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NET WORTH, EXCHANGE RATES, AND MONETARY POLICY: THE EFFECTS OF A DEVALUATION IN A FINANCIALLY FRAGILE ENVIRONMENT

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ABSTRACT

In this paper we propose an Open Economy Financial Accelerator model along the lines of Greenwald-Stiglitz (1993) close in spirit but different in many respects from the one proposed by Greenwald (1998.) The first goal of the paper is to provide a taxonomy of the effects of a devaluation in this context. The direct (first round) effect on output, taking as given net worth and interest rate, is negative for domestic firms (due to the input cost effect) and positive for exporting firms (due to a positive foreign debt effect). The indirect (second round) wealth effect (on output through net worth, taking as given the interest rate) is uncertain, depending on the relative size of the domestic and exporting firms. There is also an indirect effect on output through the response of the domestic interest rate to a devaluation due to the risk premium effect. Due to the uncertainty on the sign of most of these effects, it is difficult to assess the overall impact of a devaluation affects negatively the net worth of domestic firms, the domestic interest rate may rise (due to the risk premium effect), exerting an additional contractionary impact on output. If, on top of that, the monetary authorities force a further increase of the interest rate in an effort to curb the exchange rate, the contractionary effect will be emphasized.

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1 Introduction

The role of financial factors in the transmission of monetary shocks has been thoroughly explored in the Financial Accelerator literature.¹ There are at least three strands of this literature. The first one focuses on agency costs and the willingness to extend loans (Bernanke and Gertler, 1989, 1990; Bernanke, Gertler and Gilchrist, 2000; Carlstrom and Fuerst, 1997); the second one emphasizes the role of bankruptcy costs and induced risk aversion on the part of a firm's managers (Greenwald and Stiglitz, 1993); the third one stresses the role of asset prices and collateralizable net worth (Kiyotaki and Moore, 1997).

The theoretical underpinnings are different but all of these models predict that a change in corporate net worth is a crucial determinant of investment and production and that monetary shocks are at least in part transmitted through the impact of changes in the interest rate on net worth. If one extends the framework to the open economy, this is only part of the financial accelerator story. In this new context, in fact, also changes in the exchange rate affect net worth so that the balance sheet channel bifurcates into an interest rate/net worth channel and an exchange rate/net worth channel. Open Economy extensions of the Financial Accelerator (OEFA) framework have been proposed by Gertler, Gilchrist and Natalucci (2003) (along the lines of Bernanke-Gertler-Gilchrist and Carlstrom-Fuerst), Edison, Luangaram and Miller (1998) (along the lines of Kiyotaki and Moore). As to the Greenwald-Stiglitz approach, to the best of our knowledge there have not been open economy extensions with the notable exception of Greenwald (1998).

The stream of "twin crises" – i.e. the intertwined process of financial disruption, capital flight and exchange rate turbulence – which has afflicted emerging countries in the late '90s has brought about a new wave (the third generation) of models of currency crises (Aghion, Bacchetta and Banerjee, 2000; Caballero and Krishnamurthy, 2000; Cespedes Chang and Velasco, 2000; Christiano, Gust and Roldos, 2002; Devereux and Lane, 2003) in which

¹In the following we will use the expressions Broad Credit view or Balance Sheet channel as synonims of Financial Accelerator.

financial factors play a leading role in the propagation of the crisis. In a sense, therefore, also these models can be traced back to the OEFA framework.

Most of the models listed above end up with the normative conclusion that floating is preferable to a fixed exchange rate regime. For instance Gertler, Gilchrist and Natalucci (2003) show that welfare losses following a financial crisis are significantly larger under fixed than under floating exchange rates.

In this paper we propose an OEFA model along the lines of Greenwald-Stiglitz, close in spirit but different in many respects from the one proposed by Greenwald. The first goal of the paper is to provide a taxonomy of the effects of a devaluation in an OEFA context. There a is a wide range of different and correlated effects so that the ultimate impact on output and the macroeconomy is in uncertain and can well be negative.

Changes in the interest rate and in the exchange rate are intertwined. If a country runs a fixed exchange rate regime and a sudden collapse of confidence occurs, the central bank may be induced to manipulate the interest rate in an attempt to halt short term capital outflows. This is quite common: Italy in 1992, Thailand in 1997, Russia in 1998, Argentina in 2001 are just a few episodes. The net worth of the firms is going to suffer because of the interest rate hike. Most of the times, however, this sharp increase of interest rates is ineffective and a devaluation occurs. The transition to floating rates frees the hand of the monetary authorities, who can lower interest rates. This move is beneficial for firms'net worth while the impact of devaluation on net worth is uncertain. When the pace of devaluation becomes "unsustainable", at least in the eyes of the monetary authorities, they become anxious to halt devaluation by increasing interest rates again. Once again the firms' net worth is going to suffer. When devaluation is harmful, moreover, a restrictive monetary policy is going to be counterproductive because net worth will suffer twice.

In this paper we study the consequences of a devaluation in a financially fragile environment. In section 2 we discuss the background assumptions. Section 3 is devoted to a discussion of the first round or impact effect of a devaluation on the basis of the output-net worth relationship, whose microfoundation is described and discussed at length in the appendix. In section 5 we derive the laws of motion that describe the evolution over time of the net worth. These laws allow to assess the second round or indirect effect of a devaluation through changes in net worth. Section 6 is devoted to the determination of interest rates. Section 7 concludes.

2 The Environment

The model economy we consider is characterized by the following assumptions. In each period, there is a "large" number – say I_t – of firms which belong to one of two sectors. Firms indexed by $i = 1, 2...I_t^x$ sell their goods only on foreign markets (eXporting sector or X-sector). Firms indexed by $j = I_t^x + 1, I_t^x + 2, ...I_t$ produce only for the home market (domestic sector or H-sector). Therefore $I_t^h = I_t - I_t^x$ is the number of firms producing only for the home market.

For simplicity we assume that technology is uniform across firms. Firms produce by means of a Leontief technology whose inputs are labour and an imported intermediate good ("oil").

The production function of the generic firm therefore is $Y_k = \min(N_k, \frac{1}{\nu}O_k)$; k = i, j where Y_k, N_k, O_k represent output, employment and oil. The parameter $\nu > 0$ measures oil requirement per unit of output. Assuming that labour is always abundant, we can write $Y_k = \frac{1}{\nu}O_k$ so that labour requirement is $N_k = \frac{1}{\nu}O_k$.

Firms are price takers in a perfect competition setting. Production takes time, so that firms produce in the current period but sell only in the next one. Due to high transaction costs, there are no forward markets. Therefore the selling price which will be determined on the spot market tomorrow is uncertain today, i.e. at the moment decisions are made on production, employment, oil imports and financing both for the domestic producer and for the exporter. As to the former, we assume that the individual price (in domestic currency) of goods produced in t and sold in t+1 is $P_{jt+1} =$ $P_{t+1}u_{jt+1}$ where u_{jt+1} is a random variable – capturing an idiosyncratic shock to revenues – with $E_t(u_{jt+1}) = 1$. It follows that $E_t(P_{jt+1}) = P_{t+1}$ where P_{t+1} is the (average) market price of goods sold on domestic markets. Analogously, the price (in foreign currency) at which the exporter sells his goods on foreign markets is $P_{it+1}^x = P_{t+1}^{\$} u_{it+1}^x$ where u_{it+1}^x is a random variable with $E_t(u_{it+1}^x) = 1$ so that $E_t(P_{it+1}^x) = P_{t+1}^{\$}$ and $P_{t+1}^{\$}$ is the (average) market price of goods sold on foreign markets.

Both X and H firms succeed in selling all the output they decided to produce: Output is supply driven. In other words, the volume of demand does not constrain production. 2

Firms finance their production costs (the wage and oil bills) by means of net worth and credit. For the sake of simplicity, as a first approximation we will adopt the following

Assumption 1: The exporting sector seeks only foreign finance whose cost is the real foreign interest rate R^x ; the domestic sector asks only for domestic loans whose cost is the real domestic interest rate R.

If the firm is solvent, the loan obtained in t is reimbursed in t+1. If the firm is not able to repay the debt, it goes bankrupt. The insolvent firm leaves the market. In the present context, therefore, bankruptcy is the single most important determinant of firms'exit from the market. On the other hand, the presence of extra-profits will induce firms'entry. We will not examine the effects of industrial dynamics, however, in order to get rid of unnecessary complications at this stage of the analysis.

Firms hire labour on the domestic labour market and oil on the international oil market. The real cost of labour for H firms is the real wage

²We can provide the following rationale for this apparently restrictive assumption. Let the demand of the the i-th commodity in period t+1 be $d(P_{it+1}/P_{t+1}, \delta_{it+1})$ where P_{it+1}/P_{t+1} is the relative price of the i-th commodity and δ_{it+1} is a stochastic demand disturbance specific to the market in question. Let supply be s_{it} i.e. output produced in t and made available to the consumer in t+1. By assumption s_{it} is made up of the quantities produced by a "large number" of producers so that the contribution of each firm to total supply is negligible. In equilibrium $P_{it+1}/P_{t+1} = f(\delta_{it+1}, s_{it})$ i.e. the relative price is an increasing function of the demand disturbance, given the predetermined supply. If demand is sufficiently elastic, changes in supply do not affect the relative price significantly so that the relative price is essentially an increasing function of random demand. In our setting the random variable u_{it+1} can be conceptualized along these lines. A high realization of u_{it+1} can be thought of as a regime of high demand which drives up the relative price of the commodity in question. In this regime each firm operating in the i-th market produces a high volume of output. By construction, however, it has no difficulty in selling it. In a regime of low demand, the realization of u_{it+1} turns out to be low and may push the firm out of the market if it is "too low", i.e. if it makes the net worth of the firm negative.

