THE NATURE OF CREDIT CONSTRAINTS AND HUMAN CAPITAL

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ABSTRACT

This paper studies the nature and impact of credit constraints in the market for human capital. We derive endogenous constraints from the design of government student loan programs and from the limited repayment incentives in private lending markets. These constraints imply cross-sectional patterns for schooling, ability, and family income that are consistent with U.S. data. This contrasts with the standard exogenous constraint model, which predicts a counterfactual negative ability -- schooling relationship for low-income youth. We show that the rising empirical importance of familial wealth and income in determining college attendance (Belley and Lochner 2007) is consistent with increasingly binding credit constraints in the face of rising tuition costs and returns to schooling. Our framework also explains the recent increase in private credit for college as a market response to the rising returns to school.

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1   Introduction

Borrowing constraints have a long history in the literature on schooling and human capital investment. Because human capital cannot be repossessed in response to default, it makes for poor collateral. Since investing in human capital is most efficient when individuals are young, most students have not established a credit reputation or accumulated other forms of collateral. As a result, private financial institutions have historically offered little credit to finance higher education.[1] Even when governments have stepped in with public student lending programs, the credit offered through these programs has been quite limited, and credit constraints may still play an important role in higher education decisions.

As pointed out by Becker (1975), youth with few family resources under-invest in their human capital if they cannot obtain adequate credit. This observation has motivated a voluminous literature examining the relationship between family income (or wealth) and college attendance (e.g. Manski and Wise 1983, Cameron and Heckman 1998, 2001, Ellwood and Kane 2000, Carneiro and Heckman 2002, Belley and Lochner 2007). Using U.S. data from the 1979 and 1997 cohorts of the National Longitudinal Survey of Youth (NLSY79 and NLSY97, respectively) and controlling for individual differences in ability and family background, Belley and Lochner (2007) estimate a weak family income – college attendance relationship in the early 1980s but a much stronger positive relationship in the early 2000s.[2] This seems to suggest that borrowing constraints had little effect on schooling decisions in the early 1980s, but that their effects have grown more important in recent years. Consistent with this hypothesis, recent U.S. Department of Education studies (Berkner 2000 and Titus 2002) report that the fraction of undergraduate student borrowers who borrowed the maximum allowable amount from federal student loan programs nearly tripled from 18% in 1989-90 to 52% in 1999-2000. At the same time, however, student borrowing from private lending institutions increased from negligible amounts in the mid-1990s to $14 billion, almost 20% of all student loans distributed, in the 2004-05 academic year (College Board 2005). *Ceteris paribus*, this expansion in private lending should have helped alleviate the tight constraints imposed by government student loan (GSL) programs.

We study these patterns of college attendance and borrowing in a human capital investment model that incorporates limited borrowing opportunities from both GSL programs and private lenders. Our analysis suggests that all of these trends can be explained by the rising returns to schooling (Katz and Autor 1999, Heckman, Lochner and Todd 2008), rising costs of attendance at U.S. colleges and universities (College Board 2006), and fairly stable real borrowing limits associated with GSL programs (Kane 2007) over the last few decades. As

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[1]This has changed in recent years, as we discuss below.

[2]Using a variety of empirical approaches and data from the NLSY79, a number of researchers have concluded that borrowing constraints had little effect on college-going behavior in the early 1980s. Ellwood and Kane (2000) also estimate that family income has become a more important determinant of college-going since 1980. See Section 3 for a review of this literature.

we discuss further below, our model also implies cross-sectional correlations between ability and investment that are more in line with U.S. data than does the traditional model with exogenous borrowing constraints.

The human capital literature has paid little attention to the nature of borrowing constraints. Existing models typically assume that either interest rates increase with the amount borrowed or that there is a fixed maximum amount that individuals can borrow.[3] Both approaches neglect the link between borrowing opportunities and investment decisions that plays a key role in both GSL programs and private lending as we describe below. We show that without this link, the canonical model of exogenous borrowing constraints predicts a *negative* relationship between ability and human capital investment among constrained borrowers when the consumption intertemporal elasticity of substitution (IES) is less than one. This is troubling, since most empirical estimates of the IES are below one (Browning, Hansen, and Heckman 1999) and a strong positive ability – college attendance relationship exists for all family income (or wealth) levels in both the NLSY79 and NLSY97 (as we show in Section 3). Additionally, models with exogenous borrowing constraints offer no insights regarding the recent rise in private student lending, the interaction between private and public lending, or how lending opportunities respond to important economic and policy changes. Our framework offers insights on all of these issues.

GSL programs directly tie student credit to the level of investment — students can borrow to help finance college-related expenses only if they are enrolled in school. We show that private lenders, facing limited repayment incentives from borrowers, will also link credit limits to the level of investment, as well as observable individual characteristics that affect the returns to investment. These features of endogenously determined (or variable) borrowing limits help generate a positive relationship between ability and investment (even when the IES is less than one) while still predicting a positive relationship between family resources and investment among constrained youth.[4]

GSL programs have two distinct forms of limits: (i) a pre-specified maximum loan limit (denote this by $d_{\max}$), and (ii) an endogenous limit that restricts students from borrowing more than they spend on their education.[5] As we show, youth that would like to borrow more than they spend on their schooling (i.e. those constrained by the second limit) invest the same amount in their human capital as if they were completely unconstrained. When credit is tied directly to investment, there is no tradeoff between additional investment and consumption while in school — every additional dollar of investment can be borrowed (as

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[3]Studies assuming variable interest rates (or heterogeneous interest rates) include Becker (1975), Cameron and Taber (2004) and Card (1995). Studies assuming a fixed limit on borrowing include Belley and Lochner (2007), Caucutt and Kumar (2003), Hanushek, Leung, and Yilmaz (2003), and Keane and Wolpin (2001).

[4]We refer to these borrowing limits as 'endogenous', because they are a function of the borrower's investment behavior. We do not model the determination of these limits in the GSL system; however, borrowing limits set by private lenders are optimally determined from the incentives of borrowers to default.

[5]Under the Stafford Loan Program, students face a cumulative loan limit as well as annual borrowing limits which increase somewhat with year of post-secondary school.

long as investment remains below $d_{\max}$). This implies that consumption decisions may be severely distorted even when schooling and investment decisions are not.[6] Among youth who would like to invest more than the GSL maximum loan limit $d_{\max}$, investment is increasing in family income (or wealth) as has been observed for recent student cohorts. Most importantly, introducing the restriction that borrowing cannot exceed investment substantially reduces the set of individuals for which there is a negative ability – investment relationship.

The recent rise in private lending highlights the importance of studying how private lenders determine student credit levels. Even if human capital cannot be directly repossessed by lenders, creditors can punish defaulting borrowers in a number of ways (e.g. lowering credit scores, seizing assets, garnisheeing a fraction of labor earnings), which tend to have a greater impact on debtors with higher post-school earnings. In our lifecycle model, these mechanisms effectively link private borrowing limits for students to their abilities and human capital investments.[7] Higher ability students who invest more through education will be offered more credit by private lenders, because they can credibly commit to re-pay more given the punishments they face upon default. The dependence of credit limits on investment and ability generates a positive ability – investment relationship for *all constrained borrowers* under standard parameterizations of preferences.

Our framework incorporates the lending opportunities provided by both GSL programs and private lenders. This not only provides new insights about human capital investment behavior, but it also enables us to better understand the changing role played by private financial markets. In particular, the recent emergence and expansion of private student lending can be explained by the stability of government student loan limits over the last few decades and the recent rise in student demand for credit if: (i) GSL maximum borrowing limits were high enough to finance unrestricted levels of investment in the early 1980s, (ii) these limits are too low to cover the higher levels of investment desired today, and (iii) current students can credibly commit to re-pay more than these limits allow. The evidence on family income – college attendance patterns in the NLSY79 and NLSY97 is consistent with the first two conditions. The higher earnings potential of recent graduates, coupled with higher costs of schooling, can explain why more college students are bunching up against

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[6]Thus, evidence that family resources do not affect educational attainment or financial returns does not necessarily imply that credit constraints are non-binding. Standard empirical tests for borrowing constraints that rely on differences in educational attainment or marginal rates of return on investment by family income (or other categories used to differentiate the 'constrained' from 'unconstrained') will under-estimate the fraction of the population that is constrained as well as the adverse impacts of constraints on welfare.

[7]Our model of private lending is related to the literature on endogenous credit constraints, which has generally focused on implications for risk-sharing and asset prices in endowment economies (e.g. Alvarez and Jermann 2000, Fernandez-Villaverde and Krueger 2004, Krueger and Perri 2002, Kehoe and Levine 1993, and Kocherlakota 1996) or firm dynamics (e.g. Albuquerque and Hopenhayn 2004, Monge-Naranjo 2008). Our assumed punishments for default are similar to those employed by Livshits, MacGee, and Tertilt (2007) in their analysis of bankruptcy over the lifecycle. Andolfatto and Gervais (2006) study human capital accumulation with limited commitment, but they focus on the optimal set of intergenerational transfers and not on cross-sectional implications for investment.

GSL maximum borrowing limits (Berkner 2000 and Titus 2002). This creates new demand for private lenders to step in, offering more credit to those who can credibly commit to repay. With rising returns to schooling, commitments to repay become credible for more and more college students, suggesting that condition (iii) is likely to be met. Our lifecycle model calibrated to U.S. data generates these patterns in response to a rise in both the returns and costs of college.

The rest of the paper proceeds as follows. In the next section, we describe borrowing opportunities from U.S. GSL programs and the recent emergence of private lenders. In the third section, we discuss evidence on the relationship between ability, family income and college attendance in the U.S. and briefly survey the literature on the prevalence of credit constraints. In Section 4, we develop a simple two-period human capital investment model to analytically compare the cross-sectional implications for borrowing and investment under alternative assumptions about credit markets. Section 5 extends our framework to a lifecycle model that incorporates government subsidies for education. Qualitative results derived for the two-period model carry over to this environment. We calibrate this model using U.S. data on schooling, ability, government subsidies, and post-school earnings. This quantitative analysis shows that our model with both public and private lending does a good job reproducing observed cross-sectional human capital investment patterns for the early 1980s and 2000s. The model with exogenous constraints does not. Our model of endogenous constraints also explains both the increased effects of family income on college-going and the rising importance of private lending for recent cohorts as optimal responses to the increased costs of and financial returns to human capital investment. Section 6 concludes.

2 Available Sources of Credit

This section briefly reviews the primary sources of borrowing used for human capital investment in the U.S. We first describe key institutional features of GSL programs, which we incorporate in our endogenous constraint models below. Then, we discuss the rise of private lending for post-secondary schooling.

2.1 Government Student Loan Programs

Federal student loans are an important source of finance for higher education in the U.S., accounting for 71% of the federal student aid disbursed in 2003-04.[8] Most of these government-backed loans are provided through the Stafford Loan program, which awarded nearly $50 billion to students in the 2003-04 academic year, compared to the disbursement of $1.6 billion through the Perkins Loan program. Slightly more than $7 billion was awarded to parents of

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[8]Many other countries have similar types of government student loan programs.

undergraduate students in the form of Parent Loans for Undergraduate Students (PLUS).[9]

GSL programs generally have three important features. First, lending is directly tied to investment. Students (or parents) can only borrow up to the total cost of college (including tuition, room, board, books, supplies, transportation, computers, and other expenses directly related to schooling) less any other financial aid they receive in the forms of grants or scholarships. Thus, students cannot borrow from GSL programs to finance non-schooling related consumption goods or activities. Second, student loan programs set fixed upper limits on the total amount of credit available for each student. Students face both cumulative and annual loan limits for U.S. federal loan programs.[10] Third, loans covered by GSL programs typically have extended enforcement rules compared to unsecured private loans.

Historically, private lenders have provided the capital to student borrowers (and their parents) under the Stafford and PLUS programs, the government guaranteeing those loans with a promise to cover any unpaid amounts. Since the 1994-95 academic year, the federal government has begun to directly provide these loans to some students under the same rules and terms.[11] While Stafford loans are disbursed to students, PLUS loans can be taken out by parents to help cover the costs of their children's schooling. Another major federal student loan program, the Perkins Loan Program, provides an additional source of government funds to students most in need; however, its loan offerings depend on the level of program funding at the post-secondary institution attended by a student. In practice, Perkins loans make up a small fraction of federal student loan disbursements.

Table 1 reports loan limits (based on the dependency status and class level of each student) for Stafford and Perkins student loan programs for the period 1993-2007.[12] In recent years, dependent students could borrow up to $23,000 from the Stafford Loan Program over the course of their undergraduate careers. Independent students could borrow roughly twice that amount, although most traditional undergraduates would not fall into this category. Qualified undergraduates from low income families could receive as much as $20,000 in Perkins loans, depending on their need and post-secondary institution. It is important to note, however, that amounts offered through this program have typically been less than mandated limits.[13] Student borrowers can defer loan re-payments until six (Stafford) to nine

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[9]See The College Board (2006) for details about financial aid disbursements and their trends over time.

[10]Since 1993-94, the PLUS loan program no longer has a fixed maximum borrowing limit; however, parents still cannot borrow more than the total cost of college less other financial aid received by the student.

[11]The Stafford program offers both subsidized and unsubsidized loans. The government covers the interest on subsidized loans while students are enrolled. Unsubsidized loans accrue interest over this period; however, the student is not required to make any payments until after leaving school. To qualify for subsidized loans, students must demonstrate financial need on the basis of family income, dependency status, and the cost of the school attended. Most students under age 24 are considered dependent, and their parents' income is an important determinant of their financial need. Prior to the introduction of unsubsidized Stafford Loans in the early 1990s, Supplemental Loans to Students (SLS) were an alternative source of unsubsidized federal loans for independent students.

[12]Stafford loan limits for freshman, sophomores, and graduate students increased slightly in July, 2007.

[13]Parents that do not have an adverse credit rating can borrow up to the cost of schooling from the PLUS

Table 1: Borrowing Limits for Stafford and Perkins Student Loan Programs (1993-2007)

| | Stafford Loans | | |
	Dependent Students	Independent Students*	Perkins Loans
Eligibility Requirements	Subsidized: Financial Need** Unsubsidized: All Students		Financial Need
Undergraduate Limits:			
First Year	$2,625	$6,625	$4,000
Second Year	$3,500	$7,500	$4,000
Third-Fifth Years	$4,000	$8,000	$4,000
Cum. Total	$23,000	$46,000	$20,000
Graduate Limits:			
Annual	$18,500		$6,000
Cum. Total***	$138,500		$40,000

Notes:
* Students whose parents do not qualify for PLUS loans can borrow up to
independent student limits from Stafford program.
** Subsidized Stafford loan amounts can be no greater than the borrowing
limits for dependent students; independent students can also borrow unsubsidized
Stafford loans provided that their total (subsidized and subsidized)loan amount
is not greater than the independent student limits.
*** Cumulative graduate loan limits include loans from undergraduate loans.

(Perkins) months after leaving school.

Figure 1 shows how annual Stafford loan limits for dependent undergraduate students have evolved from 1980–81 to 2006–07 (denominated in year 2000 dollars).[14] In most years, the cumulative loan limit is equal to or slightly greater than the sum of all five annual loan limits. The jumps up reflect nominal adjustments to the limits in 1986–87 and 1993–94; otherwise, inflation has continuously eroded these limits. The entry year into college has seen the greatest erosion in real borrowing opportunities — a 44% decline from 1982–83 to 2002–03.[15] The borrowing limit for second-year students declined by roughly 25% over this

program, with repayment typically beginning within 60 days of loan disbursement. Dependent students whose parents do not qualify for PLUS loans (due to a bad credit rating) are able to borrow up to the independent student loan limits.

[14]The Consumer Price Index for All Urban Consumers (CPI-U) is used to adjust for inflation.

[15]Our NLSY79 and NLSY97 respondents made their college attendance decisions around these two periods, respectively.

Figure 1: Annual Stafford Loan Limits for Dependent Undergraduates from 1980-2006 (Year 2000 Dollars)
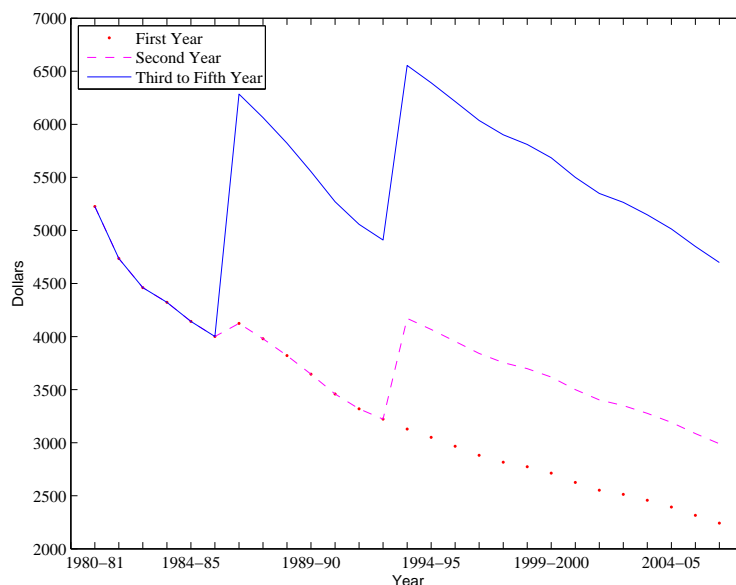
period. By contrast, third- through fifth-year undergraduates were able to borrow nearly 20% more in 2002–03 than in 1982–83 due to more substantial nominal increases in 1986–87 and 1993–94. Cumulative Stafford loan limits were almost identical in real terms in 1982–83 and 2002–03.[16]

Student loans covered by these federal programs have extended enforcement rules compared to typical unsecured private loans. Except in very special circumstances, these loans cannot generally be expunged through bankruptcy. If a suitable re-payment plan is not agreed upon with the lender once a borrower enters default, the default status will be reported to credit bureaus and collection costs (up to 25% of the balance due) may be added to the amount outstanding.[17] Up to 15% of the borrower's wages can also be garnisheed. Moreover, federal tax refunds can be seized and applied toward any outstanding balance. Other sanctions include a possible hold on college transcripts, ineligibility for further federal student loans, and ineligibility for future deferments or forbearances.[18]

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[16]Throughout most of this period, loan limits for independent undergraduates remained about twice the amounts available to dependent students. Stafford loan limits for graduate students declined by about 35% in real terms from 1986–87 to 2006–07, roughly the time our NLSY respondents would have began attending graduate school.

[17]Formally, a borrower is considered to be in default once a payment is 270 days late.

[18]Since the early 1990s, the government has also begun to punish educational institutions with high student default rates by making their students ineligible to borrow from federal lending programs.

