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# A FACTOR ANALYSIS OF BOND RISK PREMIA 

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#### Abstract

This paper uses the factor augmented regression framework to analyze the relation between bond excess returns and the macro economy. Using a panel of 131 monthly macroeconomic time series for the sample 1964:1-2007:12, we estimate 8 static factors by the method of asymptotic principal components. We also use Gibb sampling to estimate dynamic factors from the 131 series reorganized into 8 blocks. Regardless of how the factors are estimated, macroeconomic factors are found to have statistically significant predictive power for excess bond returns. We show how a bias correction to the parameter estimates of factor augmented regressions can be obtained. This bias is numerically trivial in our application. The predictive power of real activity for excess bond returns is robust even after accounting for finite sample inference problems. Forecasts of excess bond returns (or bond risk premia) are countercyclical. This implies that investors are compensated for risks associated with recessions.


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## 1 Introduction

The expectations theory of the term structure posits that variables in the information set at time $t$ should have no predictive power for excess bond returns. Consider the predictive regression

$$
r_{t+h}=a+b^{\prime} Z_{t}+e_{t h}
$$

where $r_{t+h}$ is excess returns for holding period $h$, and $Z_{t}$ is a set of predictors. Conventional tests often reject the null hypothesis that the parameter vector $b$ is zero. Some suggest that over-rejections may arise if $r$ is stationary and the variables $Z$ are highly persistent, making inference highly distorted in finite samples. For this reason, researchers often use finite sample corrections or the bootstrap to conduct inference. However, it is often the case that robust inference still points to a rejection of the null hypothesis.

For a long time, the $Z$ s found to have predictive power are often financial variables such as default premium, term premium, dividend price ratio, measures of stock market variability and liquidity. Cochrane and Piazzesi (2005) find that a linear combination of five forward spreads explains between 30 and 35 percent of the variation in next year's excess returns on bonds with maturities ranging from two to five years. Yet theory suggests that predictive power for excess bond returns should come from macroeconomic variables. Campbell (1999) and Wachter (2006) suggest that bond and equity risk premia should covary with a slowmoving habit driven by shocks to aggregate consumption. Brandt and Wang (2003) argue that risk premia are driven by shocks to inflation as well as aggregate consumption; notably, both are macroeconomic shocks.

In an effort to reconcile theory and evidence, recent work has sought to establish and better understand the relation between excess returns and macroeconomic variables. Piazzesi and Swanson (2004) find that the growth of nonfarm payroll employment is a strong predictor of excess returns on federal funds futures contracts. Ang and Piazzesi (2003) uses a no-arbitrage factor model of the term structure of interest rates that also allows for timevarying risk premia and finds that the pricing kernel is driven by a few observed macroeconomic variables and unobserved yield factors. Kozicki and Tinsley (2005) uses affine models to link the term structure to perceptions of monetary policy. Duffie (2008) finds that an 'expectations' factor unrelated to the level and the slope has strong predictive power for short term interest rates and excess returns, and that this expectations factor has a strong inverse relation with industrial production. Notably, these studies have focused on the relation between expected excess bond returns, risk premia, and a few selected macroeconomic
variables. The evidence falls short of documenting a direct relation between expected excess bond returns (bond risk premia) and the macro economy.

In Ludvigson and Ng (2007), we used a new approach. We used a small number of estimated (static) factors instead of a handful of observed predictors in the predictive regressions, where the factors are estimated from a large panel of macroeconomic data using the method of asymptotic principal components (PCA). Such a predictive regression is a special case of what is known as a 'factor augmented regression' (FAR). ${ }^{1}$ The factors enable us to substantially reduce the dimension of the predictor set while still being able to use the information underlying the variables in the panel. Furthermore, our latent factors are estimated without imposing a no-arbitrage condition or any parametric structure. Thus, our testing framework is non-structural, both from an economic and a statistical point of view. We find that latent factors associated with real economic activity have significant predictive power for excess bond returns even in the presence of financial predictors such as forward rates and yield spreads. Furthermore, we find that bond returns and yield risk premia are more countercyclical when these risk premia are constructed to exploit information in the factors.

This paper investigates the robustness of our earlier findings with special attention paid to how the factors are estimated. We first re-estimate the FAR on a panel of 131 series over a longer sample. As in our previous work, these (static) factors, denoted $\widehat{f_{t}}$, are estimated by PCA. We then consider an alternative set of factor estimates, denoted $\widehat{g}_{t}$, that differ from the PCA estimates in two important ways. First, we use a priori information to organize the 131 series into 8 blocks. Second, we estimate a dynamic factor model for each of the eight blocks using a Bayesian procedure.