 $w_t := W_t/P_t$. Assuming, for simplicity, that the price of oil in dollars is the same as the price of goods sold abroad $P_t^{\$}$, the real price of oil for H firms is $\varepsilon_t \phi_t$ (where $\phi_t := P_t^{\$}/P_t$), i.e. the real exchange rate. The average cost for H firms, therefore, is $\gamma_t^h := w_t + \varepsilon_t \phi_t \nu$. A higher exchange rate translates into higher costs of imported oil for H firms.³

Since X firms hire labour on the domestic market but sell goods and seek external finance on foreign markets, the nominal cost of labour for X firms is the nominal wage in foreign currency W_t/ε_t . The real cost of labour for X firms is the real wage evaluated at foreign prices $W_t/\varepsilon_t P_t^{\$} = w_t/\varepsilon_t \phi$. On the other hand, the nominal cost of oil in foreign currency is $P_t^{\$}$ so that the real cost of oil for X firms is unity. The average production cost for X firms, say γ_t^x , therefore is $\gamma_t^x := \frac{\gamma_t^h}{\varepsilon_t \phi_t} = \frac{w_t}{\varepsilon_t \phi_t} + \nu$. In this case, a higher exchange rate translates into lower costs for X firms.⁴ The short-run effect of a devaluation, therefore is favourable for X-firms due to lower costs in foreign currency.

The demand for (domestic) finance on the part of H-firms is the financing gap, i.e. the difference between production costs and net worth, measured in domestic currency. Since the H-sector seeks finance and sells goods only at home, the appropriate deflator of the financing gap of the H-sector is the domestic price level. The financing gap of the H-sector at constant domestic prices therefore is $\gamma_t^h Y_{it}^h - A_{it}^h$ where Y_{it}^h and A_{it}^h are output and net worth of the domestic firm and γ_t^h is increasing with the exchange rate.

On the other hand, since the X-sector seeks finance and sells goods only abroad, the appropriate deflator of the financing gap of the X-sector is the foreign price level. The financing gap of the X-sector at foreign prices there-

³In Greenwald (1998), production is carried out by means of labour alone but the nominal wage is indexed to the price of foreign goods so that in the end average cost for domestic producers is increasing with the exchange rate. In our context, if the nominal wage were indexed to a weighted average of the prices of domestic and foreign goods in domestic currency, i.e. $W_t = (1 - \theta) P_t + \theta \varepsilon_t P_t^{\$}$, the real wage would turn out to be $w_t = (1 - \theta) + \theta \varepsilon_t \phi_t$. The real wage, therefore, would be a linear function of the real exchange rate. In this case the average production cost would be $\gamma = w_t + \varepsilon_t \phi_t \nu = (1 - \theta) + (\theta + \nu) \varepsilon_t \phi_t$. Also in Greenwald's case, therefore, a devaluation would push cost up by making room for a wage increase.

⁴The average production cost at foreign prices in a Greenwald (1998) economy would be $\gamma_t^x = \frac{1-\theta}{\varepsilon_t \phi_t} + \theta + \nu$. Also in this case, the average production cost for X-firms is decreasing in the exchange rate.

fore is $\gamma_t^x Y_{it}^x - A_{it}^x$ where Y_{it}^x and A_{it}^x are output and net worth of the foreign firm and γ_t^x is decreasing with the exchange rate. X-firms, in fact, are not completely insulated from the domestic economy since they hire labour on the domestic labour market at the current wage in domestic currency. Therefore a devaluation makes the need of foreign currency to anticipate wages less acute.

3 Output and net worth

As in Greenwald and Stiglitz (1993), the problem of the firm consists in maximizing expected profits less bankruptcy costs. The solution of the problem yields optimal output (size) as a function of net worth. The relationship of output and net worth emerges quite naturally also modelling the firm as a risk averse agent as in Greenwald (1998). In a sense, the presence of expected bankruptcy costs in the objective function of the firm in Greenwald and Stiglitz (1993) plays the role of risk aversion in Greenwald (1998).

Thanks to the assumptions of linear technology and uniform density of the idiosyncratic shock, output is a linear function of net worth. The optimal supply of the i-th firm in the X-sector is (see appendix A.1 for the derivation):

$$Y_{it}^{x} = \frac{1}{R^{x} \gamma_{t}^{x}} + \frac{1}{2\gamma_{t}^{x}} A_{it}^{x} - 1 \qquad i = 1, 2, \dots, I_{t}^{x}$$
(1)

where Y_{it}^x is output and A_{it}^x is net worth of the X-firm at constant (foreign) prices. The "size" of a firm can be measured both in terms of output (the scale of production) and in term of net worth.

The impact of an increase of the exchange rate on output is:

$$\frac{\partial Y_{it}^x}{\partial \varepsilon_t} = -\frac{1}{\left(\gamma_t^x\right)^2} \left(\frac{1}{R^x} + \frac{1}{2}A_{it}^x\right) \gamma_\varepsilon^x \tag{2}$$

Since

$$\gamma_{\varepsilon}^{x} := \partial \gamma_{t}^{x} / \partial \varepsilon_{t} = -\frac{w_{t}}{\left(\varepsilon_{t}\right)^{2} \phi_{t}} < 0$$

the derivative (2), which measures the *first round* effect of a devaluation on

the output of X-firms, i.e. the effect of a devaluation *keeping the net worth constant*, is positive. In fact, an increase of the exchange rate implies a lower wage in foreign currency and a lower financing gap *(foreign debt effect)*.

The optimal supply of output/demand for labour of the j-th firm in the H-sector is (see appendix A.2 for the derivation):

$$Y_{jt}^{h} = \frac{1}{R\gamma_{t}^{h}} + \frac{1}{2\gamma_{t}^{h}}A_{jt}^{h} - 1 \qquad j = I_{t}^{x} + 1, \dots, I_{t}.$$
(3)

The impact of an increase of the exchange rate on output is:

$$\frac{\partial Y_{jt}^{h}}{\partial \varepsilon_{t}} = -\frac{1}{\left(\gamma_{t}^{h}\right)^{2}} \left(\frac{1}{R} + \frac{A_{Jt}^{h}}{2}\right) \gamma_{\varepsilon}^{h} \tag{4}$$

Since

$$\gamma_{\varepsilon}^{h} := \partial \gamma_{t}^{h} / \partial \varepsilon_{t} = \nu \phi_{t} > 0$$

the derivative (4), which measures the *first round* effect of a devaluation on output of H-firms, is negative. In fact, an increase of the exchange rate implies a higher cost of oil in domestic currency *(input cost effect)*. Since H-firms seeks finance only on the domestic credit market, the foreign debt effect is absent by construction. All in all, a devaluation hurts the H-sector as a whole and is beneficial to X-firms.⁵

Denoting averages by non indexed variables, thanks to the fact that output is a linear function of net worth, the average supply of the exporting sector and of the home sector turn out to be

$$Y_t^x = \frac{1}{R^x \gamma_t^x} + \frac{A_t^x}{2\gamma_t^x} - 1 \tag{5}$$

$$Y_t^h = \frac{1}{R\gamma_t^h} + \frac{A_t^h}{2\gamma_t^h} - 1 \tag{6}$$

⁵For simplicity, at this stage of the analysis we abstract from import competition. By construction, in this model domestic firms do not compete with foreign producers on the domestic goods market. If there were some degree of import competition, a devaluation would ease the competitive pressure on domestic firms who could increase their production and sales on domestic markets.

Therefore, the economy-wide average output is

$$Y_{t} = \omega_{t} Y_{t}^{x} + (1 - \omega_{t}) Y_{t}^{h} = \frac{\omega_{t}}{\gamma_{t}^{x}} \left(\frac{1}{R^{x}} + \frac{A_{t}^{x}}{2}\right) + \frac{1 - \omega_{t}}{\gamma_{t}^{h}} \left(\frac{1}{R} + \frac{A_{t}^{h}}{2}\right) - 1 \quad (7)$$

where $\omega_t = \frac{I_t^x}{I_t}$ is the share of X-firms in the corporate sector.⁶

Let's assume for simplicity that ω_t is unaffected by a devaluation. The impact of an increase of the exchange rate on output is:

$$\frac{\partial Y_t}{\partial \varepsilon_t} = \omega_t \frac{\partial Y_t^x}{\partial \varepsilon_t} + (1 - \omega_t) \frac{\partial Y_t^h}{\partial \varepsilon_t} = -\frac{\omega_t}{\left(\gamma_t^x\right)^2} \left(\frac{1}{R^x} + \frac{A_t^x}{2}\right) \gamma_\varepsilon^x - \frac{(1 - \omega_t)}{\left(\gamma_t^h\right)^2} \left(\frac{1}{R} + \frac{A_t^h}{2}\right) \gamma_\varepsilon^k$$
(8)

The first round or impact effect of a devaluation on aggregate output is in principle uncertain. Recalling that $\gamma_t^x = \frac{\gamma_t^h}{\varepsilon_t \phi_t}$; $\gamma_{\varepsilon}^x = -\frac{w_t}{(\varepsilon_t)^2 \phi_t}$ and $\gamma_{\varepsilon}^h = w_t \phi_t$,

– so that $\gamma_{\varepsilon}^{x} = -\frac{\gamma_{\varepsilon}^{h}}{(\varepsilon_{t}\phi_{t})^{2}}$ – rearranging and simplifying, it turns out that $\frac{\partial Y_{t}}{\partial \varepsilon_{t}} > 0$ if

$$A_t^x > \hat{A}^x := \frac{2}{R} \left(h - \frac{R}{R^x} \right) + h A_t^h \tag{9}$$

where $h := \frac{1 - \omega_t}{\omega_t} = \frac{I_t^h}{I_t^x}$ is the ratio of domestic to exporting producers. The impact effect of a devaluation on aggregate output is positive if, on average, the net worth of X-firms is "sufficiently high", i.e. bigger than a threshold \hat{A}^x . Given the interest rates R and R^x , the larger h, i.e. the number of domestic producers relative to the number of exporting firms and the larger the average size of the H-firm, the more likely it is that a devaluation will affect negatively aggregate output.