2.2 The Emergence of Private Lenders

Until the mid-1990s, few private lenders offered loans to students outside the GSL programs. In 1995-96, total non-federal student loans amounted to $1.3 billion. By 2004-05, that amount had risen to almost $14 billion (nearly 20% of all student loans distributed).[19] Private student loans generally charge higher interest rates than Stafford or Perkins loans and are, therefore, typically taken after exhausting available credit from GSL programs. Thus, the rise in borrowing from private lenders outside the Stafford and Perkins Loan Programs suggests that the GSL limits are no longer enough to satisfy many students' demands for credit. Private loans are most prevalent among graduate students (especially in professional schools) and undergraduates at high-cost private universities (Wegmann, Cunningham and Merisotis 2003).

While many private student lending programs are loosely structured like the federal GSL programs (i.e. many limit borrowing to the cost of schooling less financial aid or a fixed upper limit on total borrowing), they vary substantially in their terms and eligibility requirements. Private lending programs typically use a broader concept of schooling costs than do GSL programs, often allowing students to borrow against previous educational expenses or expenses for study abroad. Specified maximum loan limits are generally quite high, especially for students in professional schools (e.g. law, medical, or business schools); however, actual amounts offered to students vary depending on their creditworthiness, institution attended, and area of study. A cosigner with a good credit history tends to improve the terms of any loans and can affect whether a loan is offered in the first place.

3 Evidence on the Role of Ability and Family Income

In this section, we discuss the empirical relationship between family income, cognitive ability and college attendance. We review the recent literature and offer some new evidence in documenting three stylized facts on investment in human capital. First, there was a weak link between family income and college attendance in the early 1980s. Second, for very recent student cohorts, there is a much stronger relationship between family income (or wealth) and college attendance. Third, in both the early 1980s and the early 2000s, there is a strong positive relationship between college attendance and cognitive ability or achievement (as measured by scores on the Armed Forces Qualifying Test, AFQT).[20]

Many empirical studies using NLSY79 data conclude that borrowing constraints played little role in college attendance decisions in the U.S. during the early 1980s. Cameron

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[19]These figures do not include student borrowing on credit cards, which has also increased considerably over this period. See College Board (2005).

[20]This evidence is largely based on the NLSY79 and NLSY97. AFQT scores are widely used as measures of cognitive achievement by social scientists and are strongly correlated with post-school earnings conditional on educational attainment. See, e.g., Cawley, *et al.* 2000. Appendix A provides additional details.

and Heckman (1998, 1999) find that after controlling for family background, AFQT scores, and unobserved heterogeneity, family income has little effect on college enrollment rates. Carneiro and Heckman (2002) also estimate small differences in college enrollment rates and other college-going outcomes by family income after accounting for differences in family background and AFQT. Cameron and Taber (2004) find little evidence of differential returns to school that would be consistent with borrowing constraints. Keane and Wolpin (2001) estimate a structural model of schooling and work that incorporates constraints on borrowing and parental transfers that may depend on child schooling decisions. While they estimate very tight borrowing limits (much more stringent than federal student loan limits), they find little effect of borrowing constraints on educational attainment.

Much has changed since the early 1980s, when the NLSY79 respondents made their college attendance decisions. Financial returns to schooling have risen dramatically (Katz and Autor 1999, Heckman, Lochner, and Todd 2008) as have the costs of tuition, fees, room, and board at U.S. colleges and universities (College Board 2005). At the same time, real borrowing limits associated with government student loan programs have remained stable or declined (see Figure 1).

These trends appear to have increased the importance of borrowing constraints. Indeed, the fraction of all undergraduate borrowers that borrowed the maximum limit from the federal Stafford Student Loan Program went up from only 18% in 1989-90 to 52% in 1999-2000. Among dependent undergraduates, the fraction increases to nearly 70% of all borrowers in 1999-2000 (Berkner 2000 and Titus 2002). Moreover, Belley and Lochner (2007) show that family income has become a much more important determinant of college attendance for college-going decisions in the early 2000s.[21] Youth from high income families in the NLSY97 are sixteen percentage points more likely to attend college than are youth from low income families, conditional on AFQT scores, family composition, parental age and education, race/ethnicity, and urban/rural residence. This is nearly twice the effect observed in the NLSY79. The combined effects of family income and wealth are even more dramatic in the NLSY97. Comparing youth from the highest family income and wealth quartiles to those from the lowest quartiles yields an estimated difference in college attendance rates of nearly 30 percentage points after controlling for ability and family background.

Next, we examine the effects of ability (as measured by AFQT scores) on college attendance for youth from different family income (or wealth) backgrounds in both the NLSY79 and NLSY97. (See Appendix A for a detailed description of the data and variables used

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[21]Ellwood and Kane (2000) argue that college attendance differences by family income were already becoming more important by the early 1990s. Using data on youth of college-ages in the 1970s, 1980s, and 1990s (from the Health and Retirement Survey), Brown, Seshadri, and Scholz (2007) estimate that borrowing constraints limit college-going; however, they do not examine whether constraints have become more limiting in recent years. While Stinebrickner and Stinebrickner (2007) find little effect of borrowing constraints (defined by the self-reported desire to borrow more for school) on overall college dropout rates for a recent cohort of students at Berea College, they find substantial differences in dropout rates between those who are constrained and those who are not. They do not study the effects of borrowing constraints on attendance.

here.) Figure 2 shows college attendance rates by AFQT quartiles and either family income or family wealth quartiles in the NLSY79 and NLSY97. For all family income or wealth categories in both NLSY samples, we observe substantial increases in college attendance with AFQT. The difference in attendance rates between the highest and lowest ability quartiles range from .47 to .68 depending on the family income or wealth quartile. Most importantly for our theoretical analysis below, there is no indication that the effects of ability are negative for lower income youth who are most likely to be constrained, especially in the NLSY97.

Of course, AFQT scores may be correlated with other family background variables that influence college attendance decisions conditional on family resources. We, therefore, control for a host of other family background measures in addition to AFQT quartiles using ordinary least squares. Table 2 reports the estimated effects of AFQT (these estimates reflect the difference in attendance rates between the reported AFQT quartile and AFQT quartile 1) on college attendance after controlling for family background characteristics.[22] Results are reported for separate regressions by family income or wealth quartile. The estimates confirm the general patterns observed in Figure 2: ability has strong positive effects on college attendance for all family income and wealth quartiles in both NLSY samples.

4 Basic Models of Borrowing Constraints

We now develop a simple two-period model to study the impact of credit constraints on investment in human capital. We allow for some generality in preferences and skill production and derive the qualitative investment – wealth and investment – ability relationships implied by alternative forms of credit constraints. We show that these relationships depend crucially on the nature of credit constraints and evaluate the empirical relevance of the different models based on the empirical findings discussed above.

4.1 The Model

Consider two-period-lived individuals who invest in schooling in the first period and work in the second. Preferences are

$$U = u(c_0) + \beta u(c_1),\tag{1}$$

where c_t is consumption in periods $t \in \{0,1\}$, $\beta > 0$ is a discount factor and $u(\cdot)$ satisfies:

Assumption 1. $u : \mathbb{R}_+ \to \mathbb{R}$ *is strictly increasing, strictly concave, twice continuously differentiable and* $\lim_{c \searrow 0} u'(c) = +\infty$.

Each individual is endowed with initial financial assets $w \geq 0$ and ability $a > 0$. Initial assets represent all transfers from parents and other family members. Ability represents all

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[22]We control for the following: gender, race/ethnicity, mother's education, intact family during adolescence, number of siblings/children under age 18, mother's age at child's birth, urban/metropolitan area of residence during adolescence, and year of birth.

Figure 2: College Attendance by AFQT and Family Income or Wealth (NLSY79 and NLSY97)

(a) Attendance by AFQT and Family Income (NLSY79)
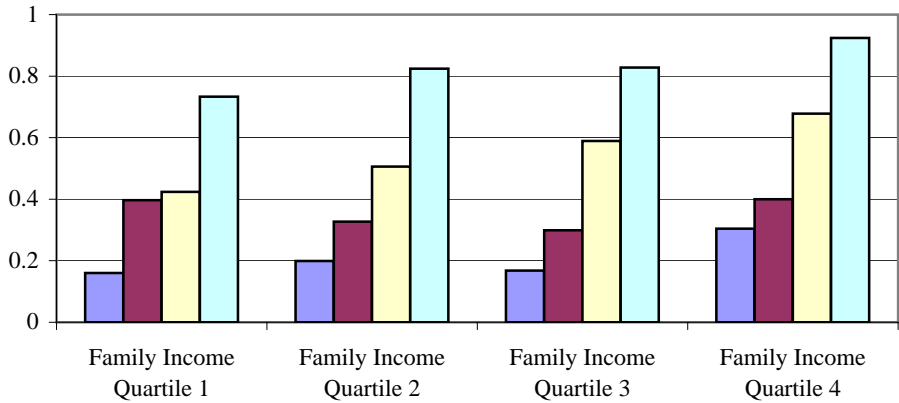
(b) Attendance by AFQT and Family Income (NLSY97)
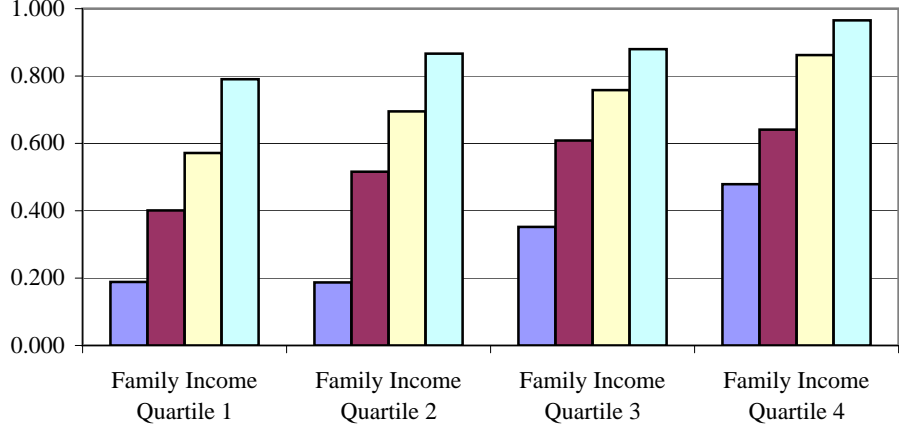
(c) Attendance by AFQT and Family Wealth (NLSY97)
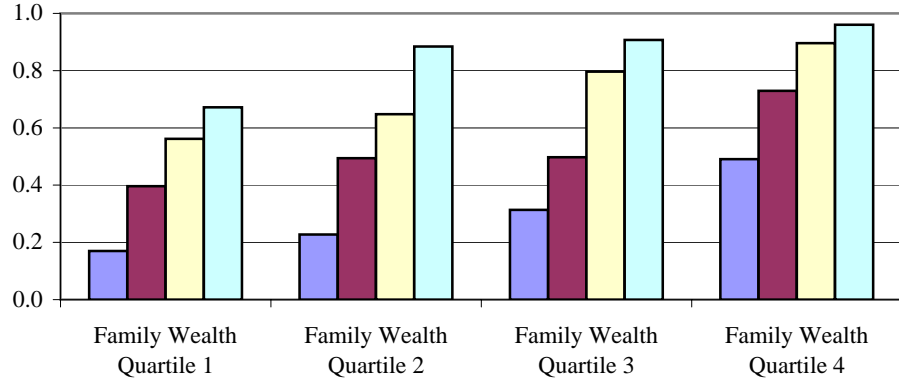
□ AFQT Quartile 1   ■ AFQT Quartile 2   □ AFQT Quartile 3   □ AFQT Quartile 4

Table 2: Estimated Effects of AFQT on College Attendance by Family Income and Wealth (NLSY79 and NLSY97)

	Effects of AFQT by Family Income Quartile:				Effects of AFQT by Family Wealth Quartile:			
	Quartile 1	Quartile 2	Quartile 3	Quartile 4	Quartile 1	Quartile 2	Quartile 3	Quartile 4
a. NLSY79								
AFQT Quartile 2	0.211	0.110	0.120	0.063				
	(0.044)	(0.050)	(0.059)	(0.065)				
AFQT Quartile 3	0.260	0.276	0.369	0.337				
	(0.058)	(0.053)	(0.060)	(0.062)				
AFQT Quartile 4	0.517	0.515	0.574	0.537				
	(0.068)	(0.058)	(0.061)	(0.062)				
Sample Size	545	556	596	591				
b. NLSY97								
AFQT Quartile 2	0.188	0.348	0.225	0.130	0.260	0.248	0.193	0.178
	(0.047)	(0.050)	(0.052)	(0.051)	(0.050)	(0.049)	(0.050)	(0.052)
AFQT Quartile 3	0.396	0.474	0.352	0.335	0.375	0.376	0.443	0.341
	(0.053)	(0.051)	(0.051)	(0.050)	(0.055)	(0.052)	(0.048)	(0.053)
AFQT Quartile 4	0.575	0.662	0.472	0.403	0.495	0.643	0.527	0.370
	(0.062)	(0.055)	(0.052)	(0.049)	(0.070)	(0.056)	(0.050)	(0.051)
Sample Size	553	597	677	702	541	573	716	666

Notes: All regressions control for gender, race/ethnicity, mother's education (HS graduate, college attendance), intact family during adolescence, number of siblings/children under 18, mother's age at child's birth, urban/metropolitan area during adolescence, and year of birth. Education measured as of age 21 (age 22 if missing at age 21). Standard errors are in parentheses

innate factors, early parental investments and other characteristics that shape the returns to investing in schooling. We take (w, a) as fixed and exogenous to focus on schooling decisions that individuals make largely on their own.

Labor earnings at $t = 1$ are $y = af(h)$, where h is schooling investment and $f(\cdot)$ satisfies:

Assumption 2. $f : \mathbb{R}_+ \to \mathbb{R}_+$ *is strictly increasing, concave, twice continuously differentiable,* $\lim_{h \searrow 0} f'(h) = +\infty$ *and* $\lim_{h \nearrow \infty} f'(h) = 0$.

Note that both a and h enhance earnings and are complementary. Assumptions 1 and 2 are standard, and we make use of them without further reference. They imply that optimal solutions in models of this section are interior (positive and finite) and determined by first order conditions.

Human capital investment, h, is in units of the consumption good. Individuals can borrow d of these units (or save, which is indicated by $d < 0$) at a gross interest rate $R > 1$. Given a, h and d, consumption in each of the periods is

$$c_0 = w + d - h, \tag{2}$$

$$c_1 = af(h) - Rd. \tag{3}$$

These sequential constraints imply the present-value lifetime budget constraint:

$$c_0 + \frac{c_1}{R} = w + \frac{af(h)}{R} - h. \tag{4}$$

4.2 Unrestricted Allocations

In the absence of financial frictions, young individuals maximize utility (1) subject to (4). This maximization can be separated into two steps. The first is to choose h to maximize the present value of lifetime net resources, $w + R^{-1}af(h) - h$. Optimal unrestricted investment, $h^U(a)$, equates the marginal return of human capital with the return of financial assets:

$$af'\left[h^U(a)\right] = R. \tag{5}$$

From this condition, $h^U(a)$ is strictly increasing in ability, a, and independent of initial assets, w.

The second step is to smooth consumption, borrowing an amount $d^U(a, w)$ to satisfy the Euler equation:

$$u'\left(w + d^U(a, w) - h^U(a)\right) = \beta R u'\left(af\left[h^U(a)\right] - Rd^U(a, w)\right). \tag{6}$$

From this condition, $d^U(a, w)$, is strictly decreasing in w and increasing in a. Optimal debt $d^U(a, w)$ is strictly increasing in ability, a, because of two forces. First, more able individuals finance a larger investment. Second, more able individuals attain higher net-lifetime resources and want to consume more during youth. The latter force implies that the relationship between borrowing and ability is steeper than the relationship between human capital investment and ability.

Lemma 1 *Let the functions $h^U(a)$ and $d^U(a, w)$ denote the unrestricted investment in human capital and borrowing. Then, $h^U(a)$ is strictly increasing in a and $d^U(a, w)$ is strictly increasing in a and strictly decreasing in w. Moreover, $\frac{\partial d^U(a,w)}{\partial a} > \frac{dh^U(a)}{da} > 0$ and $-1 < \frac{\partial d^U(a,w)}{\partial w} < 0$.*

(Proofs for all results and other analytical details for the models in Section 4 are given in Appendix B.) We make repeated use of Lemma 1 to characterize the behavior of investment with borrowing constraints.

4.3 Exogenous Borrowing Constraints

At least since Becker (1975), economists have introduced financial market imperfections in models of human capital. With imperfect access to credit, Becker shows that youth from poor families will invest less (and have higher marginal returns on schooling) than otherwise identical youth from wealthier families.

Credit constraints are typically introduced by imposing a fixed and exogenous upper bound on the amount of debt.[23] Following this approach, assume that borrowing is restricted by the exogenous constraint:

$$d \le d_0, \tag{EXC}$$

where $0 < d_0 < \infty$ is fixed and uniform for all agents. We use the superscript X for allocations in this model.

In this environment, individuals maximize utility (1) subject to (2), (3), and the borrowing constraint (EXC). This yields the following first order conditions for investment and borrowing, respectively:

$$u'\left(w + d^X(a, w) - h^X(a, w)\right) = \beta u'\left(af\left[h^X(a, w)\right] - Rd^X(a, w)\right) af'\left[h^X(a, w)\right]$$
$$u'\left(w + d^X(a, w) - h^X(a, w)\right) = \beta R u'\left(af\left[h^X(a, w)\right] - Rd^X(a, w)\right) + \lambda,$$

where λ is the LaGrange multiplier on the constraint (EXC) and is strictly positive when the constraint binds and zero otherwise. Combining these equations and re-arranging terms yields:

$$af'\left[h^X(a, w)\right] = R + \frac{\lambda}{\beta u'\left(w + d^X(a, w) - h^X(a, w)\right)} \ge R.$$

This equation clearly shows that the rate of return on human capital investment is strictly greater than the return on financial assets for those constrained by (EXC) since $\lambda > 0$. Unconstrained individuals ($\lambda = 0$) equate the marginal return on investment to that of financial assets as in equation (5) above.

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[23]See, for example, Aiyagari, Greenwood, and Seshadri (2002), Belley and Lochner (2007), Caucutt and Kumar (2003), Hanushek, Leung, and Yilmaz (2003), and Keane and Wolpin (2001). Instead, Becker (1975) assumes that individuals face an increasing interest rate schedule as a function of their investment. Becker's formulation yields similar predictions to those discussed here.

For each ability a, a threshold of assets $w_{\min}^X(a)$ defines who is constrained and who is unconstrained. Those with wealth below $w_{\min}^X(a)$ are constrained, while those with wealth above the threshold are unconstrained. The threshold is the level of w such that $d^U(a, w) = d_0$, so it is increasing in ability a. More able individuals need more initial wealth to attain higher unconstrained levels of investment and consumption. (Appendix B characterizes the threshold $w_{\min}^X(a)$ and the thresholds defined by the other constraints considered in this section.)

