alpha^2 u^2 + \sigma^2 \end{aligned} \tag{25}$$

On the other hand

$$Var\left(u_{t+1}^2 \mid u\right) = E\left(u_{t+1}^4 \mid u\right) - \left[E\left(u_{t+1}^2 \mid u\right)\right]^2$$

But

$$\begin{aligned} E\left(u_{t+1}^4 \mid u\right) &= \left[E\left(u_{t+1} \mid u\right)\right]^4 + 3\left[Var\left(u_{t+1} \mid u\right)\right]^2 + 6\left[E\left(u_{t+1} \mid u\right)\right]^2 Var\left(u_{t+1} \mid u\right) \\ &= \alpha^4 u^4 + 6\alpha^2\sigma^2 u^2 + 3\sigma^4. \end{aligned}$$

Thus, as long as $\alpha > 0$, the mean and variance of $u^2$ are both increasing in lagged $u^2$. ■

Figures 4 and 5 report the relation between mean and variance of growth at 5-year and 10-year frequencies. Wartime observations are denoted by hollow squares. A negative relation emerges for 5-year intervals with and without wars. For decades, the relation is negative only if we exclude wars. Generally, decades do not support the model well as 5-year periods and there are fewer observations at that frequency. Figure A1 of the appendix reports the entire growth-rate series in decade and 5-year form, along with the standard deviations.[2]The negative time-series relation is confirmed in Figures 7 and 8 of Ramey and Ramey (1991) for annual data.

The trade-off between growth and its variability is an equilibrium relation, however, and not one that policy can exploit. Raising $x$ would *raise* volatility, and raising the savings rate would leave volatility unchanged.

#### 3.3.1 The cross-section relation between growth and its variance.

As it stands the model has only one sector. It does not explain cross-section facts. But as I argued in Section 2.1, when firms use different technologies the model would lead us to expect that the cross-section distributions of plants' TFP should be skewed to the left. And the logic of the preceding proposition leads one to expect that, plants with low-TFP last period should have a greater variance of TFP this period. This was true in the Baily et al (1993) sample, as Table 1 and Figure 8 show, though one cannot make much of just six observations.

Less favorable is the cross-sector evidence. Imbs (2002) finds that the cross-sector correlation between growth and volatility is positive. This could happen in the model if technological opportunity, as expressed, e.g., in the parameter $\lambda$, were to vary over sectors. For instance, (10) implies that a fall in $\lambda$ raises $x$ and it raises volatility of output so that growth and volatility both rise Imbs also finds that the correlation remains substantial even after controlling for investment, suggesting that the explanation is technological, such as the one advanced here.

[2]The statistical program used required the shading of the wars to be shifted to the left by 2.5 years.