⁶In order to keep the analysis as simple as possible, we assume that the number of firms is constant. This assumption implies that bankrupt firms are replaced one-to-one with new entrants.

4 Wealth effects

In the previous section, we have discussed the (first round) effect of a devaluation on (current) output keeping net worth constant. A devaluation, however, has a *second round effect* on (future) output which can be conceptualized as a *wealth effect* because it operates through the accumulation of net worth.

Assuming, for the sake of simplicity, that firms do not distribute dividends, the net worth or equity base of the i-th firm in the X-sector in t+1 can be defined as follows

$$A_{it+1}^{x} = \pi_{it+1}^{x} = \left(u_{it+1}^{x} - R^{x}\gamma_{t}^{x}\right)Y_{it}^{x} + R^{x}A_{it}^{x}$$
(10)

Given R^x , there are three factors affecting A^x_{it+1} : (1) revenues $u^x_{it+1}Y^x_{it}$; (2) costs $\gamma^x_t Y^x_{it}$; i.e. the product of average costs γ^x_t times the scale of production Y^x_{it} ;(3) internal funds (i.e. the equity base inherited from the past A^x_{it}).Both (2) and (3) are "augmented" by the interest rate.

The wealth effect of a devaluation operates through (1) and (2). The impact of a devaluation on future net worth of X-firms in fact is:

$$\frac{\partial A_{it+1}^x}{\partial \varepsilon_t} = u_{it+1}^x \frac{\partial Y_{it}^x}{\partial \varepsilon_t} - R^x \left(\gamma_\varepsilon^x Y_{it}^x + \gamma_t^x \frac{\partial Y_{it}^x}{\partial \varepsilon_t} \right)$$

The term $u_{it+1}^x \frac{\partial Y_{it}^x}{\partial \varepsilon_t}$ is the marginal revenue of a devaluation, i.e. the change in revenues due to an increase of the exchange rate. It measures the impact effect of a devaluation on revenues. In the case of X-firms it is positive. The term $R^x \left(\gamma_{\varepsilon}^x Y_{it}^x + \gamma_t^x \frac{\partial Y_{it}^x}{\partial \varepsilon_t} \right)$ is the marginal cost. From the previous section, we know that $\gamma_{\varepsilon}^x < 0$ and $\frac{\partial Y_{it}^x}{\partial \varepsilon_t} > 0$. While the marginal revenue is always positive, in principle the marginal cost is uncertain because a devaluation implies lower average costs but higher production. We will label the impact of a devaluation on unit costs the average cost effect of a devaluation. In the case of X-firms the average cost effect is negative. The impact of a devaluation of the scale of activity will be referred to as the scale effect. In the case of X-firms the scale effect is positive. Taking into account $\frac{\partial Y_{it}^x}{\partial \varepsilon_t} = -\frac{\gamma_{\varepsilon}^x}{(\gamma_t^x)^2} \left(\frac{1}{R^x} + \frac{A_{it}^x}{2}\right) = -\frac{\gamma_{\varepsilon}^x}{\gamma_t^x} (Y_{it}^x + 1)$, the marginal cost of a devaluation boils down to $-R^x \gamma_{\varepsilon}^x > 0$. Hence, the scale effect prevails over the average cost effect and the marginal cost turns out to be positive. Therefore both the marginal revenue and the marginal cost of a devaluation are positive for X-firms. The wealth effect for X-firms is positive if the marginal revenue is greater than the marginal cost.

Recalling (2) and (1), from the expression above one gets

$$\frac{\partial A_{it+1}^x}{\partial \varepsilon_t} = \left(u_{it+1}^x - R^x \gamma_t^x\right) \frac{\partial Y_{it}^x}{\partial \varepsilon_t} - R^x \gamma_\varepsilon^x Y_{it}^x = -\gamma_\varepsilon^x \left[\frac{u_{it+1}^x}{\gamma_t^x} \left(Y_{it}^x + 1\right) - R^x\right]$$

Since $\gamma_{\varepsilon}^{x} < 0$, a devaluation has a positive wealth effect on the generic X-firm, i.e. $\frac{\partial A_{it+1}^{x}}{\partial \varepsilon_{t}} > 0$, if the expression in brackets is positive. This occurs if u_{it+1}^{x} is "not too small", i.e.if it is greater than a threshold \hat{u}_{it}^{x}

$$u_{it+1}^{x} > \hat{u}_{it}^{x} := \frac{R^{x} \gamma_{t}^{x}}{Y_{it}^{x} + 1} = \frac{2 \left(R^{x} \gamma_{t}^{x}\right)^{2}}{2 + R^{x} A_{it}^{x}}$$
(11)

When inequality (11) is satisfied a devaluation affects positively net worth and output in the future, i.e. the second round effect is positive.

A similar argument can be applied to H-firms. Since the equity base of the j-th firm in the H-sector in t+1 is

$$A_{jt+1}^{h} = \pi_{jt+1}^{h} = \left(u_{jt+1}^{h} - R\gamma_{t}^{h}\right)Y_{jt}^{h} + RA_{jt}^{h}$$
(12)

the impact of a devaluation on future net worth of H-firms is:

$$\frac{\partial A_{jt+1}^h}{\partial \varepsilon_t} = u_{jt+1}^h \frac{\partial Y_{jt}^h}{\partial \varepsilon_t} - R\left(\gamma_{\varepsilon}^h Y_{jt}^h + \gamma_t^h \frac{\partial Y_{jt}^h}{\partial \varepsilon_t}\right)$$

where $u_{jt+1}^{h} \frac{\partial Y_{jt}^{h}}{\partial \varepsilon_{t}}$ is the marginal revenue of a devaluation while $R\left(\gamma_{\varepsilon}^{h}Y_{jt}^{h} + \gamma_{t}^{h}\frac{\partial Y_{jt}^{h}}{\partial \varepsilon_{t}}\right)$ is the marginal cost. From the previous section, we know that $\gamma_{\varepsilon}^{h} > 0$ and

 $\frac{\partial Y_{jt}^{h}}{\partial \varepsilon_{t}} < 0.$ While the marginal revenue is always negative, *in principle* the marginal cost is uncertain because – in the case of a domestic firm – the average cost effect is positive and the scale effect is negative. Taking into account $\frac{\partial Y_{jt}^{h}}{\partial \varepsilon_{t}} = -\frac{\gamma_{\varepsilon}^{h}}{(\gamma_{t}^{h})^{2}} \left(\frac{1}{R} + \frac{A_{jt}^{h}}{2}\right) = -\frac{\gamma_{\varepsilon}^{h}}{\gamma_{t}^{h}} (Y_{jt}^{h} + 1)$, the marginal cost boils down to $-R\gamma_{\varepsilon}^{h} < 0$. Hence, the scale effect prevails over the average cost effect and the marginal cost turns out to be negative. Both the marginal revenue and the marginal cost of a devalutation are negative for H-firms.

Recalling (4) and (3), after rearranging, from the expression above one gets

$$\frac{\partial A_{jt+1}^{h}}{\partial \varepsilon_{t}} = -\gamma_{\varepsilon}^{h} \left[\frac{u_{jt+1}^{h}}{\gamma_{t}^{h}} \left(Y_{jt}^{h} + 1 \right) - R^{h} \right]$$

Since $\gamma_{\varepsilon}^{h} > 0$, a devaluation has a positive wealth effect, i.e. $\frac{\partial A_{jt+1}^{h}}{\partial \varepsilon_{t}} > 0$, if the expression in brackets is negative which implies that the absolute value of the (negative) marginal revenue is smaller than the absolute value of the (negative) marginal cost. This is the case if u_{jt+1}^{h} is "not too big", i.e. if it is smaller than a threshold \hat{u}_{jt}^{h} :

$$u_{jt+1}^{h} < \hat{u}_{jt}^{h} := \frac{R\gamma_{t}^{h}}{Y_{jt}^{h} + 1} = \frac{2\left(R\gamma_{t}^{h}\right)^{2}}{2 + RA_{it}^{h}}$$
(13)

Notice that the conclusion we reach concerning H-firms is symmetric with respect to the inference we can draw in the case of X-firms.