Compared with our previous work, we now use information in the large macroeconomic panel in a different way, and we estimate dynamic factors using a Bayesian method. It is thus useful to explain the motivation for doing so. The factors estimated from large panels of data are often criticized for being difficult to interpret, and organizing the data into blocks (such as output and price) provides a natural way to name the factors estimated from a block of data. At this point, we could have used PCA to estimate one static factor for each block. We could also have estimated dynamic factors using dynamic principal components, which is frequency-domain based. Whichever principal components estimator we choose, the estimates will not be precise as the number of series in each block is no longer 131 but a much smaller number. Bayesian estimation is more appropriate for the newly organized panels of

[^0]data and Bayesian estimation yields a direct assessment of sampling variability. Using an estimator that is not principal components based also allows us to more thoroughly assess whether the FAR estimates are sensitive to how the factors are estimated. This issue, to our knowledge, has not been investigated in the literature. Notably, the factors that explain most of the variation in the large macroeconomic panel of data need not be the same as the factors most important for predicting excess bond returns. Thus for each of the two sets of factor estimates, namely, $\widehat{f}_{t}$ and $\widehat{g}_{t}$, we consider a systematic search of the relevant predictors, including an out-of-sample criterion to guard against overfitting the predictive regression with too many factors. We also assess the stability of the relation between excess bond returns and the factors over the sample.

An appeal of FAR is that when $N$ and $T$ are large and $\sqrt{T} / N$ tends to zero, the estimated factors in the FAR can be treated as though they are the true but latent factors. There is no need to account for sampling error incurred when the factors are estimated. Numerous papers have studied the properties of the (static and dynamic) principal components estimators in a forecasting context. ${ }^{2}$ To date, little is known about the properties of the FAR estimates when $\sqrt{T} / N$ is not negligible. We show that principal components estimation may induce a bias in the parameter estimates of the predictive regression and suggest how a bias correction can be constructed. For our application, this bias is very small.

Our main finding is that macro factors have strong predictive power for excess bond returns and that this result holds up regardless of which method is used to estimate the factors. The reason is that both methods are capable of isolating the factor for real activity, which contributes significantly to variations in excess bond returns. However, the prior information that permits us to easily give names to the factors also constrains how information in the large panel is used. Thus, as far as predictability is concerned, the factors estimated from the large panel tend to be better predictors than the factors estimated from the eight blocks of data, for the same total number of series used in estimation. Recursive estimation of the predictive regressions finds that the macroeconomic factors are statistically significant throughout the entire sample, even though the degree of predictability varies over the 45 years considered. While the estimated bond and yield risk premia without the macro factors are acyclical, these premia are counter-cyclical when the estimated factors are used to forecast excess returns. This implies that investors must be compensated for risks associated with recessions.

Our empirical work is based on a macroeconomic panel of 131 series. This panel extends

[^1]the one used in Stock and Watson (2005), which has since been used in a number of factor analyses. ${ }^{3}$ The original data set consists of monthly observations for 132 macroeconomic time series from 1959:1-2003:12. We extend their data to 2007:12 and our panel consists of 131 series. Our empirical work uses data from 1964:1 to 2007:12.

## 2 Predictive Regressions

For $t=1, \ldots T$, let $r x_{t+1}^{(n)}$ denote the continuously compounded ( $\log$ ) excess return on an $n$-year discount bond in period $t+1$. Excess returns are defined $r x_{t+1}^{(n)} \equiv r_{t+1}^{(n)}-y_{t}^{(1)}$, where $r_{t+1}^{(n)}$ is the $\log$ holding period return from buying an $n$-year bond at time $t$ and selling it as an $n-1$ year bond at time $t+1$, and $y_{t}^{(1)}$ is the $\log$ yield on the one-year bond. That is, if $p_{t}^{(n)}$ is $\log$ price of $n$-year discount bond at time $t$, then the log yield is $y_{t}^{(n)} \equiv-(1 / n) p_{t}^{(n)}$.

A standard approach to assessing whether excess bond returns are predictable is to select a set of $K$ predetermined conditioning variables at time $t$, given by the $K \times 1$ vector $Z_{t}$, and then estimate

$$
\begin{equation*}
r x_{t+1}^{(n)}=\beta^{\prime} Z_{t}+\epsilon_{t+1} \tag{1}
\end{equation*}
$$

by least squares. For example, $Z_{t}$ could include the individual forward rates studied in Fama and Bliss (1987), the single forward factor studied in Cochrane and Piazzesi (2005), or other predictor variables based on a few macroeconomic series. Such a procedure may be restrictive when the number of eligible predictors is quite large. In particular, suppose we observe a $T \times N$ panel of macroeconomic data with elements $x_{t}=\left(x_{1 t}, x_{2 t}, \ldots x_{N t}\right)^{\prime}, t=1, \ldots, T$, where the cross-sectional dimension, $N$, is large, and possibly larger than the number of time periods, $T$. The set of eligible predictors consists of the union of $x_{t}$ and $Z_{t}$. With standard econometric tools, it is not obvious how a researcher could use the information contained in the panel because unless we have a way of ordering the importance of the $N$ series in forming conditional expectations (as in an autoregression), there are potentially $2^{N}$ possible combinations to consider. The regression