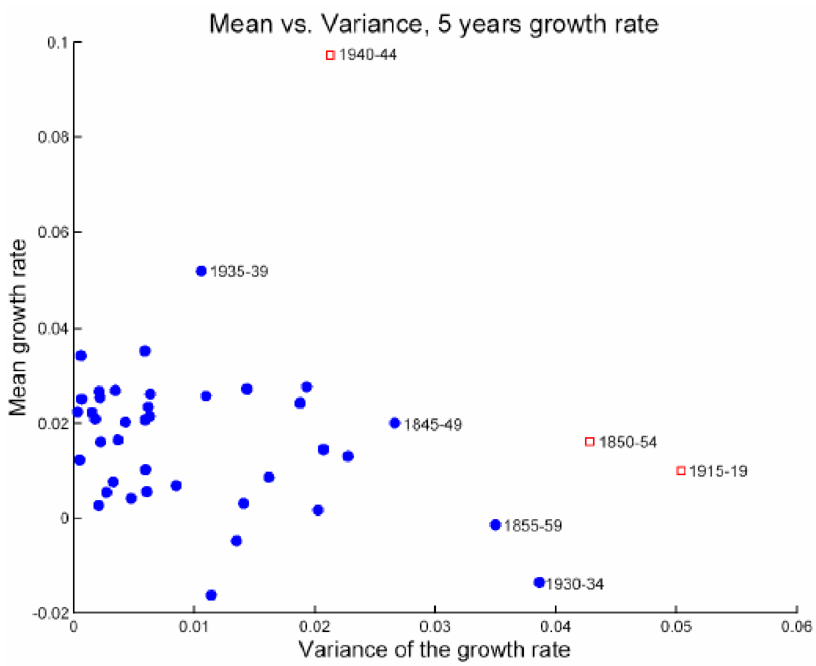



Figure 6: The five-year-interval sample

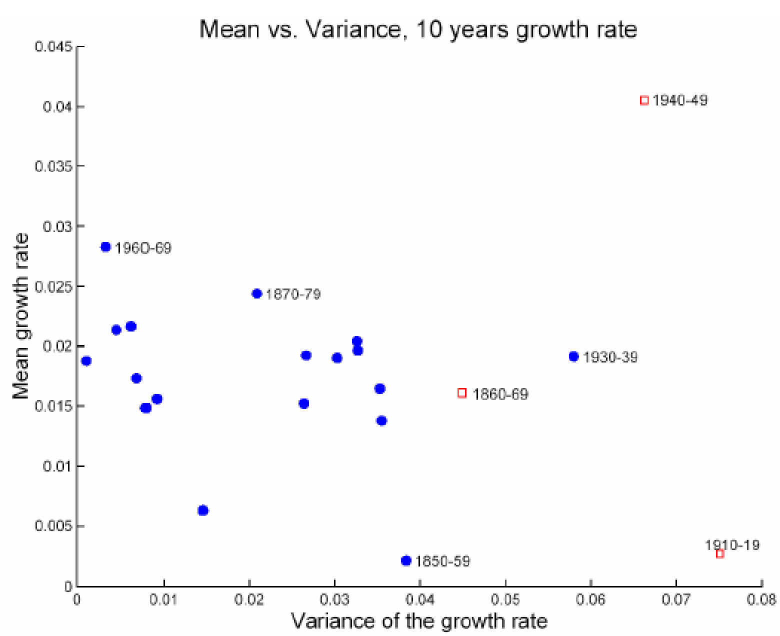



Figure 7: The Decades Sample

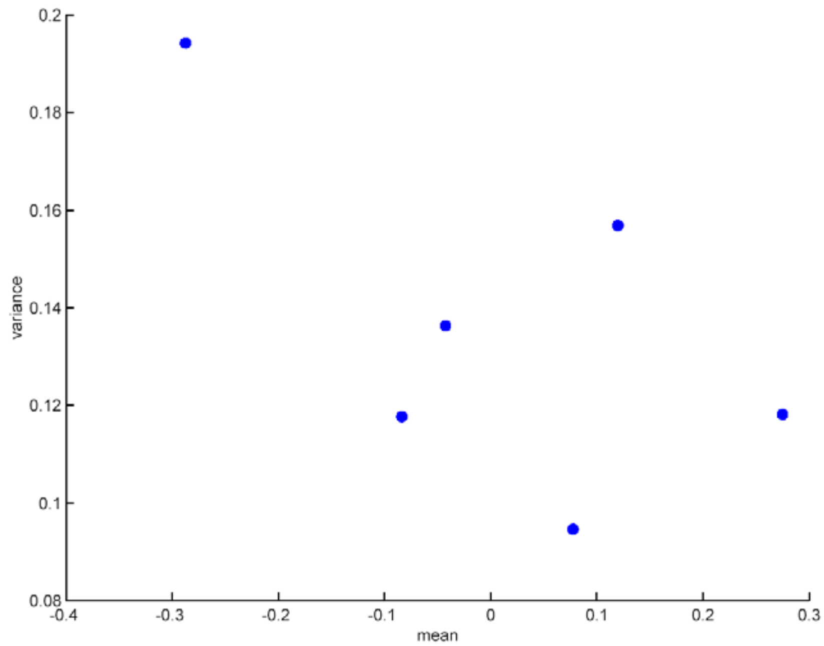



Figure 8: Mean vs. variance in the distribution of plant TFP

## 3.4 Estimates of the parameters

I shall use per-capita GDP data from 1790 until the present. This model is better suited to low frequencies because firms choose their technologies relatively infrequently. We need a long time series, at least while we deal with one country only. The 4 parameters are $\psi_0, \lambda, \theta$, and $\sigma^2$, but not all 4 are identified:

**Claim 4** *The model's likelihood function depends on* $(\lambda, \theta, \sigma^2)$ *only through the two parameters*

$$\left(\lambda\sigma^2, \theta\sigma^2\right).$$

**Proof.** The expressions in (6) and (10) do not change. From (21) the variance of $u$ is proportional to $\sigma^2$, so that the distribution of $u/\sigma$ is invariant to changes in $\sigma$. Therefore the variance of $\psi u^2$ is of order $\psi^2\sigma^4 = (\psi\sigma^2)^2$. But from (19), $\psi$ is homogeneous of degree 1 in $(\lambda, \theta)$, and this implies that the distribution of $\psi u^2$ depends only on $(\lambda\sigma^2, \theta\sigma^2)$. ■

In other words, if we double the penalties $\lambda$ and $\theta$ but halve the variance $\sigma^2$ of the innovations, the equilibrium remains the same. We shall measure $\lambda$ and $\theta$ in units of $\sigma^2$ by imposing:

$$\sigma^2 = 1.$$

I fit the model to both 5-year and 10-year frequencies. With its assumption of 100% depreciation of $k$, the model seems inappropriate for frequencies higher than that. The estimates come from data on per-capita GDP since 1790, and no other series were used. The estimates are reported in Table 1.

Let us concentrate on the last column, the five-year intervals excluding wars. The estimates of $\alpha = \theta/(\lambda+\theta)$ range between 0.52 and 0.68.

**Table 1: Parameter estimates, 5-year and 10-year periods: 1790-2000**

Standard errors in parentheses

| | 10 yrs | 10 yrs no war | 5 yrs | 5 yrs no war |
|---|---|---|---|---|
| $\psi_0$ | 0.41 (0) | 0.28 (0) | 0.49 (0) | 0.26 (0) |
| $\alpha$ | 0.64 (0.05) | 0.52 (0.10) | 0.68 (0.02) | 0.60 (0.05) |
| $\lambda$ | 5.84 (0.47) | 11.28 (2.15) | 3.42 (0.06) | 7.25 (0.32) |
| $\theta$ | 10.41 (1.73) | 12.08 (3.64) | 7.16 (0.69) | 10.87 (2.56) |
| $\delta$ | 0.29 (0.02) | 0.22 (0.03) | 0.58 (0.02) | 0.45 (0.05) |
| *N obs.* | 20 | 17 | 41 | 38 |

From (17), $\Delta \ln y_t = \psi_0 - \psi u_{t+1}^2$. Conditional on $u$, then, the percentage by which output is reduced by technological imbalance is $\psi u_{t+1}^2$. Since $\psi = \frac{1}{2}\lambda\alpha$ and since, by (21), under the steady state distribution of $u$ $Eu^2 = x^2/(1-\alpha^2)$, (recall that we cannot identify $\sigma^2$ and therefore we set $\sigma^2 = 1$), the expected loss therefore is

$$\psi E u_{t+1}^2 = \frac{1}{2\lambda\alpha}\frac{1}{1-\alpha^2} = \frac{1}{2(7.5)(0.6)}\frac{1}{0.64} = 0.17$$

Therefore the level of output is 17 percent below its maximal level – i.e., the level that would result if the technologies in question were operated at their maximal efficiency.

From this it follows that a rise in technological uncertainty (e.g., in $\sigma$ which is being held at unity in the above calculations) will give rise to lower growth as well as a lower level. Comin (2000) argued that the productivity slowdown of the 70s and 80s was the result of a rise in technological uncertainty in the 1970's which raised the demand for less productive but more flexible capital. I get a similar effect from a rise in $\sigma^2$ that reduces $x$ and, hence, TFP.

# 4 Related theory

With so much written on the subject it helps to group the papers by topic. Any model that delivers sharper downturns than recoveries is related to the present model. There many such models.