For unconstrained individuals, the trade-off between early and late consumption is determined by the return on financial assets, and investment equals $h^U(a)$. Constrained individuals exhaust their ability to bring resources to the early (investment) period (i.e. $d = d_0$). Their trade-off between early and late consumption is given by the rate of return on human capital investment, which is higher than R and increasing in ability. Constrained individuals must strike a balance between maximizing lifetime earnings and smoothing consumption over time. For them, optimal investment $h^X(a, w)$ is uniquely determined by:

$$u'\left(w + d_0 - h^X(a, w)\right) = \beta u'\left(af\left[h^X(a, w)\right] - Rd_0\right) af'\left[h^X(a, w)\right],$$

equating the marginal cost of investment in terms of early utility with the marginal benefit in terms of late utility when borrowing is set to the maximum, d_0.

We highlight three important results. First, constrained investment never exceeds unconstrained investment, i.e. $h^X(a, w) \leq h^U(a)$. This result holds for all forms of constraints considered in the paper. Second, constrained investment is strictly increasing in wealth. Third, constrained investment may be increasing or decreasing in ability depending on the intertemporal elasticity of substitution (IES), $-u'(c)/[cu''(c)]$.

Proposition 1 *Let $h^X(a, w)$ and $h^U(a)$ denote, respectively, the optimal investment with and without the constraint (EXC). If (EXC) binds, then: (i) $h^X(a, w) < h^U(a)$; (ii) $h^X(a, w)$ is strictly increasing in w; (iii) the marginal return on human capital investment, $af'\left[h^X(a, w)\right]$, is strictly greater than R and strictly decreasing in w; and (iv) if the $IES \leq 1$, then $h^X(a, w)$ is **strictly decreasing** in ability, a.*

Results (i), (ii) and (iii) are well-known and already discussed in Becker (1975). They are central to the empirical literature on credit constraints. For instance, Cameron and Heckman (1998, 1999), Ellwood and Kane (2000), Carneiro and Heckman (2002), and Belley and Lochner (2007) empirically examine if youth from lower income families acquire less schooling, conditional on family background and ability. Lang (1993), Card (1995), and Cameron and Taber (2004) explore the prediction that the marginal return on human capital investment exceeds the return on financial assets.

The more interesting and novel result is (iv). It reveals a serious shortcoming of this model that the literature has not recognized. The model predicts a *negative* relationship

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between ability and investment for an IES below one.[24] This is a serious problem, because most estimates of the IES are less than one (see Browning, Hansen, Heckman 1999) and a positive relationship between ability and investment is a robust empirical regularity. The relationship between ability and investment for constrained youth derives from two opposing effects. On the one hand, an increase in ability raises the financial returns to investment, which encourages investment. On the other hand, ability raises lifetime income, which encourages early consumption. Since constrained youth can only increase early consumption by investing less, strong preferences for smoothing (i.e. IES≤1) imply that the second effect dominates.

4.4   Government Student Loan (GSL) Programs

Consider now, credit limits that exhibit the key features of GSL programs. First, lending is *tied to investment* and cannot be used to finance non-schooling related consumption goods or activities:

$$d \leq h. \tag{TIC}$$

In the absence of other sources of credit, (TIC) is equivalent to $c_0 \geq w$. This constraint is endogenous in the sense that borrowing limits depend on the amount of investment undertaken by an individual. Second, borrowing is constrained by an upper limit $0 < d_{\max} < \infty$ for the total credit to each student:

$$d \leq d_{\max}. \tag{7}$$

This second constraint is effectively the same as the exogenous constraint above. The overall credit limits induced by GSL programs are:

$$d \leq \min \left\{ h, d_{\max} \right\}. \tag{GSLC}$$

We use the superscript G to refer to allocations under a GSL program.

To see the implications of (TIC), first assume that it is the only constraint.[25] In this case, individuals are unconstrained if their desired investment exceeds desired borrowing, i.e. $h^U(a) \geq d^U(a, w)$. These are individuals who hold wealth that exceeds the finite threshold $\tilde{w}_{\min}(a)$, which is increasing in ability a but at a slower rate than $w_{\min}^X(a)$, as shown in Appendix B. In comparison to exogenous constraint models, (TIC) is more (less) stringent than (EXC) for low (high) ability individuals in the sense that $\tilde{w}_{\min}(a) > w_{\min}^X(a)$ if $h^U(a) < d_0$, and $\tilde{w}_{\min}(a) < w_{\min}^X(a)$ if $h^U(a) > d_0$.

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[24]An $IES \leq 1$ is only a sufficient condition for a negative ability – investment relationship. More generally, the model may predict a negative relationship for IES values greater than one. While the model formally abstracts from foregone earnings, it is isomorphic to one in which foregone earnings for any given investment, h, are independent of ability. Result (iv) holds more generally in a model with foregone earnings as long as youth wage rates are not strictly decreasing in ability.

[25]This is the most appropriate model when upper borrowing limits are non-existent or set very high (e.g. PLUS program for students' parents).

If (TIC) is the only binding condition, then $d = h$ and $c_0 = w$. As such, the maximization problem becomes

$$\max_h \left\{ u\left(w\right) + \beta u\left[af\left(h\right) - Rh\right] \right\},$$

which is equivalent to maximizing late consumption, $c_1 = af(h) - Rh$. In this case, optimal investment equals the unconstrained amount $h^U(a)$.

Tying borrowing to investment removes the conflict between smoothing consumption and maximizing net lifetime resources. If (TIC) is the only constraint on credit, everyone ends up investing the unconstrained amount, $h^U(a)$, regardless of initial wealth. Only consumption decisions are distorted by the constraint. Empirical tests based on investment differences by family resources would always conclude that borrowing constraints are non-binding, even when consumption allocations are distorted. *Empirical tests must use measures of consumption over time to detect this constraint.*

Consider now the full GSL constraint (GSLC). For ease of exposition, assume that $d_{\max} = d_0$, so (EXC) coincides with (7). In this case, unconstrained individuals are those whose wealth exceeds the threshold $w_{\min}^G(a) \equiv \max\{w_{\min}^X(a), \tilde{w}_{\min}(a)\}$, which increases with ability a, because both $w_{\min}^X(a)$ and $\tilde{w}_{\min}(a)$ do. Let $\bar{a}$ denote the highest ability for which the unconstrained investment can be financed by the GSL alone, i.e. $h^U(\bar{a}) = d_{\max}$.

We now examine the relationship between investment, ability and wealth under a GSL program. Of course, those with wealth $w \geq w_{\min}^G(a)$ are unconstrained and invest $h^U(a)$. They are able to smooth consumption optimally. There are three potential groups of constrained individuals with $w < w_{\min}^G(a)$. The first group is composed of lower ability persons with $a < \bar{a}$ who are constrained by (TIC) only. They invest the unrestricted level $h^U(a)$ but would like to borrow more for consumption purposes. The second and third groups are composed of more able individuals with $a > \bar{a}$. The second group is constrained by (7) only. These individuals borrow $d_{\max}$ and invest more than this using some of their initial assets w to help finance schooling. For them, the GSL and exogenous constraint models are equivalent. Investment coincides with $h^X(a, w)$, because (TIC) is slack. The third group of very poor high ability youth is constrained by both (7) and (TIC). They borrow and invest the maximum amount, $d_{\max}$.

The previous discussion can be formally summarized as follows:

Proposition 2 *Assume that $u\left(\cdot\right)$ has IES ≤ 1. Let $d_{\max} = d_0 > 0$, and $h^G(a, w)$, $h^X(a, w)$, $h^U(a)$ be, respectively, the optimal human capital investment under the GSL, exogenous constraints, and when unconstrained. Let $\bar{a} > 0$ be defined by $h^U(\bar{a}) = d_{\max}$, and let $\hat{w} : [\bar{a}, \infty) \rightarrow \mathbb{R}_+$ be defined by $h^X\left[a, \hat{w}\left(a\right)\right] = d_{\max}$, the (possibly infinite) wealth level that leads an exogenously constrained individual with ability a to invest $d_{\max}$. Then:*

$$h^G(a, w) = \begin{cases} h^U(a) & a \leq \bar{a} \text{ or } w \geq w_{\min}^X(a) \\ h^X(a, w) & a > \bar{a} \text{ and } w < \hat{w}(a) \\ d_{\max} & \text{otherwise.} \end{cases}$$

Regardless of the IES, $h^G(a, w)$ always has a region in which it is increasing in ability, a, and independent of initial wealth, w, and may have another region in which it is constant and equal to $d_{\max}$.[26] If utility has an IES less than or equal to 1, there is a region (of middle-high abilities) in which investment decreases with ability as in the exogenous constraint model, but the additional constraint (TIC) shrinks this region.

Figures 3 and 4 illustrate the behavior of $h^G(a, w)$, $h^X(a, w)$, and $h^U(a)$ for the empirically relevant case of IES ≤ 1. These figures also display unconstrained borrowing as a function of ability for different levels of wealth. (Recall that $\bar{a}$ is the ability level satisfying $h^U(\bar{a}) = d_{\max}$, and let $\bar{w} \equiv w_{\min}^G(\bar{a})$ reflect the level of wealth below which agents of ability $\bar{a}$ are constrained.) Figure 3 displays investment and borrowing behavior for two low levels of wealth, $\bar{w}$ and a lower level $w_L < \bar{w}$. The figure reveals that under the GSL, all low-wealth individuals with ability below $\bar{a}$ invest the unrestricted level $h^U(a)$, while those with ability above $\bar{a}$ invest $d_{\max}$. Investment under the GSL for these individuals is increasing or constant in ability and independent of wealth (for any wealth $w \leq \bar{w}$). Contrast this with the behavior of $h^X(a, w_L)$, which is increasing in w and decreasing in a above a_2, the point at which $d^U(a, w_L) = d_0$. Investments are weakly higher under the GSL than under exogenous constraints.

Figure 4 illustrates investment behavior for a higher level of wealth $w_H > \bar{w}$. In this case, optimal investment is the same under the GSL and exogenous constraints up to ability level a_4. Hence, it first coincides with $h^U(a)$ until $d^U(a, w_H)$ reaches $d_{\max}$ at ability a_3; then it decreases in ability through a_4. Above this point, more able individuals are constrained to invest $d_{\max}$ by (TIC). Without this constraint, these individuals would invest less than $d_{\max}$ as in the exogenous constraint model.

Three points about investment under the GSL are worth highlighting. First, investment under the GSL equals the unconstrained level for a larger range of middle ability and low/middle wealth individuals than under exogenous constraints (e.g. individuals with wealth w_L and ability $a \in (a_2, \bar{a}]$ in Figure 3). While constraint (TIC) increases the number of constrained agents, it also encourages investment for those who would like to borrow more than they spend on schooling. This implies a positive relationship between investment and ability and no relationship between investment and wealth for a broader range of ability and wealth levels. Second, among higher ability and middle/high wealth individuals, the (TIC) restriction ensures that investment never falls below $d_{\max}$. With an IES less than one, this shrinks the range of abilities for which investment is negatively related to ability (e.g. individuals with ability $a > a_4$ in Figure 4). Third, among high ability types, investment is weakly increasing in initial assets (e.g. individuals with ability $a \in (a_3, a_4)$ in Figure 4).

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[26]If $h^X(a, w)$ is always increasing in a (e.g. for an IES sufficiently greater than one), then $h^G(a, w)$ is globally increasing in both arguments. The characterization is as follows: $h^G(a, w) = h^U(a)$, for $a \leq \bar{a}$ or $w \geq w_{\min}^X(a)$; $h^G(a, w) = d_{\max}$ for $a > \bar{a}$ and $w < \hat{w}(a)$ and $h^G(a, w) = h^X(a, w)$ otherwise. The flat region where investment equals $d_{\max}$ may not exist.

Figure 3: d^U, h^U, h^X, and h^G for low wealth individuals ($w \leq \bar{w}$)
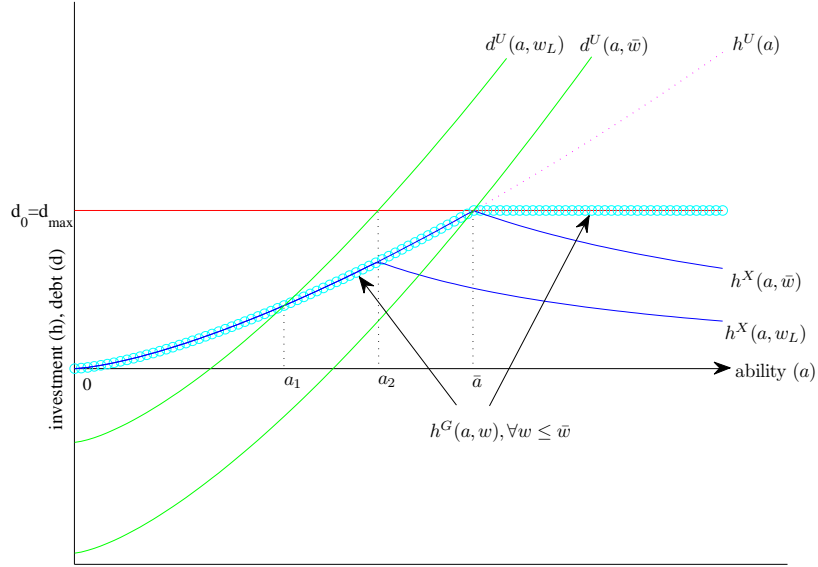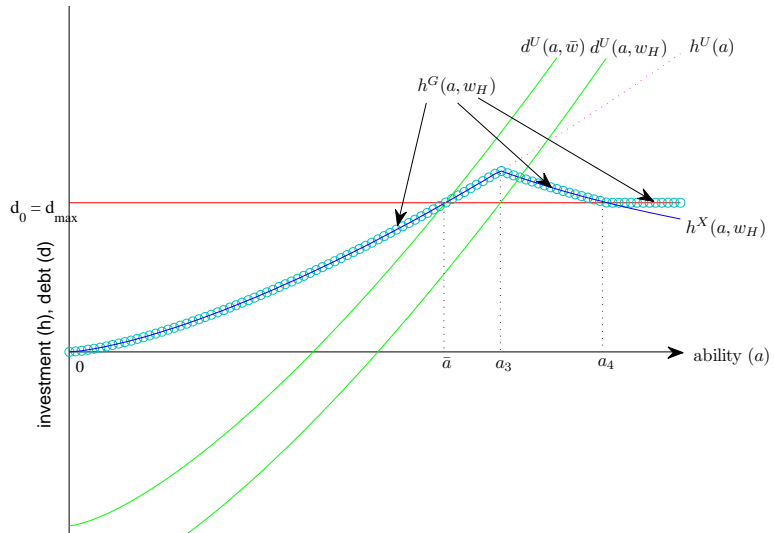

Figure 4: d^U, h^U, h^X, and h^G for high wealth individuals ($w > \bar{w}$)

Altogether, the implied investment – ability and investment – wealth relationships in the GSL model are closer to the empirical findings discussed earlier. In particular, the set of individuals whose investment declines with ability is smaller than in the traditional exogenous constraint model.

When $d_{\max} = d_0$, credit is more limited under the GSL than under the exogenous constraint, because the GSL imposes an additional restriction on borrowing. While the extra restriction (TIC) adversely affects utility and early consumption levels, it encourages investment relative to the exogenous constraint model. The following proposition compares the allocations and utility under the GSL, exogenous constraints, and unconstrained models:

Lemma 2 *Impose $d_0 = d_{\max}$ and let $\{h^m, c_0^m, c_1^m, U^m\}$, denote the optimal allocations and attained utilities in models $m = U, X, G$ for arbitrarily fixed $(a, w) \in \mathbb{R}_+^2$. Then:*

$$h^U \geq h^G \geq h^X, \qquad c_0^U \geq c_0^X \geq c_0^G, \qquad c_1^G \geq c_1^U \geq c_1^X, \qquad U^U \geq U^X \geq U^G,$$

and any of the inequalities is strict if the extra constraint between a pair of models is binding.

On one hand, the addition of (TIC) reduces the feasible set and hence attainable utility levels. On the other hand, it relieves the tension between income maximization and consumption smoothing that is inherent in the exogenous constraint model. This is particularly important for those who would like to borrow more than they want to invest. The GSL forces these individuals to invest more than they would otherwise choose and can dramatically distort their consumption profiles.

4.5 Private Lending under Limited Commitment

The inalienability of human capital and the lack of other forms of collateral are standard justifications for introducing borrowing constraints to models of human capital accumulation. Most often, however, the nature of credit limits that arise from these incentive problems is left unexplored. In this subsection, we consider constraints that arise from the limited commitment of borrowers to repay loans. In the following subsection, we consider the coexistence of private lenders that face limited commitment from borrowers with a GSL program that is perfectly enforced.

A rational borrower repays his loans if and only if the cost of repaying is lower than the cost of defaulting. The incentive to repay can be foreseen by rational lenders who, in response, limit their supply of credit.[27] Since penalties for default impose a larger monetary cost for borrowers with higher earnings and assets — only so much can be taken from someone with little to take — the credit offered to an individual is directly related to his perceived future earnings. Since earnings are determined by ability and investment, credit limits and investments will be jointly determined in equilibrium.

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[27]Gropp, Scholz, and White (1997) empirically support this form of response by private lenders.

To examine human capital in this environment, we first characterize the credit constraints that arise endogenously from limited commitment. To this end, we simply assume that lenders can punish defaulting borrowers by garnisheeing a fraction $\tilde{\kappa} \in (0, 1)$ of their earnings.[28] In Section 5.5, we incorporate additional punishments for default in a richer lifecycle model.

Given the punishment for default, repayment decisions are simple: borrowers repay (principal plus interest on a debt d) if the payment Rd is less than the punishment cost $\tilde{\kappa} a f(h)$. Foreseeing this, lenders choose to limit borrowing to:

$$d \leq \kappa a f(h), \qquad (8)$$

where $\kappa \equiv R^{-1} \tilde{\kappa} < R^{-1}$. We use the superscript L to refer to this model.

Individuals are unconstrained if their desired borrowing is compatible with its credible repayment, i.e. $d^U(a, w) \leq \kappa a f[h^U(a)]$. This condition implies that unconstrained individuals possess wealth above a finite threshold $w_{\min}^L(a)$. This threshold increases at a slower rate in a than does $w_{\min}^X(a)$, and it may even be decreasing in a if κ is large enough. Indeed, $w_{\min}^L(a)$ may be zero or negative for some values of a, in which case all individuals with those abilities are unconstrained. *In contrast with exogenous constraint models, a higher ability may reduce the likelihood that an individual is constrained.*

Consider constrained individuals who possess wealth $w < w_{\min}^L(a)$. Since (8) holds with equality, investment h determines consumption as given by $c_0(h) = w + \kappa a f(h) - h$ and $c_1(h) = a f(h)(1 - \kappa R)$. At low levels of investment, there is no tradeoff between early and late consumption. However, this is not true at higher levels, where the marginal returns to investment are lower and credit limits increase less with additional investments. Obviously, optimal investment, $h^L(a, w)$, must lie in the region where there is a trade-off. It equates the marginal cost of investing (the value of foregone early consumption) with the marginal benefit (the value of increased late consumption):

$$\{1 - \kappa a f'[h^L(a, w)]\} u'[c_0(h^L(a, w))] = \beta a f'[h^L(a, w)](1 - \kappa R) u'[c_1(h^L(a, w))].$$

With this condition, we characterize the implied ability – investment and wealth – investment relationships as follows:

Proposition 3 *Let $h^L(a, w)$ and $h^U(a)$ denote, respectively, optimal investment in human capital with credit constraints driven by limited commitment to repay loans and in the unrestricted allocation. If constraint (8) binds, then: (i) $h^L(a, w) < h^U(a)$, (ii) $h^L(a, w)$ is strictly increasing in w; (iii) a sufficient condition for $h^L(a, w)$ to be strictly increasing in a is that the IES is uniformly bounded below by $(1 - \kappa R)$; and (iv) if the IES is non-decreasing in consumption and $\beta R \geq 1$, then $h^L(a, w)$ is strictly increasing in a if $IES(c_0) \geq 1 - (1 + R)\kappa$.*

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[28] Penalty avoidance actions like re-locating, working in the informal economy, borrowing from loan sharks, or renting instead of buying a home are all costly to those who default and would contribute to $\tilde{\kappa}$.