$$
\begin{equation*}
r x_{t+1}^{(n)}=\gamma^{\prime} x_{t}+\beta^{\prime} Z_{t}+\epsilon_{t+1} \tag{2}
\end{equation*}
$$

quickly run into degrees-of-freedom problems as the dimension of $x_{t}$ increases, and estimation is not even feasible when $N+K>T$.

The approach we consider is to posit that $x_{i t}$ has a factor structure so that if these factors were observed, we would have replaced (2) by the following (infeasible) 'factor augmented

[^2]regression'
\[

$$
\begin{equation*}
r x_{t+1}^{(n)}=\alpha^{\prime} F_{t}+\beta^{\prime} Z_{t}+\epsilon_{t+1}, \tag{3}
\end{equation*}
$$

\]

where $F_{t}$ is a set of $k$ factors whose dimension is much smaller than that of $x_{t}$ but has good predictive power for $r x_{t+1}$. Equation (1) is nested within the factor-augmented regression, making (3) a convenient framework to assess the importance of $x_{i t}$ via $F_{t}$, even in the presence of $Z_{t}$. The $Z_{t}$ that we will use as benchmark is the forward rate factor used in Cochrane and Piazzesi (2005). This variable, hereafter referred to as $C P$, is a simple average of the one year yield and four forward rates.. These authors find that the predictive power of forward rates, yield spreads, and yield factors are subsumed in $C P_{t}$. To implement the regression given by (3), we need to resolve two problems. First, $F_{t}$ is latent and we must estimate it from data. Second, we need to isolate those factors with predictive power for our variable of interest, $r x_{t+1}^{(n)}$.

## 3 Estimation of Latent Factors

The first problem is dealt with by replacing $F_{t}$ with an estimated value $\widehat{F}_{t}$ that is close to $F_{t}$ in some well defined sense, and this involves making precise a model from which $F_{t}$ can be estimated. We will estimate two factor models, one static and one dynamic, using data retrieved from the Global Insight database and the Conference Board. The data are collected to incorporate as many series as that used in Stock and Watson (2005). However, one series (ao048) is no longer available on a monthly basis after 2003. Accordingly, our new dataset consists of 131 series from 1959:1 to 2007:12, though our empirical analysis starts in 1964:1 because of availability of the bond yield data. As in the original Stock and Watson data, some series need to be transformed to be stationary. In general, real variables are expressed in growth rates, first differences are used for nominal interest rates, and second $\log$ differences are used for prices. The data description is given in Appendix A. This data can be downloaded from our website http://www.econ.nyu.edu/user/ludvigsons/ Data\&ReplicationFiles.zip.

### 3.1 Static Factors

Let $N$ be the number of cross-section units and $T$ be the number of time series observations. For $i=1, \ldots N, \quad t=1, \ldots T$, a static factor model is defined as

$$
\begin{equation*}
x_{i t}=\lambda_{i}^{\prime} f_{t}+e_{i t} \tag{4}
\end{equation*}
$$

In factor analysis, $e_{i t}$ is referred to as the idiosyncratic error and $\lambda_{i}$ are the factor loadings. This is a vector of weights that unit $i$ put on the corresponding $r$ (static) common factors $f_{t}$. In finance, $x_{i t}$ is the return for asset $i$ in period $t, f_{t}$ is a vector of systematic risk, $\lambda_{i}$ is the exposure to the risk factors, and $e_{i t}$ is the idiosyncratic returns. Although the model specifies a static relationship between $x_{i t}$ and $f_{t}, f_{t}$ itself can be a dynamic vector process that evolves according to

$$
A(L) f_{t}=u_{t}
$$

where $A(L)$ is a polynomial (possibly of infinite order) in the lag operator. The idiosyncratic error $e_{i t}$ can also be a dynamic process, and $e_{i t}$ can also be cross-sectionally correlated.