*Exogenous shocks and growth.*—Jones, Manuelli and Stacchetti (1999) and Fatas (2000) study how the shock process to productivity influences growth, and Scott and Uhlig (1999) study the growth effects of a change in the volatility of investment. Acemoglu and Scott (1997) look at level effects in a model with dynamic increasing returns and show asymmetries

*Adoption and free riding.*—Chamley and Gale (1993) and Caplin and Leahy (1994) focus on incentives to delay adoption. Chalkley and Lee (1998) and Veldcamp (2002) argue cycles are asymmetric because firms can more quickly detect negative shocks than positive ones.

*Endogenous technology and cycles.*—Greenwood, Hercowitz and Huffman (1988) allow shocks to the marginal efficiency of investment, while Aghion and Howitt (1992), Barlevy (2002) and Comin and Gertler (2003) model technology as a random function of research. Martin and Rogers (2000) relate learning by doing to growth and its volatility.

*Costs of business cycles.*—Lucas (1987) argued the getting rid of cycles would yield tiny benefits. But when cycles are related to trend as is the case in Benhabib and Nishimura (1984), Shleifer (1986), Matsuyama (1999) and Ellis and Francois (2001), the question is not well posed. Caballero and Hammour (1994) argue that recessions are reallocative, for reasons similar to those I have modelled. Other related papers are Barlevy (2001), Krebs (2002), and Rampini and Eisfeldt (2003).

My defense for adding yet another model to this long list is that I solve for everything analytically.

# 5 Conclusion

This paper has explained a couple of business-cycle regularities. The left-skewed distribution of growth rates and the negative time-series relation between growth rates and their variance. It seems that firms abandon technologies long before they are perfected – current-practice TFP is 17 percent below best-practice.

The policy implication is certainly not that business cycles should be stabilized. Rather, the opposite is true, in the sense that technological adoption – and hence technological risk – are too low. Because the model assumes that there are intergenerational spillovers in technology and expertise, both sorts of investments are likely, in equilibrium, to be below their socially optimal levels.

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# 6 Appendix

Several arguments are listed in separate Appendixes.

## 6.1 The proof of Proposition 1

*Proof of* (19).—We begin with

**Lemma 5**

$$Var\left(u^2\right) = \frac{x^4\sigma^4}{1-\alpha^4}\left(2+4\frac{\alpha^2}{1-\alpha^2}\right) \tag{26}$$

**Proof.** Since

$$u_{t+1} = \alpha u_t + x\varepsilon_{t+1}, \tag{27}$$

it follows that

$$u_{t+1}^2 = \alpha^2 u_t^2 + x^2\varepsilon_{t+1}^2 + 2\alpha x\varepsilon_{t+1}u_t. \tag{28}$$

Then since $\varepsilon_{t+1}$ and $u_t$ are uncorrelated, since $Var\left(\varepsilon^2\right) = 2\sigma^4$, since $Var\left(\varepsilon_{t+1}u_t\right) = \sigma^2\sigma_u^2$ and since $\sigma_u^2 = x^2\sigma^2/\left(1-\alpha^2\right)$,

$$Var\left(u^2\right) = \frac{1}{1-\alpha^4}\left(x^4 2\sigma^4 + x^4 4\alpha^2\frac{\sigma^4}{1-\alpha^2}\right)$$

from which the claim follows ■

Substituting into (26) from (10), and using (19)

$$\begin{aligned}
\psi^2 Var\left(u^2\right) &= \left(\frac{1}{2}\lambda\alpha\right)^2\left(\frac{1}{\lambda\alpha\sigma^2}\right)^4\frac{\sigma^4}{1-\alpha^4}\left(2+4\frac{\alpha^2}{1-\alpha^2}\right) \\
&= \left(\frac{1}{\lambda\alpha\sigma^2}\right)^2\frac{1}{1-\alpha^4}\left(\frac{1}{2}+\frac{\alpha^2}{1-\alpha^2}\right),
\end{aligned}$$

which is (19).