As noted earlier, the responsiveness of credit limits to ability and investment creates a tendency for more able persons to be unconstrained. Proposition 3 shows that this responsiveness also creates a tendency for constrained investment $h^L(a,w)$ to be increasing in ability. In Section 5, we derive the implied values of κ within a more empirically plausible environment and find that for *any* reasonable values of the IES, this model produces a positive ability – investment relationship. Proposition 3 and its parallel in Section 5 show that this endogenous constraint model is qualitatively consistent with the two key cross-sectional patterns reported earlier.

Notice that the endogeneity of credit to investment reduces the marginal cost of investing for constrained individuals from 1 to $1 - \kappa a f'(h)$ units of current consumption. This should encourage more investment relative to the simple exogenous constraint model. The following proposition compares the two models at the same level of credit and shows that this is indeed the case.

Proposition 4 *Fix any* (a,w) *such that* $w < w^L_{\min}(a)$. *Let* $h^L(a,w)$ *and* $d^L(a,w)$ *denote, respectively, optimal investment and borrowing in the limited commitment model. Consider the allocations in an exogenous constraint model, where* $d_0 = d^L(a,w)$. *Then,* $w < w^X_{\min}(a)$ *and* $h^X(a,w) < h^L(a,w)$.

4.6   GSL Programs *Plus* Private Lenders

We now study the interaction of GSL programs with private sources of financing. To this end, we assume that loans from the GSL are fully enforced, while private lenders face limited repayment incentives.[29] This assumption, while strong, is motivated by the fact that GSL programs have much stronger punishments at their disposal than we observe for private unsecured lending. (See Section 2.)

With these two sources of credit, a young individual chooses human capital investments h, borrowing from the GSL d_g, and borrowing from private lenders d_p to maximize utility (1) subject to the sequential budget constraints

$$c_0 = w + d_g + d_p - h,$$
$$c_1 = af[h] - Rd_g - Rd_p,$$

the GSL lending guidelines

$$d_g \leq \min\{h, d_{\max}\},$$

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[29]Alternatively, consider the other extreme in which loans from the GSL face the same limited repayment incentives as private loans. In this case, the presence of the GSL program can only make a difference with respect to the allocations under limited commitment if some individuals default on GSL loans in equilibrium. One can show that the GSL is default-proof if $a_L \equiv \frac{Rd_{\max}}{\tilde{\kappa}f(d_{\max})}$ is less than or equal to the lower bound for the support of ability in the population and if the punishment $\tilde{\kappa}$ is an upper bound for the elasticity of the human capital production function (i.e. $f'(h)h/f(h)$). In this case, the GSL is completely redundant, and the allocations coincide with the model with only private lending.

and the repayment enforcement constraint for private lending

$$d_p \leq \kappa a f(h).$$

We refer to this case with the superscript $G + L$.

As with the previous models, for each ability level a, there is a finite threshold level of initial assets, $w_{\min}^{G+L}(a)$, above which an individual attains the unconstrained allocations. With both sources of credit, $w_{\min}^{G+L}(a) < \min\left\{w_{\min}^{G}(a), w_{\min}^{L}(a)\right\}$, so fewer individuals are constrained relative to either the GSL alone or private lenders alone. As with $w_{\min}^{L}(a)$, the threshold $w_{\min}^{G+L}(a)$ can be decreasing in a and may even be negative.

We first compare our model with both private and government lending with our model with only private lending. The introduction of a GSL program that fully enforces investments up to $d_{\max}$ leads individuals with ability levels $a \leq \bar{a}$ (i.e. $h^{U}(a) \leq d_{\max}$) to attain their unrestricted optimal investment amounts. For those with $a > \bar{a}$, the GSL ensures a minimum investment of $d_{\max}$. This investment ensures the repayment of at least $\kappa a f[d_{\max}]$ in private loans, which private lenders are willing to provide. The availability of extra resources allows for additional investment, which further increases the credit available from private sources. Individuals for whom $w_{\min}^{G+L}(a) < w < w_{\min}^{L}(a)$ invest more under private lenders and the GSL than with only private lenders.

We now compare our model with both private and government lending to the GSL alone. For those with ability $a \leq \bar{a}$, the GSL program provides enough credit to attain the unrestricted optimum investment. For them, the availability of private credit has no impact on investment, but the privately available amount $\kappa a f[h^{U}(a)]$ allows individuals with $w_{\min}^{G+L}(a) < w < w_{\min}^{G}(a)$ to optimally smooth consumption and to attain a higher level of utility. For those with $a > \bar{a}$, the GSL program does not provide sufficient funds for desired investment. At the very least, private lenders finance $\kappa a f[d_{\max}]$ in early consumption and investment. For those with $w_{\min}^{G+L}(a) < w < w_{\min}^{G}(a)$, investment is higher with private lenders and the GSL than with the GSL alone.

Proposition 5 *Let $h^{L}(a,w)$, $h^{G}(a,w)$ and $h^{L+G}(a,w)$ denote, respectively, optimal investment under private markets with limited commitment, fully enforced GSL, and with both sources simultaneously. Then: (i) $h^{L}(a,w) \leq h^{L+G}(a,w)$ and $h^{G}(a,w) \leq h^{L+G}(a,w)$. Moreover, the first inequality is strict if $w < w_{\min}^{L}(a)$ (i.e. constraint (8) binds) and the second inequality is strict if $a > \bar{a}$ and $w < w_{\min}^{G}(a)$. (ii) Let $h^{L+G}(a,w;d_{\max})$ denote optimal investment with both sources of credit and an upper GSL credit limit of $d_{\max}$. Then, $h^{L+G}(a,w;d_{\max})$ is strictly increasing in $d_{\max}$ when $a > \bar{a}$ and $w < w_{\min}^{G+L}(a)$.*

The implied relationship between human capital investment, ability, and wealth is as follows. If ability is low (i.e. $a < \bar{a}$), investment equals the unconstrained amount for any w. Among more able agents with $a \geq \bar{a}$, those with $w \geq w_{\min}^{G+L}(a)$ are unconstrained and

the rest are constrained. For the latter, $h^{L+G}(a, w)$ is less than $h^U(a)$ and increases with wealth, w. Under the conditions of Proposition 3, $h^{L+G}(a, w)$ is increasing in ability, and the model with both private lenders and the GSL is qualitatively consistent with investment patterns in the data. The existence of the GSL ensures the unconstrained optimal amount of investment for a broader range of ability and wealth levels, while the existence of private lenders delivers the empirically observed positive ability – investment relationship over the full distribution of abilities and wealth levels. Both sources of credit play an important role in determining investment.

Our preferred model for studying human capital formation includes both public and private sources of lending. First, both sources provide significant credit for higher education in the U.S. Second, incorporating the combined constraints of GSL programs and private lenders in our model produces patterns for investment, ability, and family resources that are qualitatively consistent with U.S. data. Moreover, as we now show, a calibrated model with both sources of credit performs well quantitatively.

5   A Quantitative Model

In this section, we extend our framework to a multi-period setting and incorporate education subsidies in order to explore the quantitative implications of alternative forms of borrowing constraints. After calibrating the model to match the U.S. economy, we compare the predicted cross-sectional patterns for investment, ability, and wealth under different assumptions about credit constraints. We also simulate an increase in both the returns to and costs of schooling (as observed over the 1980s and 1990s in the U.S.) in order to see whether our model with public and private lending can explain the rising importance of family resources (as a determinant of schooling) and private lending for college. Overall, the model performs well when compared against the empirical patterns discussed in Section 3.

5.1   The Model

Consider individuals with a lifespan of length $T > 1$. As of any $t_0 \in [0, T]$, the utility of an individual is

$$U(t_0) = \int_{t_0}^{T} e^{-\rho(t-t_0)} \left[\frac{c(t)^{1-\sigma}}{1-\sigma} \right] dt, \tag{9}$$

where $c(t)$ is consumption at t, $\sigma > 0$ is the inverse of the IES, and $\rho > 0$ is the discount rate.

Life is divided into three stages: "Youth", $t \in [0, 1]$, when individuals attend school and do not work; "maturity," $t \in [1, P]$, when they participate full-time in labor markets; and "retirement," $t \in [P, T]$, when they do neither and consume from accumulated savings.

At $t = 0$, individuals are endowed with financial assets $w \geq 0$ and an ability level $a > 0$.

Initial resources, w, can be seen as the present value of family transfers.[30] Ability, a, reflects genetic traits, early educational investments, and other individual characteristics that may determine the returns on investment.

For simplicity, we assume that the market interest rate equals ρ, so individuals prefer flat lifecycle consumption profiles when unconstrained. While we explicitly model different forms of credit constraints that apply during the "youth" investment period, we assume that credit markets are frictionless once individuals enter the labor market.[31] This greatly simplifies the analysis and allows us to focus on the effects of constraints most directly related to investment decisions (and to tightly link this lifecycle framework with that of our two-period model above).

Government subsidizes schooling in two ways. First, it provides all young persons with a free-of-charge investment flow $i_{pub} \geq 0$. Second, it matches every privately financed unit of investment with additional $s \geq 0$ units.[32] Hence, an individual that privately invests $x(t) \geq 0$, attains a total investment flow of

$$i(t) = i_{pub} + (1+s) x(t).\tag{10}$$

Schooling determines the total stock of human capital investment as of $t = 1$, h, with which a person enters the labor market:

$$h = \mu \int_0^1 e^{g_s(1-t)} i(t)\, dt.\tag{11}$$

Labor earnings, $y(t)$, depend on ability, a, total human capital investment, h, and labor market experience, $t-1$:

$$y(t) = \mu^{-1} a h^{\alpha} e^{g(t-1)}\tag{12}$$

for all $t \in [1, P]$. The constant g_s determines the relative productivity of investments over time during "youth". To simplify the exposition, and to abstract from tangential issues regarding the timing of investment, we assume that $g_s = \rho$. This allows us to focus on the total stock of investment, h, while ignoring the particular investment sequence that lead to h. The constant $\mu \equiv \rho / [e^{\rho} - 1]$ is introduced as a normalization to simplify some of the expressions below. Finally, $g \geq 0$ is the rate of return to labor market experience (or the rate of growth in earnings over the lifecycle).[33]

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[30]For simplicity of exposition, we will assume that human capital is produced from goods rather than time inputs. We could equivalently assume that human capital investment only requires time inputs and that an individual's total 'initial wealth', w, reflects family transfers plus the total discounted value of earnings he could receive if he worked (rather than attended school) full-time during "youth". In this case, private investment costs reflect any earnings foregone for school. Our calibration below implicitly assumes both goods and time investments are perfectly substitutable and combines these costs to determine total investment in human capital.

[31]Cameron and Taber (2004) make an analogous assumption in their framework.

[32]Given our assumptions below, the timing of 'free' investment will be irrelevant – only the total discounted value of all 'free' investment matters. All investment not provided free is subsidized at rate s.

[33]Our main theoretical results extend to the case where g is increasing in a (i.e. more able individuals have steeper wage profiles).

5.2 Unrestricted Allocations

Assuming frictionless competitive financial markets (with market interest rate ρ), an individual with ability a and initial assets w maximizes the $t_0 = 0$ value of (9) subject to the budget constraint:

$$\int_0^T e^{-\rho t} c(t) \, dt + \int_0^1 e^{-\rho t} x(t) \, dt \leq w + \int_1^P e^{-\rho t} y(t) \, dt. \tag{13}$$

Since the interest rate equals the discount rate, optimal consumption is constant over time. Also, since $g_s = \rho$, the optimal timing of investment is indeterminate. Without loss of generality, we can impose $x(t) = x \geq 0$ for all $t \in [0, 1]$, and then solve for the optimal x, given (12) and $h = i_{pub} + (1 + s) x$. Doing so, (13) simplifies to:

$$c \left[\frac{1 - e^{-\rho T}}{\rho} \right] \leq w + \frac{e^{-\rho}}{\mu} \left[a \Phi \left[i_{pub} + (1 + s) x \right]^\alpha - x \right], \tag{14}$$

where the constant Φ converts initial earnings into the present value of life-time earnings as of $t = 1$. (The expression for Φ, which depends on P, g and ρ, is shown in Appendix C.)

Optimal unconstrained investment, $h^U(a)$, maximizes the right-hand side of (14). Individuals with ability $a \leq a_0 \equiv \frac{[i_{pub}]^{1-\alpha}}{\alpha (1+s) \Phi}$ do not find it worth investing above the publicly provided amount, so $h = i_{pub}$ for them. Those with $a > a_0$ invest until the marginal return equals the (private) marginal cost. For all individuals, optimal investment in human capital is completely independent of consumption decisions and initial assets:

$$h^U(a) = \max \left\{ i_{pub}, \left[\alpha (1 + s) a \Phi \right]^{\frac{1}{1-\alpha}} \right\}. \tag{15}$$

As before, investment is solely determined by ability.

Using (14) and (15), the amount of debt that the individual carries when he enters the labor market at $t = 1$ is

$$d^U(a, w) = \left(\frac{1 - e^{-\rho}}{1 - e^{-\rho T}} \right) \mu^{-1} a \Phi \left[h^U(a) \right]^\alpha + \left(\frac{e^{-\rho} - e^{-\rho T}}{1 - e^{-\rho T}} \right) \left(\frac{x^U(a)}{\mu (1 + s)} - e^\rho w \right), \tag{16}$$

where the first term captures the fraction of life-time earnings that the agent would like to borrow and consume during youth and the second term relates debt to the gap between own resources, w, and out-of-pocket optimal investment, $x^U(a)$. From this formula, it can be verified that $-1 < \frac{\partial d^U(a,w)}{\partial w} < 0$ and $\frac{\partial d^U(a,w)}{\partial a} > \frac{\partial h^U(a)}{\partial a} \geq 0$ as in the two-period model.

5.3 Exogenous Borrowing Constraints

We now introduce exogenous credit constraints. As in the two-period model, assume that there is an upper bound on the amount of credit that an individual can accumulate while in school:

$$d \leq d_0, \tag{17}$$

where d is the accumulated amount of debt as of $t = 1$ and $0 \le d_0 < \infty$. As noted earlier, we assume that credit after $t = 1$ is unconstrained.

The budget constraint during youth is $\int_0^1 e^{-\rho t} \left[c(t) + x \right] dt \le w + e^{-\rho} d$, since own resources, w, plus debt, d, finance the flows of investment, x, and consumption, $c(t)$, for $t \in [0, 1]$. During youth, consumption will be constant, denoted c_0, since the interest rate is equal to the discount rate and the constraint (17) does not distort the intertemporal allocation of consumption within the interval $[0, 1]$. Using these results, the budget constraint during youth simplifies to

$$c_0 + x \le \mu e^\rho \left[w + e^{-\rho} d \right]. \tag{18}$$

After school (i.e. in the time interval $[1, T]$), consumption is also constant at the (potentially different) level c_1, since post-schooling financial markets are frictionless. The value of c_1 is determined by $\mu^{-1} a \Phi h^\alpha - d$, the difference between the present value of lifetime earnings and the financial liabilities carried from youth. In Appendix C, we show that $V(h, d; a)$, the person's utility as of $t = 1$, is

$$V(h, d; a) = \Theta \frac{\left[\mu^{-1} a \Phi_a h^\alpha - d \right]^{1-\sigma}}{1 - \sigma},$$

where $\Theta \equiv \left(\left[1 - e^{-\rho(T-1)} \right] / \rho \right)^\sigma > 0$. Discounted lifetime utility at $t = 0$ is

$$U(c_0, x, d; a) \equiv \frac{e^{-\rho}}{\mu} \frac{c_0^{1-\sigma}}{1 - \sigma} + e^{-\rho} V \left(\left[i_{pub} + (1 + s) x \right], d; a \right). \tag{19}$$

Individuals choose $x \ge 0$, $c_0 \ge 0$ and d to maximize $U(c_0, x, d; a)$ subject to (17) and (18). Aside from the possibility of $x = 0$, this problem is analytically identical to the corresponding problem in the two-period model, and Proposition 1 holds.

5.4 Government Student Loan Programs

As with exogenous constraints, an analysis of GSL programs in this environment is straightforward and follows that of our two-period model. Instead of (17), cumulative debt as of $t = 1$, d, is restricted to satisfy:

$$d \le \min \{ x, d_{\max} \}. \tag{20}$$

Note that borrowing is tied to out-of-pocket investment, x, and not to total investment, h.

Individuals choose $x \ge 0$, $c_0 \ge 0$, and d to maximize $U(c_0, x, d; a)$ subject to (20) and (18). Aside from the link of d to x instead of h, and the possibility of $x = 0$, this problem is analytically identical to the corresponding problem in the two-period model. Proposition 2 holds. See details in Appendix C.

5.5 Private Lending with Limited Commitment

Now, consider private loans for schooling that are subject to limited enforcement. As in the two-period model, loans are repaid if and only if the cost of defaulting is higher than the

cost of repaying. The incentives to repay at dates $t \geq 1$ define the amount of credit lenders are willing to supply during the schooling period.

We consider two penalties for default. First, defaulting borrowers are reported to credit bureaus, an action that is assumed to prevent the borrower from accessing formal credit markets for some period. This penalty does not reduce earnings, but it disrupts the ability to smooth consumption and can be quite costly if labor earnings grow quickly with age or if the IES is low. Second, the borrower forfeits a fraction $\gamma \in [0,1)$ of his labor earnings. The fraction γ encompasses direct garnishments from lenders as well as the costs of actions taken by the borrower to avoid direct penalties (e.g. working in the informal sector, renting instead of owning a house, etc.). We assume that both penalties are active for an interval of length $0 < \pi < P - 1$ that starts the moment default takes place.[34]

Consistent with our earlier assumptions about post-investment credit markets, we assume that loans contracted after schooling are fully enforceable and that loans contracted while in school can only be defaulted on at age $t = 1$. We explicitly focus on credit constraints directly related to the financing of schooling.[35]

The amount of debt that a person can credibly commit to repay depends on the discounted utility associated with default. Consider a person with ability a and human capital h that defaults at $t = 1$ on debt d. Since punishments are not reduced by partial re-payment, all defaults would be on the entire debt. During the punishment period $[1, 1 + \pi]$, consumption is $c(t) = (1 - \gamma) \mu^{-1} a h^\alpha e^{g(t-1)}$. From $t = 1 + \pi$ onwards, a fresh start allows the person to fully smooth consumption, including the time after retirement. The maximized $t = 1$ discounted utility of a person who chooses to default at the end of the schooling period is

$$V^D(h; a) = \hat{\Theta}_{\gamma, \pi} \frac{[\mu^{-1} a \Phi h^\alpha]^{1-\sigma}}{1 - \sigma},$$

where $\hat{\Theta}_{\gamma, \pi}$ is a positive constant, the expression for which is shown in Appendix C.