We estimate $f_{t}$ using the method of asymptotic principal components (PCA) originally developed by Connor and Korajzcyk (1986) for a small $T$ large $N$ environment. Letting "hats" denote estimated values, the $T \times r$ matrix $\widehat{f}$ is $\sqrt{T}$ times the $r$ eigenvectors corresponding to the $r$ largest eigenvalues of the $T \times T$ matrix $x x^{\prime} /(T N)$ in decreasing order with $\widehat{f^{\prime}} \widehat{f}=I_{r}$. The normalization is necessary as the matrix of factor loadings $\Lambda$ and $f$ are not separately identifiable. The normalization also yields $\widehat{\Lambda}=x^{\prime} \widehat{f} / T$. Intuitively, for each $t, \widehat{f_{t}}$ is a linear combinations of each element of the $N \times 1$ vector $x_{t}=\left(x_{1 t}, \ldots, x_{N t}\right)^{\prime}$, where the linear combination is chosen optimally to minimize the sum of squared residuals $x_{t}-\Lambda f_{t}$. Bai and $\operatorname{Ng}$ (2002) and Stock and Watson (2002a) showed that the space spanned by $f_{t}$ can be consistently estimated by $\widehat{f_{t}}$ defined as above when $N, T \rightarrow \infty$. The number of static factors in $x_{t}$ can be determined by the panel information criteria developed in Bai and Ng (2002). For the panel of 131 series under investigation, the $I C_{2}$ criterion finds 8 factors over the full sample of 576 observations (with the maximum number of factors set to 20).

A common criticism of the method of principal components estimator is that the factors can be difficult to interpret. Our interpretation of the factors is based on the marginal $R^{2} \mathrm{~s}$, obtained by regressing each of the 131 series on the eight factors, one at a time. Because the factors are mutually uncorrelated, the marginal $R^{2}$ is also the explanatory power of the factor in question holding other factors fixed. Extending the sample to include three more years of data did not change our interpretation of the factors. Figures 1 through 8 show the marginal R-square statistics from regressing the series number given on the x -axis onto the estimated factor named in the heading. As in Ludvigson and $\mathrm{Ng}(2007), \widehat{f}_{1}$ is a real activity factor that loads heavily on employment and output data. The second factor loads heavily on interest rate spreads, while the third and fourth factors load on prices. Factor 5 loads on interest rates (much more strongly than the interest rate spreads). Factor 6 loads predominantly on the housing variables while factor 7 loads on measures of the money supply. Factor 8 loads
on variables relating to the stock market. Thus, loosely speaking, factors 5 to 8 are more strongly related to money, credit, and finance.

While knowing that there are eight factors in the macroeconomic panel is useful information in its own right, of interest here are not the $N$ variables $x_{t}=\left(x_{1 t}, \ldots x_{N t}\right)^{\prime}$, but the scalar variable $r x_{t+1}$ which is not in $x_{t}$. Factors that are pervasive for the large panel of data need not be important for predicting $r x_{t+1}^{(n)}$. For this reason, we make a distinction between $F_{t} \subset f_{t}$ and $f_{t}$. The predictive regression of interest is

$$
\begin{equation*}
r x_{t+1}^{(n)}=\alpha_{F}^{\prime} \widehat{F}_{t}+\beta_{F}^{\prime} Z_{t}+\epsilon_{t+1} \tag{5}
\end{equation*}
$$

which has a vector of generated regressors, $\widehat{F}_{t}$.
Consistency of $\widehat{\alpha}_{F}$ follows from the fact that the difference between $\widehat{f}_{t}$ and the space spanned by $f_{t}$ vanishes at rate $\min [N, T]$, a result established in Bai and Ng (2002). ${ }^{4}$ Bai and Ng (2006a) showed that if $\sqrt{T} / N \rightarrow 0$ as $N, T \rightarrow \infty$, the sampling uncertainty from first step estimation is negligible. The practical implication is that standard errors can be computed for the estimates of $\alpha_{F}$ as though the true $F_{t}$ were used in the regression. This is in contrast to the case when $\widehat{F}_{t}$ is estimated from a first step regression with a finite number of predictors. As shown in Pagan (1984), the standard errors for $\widehat{\alpha}_{F}$ in such a case are incorrect unless they are adjusted for the estimation error incurred in the first step of $F_{t}$.

### 3.2 Dynamic factors

An advantage of the method of principal components is that it can handle a large panel of data at little computation cost, one reason being that little structure is imposed on the estimation. To be convinced that factor augmented regressions are useful in analyzing economic issues of interest, we need to show that estimates of the FAR are robust to the choice of the estimator and to the specification of the factor model. To this end, we consider an alternative way of estimating the factors with two fundamental differences.

First, we use prior information to organize the data into 8 blocks. These are (1) output, (2) labor market, (3) housing sector, (4) orders and inventories, (5) money and credit (6) bond and forex, (7) prices and (8) stock market. The largest block is the labor market which has 30 series, while the smallest group is the stock market block, which only has 4 series. The advantage of estimating the factors (which will now be denoted $g_{t}$ ) from blocks of data is that the factor estimates are easy to interpret.