*Proof of* (18).—Taking the unconditional expectation in (17),

$$\psi_0 - \psi E\left(u_{t+1}^2\right) = \ln\beta + A^* - \psi\frac{x^2\sigma^2}{1-\alpha^2}.$$

From (19) and (10), $\psi = \frac{1}{2}\lambda\alpha = \frac{1}{2x\sigma^2}$, so that $\psi\frac{x^2\sigma^2}{1-\alpha^2} = \frac{1}{2}\frac{x}{1-\alpha^2} = \frac{1}{2\lambda\alpha\sigma^2}\frac{1}{1-\alpha^2}$. So, using (17),

$$\begin{aligned}
\psi_0 - \psi E\left(u_{t+1}^2\right) &= \ln\beta + A^* - \frac{1}{2\lambda\alpha\sigma^2}\frac{1}{1-\alpha^2} \\
&= \ln\beta + \frac{1}{\delta\alpha\lambda\sigma^2} - \frac{1}{2\lambda\alpha}\frac{\sigma^2}{1-\alpha^2} \\
&= \ln\beta + \frac{1}{\alpha\lambda\sigma^2}\left(\frac{1}{\delta} - \frac{1}{2}\frac{1}{1-\alpha^2}\right)
\end{aligned}$$

i.e., (18).

## 6.2 Estimation procedure

To estimate $\psi_0$ defined in (19) we use the consistent estimate

$$\hat{\psi}_0 = \max_t \Delta\ln y_t. \tag{29}$$

For the other parameters we proceed as follows: (17) says that

$$\Delta\log y_t = \psi_0 - \psi u_{t+1}^2. \tag{30}$$

so that $u_{t+1}^2 = \frac{\Delta\log y_t - \psi_0}{-\psi}$. Substituting into (30) for $u_{t+1}^2$ from (28),

$$\frac{\Delta\log y_t - \psi_0}{-\psi} = \alpha^2\frac{\Delta\log y_{t-1} - \psi_0}{-\psi} + x^2\varepsilon_{t+1}^2 + 2\alpha x\varepsilon_{t+1}u_t$$

Then

$$\Delta \log y_t - \psi_0 = \alpha^2 \left(\Delta \log y_{t-1} - \psi_0\right) - \psi \eta_{t+1}, \tag{31}$$

where

$$\eta_{t+1} = x^2 \varepsilon_{t+1}^2 + 2\alpha x \varepsilon_{t+1} u_t.$$

Now

$$\begin{aligned}
E\left(-\psi \eta_{t+1} \Delta \log y_{t-1}\right) &= \psi^2 E\left(\eta_{t+1} u_t^2\right) \\
&= \psi^2 \left[E\left(x^2 \varepsilon_{t+1}^2 u_t^2\right) + 2E\left(\alpha x \varepsilon_{t+1} u_t^3\right)\right] \\
&= \psi^2 x^2 E\left(u_t^2\right) E\left(\varepsilon_{t+1}^2\right) \\
&= \psi^2 x^2 \frac{x^2 \sigma^2}{1-\alpha^2} \sigma^2 \\
&= \psi^2 \frac{x^4 \sigma^4}{1-\alpha^2}
\end{aligned}$$

Therefore the expectation of the OLS estimate $\hat{\alpha}^2$ in (31) is

$$\begin{aligned}
E\left(\hat{\alpha}^2\right) &= \alpha^2 + \frac{E\left(-\psi \eta_{t+1} \Delta \log y_{t-1}\right)}{E\left(\left(\Delta \log y_{t-1}\right)^2\right)} \\
&= \alpha^2 + \psi^2 \frac{x^4 \sigma^4}{1-\alpha^2} \frac{1}{E\left(\left(\Delta \log y_{t-1}\right)^2\right)}.
\end{aligned}$$

Now

$$E\left(\left(\Delta \log y_{t-1} - \psi_0\right)^2\right) = \psi^2 E\left(u_t^4\right) = 3\psi^2 \frac{x^4 \sigma^4}{1-\alpha^4}.$$

Therefore

$$E\left(\hat{\alpha}^2\right) = \alpha^2 + \psi^2 \frac{x^4 \sigma^4}{1-\alpha^2} \frac{1}{3\psi^2 \frac{x^4 \sigma^4}{1-\alpha^4}} = \alpha^2 + \frac{1+\alpha^2}{3} = \frac{4}{3}\alpha^2 + \frac{1}{3}$$

so that OLS estimates are biased upward.