Let's assess now the impact of a devaluation on *average net worth*. The average equity base of the X-sector in t+1 is

$$A_{t+1}^{x} = \pi_{t+1}^{x} = \left[E\left(u_{it+1}^{x}\right) - R^{x}\gamma_{t}^{x} \right] Y_{t}^{x} + R^{x}A_{t}^{x} = \left(1 - R^{x}\gamma_{t}^{x}\right)Y_{t}^{x} + R^{x}A_{t}^{x} \quad (14)$$

Therefore the impact of a devaluation on future average net worth of X-firms is:

$$\frac{\partial A_{t+1}^x}{\partial \varepsilon_t} = \left(1 - R^x \gamma_t^x\right) \frac{\partial Y_t^x}{\partial \varepsilon_t} - R^x \gamma_\varepsilon^x Y_t^x = -\gamma_\varepsilon^x \left(\frac{Y_t^x + 1}{\gamma_t^x} - R^x\right)$$

The expression in brackets is positive and therefore $\frac{\partial A_{t+1}^x}{\partial \varepsilon_t} > 0$ if

$$1 > \hat{u}_t^x = \frac{2\left(R^x \gamma_t^x\right)^2}{2 + R^x A_t^x} = \frac{2\left[R^x \left(\frac{w_t}{\varepsilon_t \phi_t} + \nu\right)\right]^2}{2 + R^x A_t^x}$$
(15)

According to (15), a devaluation has a positive wealth effect on the average X-firm if the expected relative price (which is equal to one by construction) is greater than a threshold \hat{u}_t^x which ensures that the average marginal revenue $\frac{\partial Y_t^x}{\partial \varepsilon_t}$ is greater than the average marginal cost. The threshold is decreasing with the exchange rate and the equity base. The smaller the exchange rate and the "size" of the average X-firm the less likely it is that the wealth effect is positive for X-firms.

A similar argument can be applied to H-firms. The *average* equity base of the H-sector in t+1 is

$$A_{t+1}^{h} = \pi_{t+1}^{h} = \left(1 - R\gamma_{t}^{h}\right)Y_{t}^{h} + RA_{t}^{h}$$
(16)

The impact of a devaluation on future average net worth of H-firms is:

$$\frac{\partial A_{t+1}^h}{\partial \varepsilon_t} = \left(1 - R\gamma_t^h\right) \frac{\partial Y_t^h}{\partial \varepsilon_t} - R\gamma_\varepsilon^h Y_t^h$$

After rearranging, from the expression above one gets

$$\frac{\partial A_{t+1}^h}{\partial \varepsilon_t} = -\gamma_{\varepsilon}^h \left(\frac{Y_t^h + 1}{\gamma_t^h} - R \right)$$

The expression in brackets is negative and therefore $\frac{\partial A_{t+1}^h}{\partial \varepsilon_t} > 0$ if

$$1 < \hat{u}_{t}^{h} = \frac{2\left(R\gamma_{t}^{h}\right)^{2}}{2 + RA_{t}^{h}} = \frac{2R^{2}\left(w_{t} + \varepsilon_{t}\phi_{t}\nu\right)^{2}}{2 + RA_{t}^{h}}$$
(17)

According to (17), a devaluation has a positive wealth effect on the average H-firm if the expected relative price (equal to one by construction) is smaller than a threshold \hat{u}_t^h which ensures that the absolute value of the average marginal revenue $\frac{\partial Y_t^h}{\partial \varepsilon_t}$ is smaller than the absolute value of the average marginal cost. The threshold is increasing with the exchange rate and decreasing with the equity base. The higher the exchange rate and the smaller the "size" of the average H-firm the more likely it is that the wealth effect is positive for H-firms.

At a point in time an economy can be identified by the triple $(\varepsilon_t, A_t^h, A_t^x)$. Inequalities (15) and (17) can be used to characterize points of the equity base - exchange rate space in terms of the sign of the wealth effect on X and H firms respectively. This characterization is carried out in figure 1.

On the x-axis we measure the exchange rate ε_t . On the y-axis we measure the equity base of the average X-firm (A_t^x) and H-firm (A_t^h) . The downward

sloping line has equation $1 = \frac{2\left[R^x \left(\frac{w_t}{\varepsilon_t \phi_t} + \nu\right)\right]^2}{2 + R^x A_t^x}$. Points lying above the line satisfy inequality (15) and therefore imply $\frac{\partial A_{t+1}^x}{\partial \varepsilon_t} > 0$. The opposite is true for points lying below the line.

The upward sloping line has equation $1 = \frac{2R^2 (w_t + \varepsilon_t \phi_t \nu)^2}{2 + RA_t^h}$. Points lying below the line satisfy inequality (17) and therefore imply $\frac{\partial A_{t+1}^h}{\partial \varepsilon_t} > 0$. The opposite is true for points lying above the line.

We can partion the positive orthant, therefore, in four regions. The northern region (N) is characterized by $\frac{\partial A_{t+1}^x}{\partial \varepsilon_t} > 0$ and $\frac{\partial A_{t+1}^h}{\partial \varepsilon_t} < 0$. In words, the wealth effect is positive for X-firms and negative for H-firms. This is reminded by enclosing the sign of the wealth effect (for exporting and domestic firms respectively) in brackets associated to the letter indentifying the region. The same criterion has been employed to characterize the eastern region (E), the southern region (S) and the western region (W).

Let's assume that, in period zero, $A_0^h > A_0^x$, i.e. on average H-firms are bigger than X-firms. If the current exchange rate is relatively low (e.g. ε_0) a devaluation has a negative wealth effects for H-firms (see point H in the N region), and a negative wealth effects for X-firms too (see point X in the S



Figure 1: Characterization of the equity base-exchange rate space in terms of wealth effects.

region). If the exchange rate were very low, i.e. close to the origin, points H and X would end up in region W where the wealth effect is negative for both types of firms. Only at very high levels of the exchange rate the wealth effect would be positive for both types of firms (region W). In other words, in this case the wealth effect of a devaluation is negative for both firms "most of the times".

Of course the opposite would be true is $A_0^x > A_0^h$. The second round effect of a devaluation, therefore, depends not only on the relative importance of the X and H sectors but also on the relative average size of X and H firms.

5 Laws of motion

In a macrodynamic framework, the effects of a devaluation are not limited to the impact and the second round effects. The repercussions of a devaluation will be felt on net worth – and therefore on production – in all the subsequent periods. Assuming that the equilibrium is stable, with the passing of time the effect will be smaller and smaller and tend asymptotically to zero. In order to assess the "long run" wealth effect of a devaluation, we must study the law of motion of net worth.

As to X-firms, substituting (1) into (10) and rearranging we get

$$A_{it+1}^x = \Gamma_{i0}^x + \Gamma_{i1}^x A_{it}^x \tag{18}$$

where

$$\Gamma_{i0}^{x} = \left(u_{it+1}^{x} - R^{x}\gamma_{t}^{x}\right)\left(\frac{1}{R^{x}\gamma_{t}^{x}} - 1\right)$$

$$\Gamma_{i1}^{x} = \left(u_{it+1}^{x} - R^{x}\gamma_{t}^{x}\right)\frac{1}{2\gamma_{t}^{x}} + R^{x}$$

(18) is a linear first order difference equation subject to a stochastic disturbance which describes the law of motion of the individual state variable, i.e. the net worth of the i-th exporting firm.

Recalling that, by assumption, $E(u_{it+1}^x) = 1$, summation and averaging

across firms yields:

$$A_{t+1}^x = \Gamma_0^x + \Gamma_1^x A_t^x \tag{19}$$

$$\Gamma_0^x = \frac{(1 - R^x \gamma_t^x)^2}{R^x \gamma_t^x}$$

$$\Gamma_1^x = \frac{1 + R^x \gamma_t^x}{2\gamma_t^x}$$

(19) is a linear first order difference equation which describes the law of motion of the *average* net worth of the X-sector. ⁷

We assume $\Gamma_1^x < 1$, i.e. $\frac{1}{\gamma_t^x} + R^x < 2$ in order to assure stability of the equilibrium. The steady state of the average net worth of the X-sector is:

$$A_s^x = \frac{\Gamma_0^x}{1 - \Gamma_1^x} = \frac{2\left(1 - R^x \gamma_t^x\right)^2}{R^x \left[(2 - R^x) \gamma_t^x - 1\right]}$$
(20)

It is clear from (20) that a devaluation will impact positively on A_s^x – i.e. $\partial A_s^x/\partial \varepsilon_t > 0$ – by reducing unit costs γ_t^x .

Average output of the exporting sector in the steady state will be:

$$Y_s^x = \frac{1}{R^x \gamma_t^x} + \frac{A_s^x}{2\gamma_t^x} - 1$$

Therefore

$$\frac{\partial Y_s^x}{\partial \varepsilon_t} = -\frac{\gamma_\varepsilon^x}{\left(\gamma_t^x\right)^2} \left(\frac{1}{R^x} + \frac{A_s^x}{2}\right) + \frac{1}{2\gamma_t^x} \frac{\partial A_s^x}{\partial \varepsilon_t} > 0$$

The long run effects of a devaluation on output of the X-firms consists of two parts. The firs term in the sum is the impact effect, which is positive as shown in section (3). The second term is the sum of the second round effect and all the subsequent effects (which we will refer to as the long run wealth

⁷Due to the linearity of (18), averaging across firms yields a linear law of motion of the average net worth. If the individual law of motion were non linear, also higher moments of the distribution would show up in the law of motion of the average state variable. For instance, if the individual law of motion were concave (convex), the variance of the state variable would affect negatively (positively) the law of motion of the mean of the state variable.



Figure 2: Long run wealth effects of a devaluation on the average X-firm

effect) which is positive too.

Figure 2 illustrates the point. In the upper panel, the thin line represents the phase diagram of equation (19) for a given level of the exchange rate, say ε_0 . A devaluation – i.e. an increase of the exchange rate to ε_1 – makes the phase diagram shift and rotate upward (bold line) because both the intercept and the slope of the phase diagram are decreasing with the average cost, which in turn is decreasing with the exchange rate. The steady state equity base goes up from A_0^x to A_1^x .