Rational lenders foresee the repayment incentives of borrowers and restrict credit to avoid triggering default. Given penalties (π, γ) a borrower with ability a and human capital investment h is better off repaying a level of debt d when $V^D(h; a) \leq V(h, d; a)$. In our setting, this is equivalent to:

$$d \leq \kappa \mu^{-1} a \Phi [i_{pub} + (1 + s) x]^\alpha, \tag{21}$$

where $\kappa \equiv 1 - \left[\hat{\Theta}_{\gamma, \pi} / \Theta \right]^{\frac{1}{1-\sigma}} \geq 0$ incorporates the effects of both the garnishment and distortions to consumption profiles. Notice that credit limits are proportional to labor earnings, just as we imposed in the two-period model. However, in this model, the value of κ is determined by preferences (ρ, σ) and institutions (γ, π). Below we show that κ can be large even if wage garnishments, γ, are negligible.

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[34]Livshits, MacGee, and Tertilt (2007) make a similar set of assumptions in modelling U.S. bankruptcy regulations.

[35]Monge-Naranjo (2007) considers a continuous time model in which the agent can default in any period.

Some aspects of the determination of κ are worth mentioning. First, κ is increasing in γ and π. The option to default is less tempting with harsher punishments. Second, $\kappa > 0$ as long as $\pi > 0$, even if $\gamma = 0$. The exclusion from financial markets alone suffices to sustain lending. Third, if $\pi = 0$, the model boils down to an exogenous constraint model with $d_0 = 0$, since no lending can be sustained in equilibrium, i.e. $\kappa = 0$.

With credit limits determined by limited commitment, a person chooses investment, consumption and borrowing ($x \geq 0$, $c_0 \geq 0$ and d) to maximize $U(c_0, x, d; a)$ subject to (18) and (21). Given the endogenously determined κ and ignoring the possibility of $x = 0$, this problem is analytically identical to the two-period case and a parallel to Proposition 3 holds.

Proposition 6 *Let ability and financial assets of a young individual be (a, w), and let $h^L(a, w)$ and $h^U(a)$ indicate, respectively, the optimal investments in human capital with private lenders with limited commitment and in the unrestricted allocation. If $a \leq a_0$, then $h^L(a, w) = h^U(a) = i_{pub}$. If instead $a > a_0$ and constraint (21) binds, then: (i) $h^L(a, w) < h^U(a)$; (ii) $h^L(a, w)$ is strictly increasing in w; and (iii) $h^L(a, w)$ is strictly increasing in a if $\kappa \geq \underline{\kappa}(\sigma) \equiv [(\sigma - 1)/\sigma] [(1 - e^{-\rho})/(1 - e^{-\rho T})]$.*

This proposition is central to our quantitative analysis of credit constraints, given its empirically verifiable predictions.

Recall from our discussion in Section 4.3 that strong preferences for smooth consumption (i.e. a high σ or low IES) generate a negative ability – investment relationship when credit constraints are exogenously fixed. This tendency also exists when constraints are endogenous since $\partial \underline{\kappa}(\sigma)/\partial \sigma \geq 0$, which implies that a stronger link between investment and credit limits (i.e. a larger κ) is needed to generate a positive ability – investment relationship as preferences for smooth consumption become stronger. However, a greater preference for smooth consumption profiles also makes the default punishment of exclusion from credit markets more painful. Private lenders will be willing to offer more credit if the cost of defaulting is higher. Thus, κ is also increasing in σ under limited commitment. Below, we show that for empirically plausible punishment parameters (γ, π) the effect of σ on κ often dominates its effect on $\underline{\kappa}(\sigma)$, and a higher value of σ (or lower IES) makes it more rather than less likely that condition (iii) of this proposition holds. Most importantly, we show that $\kappa > \underline{\kappa}(\sigma)$ for empirically plausible parameters, so that the model implies a positive relationship between investment and ability for all ability and initial wealth levels.

5.6   GSL Programs *Plus* Private Lenders

Finally, we consider the coexistence of a GSL program with private lenders. As before, we assume that the repayment of loans from the GSL program is fully enforced. As noted earlier, this assumption is in line with the fact that GSL programs in the U.S. are better protected against default than private unsecured loans. First, private loans can be cleared

in bankruptcy proceedings while GSL loans cannot. This implies a longer (potentially un-limited) punishment period for government loans. Second, wage garnishments of up to 15% are explicitly incorporated in GSL programs, whereas no explicit rate exists for private un-secured loans. Third, GSL programs include a wide array of additional punishments for those who default that have no counterpart in the private sector (e.g. governments can seize income tax returns for those defaulting on a GSL program loan).

With both public and private sources of credit, a young person with (a, w) chooses out-of-pocket investment x, consumption during youth c_0, borrowing from GSL d_g, and borrowing from private lenders d_p to maximize $U(c_0, x, d; a)$ subject to (18) and

$$d \leq d_p + d_g \tag{22}$$

$$d_p \leq \kappa \mu^{-1} a \Phi \left[i_{pub} + (1 + s) x \right]^\alpha, \tag{23}$$

$$d_g \leq \min \left\{ x, d_{\max} \right\}. \tag{24}$$

The threshold level of initial assets $w_{\min}^{G+L}(a)$ above which individuals with ability a are unconstrained is given in Appendix C. As with the two-period model, this threshold is lower and fewer people are constrained than under the GSL alone or under private markets alone. Proposition 5 of the two-period model also applies in this setting.

5.7 Calibration

We now calibrate parameter values to explore the quantitative implications of the model. Some of the parameters are estimated using data on earnings and educational attainment from the random sample of males in the NLSY79. Other parameter values are calibrated to replicate features of the U.S. economy. We use AFQT quartiles to measure ability. All dollar amounts are denominated in 1999 dollars using the Consumer Price Index (CPI-U). Table 3 reports the value of all parameters used in our baseline simulations.

We assume that youth (investment period) begins at age 16 and ends at age 24. Maturity (labor market participation period) runs from age 24 until age 65. Retirement runs from age 65 until death at age 80. Since youth lasts one period in the model, each interval of unit length corresponds to 8 years of life. Dates for retirement and death are, respectively, $P = (65 - 16)/8 = 6.125$, and $T = (80 - 16)/8 = 8$.

To match an annual interest rate of 4%, we set $\rho = 8 \times ln(1.04) = 0.319$ for the value of the discount rate in the model. We set $\sigma = 2$ as our baseline value to match an IES of 0.5, an intermediate value in the estimates in Browning, Hansen, and Heckman (1999). Values of σ inside the interval $[1.5, 3]$ yield similar results.

Based on U.S. bankruptcy regulations, we set $\pi = 7/8 = 0.875$ (7 year penalty period) as the baseline length of penalties. Also, we set $\gamma = 0.1$ for the fraction of lost earnings for individuals who default. Under the GSL program guidelines, defaulting borrowers face an explicit 15% wage garnishment. For private unsecured loans, an explicit garnishment rule

Table 3: Baseline Model Parameters

Calibrated Parameters			Estimated Parameters (from Log Earnings)		
Parameter	Value	To match:	Parameter	Value	Estimates for:
P	6.125	Retirement at 65	g	0.369	Experience
T	8	Lifespan of 80	α	0.432	Schooling investment
π	0.875	7 yrs., U.S. Bankruptcy			
ρ	0.3138	Annual rate = 4%			Ability Levels:
σ	2	IES = 0.5	$\tilde{a}_1$	106.70	AFQT quartile 1
γ	0.1	Garnishment & other costs	$\tilde{a}_2$	137.83	AFQT quartile 2
i_{pub}	65,239	Educ. costs through grade 9	$\tilde{a}_3$	157.38	AFQT quartile 3
s	1.19	Educ. subsidy grades 10+ = 54%	$\tilde{a}_4$	158.29	AFQT quartile 4

does not exist. However, actual costs of default — either via direct penalties or via avoidance actions — extend beyond simple garnishments (e.g. individuals may end up suboptimally employed, renting instead of owing a house, and paying subprime interest rates for short-term transactions, etc.) Since the implied κ varies little with γ, our results are not sensitive to reasonable variations in this parameter.

We assume all investment through age 16 is publicly provided for free. After age 16, schooling entails direct costs (i.e. tuition and public expenditures on primary, secondary, and post-secondary schooling) and indirect costs (i.e. foregone earnings). To compute direct costs we use an annual government expenditure of $5,928$ for primary and secondary schooling. Annual direct expenditures for college and graduate education are assumed to equal $16,838$.[36] (College expenditures include government expenditures as well as tuition and fees paid by students.) We set $i_{pub} = \$65,239$, which is equal to the discounted value of all direct schooling expenditures through grade nine.[37] This is consistent with our focus on investments made from age 16 onwards.

Because of the laws on compulsory schooling and minimum work age, we only include foregone earnings as part of investment costs for grades ten and above. To estimate foregone earnings, we use data from the NLSY79 to regress log earnings on indicators for each possible year of completed schooling from grade 10 through six years of post-secondary studies, indicators for AFQT quartiles, total years of potential work experience and experience-squared.[38] From this regression, we compute foregone earnings for $S \geq 10$ years of schooling

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[36]Annual expenditure for education through grade twelve (for college and graduate education) is the average of annual current expenditures per pupil for public primary and secondary schools (for all two and four year colleges) over the academic years 1979-80 through 1988-89 as reported in Table 170 (Table 342) of the *Digest of Education Statistics, 1999*. These years roughly reflect the years our NLSY79 sample respondents made their final schooling decisions.

[37]We use a 4% annual discount rate, reporting the value discounted to the end of grade nine. Less than 0.2% of our NLSY79 sample acquired less than 10 years of school.

[38]This regression uses all available earnings observations for male respondents with at least nine years of

using the predicted earnings of someone with nine years of completed schooling, $S - 10$ years of potential work experience, and the desired AFQT quartile. These foregone earnings estimates are included in our estimates of total schooling expenditures and are reported in Table D1 of the Appendix.

We calibrate the government subsidy rate s as follows. We assume that the private costs of investment are foregone earnings plus a fraction of post-secondary tuition and other direct costs. Table 333 of the *Digest of Education Statistics (2003)* reports that tuition and fees accounted for 20% of total current-fund revenue for degree-granting higher education institutions in 1980. Assuming a ratio of 0.20 for private to total direct expenditures for college, the subsidy rates for investment education beyond i_{pub} range from 0.47 to 0.6 for completed schooling levels 12–16 depending on the AFQT quartile and completed years of schooling. As our baseline, we use a subsidy rate of 54% (i.e. $s = 1.19$) based on the average government subsidy rate for individuals completing 2 years of college; however, our results are very similar when we use other reasonable values.

We estimate the parameters g and α of the earnings equation using data from the NLSY79. From the model, the wage earnings of someone with ability a who invested h and has been working τ periods is $y(\tilde{a}, h, \tau) = \tilde{a} h^\alpha e^{g\tau}$ where $\tilde{a} \equiv a/\mu$. This is log-linear, so we regress log earnings for individual i on AFQT quartile indicators (A_i), estimated total schooling expenditures (h_i) as reported in Table D1, and years of experience ($\tau = age - 24$):

$$ln[y_{i\tau}] = \beta_0' A_i + \beta_1 ln(h_i) + \beta_2 \tau + \nu_{i\tau},$$

where $\nu_{i\tau}$ is a mean zero idiosyncratic earnings shock, i.e. $E(\nu_{i\tau}|A_i, h_i, \tau) = 0$. The implied estimates for α and g are, respectively, $\hat{\alpha} = \hat{\beta}_1$ and $\hat{g} = 8\hat{\beta}_2$ (recall that a unit time interval in the model corresponds to 8 years in the data.) Even though $\nu_{i\tau}$ is mean zero, $E[e^{\nu_{i\tau}}] > 1$, so we adjust the coefficient vector $\hat{\beta}_{0q}$ on AFQT quartiles using the sample average $\overline{e^{\hat{\nu}_{i\tau}}}$. That is, our ability estimate for quartile q is $\tilde{a}_q = e^{\hat{\beta}_{0q}} \overline{e^{\hat{\nu}_{i\tau}}}$. These ability estimates range from 106.7 for the least able to 158.3 for the most able, suggesting that, for the same schooling, the most able are on average about 50% more productive than are the least able.

Ideally, we would like to specify a joint distribution of wealth and ability to simulate our model and compute the distribution of investment in the economy. Unfortunately, this is not feasible. We do not directly observe the assets available to youth, because they not only depend on their parents net worth and income but also on intra-family transfers, which are not always observed. Modelling these transfers is beyond the scope of this paper. Instead, we analyze investment behavior for our estimated ability types and a range of potential wealth levels. While we cannot compare moments for investment implied by the model with those

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completed schooling when they were ages 16-24 and no longer enrolled in school. Potential work experience is measured as age - years of completed schooling - 6. The estimates (available upon request) suggest that earnings for these young workers are generally increasing in years of completed schooling and increasing and concave in potential work experience.

observed in the data, we can explore whether investment is increasing in ability and wealth over reasonable ranges of wealth, how investment behavior depends on the type of constraints we assume, and how investment and borrowing (as functions of ability and wealth) respond to changes in the economy.

Because foregone earnings are an important part of investment expenditures in our calibration, an individual's initial wealth, w, includes at least the amount he could earn if he left school after grade 9 and began working. This amount depends on ability, since foregone earnings depend on ability (see Appendix Table D1). The relevant range of initial wealth, therefore, begins at \$52,000 for the least able, \$74,000 for AFQT quartile 2, and \$80–84,000 for the top two quartiles. Any wealth levels above these amounts must come from parents or other outside sources.

5.8   Baseline Simulations

We are primarily interested in the implied cross-sectional relationship between ability, wealth, and investment in our model with both the GSL and private lenders. We begin with a discussion of borrowing constraints in this environment and then discuss investment behavior.

Figure 5 shows a very strong result: for any value of $\sigma \geq 0$, the limited commitment model implies a positive relationship between investment and ability given our values for γ and π.[39] The figure displays the value of κ (the fraction of future earnings that can be borrowed from private lenders) associated with different values of σ under our baseline parameterization (thick green line) and under alternative assumptions about punishments γ and π. The figure also displays $\underline{\kappa}(\sigma)$ as defined in Proposition 6 (dashed line). When $\kappa \geq \underline{\kappa}(\sigma)$, investment is increasing in ability. Notice, κ exceeds $\underline{\kappa}(\sigma)$ for any value of σ under our baseline parameters. This is also true if the only penalty for default is a seven-year exclusion from financial markets ($\pi = 7/8$, $\gamma = 0$) or if the exclusion period lasts only one year ($\pi = 1/8$) and $\gamma = 0.1$. When no penalties for default exist (i.e. $\pi = \gamma = 0$), the limited commitment model is equivalent to an exogenous constraint model with $d_0 = 0$, and there is only a positive relationship between ability and investment if $0 \leq \sigma < 1$ (IES > 1).

As discussed earlier, both κ and $\underline{\kappa}(\sigma)$ are increasing in σ. Over most of the empirically relevant range where $\sigma > 1$ (IES < 1), an increase in σ makes it more rather than less likely that the condition in Proposition 6 is met and investment is increasing in ability.

Figure 6 shows the implied borrowing constraints as a function of individual investment (not including government subsidy amounts) for the GSL program. The figure also shows the private lending constraints as a function of individual investment for all four ability groups. While offering sizeable loans to students, private lenders do not offer as much as the GSL program over a wide range of investment amounts. At any level of investment, the difference in private lending limits between the most and least able is sizeable, ranging from about

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[39]Since the IES equals the inverse of σ, the empirically relevant range is $\sigma \geq 1$.

Figure 5: Sufficient Condition for a positive ability-investment relationship in the L model.
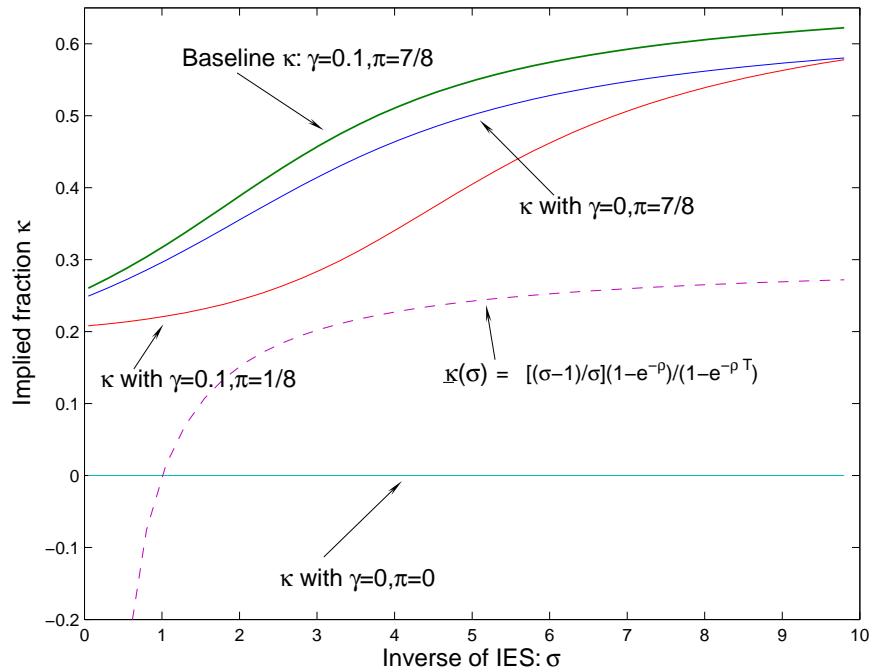
$5,000 to $10,000. These limits are also strongly increasing in investment (more so for the most able).

The amount of borrowing from private lenders as a function of ability and initial assets is shown in Figure 7 for our baseline economy with both private lending and a GSL program. We assume that individuals borrow first from the GSL program and then, when this source is exhausted, they may borrow from private lenders. As one can see, our calibrated model implies that private borrowing should be negligible during the baseline period (i.e. for NLSY79 cohorts). As noted earlier, initial assets for all individuals are at least as high as the potential earnings they would receive if they quit school after grade 9: $52,000 for the lowest AFQT quartile, $74,000 for the second lowest, and $80–84,000 for the top two quartiles. For all ability quartiles, borrowing from private lenders is zero for asset levels above potential school-period earnings.