[^3]Second, we estimate a dynamic factor model specified as

$$
\begin{equation*}
x_{i t}=\beta_{i}^{\prime}(L) g_{t}+e_{x i t} \tag{6}
\end{equation*}
$$

where $\beta_{i}(L)=\left(1-\lambda_{i 1} L-\ldots-\lambda_{i s} L^{s}\right)$ is a vector of dynamic factor loadings of order $s$ and $g_{t}$ is a vector of $q$ 'dynamic factors' evolving as

$$
\psi_{g}(L) g_{t}=\epsilon_{g t}
$$

where $\psi_{g}(L)$ is a polynomial in $L$ of order $p_{G}, \epsilon_{g t}$ are iid errors. Furthermore, the idiosyncratic component $e_{x i t}$ is an autoregressive process of order $p_{X}$ so that

$$
\psi_{x}(L) e_{x i t}=\epsilon_{x i t}
$$

This is the factor framework used in Stock and Watson (1989) to estimate the coincident indicator with $N=4$ variables. Here, our $N$ can be as large as 30 .

The dimension of $g_{t}$, (which also equals the dimension of $\epsilon_{t}$ ), is referred to as the number of dynamic factors. The main distinction between the static and the dynamic model is best understood using a simple example. The model $x_{i t}=\beta_{i 0} g_{t}+\beta_{i 1} g_{t-1}+e_{i t}$ is the same as $x_{i t}=\lambda_{i 1} f_{1 t}+\lambda_{i 2} f_{2 t}$ with $f_{1 t}=g_{t}$ and $f_{2 t}=g_{t-1}$. Here, the number of factors in the static model is two but there is only one factor in the dynamic model. Essentially, the static model does not take into account that $f_{t}$ and $f_{t-1}$ are dynamically linked. Forni, Hallin, Lippi, and Reichlin (2005) showed that when $N$ and $T$ are both large, the space spanned by $g_{t}$ can also be consistently estimated using the method of dynamic principal components originally developed in Brillinger (1981). Boivin and Ng (2005) finds that static and dynamic principal components have similar forecast precision, but that static principal components are much easier to compute. It is an open question whether to use the static or the dynamic factors in predictive regressions though the majority of factor augmented regressions use the static factor estimates. Our results will shed some light on this issue.

We estimate a dynamic factor model for each of the eight blocks. Given the definition of the blocks, it is natural to refer to $g_{1 t}$ as an output factor, $g_{7 t}$ as a price factor, and so on. However, as some blocks have a small number of series, the (static or dynamic) principal components estimator which assumes that $N$ and $T$ are both large will give imprecise estimates. We therefore use the Bayesian method of Monte Carlo Markov Chain (MCMC). MCMC samples a chain that has the posterior density of the parameters as its stationary distribution. The posterior mean computed from draws of the chain are then unbiased for $g_{t}$. For factor models, Kose, Otrok, and Whiteman (2003) uses an algorithm that involves
inversion of $N$ matrices that are of dimension $T \times T$, which can be computationally demanding. The algorithms used in Aguilar and West (2000), Geweke and Zhou (1996) and Lopes and West (2004) are extensions of the MCMC method developed in Carter and Kohn (1994) and Fruhwirth-Schnatter (1994). Our method is similar and follows the implementation in Kim and Nelson (2000) of the Stock-Watson coincident indicator closely. Specifically, we first put the dynamic factor model into a state-space framework. We assume $p_{X}=p_{G}=1$ and $s_{g}=2$ for every block. For $i=1, \ldots N_{b}$ (the number of series in block $b$ ), let $x_{i b t}$ be the observation for unit $i$ of block $b$ at time $t$. Given that $p_{X}=1$, the measurement equation is

$$
\left(1-\psi_{b i} L\right) x_{b i t}=\left(1-\psi_{b i} L\right)\left(\beta_{b i 0}+\beta_{b i 1} L+\beta_{b i 2} L^{2}\right) g_{b t}+\epsilon_{X b i t}
$$