In the lower panel the thin line represent equation (5) when the exchange

rate is ε_0 . A devaluation makes the line shift and rotate upward (bold line) because both the intercept and the slope are decreasing with the average cost, which in turn is decreasing with the exchange rate. The *impact effect* of the devaluation on output is measured by the vertical distance between points S_0 and B. The second round effect is measured by the vertical distance between the coordinates of points B and C on the y-axis. In fact, the devaluation makes net worth go up from A_0^x to A_C^x . After the second round a sequence of adjustments of net worth and production occurs along the dynamic path of the economy towards the new long run equilibrium.

Let's focus now on H-firms. Substituting (3) into (12) and rearranging one gets

$$A_{jt+1}^{h} = \Gamma_{j0}^{h} + \Gamma_{j1}^{h} A_{jt}^{h}$$
(21)

where

$$\Gamma_{j0}^{h} = \left(u_{jt+1}^{h} - R\gamma_{t}^{h}\right) \left(\frac{1}{R\gamma_{t}^{h}} - 1\right)$$

$$\Gamma_{j1}^{h} = \left(u_{jt+1}^{h} - R\gamma_{t}^{h}\right) \frac{1}{2\gamma_{t}^{h}} + R$$

Summation and averaging across firms yields:

$$A_{t+1}^h = \Gamma_0^h + \Gamma_1^h A_t^h \tag{22}$$

$$\Gamma_0^h = \frac{\left(1 - R\gamma_t^h\right)^2}{R\gamma_t^h}$$

$$\Gamma_1^h = \frac{1 + R\gamma_t^h}{2\gamma_t^h}$$

(22) is a linear first order difference equation which describes the law of motion of the average net worth of the H-sector.

We assume $\Gamma_1^h < 1$, i.e. $\frac{1}{\gamma_t^h} + R < 2$ in order to assure stability of the

equilibrium. The steady state of the average net worth of the H-sector is:

$$A_{s}^{h} = \frac{\Gamma_{0}^{h}}{1 - \Gamma_{1}^{h}} = \frac{2\left(1 - R\gamma_{t}^{h}\right)^{2}}{R\left[(2 - R)\gamma_{t}^{h} - 1\right]}$$
(23)

It is clear from (23) that a devaluation will impact negatively on A_s^h – i.e. $\partial A_s^h / \partial \varepsilon_t < 0$ – by increasing unit costs γ_t^h .

Average output of the home sector in the steady state will be:

$$Y_s^h = \frac{1}{R\gamma_t^h} + \frac{A_s^h}{2\gamma_t^h} - 1$$

Hence

$$\frac{\partial Y^h_s}{\partial \varepsilon_t} = -\frac{\gamma^h_\varepsilon}{\left(\gamma^h_t\right)^2} \left(\frac{1}{R} + \frac{A^h_s}{2}\right) + \frac{1}{2\gamma^h_t} \frac{\partial A^h_s}{\partial \varepsilon_t} < 0$$

The long run effects of a devaluation on output of the H-firms is negative. Of course the adjustment of net worth and output of the average H-firm following a devaluation could be represented graphically adopting the same procedure followed in figure 2. In the case of the H-firm both the phase diagram and the relationshipe between output and net worth would shift and rotate downward.

Therefore, the economy-wide average output in the steady state is

$$Y_{s} = \omega_{t} Y_{s}^{x} + (1 - \omega_{t}) Y_{s}^{h} = \frac{\omega_{t}}{\gamma_{t}^{x}} \left(\frac{1}{R^{x}} + \frac{A_{s}^{x}}{2}\right) + \frac{1 - \omega_{t}}{\gamma_{t}^{h}} \left(\frac{1}{R} + \frac{A_{s}^{h}}{2}\right) - 1 \quad (24)$$

where $\omega_t = \frac{I_t^x}{I_t}$ is the share of X-firms in the corporate sector. Assuming for simplicity that ω_t is unaffected by a devaluation, the impact of an increase of the exchange rate on output in the long run is: $\frac{\partial Y_s}{\partial \varepsilon_t} = \omega_t \frac{\partial Y_s^x}{\partial \varepsilon_t} + (1 - \omega_t) \frac{\partial Y_s^h}{\partial \varepsilon_t}$. It is easy to see that $\frac{\partial Y_s}{\partial \varepsilon_t} > 0$ if $-\frac{\gamma_{\varepsilon}^x}{\gamma_t^x} \left(\frac{1}{R^x} + \frac{A_s^x}{2}\right) + \frac{1}{2} \frac{\partial A_s^x}{\partial \varepsilon_t} > \frac{(1 - \omega_t) \gamma_t^x}{\omega_t \gamma_t^h} \left[\frac{\gamma_{\varepsilon}^h}{\gamma_t^h} \left(\frac{1}{R} + \frac{A_s^h}{2}\right) - \frac{1}{2} \frac{\partial A_s^h}{\partial \varepsilon_t}\right]$

6 Interest rates and bankruptcy risk

So far, we have dealt with interest rates as if they were exogenous. In this section we relax this assumption. Following Greenwald (1998) we assume that both in the country considered and in the rest of the world, the interest rate can be modelled as the sum of a risk-free interest rate – for simplicity uniform across countries – and a risk premium. The foreign interest rate is $R^x = R_f + \beta^x \xi$ where R_f is the risk free interest rate, ξ is a generic risk (to be specified in the following) and β^x is the "international beta". Symmetrically, the domestic interest rate is $R = R_f + \beta^h \xi$ where β^h is the "domestic beta". Assuming perfect capital mobility, the relationship between the domestic and foreign interest rates turns out to be

$$R = R^x + \beta \xi \tag{25}$$

with $\beta = \beta^h - \beta^x$ with $\beta^h > \beta^x$.

Given the focus of the present paper, we assume that the domestic interest rate should compensate for the *risk of bankruptcy*. Hence ξ is related to (for simplicity, in the following it will coincide with) the probability of bankruptcy of H-firms evaluated *in the aggregate*, i.e. at the level of the macroeconomy, say $F.^8$

In the appendix A.2, we define the following profit function for the generic H firm:

$$\pi_{jt+1}^{h} := \left(u_{jt+1}^{h} - R\gamma_{t}^{h} \right) Y_{jt}^{h} + RA_{jt}^{h}$$
(26)

where u_{jt+1}^{h} is an idiosyncratic shock distributed as a uniform r.v. with support (0,2). Recall that $\gamma_{t}^{h} = w_{t} + \varepsilon_{t}\phi_{t}\nu$ is the average cost. The i-th firm goes bankrupt if $\pi_{jt+1}^{h} < 0$, i.e. $u_{jt+1}^{h} < R\left(\gamma_{t}^{h} - \frac{A_{jt}^{h}}{Y_{jt}^{h}}\right) \equiv \bar{u}_{jt+1}^{h}$. The probability of bankruptcy at the individual level, therefore, can be defined as

⁸Also X-firms can go bankrupt but their default in principle affects the interest rate on foreign debt since they seek finance only on foreign financial markets.

$$F_{j} := \Pr\left(u_{jt+1}^{h} < \bar{u}_{jt+1}^{h}\right) = \frac{\bar{u}_{jt+1}^{h}}{2} = \frac{R}{2} \left(\gamma_{t}^{h} - \frac{A_{jt}^{h}}{Y_{jt}^{h}}\right)$$
(27)

In order to define F, a measure of the probability of bankruptcy for the macroeconomy, we focus on the average *H*-firm, i.e.

$$F = \frac{R}{2} \left(\gamma_t^h - \frac{A_t^h}{Y_t^h} \right) = \frac{1}{2} R \gamma_t^h \lambda_t^h$$
(28)

where

$$\lambda_t^h = 1 - \frac{A_t^h}{\gamma_t^h Y_t^h} \tag{29}$$

is the *leverage ratio*. In fact in our context (average) debt is defined as $D_t^h := \gamma_t^h Y_t^h - A_t^h$ so that λ_t^h turns out to be the proportion of total cost which is financed by means of debt. Apart from a scale factor, therefore, the probability of bankruptcy can be thought of as the product of the interest rate R times the average cost γ_t^h times the leverage ratio λ_t^h .

Notice that $\gamma_t^h \lambda_t^h = \frac{D_t^h}{Y_t^h}$. Hence, $F = \frac{RD_t^h}{2Y_t^h}$. Apart from the scale factor, therefore, the probability of bankruptcy can be conceived also as the ratio of debt service (principal and interest) RD_t^h to output.

Substituting (5) into (29) and rearranging we get

$$\lambda_t^h = \frac{2\left(1 - R\gamma_t^h\right) - RA_t^h}{2\left(1 - R\gamma_t^h\right) + RA_t^h} = \frac{1 - \alpha_t^h}{1 + \alpha_t^h} = \lambda\left(\alpha_t^h\right)$$
(30)

where

$$\alpha_t^h := \frac{A_t^h}{2\left(\frac{1}{R} - \gamma_t^h\right)} = \alpha\left(R, A_t^h, \gamma_t^h\right)$$

It is easy to see that $\lambda_{\alpha}^{h} = \frac{\partial \lambda_{t}^{h}}{\partial \alpha_{t}^{h}} < 0$ and $\alpha_{R}^{h} > 0, \alpha_{A}^{h} > 0, \alpha_{\gamma}^{h} > 0$ (see appendix A.3).