For all ability types, individuals with initial assets above $40,000 (far below potential school-period earnings) invest the unconstrained optimal amounts. As a result, investment is increasing in ability and independent of initial assets, consistent with reported schooling patterns in the NLSY79 data. Optimal total human capital investment ranges from roughly $85,000 for the least able to $130,000 for the second ability quartile to around $165,000 for the top two ability quartiles.[40]

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[40]With private lending markets alone, the most able are constrained up through initial asset levels of around $85,000. With the GSL alone or with exogenous constraints, the most able are constrained through initial asset levels of around $70,000. In all cases, investment is unconstrained for asset levels above potential

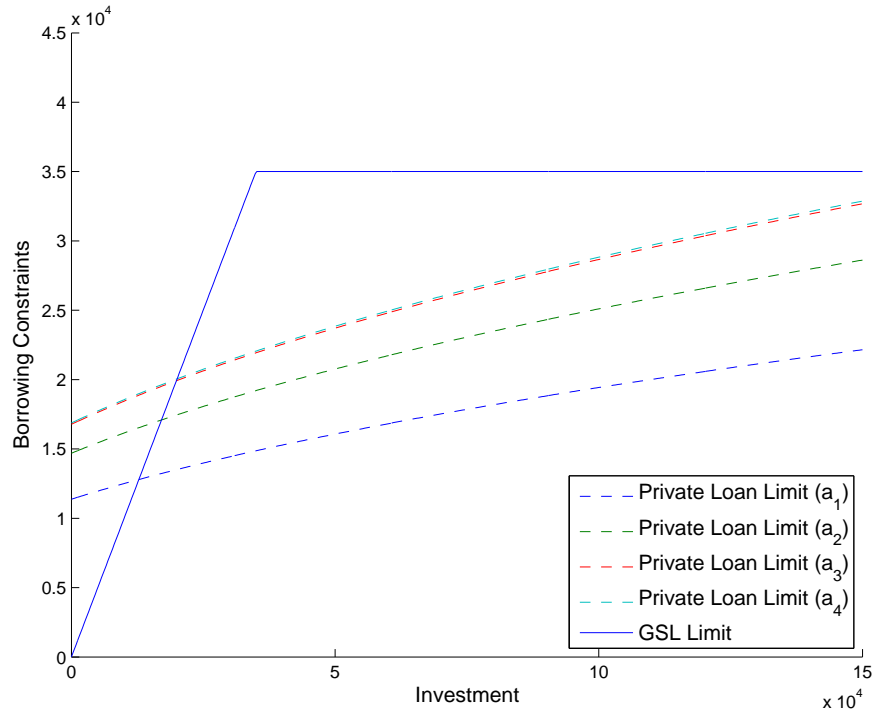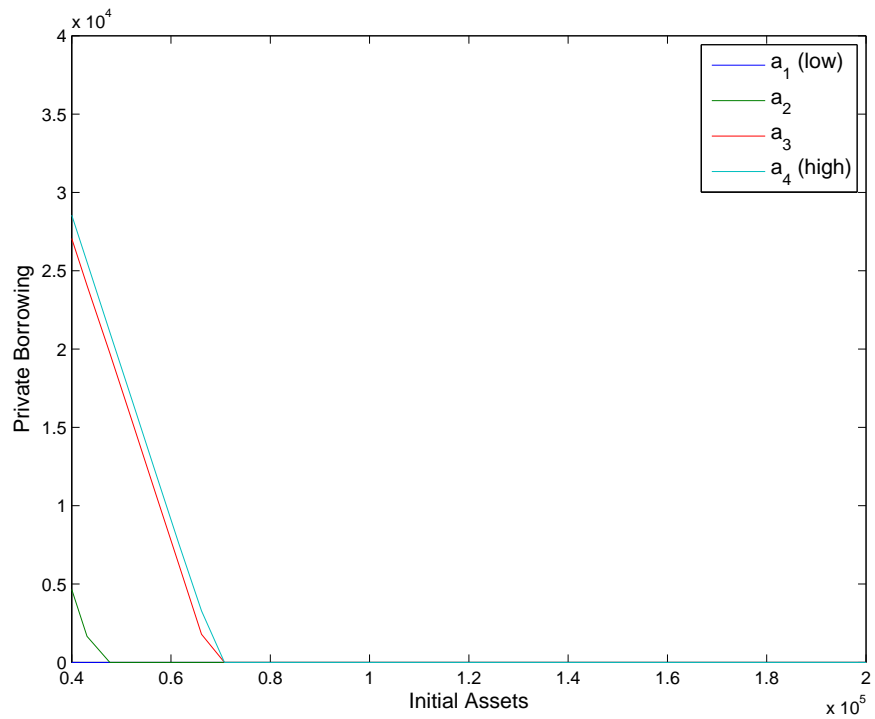Figure 6: GSL and Private Lending Constraints (Baseline Economy)

Figure 7: Private Borrowing (Baseline Economy)

It is noteworthy that the investment amounts implied by the model are fairly close to average total expenditures by AFQT quartile in the NLSY79 data, even though we did not target these values.[41] This external validity provides additional confidence in our model and the baseline parameterization.

5.9 A Rise in the Costs of and Returns to Schooling

We now simulate the effects of an increase in the costs of and returns to schooling — two major economic changes that took place between the early 1980s and early 2000s. We aim to see whether the model can reproduce the observed rise in private lending as well as the increased effects of family income on educational attainment. We also compare the investment and consumption allocations under different assumptions about credit markets. This sheds light on the importance of a GSL program for investment, as well as the role played by private lenders today. We also compare these environments with the standard model, which assumes borrowing constraints are exogenous.

We model an increase in the wage returns to education by assuming that α increases by 0.02 (from the baseline estimate of 0.432). This change produces a modest increase in the college – high school log wage differential. We model the rise in net tuition costs by assuming that the government subsidy rate, s, falls from 1.19 to 1.05. This reduction reflects the increased importance of tuition and fees as a fraction of total current-fund revenue for public and private universities in the U.S. Finally, we incorporate the stability of maximum GSL loan limits by assuming that $d_{\max}$ remains unchanged at $35,000$.

Figure 8 graphs the new private lending limits against the unchanged GSL limits. For all investment and ability levels, private credit limits increase by at least $3,000 over the baseline amounts, with much larger increases at higher investment amounts. This increase is entirely driven by the increased return to investment, which raises the monetary costs of default and, therefore, the amount students can commit to repay.

Youth wish to borrow more in response to increases in the costs and returns to investment. With no increase in credit available from the GSL, private lenders must cover the increased demand for credit. Indeed, Figure 9 shows that private lending increases substantially. While the least able youth still do not borrow from private lenders, high ability youth with low asset levels now borrow as much as $50,000.

Figure 10 shows investment behavior under the higher assumed costs and returns to school (i.e. $\alpha = 0.453$ and $s = 1.05$) for the four different models of credit constraints (GSL plus private lenders, private lenders alone, GSL alone, and exogenous constraints with $d_0 = d_{\max}$). First, panel (a) considers investment in our preferred model with a GSL program

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school-period earnings.

[41]Combining the total costs by AFQT quartile and schooling level reported in Table D1 with the distribution of educational attainment by AFQT in the NLSY79, we obtain average total investment amounts ranging from $88,000 for the least able to $178,000 for the most able.

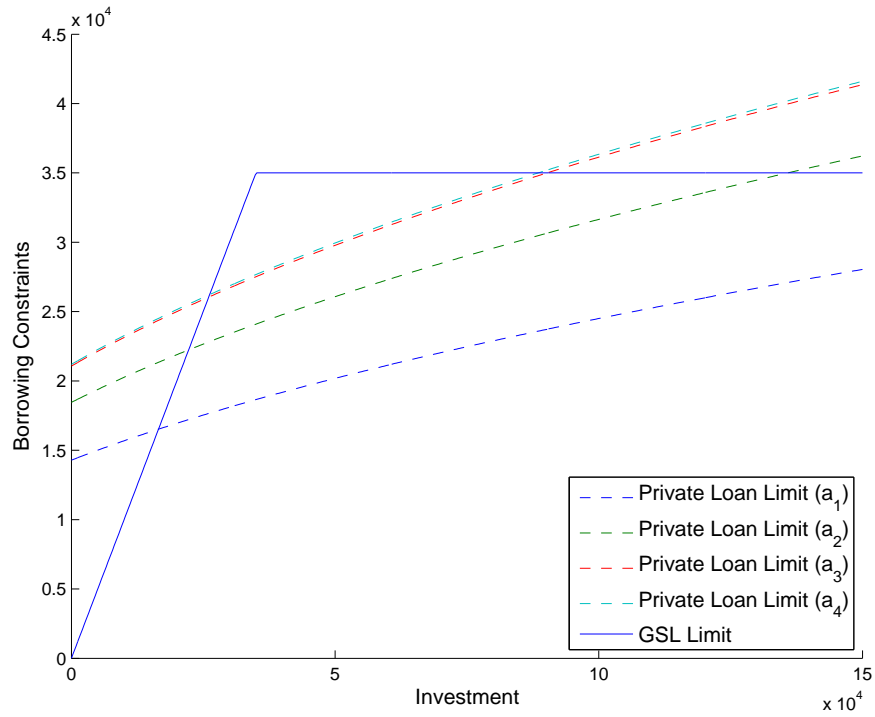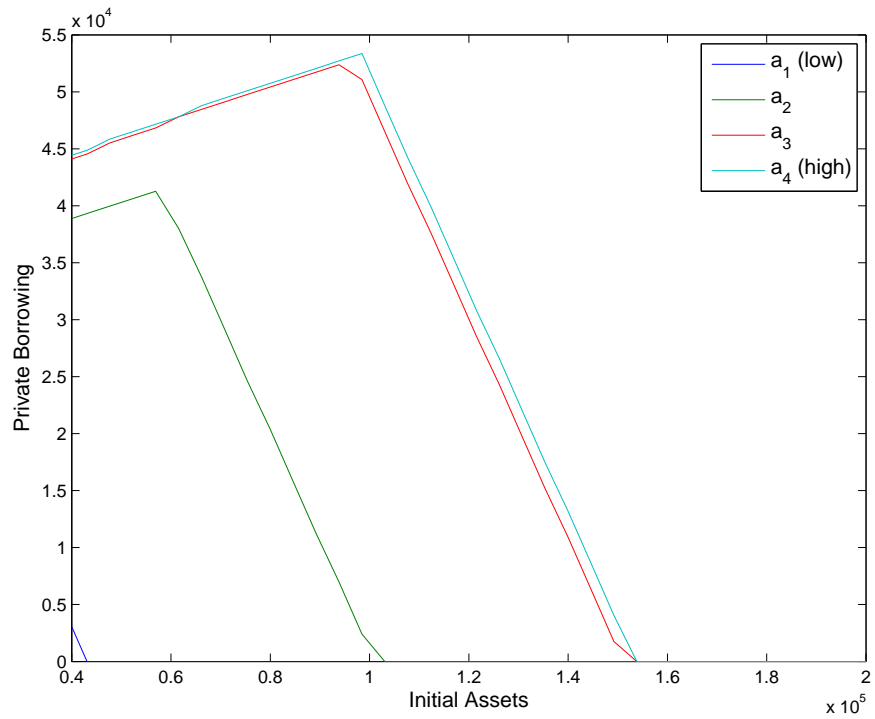Figure 8: 'Year 2000' GSL and Private Lending Constraints

Figure 9: Private Borrowing ('Year 2000')

and private lending. Investment increases noticeably relative to the baseline economy. This increase would have been even greater if private costs had not risen as well. Borrowing constraints now appear to be binding for a broad range of initial asset levels among the higher ability types. Individuals in the top two ability quartiles with assets below $100,000 ($15-20 thousand above potential school-period earnings) are constrained. Individuals in the second lowest ability quartile with initial assets below $60,000 (less than potential school-period earnings) are constrained. The least able are unconstrained for all reported levels of wealth. Consistent with the NLSY97 data, family resources have become more important for investment among a broader set of individuals.

Now, contemplate eliminating either source of credit. Panel (b) of Figure 10 considers private lending alone, while panel (c) considers the GSL program alone. In both cases, individuals from a much broader range of initial assets and abilities are constrained and invest less than in panel (a) when both sources are present. For most initial asset and ability levels, the private lending market and GSL yield fairly similar investment levels; however, this is not true for those with very low initial assets (amounts below potential school-period earnings). Under the GSL, these youth would invest the maximum amount they can borrow, $35,000, above the publicly provided amount. Investing less does not provide them with any more consumption while in school, since they would also be required to borrow less. Private lenders do not impose this tight restriction, so very poor youth would invest less even though they may be able to borrow more than under the GSL.

Notice that both models which incorporate private lending (panels a and b) imply a positive relationship between ability and investment for all levels of assets; although, investment is quite similar across ability types for very low asset levels (below potential school-period earnings). This is not the case for the GSL alone (panel c), since the upper limit on borrowing is the binding constraint for a broad range of initial asset levels and ability types. The perverse relationship between ability and optimal investment is even worse for the exogenous constraint model as shown in panel (d).

Finally, we show consumption during the investment period under all four credit market assumptions in Figure 11. Consumption is substantially higher when both the GSL and private lending markets are available than when either is not. As expected from our discussion of the GSL program in Section 4, consumption while in school is quite low for those with low initial assets. The fact that borrowing cannot be used to finance consumption under the GSL can be quite costly for the poor in the absence of private lending. All other forms of credit constraints allow for more intertemporal consumption smoothing, even if it is at the expense of lower investment.

Figure 10: Total Investment ('Year 2000') with Different Credit Market Assumptions

(a) GSL and private lending

(b) Private Lending

(c) GSL

(d) Exogenous borrowing limits
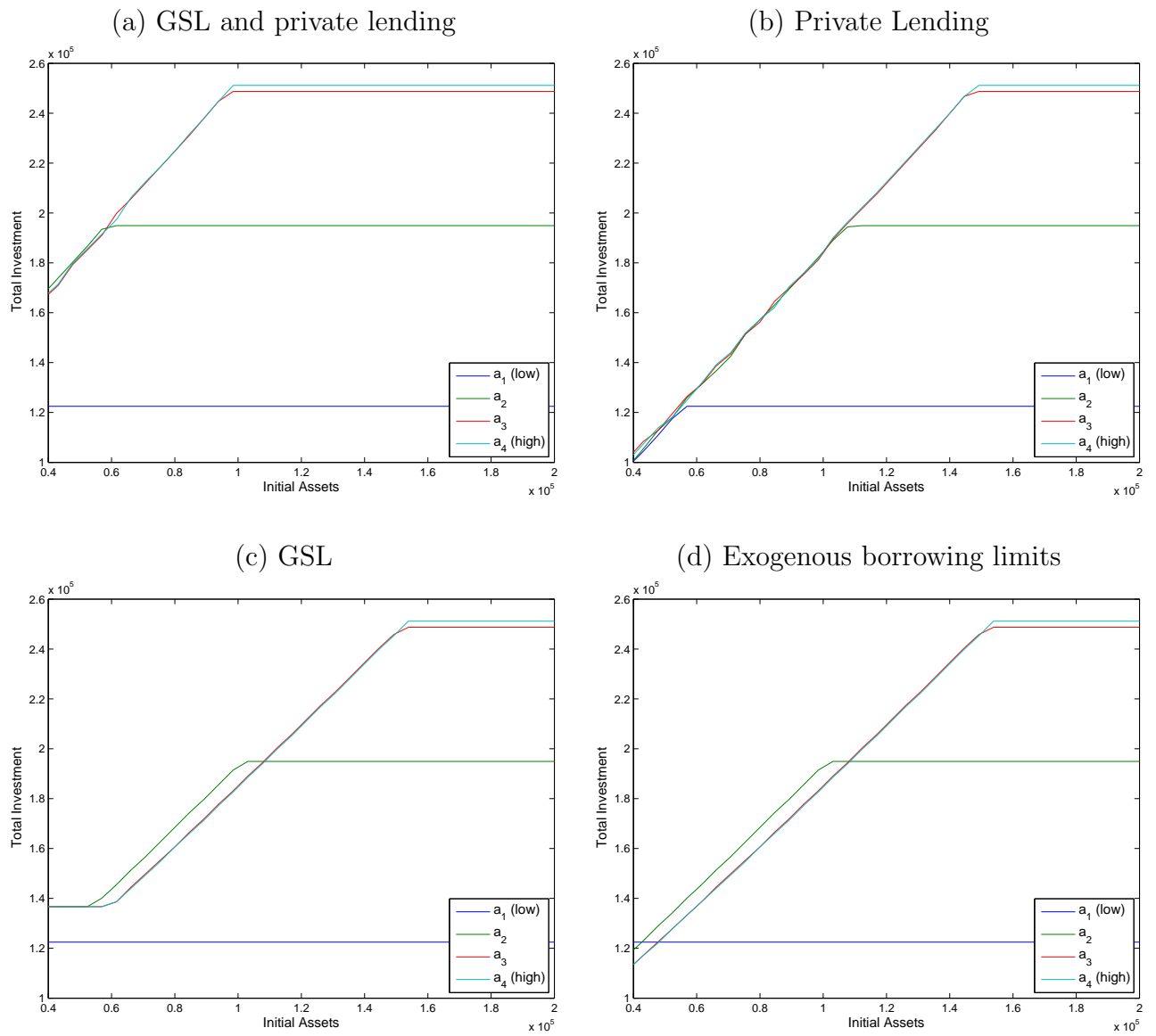
Figure 11: Consumption during Investment Period ('Year 2000') with Different Credit Market Assumptions