For the second restriction on moments,

$$-\psi = \frac{E\left(\Delta \log y_t\right) - \psi_0}{E\left(u_{t+1}^2\right)} = \left(1-\alpha^2\right) \frac{E\left(\Delta \log y_t\right) - \psi_0}{x^2 \sigma^2}.$$

From (19) and (10) $\psi = \frac{1}{2}\lambda\alpha = \frac{1}{2x\sigma^2}$, and so $\psi \frac{x^2\sigma^2}{1-\alpha^2} = \frac{1}{2}\frac{x}{1-\alpha^2} = \frac{1}{2\lambda\alpha\sigma^2}\frac{1}{1-\alpha^2}$. Therefore, these two moment conditions are

$$m(\alpha, \lambda) = \begin{cases} 0 = E\left(\Delta \log y_t - \psi_0 + \frac{1}{2\lambda\alpha\sigma^2}\frac{1}{1-\alpha^2}\right), \text{ and} \\ 0 = E\left(\Delta \log y_t - \psi_0 - \left(\frac{4}{3}\alpha^2 + \frac{1}{3}\right)\left(\Delta \log y_{t-1} - \psi_0\right)\right)\left(\Delta \log y_t - \psi_0\right) = 0. \end{cases}$$

So our GMM procedure criterion minimizes

$$\hat{m}(\alpha, \lambda)' W^{-1} \hat{m}(\alpha, \lambda)$$

where $\hat{m}(\beta)$ is the empirical counterpart of $m(\alpha, \lambda)$ and $W^{-1}$ is the optimal weighting matrix as in Hansen (1982). Having estimated $\alpha$, we then obtain $\delta$ as follows: Using (17) and (10),

$$\hat{\delta} = \frac{1}{A^*}\frac{1}{\lambda\alpha} = \left(\frac{1}{\psi_0 - T\ln\beta}\right)\frac{1}{\lambda\alpha},$$

where $\beta = 0.95$ and $T = 5$ or $T = 10$, depending on whether the time-interval is five or ten years. Table 1 reports the estimates.

## 6.3 Derivation of optimal savings

Let us now analyze the savings problem defined in (13) and derive the optimal consumption rule expressed in (14). To save space I do it only under the assumption that $A = A^*$ so that we can drop $A$ from the vector of states.

**Lemma 6** *(13) has a solution of the form*

$$w(n, u) = W(u) + \frac{1}{1-\beta}\ln n,$$

*where*

$$W(u) = \max_{\xi}\left\{\ln(Z(u) - \xi) + \frac{\beta}{1-\beta}\ln\xi + \beta\int W(u)\, d\Phi(u', u)\right\}. \tag{32}$$

**Proof.** We can change variables and let $\xi = n'/n$ so that substituting into the RHS an equation of the form $w(n, u) = W(u) + B\ln n$, (13) becomes

$$\begin{aligned} w(n, u) &= \max_{\xi}\left\{\ln(nZ[u] - n\xi) + \beta[B\ln n + B\ln\xi] + \beta\int W(u')\, d\Phi(u', u)\right\} \\ &= \ln n + \beta[B\ln n] + \max_{\xi}\left\{\ln(Z - \xi) + \beta B\ln\xi + \beta\int W(u')\, d\Phi\right\} \end{aligned}$$

which works if

$$B = 1 + \beta B,$$

i.e., if $B = 1/(1-\beta)$. Since the right hand side is a contraction operator on a complete metric space, there exists exactly one function $W(u)$ such that (32 holds. ■

Then the FOC for $\xi$ in (32) is

$$-\frac{1}{Z-\xi} + \frac{\beta}{1-\beta}\frac{1}{\xi} = 0, \tag{33}$$

**Proposition 7** *Optimal consumption is*

$$c = (1-\beta)Zn.$$

**Proof.** (12) implies

$$\frac{c}{n} = Z - \xi. \tag{34}$$

Since shares are one-period, consumer wealth is the same as aggregate output. We posit consumption to be a constant fraction of wealth

$$c = \omega Z n,$$

Together with (34) this implies

$$\xi = Z - \frac{c}{n} = Z\left(1 - \omega\right).$$

Substituting for $\xi$ into (33), we find that it holds if and only if

$$\omega = 1 - \beta.$$

■