We assume: (i) $\frac{1}{R} > \gamma_t^{h 9}$ so that $\alpha_t^h > 0$. In order to assure non-negativity

 $[\]overline{{}^{9}\text{Since }E_{t}\left(\pi_{jt+1}^{h}\right)=\left(1-R\gamma_{t}\right)Y_{it}^{h}+RA_{it}^{h}, \text{ the inequality }1-R\gamma_{t}>0 \text{ is a sufficient condition for }E_{t}\left(\pi_{jt+1}^{h}\right)>0 \text{ which can be thought of as a necessary condition for a firm }$

of λ_t^h and F, moreover, we impose the restriction (ii) $A_t^h \leq 2\left(\frac{1}{R} - \gamma_t^h\right)$ which implies $1 \geq \alpha_t^h$. The probability of bankruptcy is smaller than unity because $R\gamma_t^h < 1$ thanks to (i) and $\lambda_t^h < 1$ thanks to (ii).

It is easy to conclude therefore that the leverage ratio is decreasing in a non-linear way with the interest rate $(\lambda_R^h = \lambda_\alpha^h \alpha_R^h < 0)$, the average cost $(\lambda_\gamma^h = \lambda_\alpha^h \alpha_\gamma^h < 0)$ and the equity base $(\lambda_A^h = \lambda_\alpha^h \alpha_A^h < 0)$ (see appendix A.3).¹⁰

As to the probability of bankruptcy it is obvious that

$$F = \frac{1}{2}R\gamma_t^h \frac{2\left(1 - R\gamma_t^h\right) - RA_t^h}{2\left(1 - R\gamma_t^h\right) + RA_t^h} = F\left(R, \gamma_t^h, A_t^h\right)$$

F is clearly decreasing with net worth $(F_A = \frac{1}{2}R\gamma_t^h\lambda_A^h < 0)$ while it is a non monotonic function of the interest rate and the average cost. In fact from the definition (28) follows

$$F_R = \frac{1}{2} \gamma_t^h \left(\lambda_t^h + R \lambda_R^h \right) \tag{31}$$

The sum in brackets in (31) measures the change in the product $R\lambda_t^h = RD_t^h/\gamma_t^hY_t^h$ (i.e. the ratio of debt service to average cost) caused by a change of the interest rate. An increase of the interest rate affects $R\lambda_t^h$ in two contrasting ways: the impact effect is obviously positive (because R goes up by assumption) but the indirect effect is negative because the leverage ratio goes down $(\lambda_R^h = \lambda_\alpha^h \alpha_R^h < 0)$. We can rewrite (31) as

$$F_R = \frac{1}{2} \gamma_t^h \lambda_t^h \left(1 + \eta_{\lambda R}^h \right)$$

where

$$\eta^h_{\lambda R} := \frac{R\lambda^h_R}{\lambda^h_t} < 0$$

is the elasticity of the leverage ratio with respect to the interest rate. This elasticity is negative since $\lambda_R^h < 0$. If elasticity is smaller than one in absolute value, i.e. $-\eta_{\lambda R}^h < 1$, then $F_R > 0$ i.e. the impact effect prevails over the

to start production.

¹⁰Since the scale of production is increasing with net worth, both output and net worth can be a measure of the size of the average firm. Therefore, the leverage ratio is decreasing with size: the greater the size of the average domestic firm, the smaller the leverage.

indirect effect. The opposite holds true if elasticity is greater than one in absolute value.

A similar argument applies to the response of the probability of bankruptcy to a change in average cost. In fact from the definition (28) follows

$$F_{\gamma} = \frac{1}{2} R \left(\lambda_t^h + \gamma_t^h \lambda_{\gamma}^h \right) \tag{32}$$

The sum in brackets in (32) measures the change in the debt to output ratio $\gamma_t^h \lambda_t^h = \frac{D_t^h}{Y_t^h}$ caused by a change in average cost. The impact effect of an increase of the average cost on $\gamma_t^h \lambda_t^h$ is obviously positive (because γ_t^h goes up by assumption) but the indirect effect is negative because the leverage ratio goes down $(\lambda_{\gamma}^h = \lambda_{\alpha}^h \alpha_{\gamma}^h < 0)$. We can rewrite (32) as

$$F_{\gamma} = \frac{1}{2} R \lambda_t^h \left(1 + \eta_{\lambda\gamma}^h \right)$$

where

$$\eta^h_{\lambda\gamma} := \frac{\gamma^h_t \lambda_\gamma}{\lambda^h_t} < 0$$

is the elasticity of the leverage ratio with respect to the average cost. This elasticity is negative since $\lambda_{\gamma}^{h} < 0$. If elasticity is smaller than one in absolute value, i.e. $-\eta_{\lambda\gamma}^{h} < 1$, then $F_{\gamma} > 0$ i.e. the impact effect prevails over the indirect effect. The opposite holds true if elasticity is greater than one in absolute value. In figure 6 we plot F as a function of γ_{t}^{h} . It is a hump shaped curve. The upward (downward) sloping branch of the function is characterized by elasticity smaller (greater) than one in absolute value. Higher values of A_{t}^{h} bring about shallower hump shaped functions. For each level of the average cost, in fact, as net worth goes up, the probability of bankruptcy goes down.



Fig. 3: The probability of bankruptcy F as a function of average cost γ

Since the average cost is increasing with the exchange rate, the probability of bankruptcy turns out to be also a non monotonic function of the exchange rate:

$$F_{\varepsilon} = F_{\gamma} \gamma_{\varepsilon}^{h} = \frac{1}{2} R \lambda_{t}^{h} \left(1 + \eta_{\lambda\gamma}^{h} \right) \gamma_{\varepsilon}^{h}$$

In our context $\xi \approx F$. Therefore (25) becomes

$$R = R^{x} + \beta F\left(R, \gamma_{t}^{h}, A_{t}^{h}\right)$$

$$(33)$$

In order to simplify the analysis we linearize F (see appendix A.3):

$$F \approx \phi_0 + \phi_1 R + \phi_2 \gamma_t^h + \phi_3 A_t^h \tag{34}$$

where the sign of ϕ_0, ϕ_1 and ϕ_2 is uncertain while $\phi_3 < 0.$ Substituting (34) into (33) and solving for R one gets

$$R = \frac{1}{1 - \beta \phi_1} \left[R^x + \beta \left(\phi_0 + \phi_2 \gamma_t^h + \phi_3 A_t^h \right) \right]$$
(35)

We assume that $1 - \beta \phi_1 > 0$. This condition is always satisfied if $\phi_1 < 0$, i.e. if the probability of bankruptcy is decreasing with the interest rate. If,

on the other hand, $\phi_1 > 0$, i.e. the probability of bankruptcy is increasing with the interest rate, the condition is satisfied if $\phi_1 < \frac{1}{\beta}$.

According to (35) the domestic interest rate is a mark-up $\frac{1}{1-\beta\phi_1}$ over the international interest rate "aumented" by the average cost and net worth. It is easy to see that the domestic interest rate is decreasing with net worth: $\frac{\partial R}{\partial A_t^h} = \frac{\beta\phi_3}{1-\beta\phi_1} < 0$. On the other hand, since

$$\frac{\partial R}{\partial \gamma^h_t} = \frac{\beta \phi_2}{1-\beta \phi_1}$$

and $\phi_2 = F_{\gamma}$ (computed in a predetermined initial condition), the domestic interest rate is increasing (decreasing) with unit costs if $\phi_2 > 0$ ($\phi_2 < 0$), i.e. for relatively low (high) levels of the average cost.

An increase in γ_t may be due, among other things, to a devaluation. The impact of a devaluation on the domestic interest rate:

$$\frac{\partial R}{\partial \varepsilon_t} = \frac{\partial R}{\partial \gamma^h_t} \gamma^h_\varepsilon = \frac{\beta \phi_2}{1 - \beta \phi_1} \gamma^h_\varepsilon$$

captures the risk premium effect of a devaluation. Of course the risk premium effect is positive, i.e. a devaluation pushes up the domestic interest rate, if $\phi_2 > 0$; it is negative otherwise.

Taking into account the determinants of the domestic interest rate examined in the present section, the first and second round effects of a devaluation on H-output are more complicated than assumed in sections 3 and 4. For instance, an increase of the exchange rate, keeping net worth constant, makes output of the H-firms shrink because of the increase in unit cost. The reduction in output can be even bigger if the interest rate goes up as a consequence of the increased risk of bankruptcy. This occurs if $\phi_2 > 0$. If the opposite holds true, the devaluation makes the domestic interest go down, an effect which offsets, at least in part, the reduction in the scale of production due to increased unit costs. In symbols, from (3) follows

$$\frac{\partial Y_{jt}^{h}}{\partial \varepsilon_{t}} = -\frac{\gamma_{\varepsilon}^{h}}{\left(\gamma_{t}^{h}\right)^{2}} \left[\frac{1}{R}\left(\eta_{R\gamma}+1\right) + \frac{A_{Jt}^{h}}{2}\right]$$
(36)

The derivative (36) differs from the derivative (4) computed in section 3, i.e. keeping net worth and the interest rate constant, because of the term $\eta_{R\gamma}$ which measures the elasticity of the domestic interest rate to average cost and can be either positive or negative. In section 3, $\eta_{R\gamma} = 0$ by construction.