(a) GSL and private lending

(b) Private Lending

(c) GSL

(d) Exogenous borrowing limits

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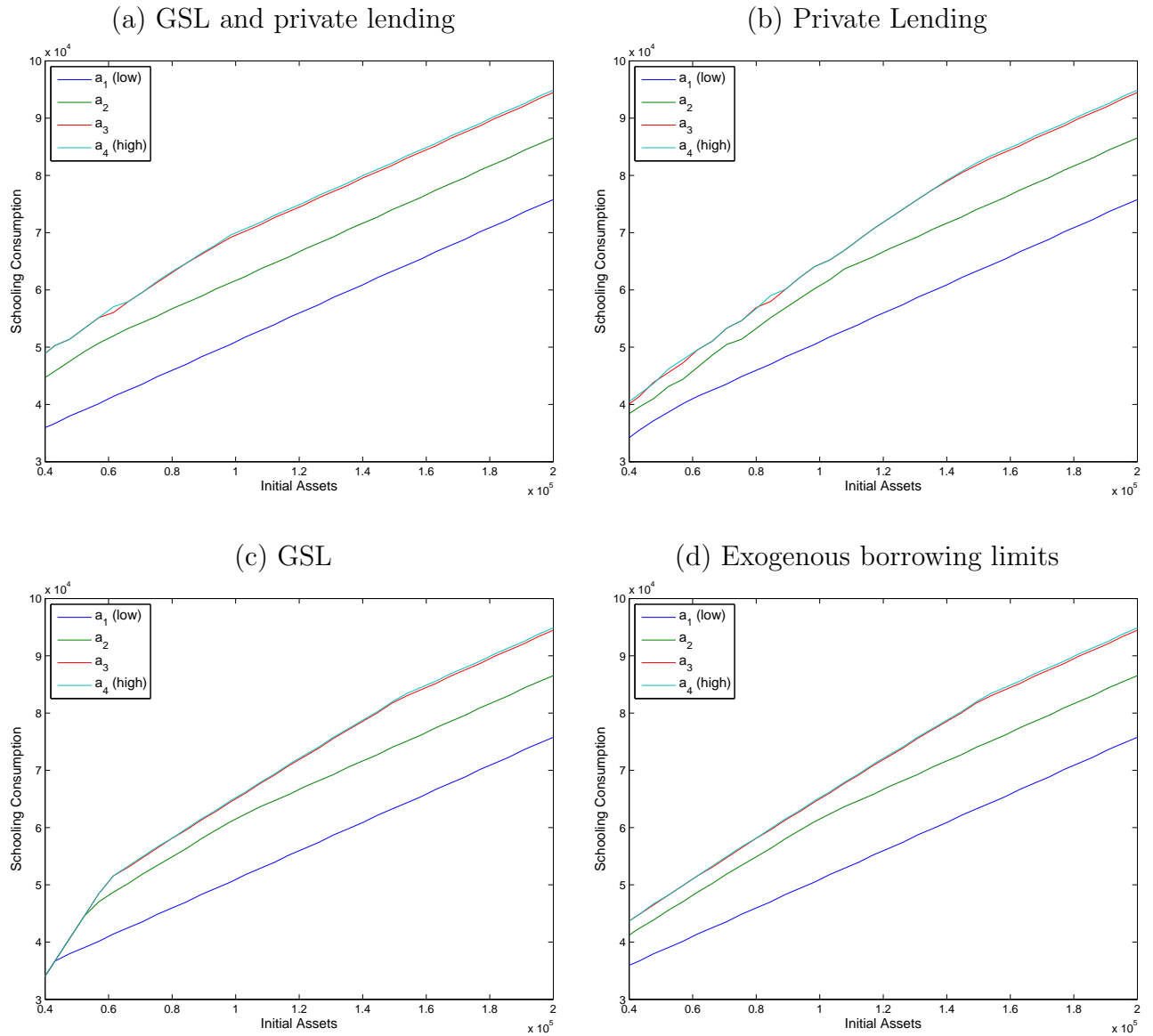
6 Conclusions

This paper develops a lifecycle human capital investment model that incorporates the borrowing opportunities of GSL programs and private lenders who face limited commitment by borrowers. Both types of lenders directly link credit to investment behavior, with private lenders further linking credit to observable borrower characteristics that determine investment productivity. These links are absent in previous models and we show that they play a central role in determining human capital investment behavior.

We draw three broad lessons. First, our model with endogenous borrowing constraints is consistent with empirical studies in that it implies positive effects of both family income and ability on schooling attainment among constrained borrowers. In contrast, under empirically plausible assumptions, a standard exogenous constraint model predicts a negative ability – schooling relationship for constrained borrowers. Second, the direct link between credit and investment inherent in GSL programs breaks the tradeoff between income maximization and consumption smoothing for some constrained borrowers. As a result, students constrained by GSL limits from borrowing more then they invest will choose to invest the unconstrained optimal amount. Previous empirical tests based only on educational attainment (or the marginal returns on investment) cannot detect this constraint. Third, our model is able to reproduce the increased effect of family income on college attendance, the increased fraction of students borrowing the maximum amount from GSL programs, and the increased student borrowing from private lenders over the last few decades as an equilibrium response to rising college costs and returns.

It is important to consider both GSL programs and private lenders when modelling human capital investment decisions. The features of GSL programs allow for the possibility that some student borrowers invest the optimal unconstrained amount even if they are constrained. For them, the existence of a private loan market allows for better smoothing of consumption over time. The presence of private lending generates a positive relationship between ability and investment for individuals from all income backgrounds – a robust empirical pattern. The co-existence of private and public sources of credit yields some important interactions. First and foremost, investment is higher when both sources are available than when only one or the other exists. Our quantitative analysis suggests that many more persons would be constrained in the absence of either GSL programs or a private student loan market. We also show that private loan limits should depend positively on the better enforced GSL credit limits. An increase in GSL loan limits may crowd out some borrowing from private lenders, but it will not cause private lenders to offer less credit.

We use an analytically tractable model to show that the main features of GSL programs and private lending under limited commitment are important for explaining observed investment patterns. An obvious next step is to introduce uncertainty about the returns to investment. While we do not expect such an extension to alter our predictions about the

relationship between investment, ability and family resources in any important way, incorporating uncertainty opens new and interesting areas of inquiry. With uncertainty about labor market success, the option of default provides insurance against adverse outcomes. Private lenders and governments must strike a balance between providing this insurance to borrowers and enforcing repayment. This defines an interesting optimal lending and enforcement policy, which may be complicated by the fact that students possess private information about their own abilities or willingness to study. Additionally, the existence of labor market uncertainty generally implies default by some agents in equilibrium. This makes it possible to study which agents are most likely to default and how economic changes and public policies affect default behavior. We view our framework as a natural starting point for these types of analysis.

We also suggest that future empirical efforts to estimate school-choice models consider the types of endogenous constraints and punishments we emphasize here. With reliable data on schooling, borrowing, earnings, and loan repayment (an admittedly tall order), structural estimation may be able to identify more general punishment strategies than we have assumed in this paper. Such an analysis would provide important new insights about the role of borrowing constraints, who is likely to be constrained, and how higher education policies and economic changes affect schooling and borrowing decisions.

Appendices

A   NLSY79 and NLSY97 Data

The NLSY79 is a random survey of American youth ages 14-21 at the beginning of 1979, while the NLSY97 samples youth ages 12-16 at the beginning of 1997.[42] Since the oldest respondents in the NLSY97 recently turned age 24 in the 2004 wave of data, we analyze college attendance as of age 21 in both samples.

Individuals are considered to have attended college if they *attended* at least 13 years of school by the age of 21.[43] For the 1979 cohort, we use average family income when youth are ages 16-17, excluding those not living with their parents at these ages. In the NLSY97 data, we use household income and net wealth reported in 1997 (corresponding to ages 13-17), dropping individuals not living with their parents that year.[44] We use AFQT as a measure of cognitive ability. It is a composite score from four subtests of the Armed Services Vocational Aptitude Battery (ASVAB) used by the U.S. military: arithmetic reasoning, word knowledge, paragraph comprehension, and numerical operations. These tests are taken by respondents in both the NLSY79 and NLSY97 during their teenage years as part of the survey process. We categorize individuals according to their family income, family net wealth (in NLSY97), and AFQT score quartiles.[45]

Our multivariate analysis controls for a host of family background variables. For both cohorts, we control for maternal education by categorizing mothers as high school dropouts, those who completed high school or more, and those who completed at least one year of college. We account for family structure in the NLSY79 by controlling for the number of siblings the youth reported in 1979. For the NLSY97, we control for the number of household members under the age of 18 as of the 1997 survey date. Additional family structure information is provided by an indicator variable for whether both parents are present in the home at age 14 in the NLSY79 and in 1997 (i.e. ages 13-17) in the NLSY97. Family residence in an urban (metropolitan) area at age 14 (age 12) is accounted for with the 1979 (1997) cohort. We control for the mother's age at birth as well as gender and race (blacks, hispanics and whites for the NLSY79; blacks, hispanics, other non-whites, and whites for the NLSY97 data). Finally, we allow for differences by year of birth.

B   Proofs and Other Aspects of the Two-Period Model

B.1   The set of constrained individuals

For each ability level a, the various forms of credit constraints define a threshold wealth level below which the agent is constrained (and above which he is not). We now characterize those thresholds.

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[42]See Belley and Lochner (2007) for additional details on the sample and variables used in this paper.

[43]Schooling attainment by age 22 is used if it is missing or unavailable at age 21 (fewer than 10% of all respondents in both surveys).

[44]Family income includes government transfers (e.g. welfare and unemployment insurance), but it does not subtract taxes. Net wealth measures the value of all assets (e.g. home and other real estate, vehicles, checking and savings, and other financial assets) less any loans and credit card debt.

[45]Since AFQT percentile scores increase with age in the NLSY79, we determine an individual's quartile based on year of birth. AFQT percentile scores in the NLSY97 have already been adjusted to account for age differences.

Exogenous Constraints: The threshold $w_{\min}^X(a)$ is defined by $d^U\left(a, w_{\min}^X(a)\right) = d_0$, so it is increasing in a. Note that $w_{\min}^X(a) \geq h^U(a) - d_0$, the wealth level needed to finance $h^U(a)$ given maximum borrowing. Consumption smoothing further implies that $w_{\min}^X(a)$ is steeper than $h^U(a)$ as a function of a, since $\frac{dw_{\min}^X(a)}{da} = \frac{\partial d^U\left(a, w_{\min}^X\right)}{\partial a} / \frac{\partial d^U\left(a, w_{\min}^X\right)}{\partial w} > \frac{\partial d^U\left(a, w_{\min}^X\right)}{\partial a} > \frac{dh^U(a)}{da} > 0$ by implicit differentiation.

GSL Programs: The threshold $w_{\min}^G(a) \equiv \max\{w_{\min}^X(a), \tilde{w}_{\min}(a)\}$, where $\tilde{w}_{\min}(a)$ is defined by $h^U(a) = d^U(a, \tilde{w}_{\min}(a))$. It is increasing in a because $d^U(\cdot, w)$ is steeper than $h^U(\cdot)$. To see that $w_{\min}^X(a)$ is steeper than $\tilde{w}_{\min}(a)$, use implicit differentiation to obtain $\frac{d\tilde{w}_{\min}(a)}{da} = \frac{dw_{\min}^X(a)}{da} + \frac{\partial h^U}{\partial a} / \frac{\partial d^U}{\partial w} < \frac{dw_{\min}^X(a)}{da}$.

Private Lending with Limited Commitment: The threshold $w_{\min}^L(a)$ is defined by $d^U(a, w_{\min}^L(a)) = \kappa a f\left[h^U(a)\right]$. This threshold increases at a slower rate in a than does $w_{\min}^X(a)$, and it may even be decreasing in a if κ is large enough. To see this, use implicit differentiation and obtain $\frac{dw_{\min}^L}{da} = \frac{dw_{\min}^X}{da} + \left[\kappa\left(f(h^U) + R\frac{\partial h^U}{\partial a}\right)\right] / \frac{\partial d^U}{\partial w} < \frac{dw_{\min}^X}{da}$ because $\frac{\partial d^U}{\partial w} < 0$.

GSL Programs Plus Private Lenders: The threshold $w_{\min}^{G+L}(a)$ is defined by $d^U\left(a, w_{\min}^{G+L}(a)\right) = \kappa a f\left(h^U(a)\right) + \min\left\{h^U(a), d_{\max}\right\}$. Direct inspection implies that $w_{\min}^{G+L}(a) < \min\left\{w_{\min}^G(a), w_{\min}^L(a)\right\}$. As with $w_{\min}^L(a)$, the threshold $w_{\min}^{G+L}(a)$ can be decreasing in a and may even be negative.

B.2   Proofs

Proof of Lemma 1. Implicit differentiation of (5) yields $\frac{dh^U(a)}{da} = -\frac{f'[h^U(a)]}{af''[h^U(a)]} > 0$. Using expression (6), define

$$F \equiv u'\left[w + d - h^U(a)\right] - \beta R u'\left[af\left[h^U(a)\right] - Rd\right] = 0.$$

From the implicit function theorem $\frac{\partial d^U(a,w)}{\partial a} = -\frac{\partial F}{\partial a} / \frac{\partial F}{\partial d}$, then

$$\frac{\partial d^U(a,w)}{\partial a} = \frac{\partial h^U(a)}{\partial a} + \beta R \frac{u''\left[af\left[h^U(a)\right] - Rd\right] f\left[h^U(a)\right]}{u''\left[w + d - h^U(a)\right] + \beta R^2 u''\left[af\left[h^U(a)\right] - Rd\right]} > \frac{\partial h^U(a)}{\partial a} > 0.$$

where we have used $af'\left[h^U(a)\right] = R$. Similarly,

$$\frac{\partial d^U(a,w)}{\partial w} = -\frac{u''\left[w + d - h^U(a)\right]}{u''\left[w + d - h^U(a)\right] + \beta R^2 u''\left[af\left[h^U(a)\right] - Rd\right]} = -\left(\frac{1}{1 + \beta R^2 \frac{u''[af[h^U(a)] - Rd]}{u''[w + d - h^U(a)]}}\right).$$

Since the denominator is greater than one, the argument is complete. ∎

Proof of Proposition 1. From the FOC define

$$F \equiv -u'(w + d_0 - h) + \beta a f'[h] u'[af(h) - Rd_0] = 0.$$

From the second order condition $\partial F / \partial h < 0$ and then, from implicit differentiation $sign\left\{\frac{\partial h}{\partial w}\right\} = sign\left\{\frac{\partial F}{\partial w}\right\}$ and $sign\left\{\frac{\partial h}{\partial a}\right\} = sign\left\{\frac{\partial F}{\partial a}\right\}$. First, $\frac{\partial h}{\partial w} > 0$ since $\frac{\partial F}{\partial w} = -u''(w + d_0 - h) > 0$. Second,

$$\frac{\partial F}{\partial a} = \beta f'[h] u'[af(h) - Rd_0]\left\{1 + af(h)\frac{u''[af(h) - Rd_0]}{u'[af(h) - Rd_0]}\right\}$$

$$< \beta f'[h] u'[af(h) - Rd_0]\left\{1 + [af(h) - Rd_0]\frac{u''[af(h) - Rd_0]}{u'[af(h) - Rd_0]}\right\}$$

$$= \beta f'[h] u'[af(h) - Rd_0]\left\{1 - 1/\eta[af(h) - Rd_0]\right\},$$

where the first results from direct derivation, the second from $u' > 0$, $u'' < 0$, $f' > 0$, and $d_0 > 0$, and the third uses the definition of IES$\equiv \eta\left(\cdot\right)$. If $\eta\left(c\right) \leq 1$ for all $c > 0$, then the right-hand-side of the last line is non-positive and $\frac{\partial F}{\partial a} < 0$. ∎

Proof of Proposition 2. Using the FOC of the exogenous constraint model,

$$\hat{a}\left(w\right) \equiv \sup\left\{\hat{a} : u'\left(w\right) \geq \beta \hat{a} f'\left[d_{\max}\right] u'\left[\hat{a} f\left(d_{\max}\right) - R d_{\max}\right]\right\},$$

which in principle could be $+\infty$. If $u\left(c\right) = c^{1-\sigma}/\left(1-\sigma\right)$, then a finite $\hat{a}\left(w\right)$ would be given by

$$\hat{a} : w\left(\beta f'\left[d_{\max}\right]\right)^{\frac{1}{\sigma}} = \left(\hat{a}\right)^{\frac{\sigma-1}{\sigma}} f\left(d_{\max}\right) - R d_{\max}\left(\hat{a}\right)^{\frac{-1}{\sigma}}.$$

If $\sigma > 1$ (IES < 1), the RHS is strictly increasing and unbounded and, hence, $\hat{a}\left(w\right)$ is finite. The rest is direct upon examination of optimality conditions under the three different cases. ∎

Proof of Lemma 2. Straightforward and omitted. ∎

Proof Proposition 3. The non-monotonicity of $c_0\left(h\right)$ is driven by the Inada condition on $f\left(\cdot\right)$. For low values of h, $c_0\left(h\right)$ is increasing since $\kappa a f'\left[h\right] > 1$, i.e. borrowing limits increase more than the cost of investment. For each a, let $h^O\left(a\right)$ be the level that maximizes $c_0(h)$, i.e. $\kappa a f'\left[h^O\left(a\right)\right] = 1$. Since $\kappa < R^{-1}$, then $h^O\left(a\right) < h^U\left(a\right)$. Obviously $h^L\left(a,w\right) > h^O\left(a\right)$ as otherwise consumption in both periods could be increased by increasing h. From the first order condition, define

$$F \equiv \left(\kappa a f'\left(h\right) - 1\right) u'\left(w + \kappa a f\left(h\right) - h\right) + \beta a f'\left(h\right)\left(1 - \kappa R\right) u'\left[a f\left(h\right)\left(1 - \kappa R\right)\right] = 0.$$

We first prove (i). Contrary to the hypothesis, assume that the agent is constrained and $h^L\left(a,w\right) > h^U\left(a\right)$. Then $a f\left[h^L\left(a,w\right)\right] < R$ and

$$
\begin{aligned}
\beta R\left(1 - \kappa R\right) u'\left[c_1\right] &> \beta a f'\left(h\right)\left(1 - \kappa R\right) u'\left[c_1\right] \\
&= \left(1 - \kappa a f'\left(h\right)\right) u'\left(c_0\right) \\
&> \left(1 - \kappa R\right) u'\left(c_0\right),
\end{aligned}
$$

where the second line uses $F = 0$ and the third uses $a f\left[h^L\left(a,w\right)\right] < R$ again. Hence $\beta R u'\left[c_1\right] > u'\left(c_0\right)$, and the agent could not have been constrained. Part (ii) is direct from implicit derivation as in Proposition **1**. We now prove (iii). From the second order condition $\partial F/\partial h < 0$ and sign$\{\partial h/\partial a\}$=sign$\{\partial F/\partial a\}$. After some simplification:

$$\frac{\partial F}{\partial a} = \kappa f'\left(h\right) u'\left(c_0\right) + \left(1 - \kappa a f'\left(h\right)\right) \kappa f\left(h\right)\left[-u''\left(c_0\right)\right] + \beta\left(1 - \kappa R\right) f'\left(h\right)\left\{u'\left[c_1\right] + c_1 u''\left[c_1\right]\right\}.$$

The first two terms are always positive, while the third term can be either positive or negative. Multiply and divide $\partial F/\partial a$ by $u'\left(c_1\right)$, to obtain

$$
\begin{aligned}
\frac{\partial F}{\partial a} &= u'\left(c_1\right)\left[\left(\frac{u'\left(c_0\right)}{u'\left(c_1\right)}\right)\kappa f'\left(h\right) + \left(1 - \kappa a f'\left(h\right)\right)\kappa\left(\frac{-u''\left(c_0\right)}{\frac{\left(1-\kappa a f'\left(h\right)\right)}{\beta\left(1-\kappa R\right) a f'\left(h\right)} u'\left(c_0\right)}\right) f\left(h\right) + \beta\left(1 - \kappa R\right) f'\left(h\right)\left[1 - \sigma\left(c_1\right)\right]\right] \\
&= u'\left(c_1\right) f'\left(h\right)\left[\left(\frac{u'\left(c_0\right)}{u'\left(c_1\right)}\right)\kappa + \kappa\beta\frac{c_1}{c_0}\frac{1}{\eta\left(c_0\right)} + \beta\left(1 - \kappa R\right)\left[1 - \frac{1}{\eta\left(c_1\right)}\right]\right], \\
&\geq \beta u'\left(c_1\right) f'\left(h\right)\left[\kappa\frac{c_1}{c_0}\frac{1}{\eta\left(c_0\right)} + 1 - \frac{1}{\eta\left(c_1\right)}\left(1 - \kappa R\right)\right],
\end{aligned}
$$