or more compactly,

$$
x_{b i t}^{*}=\beta_{i}^{*}(L) g_{b t}+\epsilon_{X b i t} .
$$

Given that $p_{G}=1$, the transition equation is

$$
g_{b t}=\psi_{g b} g_{b t-1}+\epsilon_{g b t} .
$$

We assume $\epsilon_{X b i t} \sim N\left(0, \sigma_{X b i}^{2}\right)$ and $\epsilon_{g b} \sim N\left(0, \sigma_{g b}^{2}\right)$. We use principal components to initialize $g_{b t}$. The parameters $\beta_{b}=\left(\beta_{b 1}, \ldots \beta_{b, N b}\right), \psi_{X b}=\psi_{X b 1}, \ldots \psi_{X b, N b}$ are initialized to zero. Furthermore, $\sigma_{X b}=\left(\sigma_{X b 1}, \ldots \sigma_{X b, N_{b}}\right), \psi_{g b}$, and $\sigma_{g b}^{2}$ are initialized to random draws from the uniform distribution. For $b=1, \ldots 8$ blocks, Gibbs sampling can now be implemented by successive iteration of the following steps:
i draw $g_{b}=\left(g_{b 1}, \ldots g_{b T}\right)^{\prime}$ conditional on $\beta_{b}, \psi_{X b}, \sigma_{X b}$ and the $T \times N_{b}$ data matrix $x_{b}$.
ii draw $\psi_{g b}$ and $\sigma_{g b}^{2}$ conditional on $g_{b}$.
iii for each $i=1, \ldots N_{b}$, draw $\beta_{b i}, \psi_{X b i}$ and $\sigma_{X b i}^{2}$ conditional on $g_{b}$ and $x_{b}$.
We assume normal priors for $\beta_{b i}=\left(\beta_{i 0}, \beta_{i 1}, \beta_{i 2}\right), \psi_{X b i}$ and $\psi_{g b}$. Given conjugacy, $\beta_{b i}, \psi_{X b i}, \psi_{g b}$, are simply draws from the normal distributions whose posterior means and variances are straightforward to compute. Similarly, $\sigma_{g b}^{2}$ and $\sigma_{X b i}^{2}$ are draws from the inverse chi-square distribution. Because the model is linear and Gaussian, we can run the Kalman filter forward to obtain the conditional mean $g_{b T \mid T}$ and conditional variance $P_{b T \mid T}$. We then draw $g_{b T}$ from its conditional distribution, which is normal, and proceed backwards to generate draws $g_{b t \mid T}$ for $t=T-1, \ldots 1$ using the Kalman filter. For identification, the loading on the first series in each block is set to 1 . We take 12,000 draws and discard the first 2000.

The posterior means are computed from every 10th draw after the burn-in period. The $\widehat{g}_{t} \mathrm{~S}$ used in subsequent analysis are the means of these 1000 draws.

As in the case of static factors, not every $g_{b t}$ need to have predictive power for excess bond returns. Let $G_{t} \subset g_{t}=\left(g_{1 t}, \ldots g_{8 t}\right)$ be those that do. The analog to (5) using dynamic factors is

$$
\begin{equation*}
r x_{t+1}^{(n)}=\alpha_{G}^{\prime} \widehat{G}_{t}+\beta_{G}^{\prime} Z_{t}+\epsilon_{t+1}, \tag{7}
\end{equation*}
$$

We have now obtained two sets of factor estimates using two distinct methodologies. We can turn to an assessment of whether the estimates of the predictive regression are sensitive to how the factors are estimated.

### 3.3 Comparison of $\widehat{f}_{t}$ and $\widehat{g}_{t}$

Table 1 reports the first order autocorrelation coefficients for $f_{t}$ and $g_{t}$. Both sets of factors exhibit persistence, with $\widehat{f}_{1 t}$ being the most correlated of the eight $\widehat{f}_{t}$, and $\widehat{g}_{3 t}$ being the most serially correlated amongst the $\widehat{g}_{t}$. Table 2 reports the contemporaneous correlations between $\widehat{f}$ and $\widehat{g}$. The real activity factor $\widehat{f}_{1}$ is highly correlated with the $\widehat{g}_{t}$ estimated from output, labor and manufacturing blocks. $\widehat{f}_{2}, \widehat{f}_{4}$, and $\widehat{f}_{5}$ are correlated with many of the $\widehat{g}$, but the correlations with the bond/exchange rate seem strongest. $\widehat{f}_{3}$ is predominantly a price factor, while $\widehat{f}_{8}$ is a stock market factor. $\widehat{f}_{7}$ is most correlated with $\widehat{g}_{5}$, which is a money market factor. $\widehat{f}_{8}$ is highly correlated with $\widehat{g}_{8}$, which is estimated from stock market data.

The contemporaneous correlations reported in Table 2 does not give a full picture of the correlation between $\widehat{f}_{t}$ and $\widehat{g}_{t}$ for two reasons. First, the $\widehat{g}_{t}$ are not mutually uncorrelated, and second, they do not account for correlations that might occur at lags. To provide a sense of the dynamic correlation between $\widehat{f}$ and $\widehat{g}_{t}$, we first standardize $\widehat{f}_{t}$ and $\widehat{g}_{t}$ to have unit variance. We then consider the regression

$$
\widehat{f}_{r t}=a+A_{r .0} \widehat{g}_{t}+\sum_{i=1}^{p-1} A_{r . i} \Delta \widehat{g}_{t-i}+e_{i t}
$$