7 Conclusions

In a financially fragile environment, i.e. an economy in which production decisions depend on the availability and the cost of internal and external funds, a devaluation can affect output by means of a wide range of effects. In this paper we have provided a tentative taxonomy which can be summarized as follows:

- 1. direct (first round) effect on output, taking as given net worth and interest rate:
 - a negative *input cost effect* on H firms,
 - a positive *foreign debt* effect on X firms.
- 2. indirect (second round) *wealth effect* (on output through net worth, taking as given the interest rate) on X and H firms. The sign of this effect depends on the size of the average X an H firm.
- 3. indirect effect on output of H firms through the response of the domestic interest rate to a devaluation due to the *risk premium* effect. The sign of this effect depends on the elasticity of the interest rate to the average cost of H-firms.

Due to the uncertainty surrounding the sign of most of these effects, it is difficult to assess the overall effect of a devaluation. One cannot rule out, however, an economywide *contractionary effect of a devaluation*. The likelihood and extent of this case is an interesting question to be answered on empirical ground in future research. In the paper, however, we have shown – albeit only at a speculative level – that this occurrence cannot be confined to an analytical curiosum.

If the devaluation affects negatively the net worth of H-firms, the domestic interest rate, which incorporates a premium for the risk of bankruptcy, may rise (due to the risk premium effect), exerting an additional contractionary impact on output. If, on top of that, the monetary authorities force a further increase of the interest rate in an effort to curb the exchange rate, the contractionary effect will be emphasized.

In this paper, we have dealt with the exchange rate as if it were exogenous. In future research developments we want to endogenize the exchange rate in general equilibrium variant of the present framework .We conjecture that the equilibrium exchange rate may be a function not only of the domestic and international interest rates but also of the financial conditions of firms as captured by net worth. A policy move which leads to a devaluation, therefore, can feed back on the exchange rate through its impact on net worth.

A Appendix

A.1 The maximization problem of the X-firm

X-firms sell only on foreign goods markets and seek finance only on foreign credit markets. Total revenues are $P_{it+1}^x Y_{it}^x$ where P_{it+1}^x is the price (in dollars) of goods sold by the i-th firm on foreign markets and Y_{it}^x is production. Total costs (also in dollars) are $\frac{W_t}{\varepsilon_t} N_{it}^x + P_t^{\$} O_{it}^x$ where W_t is the nominal wage in euros – so that $\frac{W_t}{\varepsilon_t}$ is the nominal wage in dollars – N_{it}^x is employment, $P_t^{\$}$ is the price of oil, O_{it}^x is oil used up in production. Therefore, the profit of the i-th firm in the exporting sector at current foreign prices in t+1 is

$$\Pi_{it+1}^{x} = P_{it+1}^{x} Y_{it}^{x} - (1+i^{x}) \left(\frac{W_{t}}{\varepsilon_{t}} N_{it}^{x} + P_{t}^{\$} O_{it}^{x} - Z_{it}^{x} \right)$$
(37)

where i^x is the nominal interest rate on foreign debt and Z_{it}^x is net worth in dollars so that $\frac{W_t}{\varepsilon_t}N_{it}^x + P_t^{\$}O_{it}^x - Z_{it}^x$ is the financing gap in foreign currency.

The individual price P_{it+1}^x is uncertain. We assume that

$$P_{it+1}^x = P_{t+1}^{\$} u_{it+1}^x$$

where u_{it+1}^x is a random variable with $E_t(u_{it+1}^x) = 1$ so that $E_t(P_{it+1}^x) = P_{t+1}^{\$}$ where $P_{t+1}^{\$}$ is the market price of goods sold on foreign markets in t+1.

Dividing (37) by $P_{t+1}^{\$}$ after some manipulation we obtain the following expression for profit at constant foreign prices:

$$\pi_{it+1}^{x} := \frac{\Pi_{it+1}^{x}}{P_{t+1}^{\$}} = u_{it+1}^{x} Y_{it}^{x} - R^{x} \left(\frac{w_{t}}{\varepsilon_{t} \phi_{t}} N_{it}^{x} + O_{it}^{x} - A_{it}^{x} \right)$$
(38)

where $u_{it+1}^x := \frac{P_{it+1}^x}{P_{t+1}^{\$}}$ can be interpreted as the real average revenue (in terms of foreign goods) of the firm, $R^x := \frac{(1+i^x) P_t^{\$}}{P_{t+1}^{\$}}$ is the real foreign interest rate, $w_t := \frac{W_t}{P_t}$ is the real wage (in terms of domestic goods, i.e the nominal wage in domestic currency deflated by the domestic price level), $\varepsilon_t \phi_t := \varepsilon_t \frac{P_t^{\$}}{P_t}$ is the real exchange rate in t, $A_{it}^x := \frac{Z_{it}^x}{P_t^{\$}}$ is real net worth (in terms of foreign goods).

Thanks to the assumption on technology, we can rewrite (38) as follows:

$$\pi_{it+1}^x := \left[u_{it+1}^x - R^x \left(\frac{w_t}{\varepsilon_t \phi_t} + \nu \right) \right] Y_{it}^x + R^x A_{it}^x \tag{39}$$

The expression $w_t + \varepsilon_t \phi_t \nu$ represents the real average production cost in terms of domestic goods, i.e. the nominal average production cost in domestic currency deflated by the domestic price level, and will be denoted by γ_t^h . Dividing γ_t^h by the real exchange rate one gets the *real average production* cost in terms of foreign goods, say γ_t^x , i.e. the nominal average production cost in foreign currency deflated by the foreign price level:

$$\gamma_t^x := \frac{\gamma_t^h}{\varepsilon_t \phi_t} = \frac{w_t + \varepsilon_t \phi_t \nu}{\varepsilon_t \phi_t} = \frac{w_t}{\varepsilon_t \phi_t} + \nu$$

which is the expression in parentheses in (39). Both the numerator (i.e. the average cost at constant domestic prices) and the denominator (i.e. the real exchange rate) of the expression above are increasing with the nominal exchange rate. The higher the exchange rate, the higher will be the "oil bill" an therefore the production cost in domestic currency (*input cost effect* of a devaluation). At the same time, the higher the exchange rate, the lower will be the financing gap in dollars (*foreign debt effect*) because wages are paid in domestic currency but "financed" in foreign currency: given the nominal wage in domestic currency, the higher the exchange rate, the lower will be the dollar value of the nominal wage and the financing gap. The real average production cost in terms of foreign goods, however, is decreasing with the nominal exchange rate. For X-firms who raise funds only on foreign financial markets, the foreign debt effect more than offset the input cost effect.

The i-th firm goes bankrupt in period t if $\pi_{it+1}^x < 0$, i.e.

$$u_{it+1}^x < R^x \left[\left(\frac{w_t}{\varepsilon_t \phi_t} + \nu \right) - \frac{A_{it}^x}{Y_{it}^x} \right] \equiv \bar{u}_{it+1}^x \tag{40}$$

where \bar{u}_{it+1}^x is the critical threshold of the average revenue. According to (40) if there is a negative shock and the average revenue of the firm falls below the threshold, the firm goes bankrupt.

Let's assume that u_{it+1}^x is distributed as a uniform r.v. with support (0,2) so that $E_t(u_{it+1}^x) = 1$. The probability of bankruptcy can be expressed as follows:

$$\Pr\left(u_{it+1}^x < \bar{u}_{it+1}^x\right) = \frac{\bar{u}_{it+1}^x}{2} = \frac{R^x}{2} \left[\left(\frac{w_t}{\varepsilon_t \phi_t} + \nu\right) - \frac{A_{it}^x}{Y_{it}^x} \right]$$
(41)

The probability of bankruptcy is increasing with the interest rate and the real wage and decreasing with the real exchange rate and net worth per unit of output.

Finally we assume that bankruptcy is costly and the cost of bankruptcy is an increasing quadratic function of the scale of production, i.e. $CB_i = (Y_{it}^x)^2$.

The objective function of the firm V_{it+1}^x is the difference between expected profit $E_t(\pi_{it+1}^x)$ and bankruptcy (or borrower's) risk, i.e. bankruptcy cost in case bankruptcy occurs $CB_i \Pr\left(u_{it+1}^x < \bar{u}_{it+1}^x\right)$:

$$V_{it+1}^{x} = E_{t}\left(\pi_{it+1}^{x}\right) - CB_{i} \operatorname{Pr}\left(u_{it+1}^{x} < \bar{u}_{it+1}^{x}\right) = \left[1 - R^{x}\left(\frac{w_{t}}{\varepsilon_{t}\phi_{t}} + \nu\right)\right] Y_{it}^{x} + R^{x} A_{it}^{x} 42$$
$$- (Y_{it}^{x})^{2} \frac{R^{x}}{2} \left[\left(\frac{w_{t}}{\varepsilon_{t}\phi_{t}} + \nu\right) - \frac{A_{it}^{x}}{Y_{it}^{x}}\right]$$
(43)

The FOC:

$$1 = \underbrace{R^{x}\left(\frac{w_{t}}{\varepsilon_{t}\phi_{t}} + \nu\right)}_{\text{M arg.Prod.Cost}} + \underbrace{R^{x}\left[\left(\frac{w_{t}}{\varepsilon_{t}\phi_{t}} + \nu\right)Y_{it}^{x} - \frac{A_{it}^{x}}{2}\right]}_{\text{M arg.Bankrutcy Cost}}$$

can be interpreted as follows: the expected marginal real revenue (equal to 1) by assumption) must be equal to the real marginal cost, which in turn is the sum of the marginal production cost and the marginal bankruptcy cost. The marginal production cost is equal to the real average cost in terms of foreign goods multipled by the interest rate $R^x \gamma_t^x$. The marginal bankruptcy cost consists of two parts. The first one is positive and increasing with output: $R^x \gamma_t^x Y_{it}^x$. The second part is negative and increasing in absolute value with net worth: $-R^x \frac{A_{it}^x}{2}$. From the FOC we obtain

$$Y_{it}^{x} = \phi_{t} \varepsilon_{t} \frac{(1/R^{x}) + (A_{it}^{x}/2)}{w_{t} + \varepsilon_{t} \phi_{t} \nu} - 1$$
(44)

In the following we will write:

$$Y_{it}^x = \frac{1}{R^x \gamma_t^x} + \frac{1}{2\gamma^x} A_{it}^x - 1$$

A.2 The maximization problem of the H-firm

Profit of the j-th firm producing for the Home market (H-sector for short) at current prices in t+1 is

$$\Pi_{jt+1}^{h} = P_{jt+1}Y_{jt}^{h} - (1+i)\left(W_{t}N_{jt}^{h} + \varepsilon_{t}P_{t}^{\$}O_{jt}^{h} - Z_{jt}^{h}\right)$$
(45)

where P_{jt+1} is the price of the non-tradables produced by the i-th firm, *i* is the interest rate on the domestic market for loans. The meaning of the remaining symbols is straightforward.