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where the second uses $c_1 = (1 - \kappa R) \, af(h)$ and the definition of $\eta(\cdot)$, the IES. The last line follows from $u'(c_0)/u'(c_1) \geq \beta R$ and then simplifying. Since $\kappa \frac{c_1}{c_0} \frac{1}{\eta(c_0)}$ is non-negative, the condition $\eta(c_1) > 1 - \kappa R$ implies that $\partial F/\partial a > 0$ which completes the proof of part (iii). Finally, we prove (iv). If $\beta R \geq 1$ then $c_1 \geq c_0$, and with $\eta(\cdot)$is non-decreasing, then $\eta(c_1) \geq \eta(c_0)$. Therefore,

$$\frac{\partial F}{\partial a} \geq \beta u'(c_1) f'(h) \left\{ 1 - \frac{1}{\eta(c_0)} \left[1 - \kappa(1+R) \right] \right\},$$

which is strictly positive if $\kappa \geq [1 - \eta(c_0)]/(1+R)$. ∎

Proof of Proposition 4. The first part is trivial since $w < w^L_{\min}(a)$ implies that $d^U(a,w) > d^L(a,w)$ and, since $d_0 = d^L(a,w)$, $d^U(a,w) > d_0$, which implies $w < w^X_{\min}(a)$. To shorten notation, we suppress the dependence of the endogenous variables on (a,w). Contrary to the statement, assume that $h^L \leq h^X$. Then $c^X_0 = w + d^X - h^X \leq w + d_0 - h^X \leq w + d^L - h^L \leq c^L_0$, which implies that $u'(c^X_0) \geq u'(c^L_0)$. Similarly, $c^X_1 = a \left[h^X \right]^\alpha - Rd^X \geq a \left[h^L \right]^\alpha - Rd^L = c^L_1$, which implies that $u'(c^X_1) \leq u'(c^L_1)$. From the FOC of the L and X models:

$$\left(1 - \kappa \alpha a \left[h^L \right]^{\alpha-1} \right) u'(c^L_0) = \beta \alpha a \left[h^L \right]^{\alpha-1} [1 - \kappa R] \, u'(c^L_1).$$

Then,

$$u'(c^L_0) > \beta \left[\alpha a \left[h^L \right]^{\alpha-1} \right] u'(c^L_1) \geq \beta \left[\alpha a \left[h^X \right]^{\alpha-1} \right] u'(c^L_1) \geq \beta \left[\alpha a \left[h^X \right]^{\alpha-1} \right] u'(c^X_1) = u'(c^X_0),$$

a contradiction. The first inequality follows from the fact that $h^L < h^U$, which implies that $\left(1 - \kappa \alpha a \left[h^L \right]^{\alpha-1} \right) < (1 - \kappa R)$. The second inequality follows from the hypothesis that $h^L \leq h^X$ and the third from $u'(c^X_1) \leq u'(c^L_1)$. The last equality follows from the FOC of the X model. ∎

Proof of Proposition 5. The fact that $h^{L+G}(a,w;d_{\max}) \leq h^U(a)$ follows from the same arguments as before. Define $F(h, d_{\max})$ as

$$F \equiv \left(\kappa af'(h) - 1 \right) u' \left[w + d_{\max} + \kappa af[h] - h \right] + \beta af'(h)(1 - \kappa R) u' \left[af(h)(1 - \kappa R) - Rd_{\max} \right].$$

The first order condition that determines $h^{L+G}(a,w;d_{\max})$ is $F = 0$ and $\partial h^{L+G}(a,w;d_{\max})/\partial d_{\max} = - \left[\partial F/\partial d_{\max} \right]/\left[\partial F/\partial h \right]$. Since h is optimally chosen, $\partial F/\partial h < 0$ and $sign \left\{ \partial h^{L+G}(a,w;d_{\max})/\partial d_{\max} \right\} = sign \left\{ \partial F/\partial d_{\max} \right\}$. To prove (ii):

$$\frac{\partial F}{\partial d_{\max}} = \left[1 - \kappa af'(h) \right] \left[-u''(c_0) \right] + \beta af'[h](1 - \kappa R) R \left[-u''(c_1) \right] > 0.$$

This implies that $h^L(a,w) = h^{L+G}(a,w;0) < h^{L+G}(a,w;d_{\max})$ for $d_{\max} > 0$, proving (i). ∎

C Proofs and Other Aspects of the Quantitative Model

C.1 Unrestricted Allocations

Given (a,w) an individual maximizes the $t_0 = 0$ value of utility (9) subject to

$$\int_0^T e^{-\rho t} c(t) \, dt + \int_0^1 e^{-\rho t} x(t) \, dt \leq w + \int_1^P e^{-\rho t} y(t) \, dt. \tag{25}$$

The definition of Φ is

$$\Phi \equiv \begin{cases} \left[e^{(g-\rho)(P-1)} - 1\right] / [g - \rho] & \text{if } g \neq \rho \\ P - 1 & g = \rho. \end{cases}$$

Optimal out-of-pocket investment is

$$x^U (a) = \arg \max_{x \geq 0} \left\{ w + \frac{e^{-\rho}}{\mu} \left[a\Phi \left[i_{pub} + (1+s)\, x\right]^\alpha - x\right] \right\}.$$

Since total schooling investment is given by (15), it clear that if $a \leq a_0 \equiv \frac{[i_{pub}]^{1-\alpha}}{\alpha(1+s)\Phi}$, then $x^U (a) = 0$ and $h^U (a) = i_{pub}$. Finally, the optimal unconstrained consumption is constant and equal to

$$c^U (t, a, w) = \frac{\rho}{1 - e^{-\rho T}} \left\{ w + \frac{e^{-\rho}}{\mu} \left[a\Phi \left[i_{pub} + (1+s)\, x^U (a)\right]^\alpha - x^U (a)\right] \right\}.$$

C.2 Exogenous Constraint Model

The expression for $w^X (a)$ is given by

$$w^X (a) = \left(\frac{1 - e^{-\rho}}{\mu \left(1 - e^{-\rho(T-1)}\right)}\right) a\Phi \left[h^U (a)\right]^\alpha + \left(\frac{h^U (a) - i_{pub}}{\mu (1+s)}\right) - d_0 \left(\frac{1 - e^{-\rho T}}{1 - e^{-\rho(T-1)}}\right).$$

Everything else is the same as in the basic model.

C.3 GSL Model

The threshold level of initial assets $w^G_{\min}(a)$ above which individuals with ability a are unconstrained satisfies $d^U \left(a, w^G_{\min}(a)\right) = \min \left\{d_{\max}, \max \left\{0, \left(h^U (a) - i_{pub}\right) / (1+s)\right\}\right\}$, where $d^U (a, w)$ is given by expression (16) and $h^U (a)$ by expression (15).

C.4 Private Lending with Limited Commitment

The highest discounted utility that can be attained by an individual that defaults at $t = 1$ is $V^D (a, h) = \hat{\Theta}_{\gamma, \pi} \frac{\left[\mu^{-1} a \Phi h^\alpha\right]^{1-\sigma}}{1-\sigma}$, where

$$\hat{\Theta}_{\gamma, \pi} \equiv \left(\frac{1 - \gamma}{\Phi}\right)^{1-\sigma} \left(\frac{e^{[g(1-\sigma)-\rho]\pi} - 1}{g(1-\sigma) - \rho}\right) + e^\rho \left(\frac{e^{-\rho(1+\pi)} - e^{-\rho T}}{\rho}\right)^\sigma \left(\frac{e^{(g-\rho)P} - e^{(g-\rho)(1+\pi)}}{\Phi(g - \rho)}\right)^{1-\sigma}.$$

Claims about κ follow directly from: (i) $\hat{\Theta}_{\gamma, \pi} < \Theta$ for $\gamma, \pi > 0$; (ii) $\hat{\Theta}_{\gamma, \pi}$ is decreasing in γ; (iii) for all $\gamma \in (0, 1)$, $\hat{\Theta}_{\gamma, \pi}$ converges to Θ as $\pi \to 0$.

As of $t = 0$, the maximization problem consists of choosing a consumption c_0 for all $t \in [0, 1]$, and investment and borrowing levels (x, d), such that

$$[BC] : \frac{e^{-\rho}}{\mu} [c_0 + x] \leq w + e^{-\rho} d, \tag{26}$$

$$[CC] : d \leq \kappa \left[\mu^{-1} a \Phi \left[i_{pub} + (1+s)\, x\right]\right]^\alpha. \tag{27}$$

Aside from government subsidies (s, i_{pub}) and the determination of Θ, Φ, and κ, this problem is equivalent to the two-period model of Section 4.5.

The value $w_{\min}^L(a)$ defined by $d^U\left(a, w_{\min}^L(a)\right) = \kappa\mu^{-1}a\Phi_a\left[h^U(a)\right]^\alpha$ is the threshold of wealth above which an agent is unconstrained. It is equal to

$$
w_{\min}^L(a) = \begin{cases}
a\Phi\left[i_{pub}\right]^\alpha\left[\dfrac{\left(1-e^{-\rho}\right)-\kappa\left(1-e^{-\rho T}\right)}{\mu\left(1-e^{-\rho(T-1)}\right)}\right] & \text{for } a \leq a_0 \\[2em]
h^U(a)\left[\dfrac{1-\kappa-(1-\alpha)e^{-\rho}+(\kappa-\alpha)e^{-\rho T}}{\mu\alpha(1+s)\left(1-e^{-\rho(T-1)}\right)}\right] - \dfrac{e^{-\rho}}{\mu}\left(\dfrac{i_{pub}}{1+s}\right) & \text{for } a > a_0.
\end{cases}
$$

Individuals with $w \geq w_{\min}^L(a)$ attain the unrestricted allocations. For those with $w < w_{\min}^L(a)$, constraint (21) holds with equality and we can use it to eliminate d. With this, the problem becomes

$$
\max_{\{x:x\geq 0\}}\left\{\frac{e^{-\rho}}{\mu}\frac{\left[e^\rho\mu w + \kappa a\Phi_a\left[i_{pub} + (1+s)x\right]^\alpha - x\right]^{1-\sigma}}{1-\sigma} + e^{-\rho}\Theta\frac{\left[(1-\kappa)\mu^{-1}a\Phi_a\left[i_{pub} + (1+s)x\right]^\alpha\right]^{1-\sigma}}{1-\sigma}\right\}.
$$

Proof of Proposition 6. To shorten notation define:

$$
\begin{aligned}
A &\equiv a\Phi, & c_0 &\equiv e^\rho\mu w + \kappa Ah^\alpha - x, \\
m_1 &\equiv (1-\kappa)\mu^{-1}Ah^\alpha, & \delta &\equiv \alpha Ah^{\alpha-1}(1+s).
\end{aligned}
$$

Optimality requires that either $F < 0$ and $x = 0$, or $F = 0$ and $x > 0$, where

$$
F \equiv [\kappa\delta - 1][c_0]^{-\sigma} + \Theta[m_1]^{-\sigma}(1-\kappa)\delta.
$$

We first prove part (i). If the credit constraint binds, then $[c_0]^{-\sigma} > \Theta[m_1]^{-\sigma}$. If $F < 0$, then $h^L(a, w) = i_{pub}$, and the result is trivial. If $F = 0$, then $[1 - \kappa\delta] < (1-\kappa)\delta$, implying that $\delta > 1$. For the unconstrained case define

$$
\begin{aligned}
c_0^U(a, w) &= \mu e^\rho w + \mu d^U(a, w) - x^U(a), \\
m_1^U(a, w) &\equiv \mu^{-1}A\left[h^U(a)\right]^\alpha - d^U(a, w), \\
\delta^U(a) &\equiv \alpha A\left[h^U(a)\right]^{\alpha-1}(1+s).
\end{aligned}
$$

Given that $\left[c_0^U(a, w)\right]^{-\sigma} = \Theta\left[m_1^U(a, w)\right]^{-\sigma}$, the first order condition implies that $\delta^U(a) \leq 1$. Thus, $\delta > \delta^U(a)$ and hence $h^L(a, w) < h^U(a)$. We now prove part (ii). From maximization, we have the condition $\frac{\partial F}{\partial h} < 0$ and therefore $sign\left\{\frac{\partial h}{\partial A}\right\} = sign\left\{\frac{\partial F}{\partial A}\right\}$. The latter derivative is

$$
\frac{\partial F}{\partial A} = \left[\alpha\kappa Ah^{\alpha-1}(1+s) - 1\right]\left\{-\sigma[c_0]^{-\sigma-1}\frac{\partial c_0}{\partial A}\right\} + \alpha\kappa h^{\alpha-1}(1+s)[c_0]^{-\sigma}
$$

$$
+ \Theta[m_1]^{-\sigma}\alpha(1-\kappa)h^{\alpha-1}(1+s) + \alpha(1-\kappa)Ah^{\alpha-1}(1+s)\left\{-\sigma\Theta[m_1]^{-\sigma-1}\frac{\partial m_1}{\partial A}\right\}
$$

First, from the first order condition $\left[\alpha\kappa Ah^{\alpha-1}(1+s) - 1\right][c_0]^{-\sigma} = -\Theta[m_1]^{-\sigma}\alpha(1-\kappa)Ah^{\alpha-1}(1+s)$ and then taking $\Theta[m_1]^{-\sigma}\alpha h^{\alpha-1}(1+s) > 0$ as a common factor gives

$$
\frac{\partial F}{\partial A} = \left\{\Theta[m_1]^{-\sigma}\alpha h^{\alpha-1}(1+s)\right\}\left\{\sigma\kappa\mu\frac{m_1}{c_0} + \kappa\frac{[c_0]^{-\sigma}}{\Theta[m_1]^{-\sigma}} + (1-\kappa) - \sigma(1-\kappa)\right\}
$$

$$
= \left\{\Theta[m_1]^{-\sigma}\alpha h^{\alpha-1}(1+s)\right\}\left\{\sigma\kappa\mu\frac{m_1}{c_0} + \kappa\frac{[c_0]^{-\sigma}}{\Theta[m_1]^{-\sigma}} + (1-\kappa) - \sigma + \sigma\kappa\right\}
$$

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where we also have multiplied and divided by μ and used the definition of m_1. For constrained individuals, we have that $\frac{[c_0]^{-\sigma}}{\Theta[m_1]^{-\sigma}} \geq 1$, and therefore $\frac{m_1}{c_0} \geq \Theta^{\frac{1}{\sigma}}$. With these inequalities we can find a lower bound to $\partial F / \partial A$:

$$
\frac{\partial F}{\partial A} \geq \left\{ \Theta\left[m_1\right]^{-\sigma} \alpha h^{\alpha-1}\left(1+s\right)\right\} \left\{ \sigma\kappa \left(\frac{1-e^{-\rho T} e^{\rho}}{e^{\rho}-1}\right) + 1 - \sigma\left(1-\kappa\right)\right\}
$$
$$
= \left\{ \Theta\left[m_1\right]^{-\sigma} \alpha h^{\alpha-1}\left(1+s\right)\right\} \left\{ 1 - \sigma\left[1 - \kappa\left(\frac{1-e^{-\rho T}}{1-e^{-\rho}}\right)\right]\right\},
$$

where in the first line we have used the expressions for Θ and μ and the second we have simplified. As claimed in the text, the last expression is positive if $\kappa \geq \left[(\sigma-1)/\sigma\right]\left[\left(1-e^{-\rho}\right)/\left(1-e^{-\rho T}\right)\right]$. ∎

C.5 GSL Programs Plus Private Lenders

The threshold level of initial assets $w_{\min}^{G+L}(a)$ above which individuals with ability a are unconstrained satisfies

$$
d^U\left(a, w_{\min}^{G+L}(a)\right) = \kappa\mu^{-1} a\Phi\left[h^U\left(a\right)\right]^{\alpha} + \min\left\{d_{\max}, \max\left\{0, \left(h^U\left(a\right) - i_{pub}\right)/\left(1+s\right)\right\}\right\},
$$

where $d^U\left(a, w\right)$ is given by expression (16) and $h^U\left(a\right)$ by expression (15). Since borrowers combine both sources of credit, $w_{\min}^{G+L}(a) \leq \min\left\{w_{\min}^G(a), w_{\min}^L(a)\right\}$, where $w_{\min}^L(a)$ is the threshold under private lending alone and $w_{\min}^G(a)$ is the threshold under the GSL alone.

Table D1: Educational Expenditures by Year of Schooling and AFQT Quartile (1999 Dollars)

Years of School	Direct Expenditures	Foregone Earnings by AFQT Quartile:				Total Costs by AFQT Quartile:			
		Quart. 1	Quart. 2	Quart. 3	Quart. 4	Quart. 1	Quart. 2	Quart. 3	Quart. 4
8	59,075	0	0	0	0	59,075	59,075	59,075	59,075
9	65,239	0	0	0	0	65,239	65,239	65,239	65,239
10	71,167	2,197	3,080	3,526	3,353	73,364	74,246	74,693	74,520
11	76,866	5,058	7,088	8,116	7,717	81,924	83,955	84,982	84,584
12	82,347	8,638	12,106	13,862	13,181	90,985	94,453	96,208	95,527
13	97,315	12,948	18,147	20,778	19,757	110,263	115,462	118,093	117,072
14	111,708	17,936	25,138	28,783	27,369	129,645	136,846	140,491	139,077
15	125,548	23,489	32,919	37,692	35,841	149,036	158,467	163,240	161,388
16	138,855	29,431	41,247	47,228	44,908	168,286	180,102	186,083	183,763
17	151,650	35,547	49,818	57,042	54,240	187,197	201,468	208,692	205,890
18	163,953	41,599	58,301	66,754	63,475	205,552	222,254	230,707	227,428
19	175,783	47,359	66,373	75,996	72,263	223,142	242,156	251,780	248,046
20	187,158	52,629	73,759	84,454	80,306	239,787	260,918	271,612	267,464

Notes:

1) Direct expenditures assume average expenditure per pupil in primary and secondary schooling through grade 12. Additional expenditures for higher grades are taken from average expenditures per student in all colleges and universities. Expenditures based on averages for school years 1979-80 to 1988-89. (Source: Tables 170 and 342, Digest of Education Statistics, 1999.)

2) Foregone earnings are calculated from regression of log(earnings) on AFQT quartile, education indicators, experience and experience-squared. Foregone earnings are based on someone with 9 years of schooling and the corresponding level of experience. Sample includes not enrolled youth ages 16-24.

3) Expenditures are discounted at a 4% annual interest rate to grade 10.

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