where for $r=1, \ldots 8$ and $i=0, \ldots p-1, A_{r . i}$ is a $8 \times 1$ vector of coefficients summarizing the dynamic relation between $\widehat{f}_{r t}$ and lags of $\widehat{g}_{t}$. The coefficient vector $A_{r .0}$ summarizes the long run relation between $\widehat{g}_{t}$ and $\widehat{f}_{t}$. Table 3 reports results for $p=4$, along with the $R^{2}$ of the regression. Except for $\widehat{f}_{6}$, the current value and lags of $\widehat{g}_{t}$ explain the principal components quite well. While it is clear that $\widehat{f}_{1}$ is a real activity factor, the remaining $\widehat{f_{\mathrm{s}}}$ tend to load on variables from different categories. Tables 2 and 3 reveal that $\widehat{g}_{t}$ and $\widehat{f_{t}}$ reduce the dimensionality of information in the panel of data in different ways. Evidently, the $\widehat{f}_{t} \mathrm{~S}$
are weighted averages of the $\widehat{g}_{t} s$ and their lags. This can be important in understanding the results to follow.

## 4 Predictive Regressions

Let $\widehat{H}_{t} \subset \widehat{h}_{t}$, where $\widehat{h}_{t}$ is either $\widehat{f}_{t}$ or $\widehat{g}_{t}$. Our predictive regression can generically be written as

$$
\begin{equation*}
r x_{t+1}^{(n)}=\alpha^{\prime} \widehat{H}_{t}+\beta^{\prime} C P_{t}+\epsilon_{t+1} \tag{8}
\end{equation*}
$$

Equation (8) allows us to assess whether $\widehat{H}_{t}$ has predictive power for excess bond returns, conditional on the information in $C P_{t}$. In order to assess whether macro factors $\widehat{H}_{t}$ have unconditional predictive power for future returns, we also consider the restricted regression

$$
\begin{equation*}
r x_{t+1}^{(n)}=\alpha^{\prime} \widehat{H}_{t}+\epsilon_{t+1} . \tag{9}
\end{equation*}
$$

Since $\widehat{F}_{t}$ and $\widehat{G}_{t}$ are both linear combinations of $x_{t}=\left(x_{1 t}, \ldots x_{N t}\right)^{\prime}$, say $F_{t}=q_{F}^{\prime} x_{t}$ and $G_{t}=q_{G}^{\prime} x_{t}$, we can also write (8) as

$$
r x_{t+1}^{(n)}=\alpha^{* \prime} x_{t}+\beta^{\prime} C P_{t}+\epsilon_{t+1}
$$

where $\alpha^{* \prime}=\alpha_{F}^{\prime} q_{F}^{\prime}$ or $\alpha_{G}^{\prime} q_{G}^{\prime}$. The conventional regression (1) puts a weight of zero on all but a handful of $x_{i t}$. When $\widehat{H}_{t}=\widehat{F}_{t}, q_{F}$ is related to the $k$ eigenvectors of $x x^{\prime} /(N T)$ that will not, in general, be numerically equal to zero. When $\widehat{H}_{t}=\widehat{G}_{t}, q_{G}$ and thus $\alpha^{*}$ will have many zeros since each column of $\widehat{G}_{t}$ is estimated using a subset of $x_{t}$. Viewed in this light, a factor augmented regression with PCA down-weights unimportant regressors. A FAR estimated using blocks of data sets put some but not all coefficients on $x_{t}$ equal to zero. A conventional regression is most restrictive as it constrains almost the entire $\alpha^{*}$ vector to zero.

As discussed earlier, factors that are pervasive in the panel of data $x_{i t}$ need not have predictive power for $r x_{t+1}^{(n)}$, which is our variable of interest. In Ludvigson and Ng (2007), $\widehat{H}_{t}=\widehat{F}_{t}$ was determined using a method similar to that used in Stock and Watson (2002b). We form different subsets of $\widehat{f_{t}}$, and/or functions of $\widehat{f}_{t}$ (such as $\widehat{f}_{1 t}^{2}$ ). For each candidate set of factors, $\widehat{F}_{t}$, we regress $r x_{t+1}^{(n)}$ on $\widehat{F}_{t}$ and $C P_{t}$ and evaluate the corresponding in-sample BIC and $\bar{R}^{2}$. The in-sample BIC for a model with $k$ regressors is defined as

$$
B I C_{i n}(k)=\widehat{\sigma}_{k}^{2}+k \frac{\log T}{T}
$$

where $\widehat{\sigma}_{k}^{2}$ is the variance of the regression estimated over the entire sample. To limit the number of specifications we search over, we first evaluate $r$ univariate regressions of returns
on each of the $r$ factors. Then, for only those factors found to be significant in the $r$ univariate regressions, we evaluate whether the squared and the cubed terms help reduce the $B I C$ criterion further. We do not consider other polynomial terms, or polynomial terms of factors not important in the regressions on linear terms.