Dividing (45) by P_{t+1} and using the usual assumptions, we obtain the following function of profit at constant (domestic) prices:

$$\pi_{jt+1}^{h} := \left[u_{jt+1}^{h} - R \left(w_t + \varepsilon_t \phi_t \nu \right) \right] Y_{jt}^{h} + RA_{jt}^{h}$$

$$\tag{46}$$

The i-th firm goes bankrupt in period t if $\pi_{jt+1}^h < 0$, i.e.

$$u_{jt+1}^{h} < R\left(w_t + \varepsilon_t \phi_t \nu - \frac{A_{jt}^{h}}{Y_{jt}^{h}}\right) \equiv \bar{u}_{jt+1}^{h}$$

$$\tag{47}$$

Let's assume that u_{jt+1}^{h} is distributed as a uniform r.v. with support (0,2) so that $E_t(u_{jt+1}^{h}) = 1$. The probability of bankruptcy, therefore, can be expressed as follows:

$$\Pr\left(u_{jt+1}^{h} < \bar{u}_{jt+1}^{h}\right) = \frac{\bar{u}_{jt+1}^{h}}{2} = \frac{R}{2} \left(w_{t} + \varepsilon_{t}\phi_{t}\nu - \frac{A_{jt}^{h}}{Y_{jt}^{h}}\right)$$
(48)

Finally we assume that bankruptcy is costly and the cost of bankruptcy is an increasing qaudratic function of the scale of production, i.e. $CB_i = (Y_{jt}^h)^2$.

The objective function of the firm V_{jt+1}^h is the difference between expected profit $E_t(\pi_{jt+1}^h)$ and bankruptcy (or borrower's) risk, i.e. bankruptcy cost in case bankruptcy occurs $CB_j \Pr(u_{jt+1}^h < \bar{u}_{jt+1}^h)$:

$$V_{jt+1}^{h} = E_{t} \left(\pi_{jt+1}^{h} \right) - CB_{j} \operatorname{Pr} \left(u_{jt+1}^{h} < \bar{u}_{jt+1}^{h} \right) =$$

$$= \left[1 - R \left(w_{t} + \varepsilon_{t} \phi_{t} \nu \right) \right] Y_{jt}^{h} + RA_{jt}^{h} - \left(Y_{jt}^{h} \right)^{2} \frac{R}{2} \left(w_{t} + \varepsilon_{t} \phi_{t} \nu \right) + \frac{R}{2} Y_{jt}^{h} \mathfrak{A}_{jt}^{h}$$
(49)

From the FOC we obtain

$$Y_{jt}^{h} = \frac{(1/R) + (A_{jt}^{h}/2)}{w_{t} + \varepsilon_{t}\phi_{t}\nu} - 1$$
(51)

In the text we will write:

$$Y_{jt}^h = \frac{1}{R\gamma_t^h} + \frac{1}{2\gamma_t^h}A_{jt}^h - 1$$

A.3 The leverage ratio and the probability of bankruptcy

The leverage ratio is defined (see (30)) as

$$\lambda_t^h = \frac{1 - \alpha_t^h}{1 + \alpha_t^h} = \lambda \left(\alpha_t^h \right)$$

where

$$\alpha_t^h := \frac{A_t^h}{2\left(\frac{1}{R} - \gamma_t^h\right)} = \alpha\left(R, A_t^h, \gamma_t^h\right)$$

In order to save on notation, let's define $\delta_t^h := \frac{1}{R} - \gamma_t^h$. Therefore

$$\alpha_A^h = \frac{1}{2\delta_t^h} > 0; \quad \alpha_R^h = \frac{A_t^h}{2\left(R\delta_t^h\right)^2} > 0; \quad \alpha_\gamma^h = \frac{A_t^h}{2\left(\delta_t^h\right)^2} > 0$$

Moreover

$$\lambda_{\alpha}^{h} := \frac{d\lambda_{t}^{h}}{d\alpha_{t}^{h}} = -\frac{2}{\left(1 + \alpha_{t}^{h}\right)^{2}} = -\frac{8\delta_{t}^{h}}{\left(2\delta_{t}^{h} + A_{t}^{h}\right)^{2}} < 0$$

Hence

$$\begin{split} \lambda_A^h &= \lambda_\alpha^h \alpha_A^h = -\frac{4}{\left(2\delta_t^h + A_t^h\right)^2} < 0\\ \lambda_R^h &= \lambda_\alpha^h \alpha_R^h = -\frac{4A_t^h}{R^2\delta_t^h \left(2\delta_t^h + A_t^h\right)^2} < 0\\ \lambda_\gamma^h &= \lambda_\alpha^h \alpha_\gamma^h = -\frac{4A_t^h}{\delta_t^h \left(2\delta_t^h + A_t^h\right)^2} < 0 \end{split}$$

As to bank ruptcy, from the definition $F=\frac{1}{2}R\gamma_t^h\lambda_t^h$ follows

$$F_{A} = \frac{1}{2}R\gamma_{t}^{h}\lambda_{A}^{h} = -\frac{2R\gamma_{t}^{h}}{\left(2\delta_{t}^{h} + A_{t}^{h}\right)^{2}} < 0$$

$$F_{R} = \frac{1}{2}\gamma_{t}^{h}\left(\lambda_{t}^{h} + R\lambda_{R}^{h}\right)$$

$$F_{\gamma} = \frac{1}{2}R\left(\lambda_{t}^{h} + \gamma_{t}^{h}\lambda_{\gamma}^{h}\right)$$

The sign of F_R and F_{γ} is uncertain. From $F_R = \frac{1}{2}\gamma_t^h \left(\lambda_t^h + R\lambda_R^h\right)$ follows

$$F_R = \frac{1}{2} \gamma_t^h \lambda_t^h \left(1 + \eta_{\lambda R}^h \right)$$

where

$$\eta^h_{\lambda R} := \frac{R\lambda^h_R}{\lambda^h_t} < 0$$

is the *elasticity of the leverage ratio* with respect to the interest rate.

A similar argument applies to the response of the probability of bankruptcy to a change in average cost. In fact we can write

$$F_{\gamma} = \frac{1}{2} R \lambda_t^h \left(1 + \eta_{\lambda\gamma}^h \right)$$

where

$$\eta^h_{\lambda\gamma} := \frac{\gamma^h_t \lambda_\gamma}{\lambda^h_t} < 0$$

is the *elasticity of the leverage ratio* with respect to the average cost.

We linearize the probability of bankruptcy in a neighborhood of arbitrary

initial conditions R_0, γ_0^h, A_0^h as follows:

$$F \approx F_{0} + F_{R}(R_{0})[R - R_{0}] + F_{\gamma}(\gamma_{0}^{h})[\gamma_{t}^{h} - \gamma_{0}^{h}] + F_{A}(A_{0}^{h})[A_{t}^{h} - A_{0}^{h}]$$

where $F_0 := F(R_0, \gamma_0^h, A_0^h)$ and $F_x(x_0)$ is the derivative of F with respect to the generic variable x measured in x_0 . Rearranging, from the expression above we obtain

$$F \approx \phi_0 + \phi_1 R + \phi_2 \gamma_t^h - \phi_3 A_t^h$$

where

$$\begin{split} \phi_{0} &= F_{0} - \left[R_{0}F_{R}\left(R_{0}\right) + \gamma_{0}^{h}F_{\gamma}\left(\gamma_{0}^{h}\right) + A_{0}^{h}F_{A}\left(A_{0}^{h}\right) \right] \\ \phi_{1} &= F_{R}\left(R_{0}\right) = \frac{1}{2}\gamma_{0}^{h}\lambda_{0}^{h} + \frac{1}{2}R_{0}\gamma_{0}^{h}\lambda_{R}^{h}\left(R_{0}\right) \\ \phi_{2} &= F_{\gamma}\left(\gamma_{0}^{h}\right) = \frac{1}{2}R\lambda_{0}^{h} + \frac{1}{2}R_{0}\gamma_{0}^{h}\lambda_{\gamma}^{h}\left(\gamma_{0}^{h}\right) \\ \phi_{3} &= -\frac{2R_{0}\gamma_{0}^{h}}{\left(2\delta_{0}^{h} + A_{0}^{h}\right)^{2}} \end{split}$$

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