In this paper, we again use the BIC to find the preferred set of factors, but we perform a systematic and therefore much larger search. Instead of relying on results from preliminary univariate regressions to guide us to the final model, we directly search over a large number models with different numbers of regressors. We want to allow excess bond returns to be possibly non-linear in the eight factors and hence include the squared terms as candidate regressors. If we additionally include all the cubic terms, and given that we have eight factors and CP to consider, we would have over thirteen million ( $2^{27}$ ) potential models. As a compromise, we limit our candidate regressor set to eighteen variables: $\left(\widehat{f}_{1 t}, \ldots f_{8 t}, \widehat{f}_{1 t}^{2}, \ldots f_{8 t}^{2}, \widehat{f}_{1 t}^{3}, C P_{t}\right)$. We also restrict the maximum number of predictors to eight. This leads to an evaluation of 106762 models. ${ }^{5}$

The purpose of this extensive search is to assess the potential impact on the forecasting analysis of fishing over large numbers of possible predictor factors. As we show, the factors chosen by the larger, more systematic, search are the same as those chosen by the limited search procedure used in Ludvigson and Ng (2007). This suggests that data-mining does not in practice unduly influence the findings in this application, since we find that the same few key factors always emerge as important predictor variables regardless of how extensive the search is.

It is well known that variables found to have predictive power in-sample do not necessarily have predictability out-of-sample. As discussed in Hansen (2008), in-sample overfitting generally leads to a poor out-of-sample fit. One is less likely to produce spurious results based on an out-of-sample criterion because a complex (large) model is less likely to be chosen in an out-of-sample comparison with simple models when both models nests the true model. Thus, when a complex model is found to outperform a simple model out of sample, it is stronger evidence in favor of the complex model. To this end, we also find the best amongst 106762 models as the minimizer of the out-of-sample BIC. Specifically, we split the sample at $t=T / 2$. Each model is estimated using the first $T / 2$ observations. For $t=T / 2+1, \ldots T$, the values of predictors in the second half of the sample are multiplied into the parameters estimated using the first half of the sample to obtain the fit, denoted

[^4]$\widehat{r} x_{t+12}$. Let $\widetilde{e}_{t}=r x_{t+12}-\widehat{r} x_{t+12}$ and $\widetilde{\sigma}_{k}^{2}=\frac{1}{T / 2} \sum_{t=T / 2+1}^{T} \widetilde{e}_{t}^{2}$ be the out-of-sample error variance corresponding to model $j$. The out-of-sample BIC is defined as
$$
B I C_{o u t}(j)=\log \widetilde{\sigma}_{j}^{2}+\frac{\operatorname{dim}_{j} \log (T / 2)}{T / 2}
$$
where $\operatorname{dim}_{j}$ is the size of model $j$. By using an out-of-sample BIC selection criterion, we guard against the possibility of spurious overfitting. Regressors with good predictive power only over a subsample will not likely be chosen. As the predictor set may differ depending on whether the CP factor is included (ie. whether we consider (8) or (9)), the two variable selection procedures are repeated with CP excluded from the potential predictor set. Using the predictors selected by the in- and the out-of-sample BIC, we re-estimate the predictive regression over the entire sample. In the next section, we show that the predictors found by this elaborate search are the same handful of predictors found in Ludvigson and Ng (2007) and that these handful of macroeconomic factors have robust significant predictive power for excess bond returns beyond the CP factor.

We also consider as predictor a linear combination of $\widehat{h}_{t}$ along the lines of Cochrane and Piazzesi (2005). This variable, denoted $\widehat{H} 8_{t}$ is defined as $\widehat{\gamma}^{\prime} \widehat{h}_{t}^{+}$where $\widehat{\gamma}$ is obtained from the following regression:

$$
\begin{equation*}
\frac{1}{4} \sum_{n=2}^{5} r x_{t+1}^{n}=\gamma_{0}+\gamma^{\prime} \widehat{h}_{t}^{+} \tag{10}
\end{equation*}
$$

with $\widehat{h}_{t}^{+}=\left(\widehat{h}_{1 t}, \ldots \widehat{h}_{8 t}, \widehat{h}_{1 t}^{3}\right)$. The estimates are as follows:

|  | $h_{t}=\widehat{f}_{t}$ |  | $h_{t}=\widehat{g}_{t}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\widehat{\gamma}$ | $t_{\widehat{\gamma}}$ | $\widehat{\gamma}$ | $t_{\widehat{\gamma}}$ |
| $h_{1}$ | -1.681 | -4.983 | 0.053 | 0.343 |
| $h_{2}$ | 0.863 | 3.009 | -1.343 | -2.593 |
| $h_{3}$ | -0.018 | -0.203 | -0.699 | -1.891 |
| $h_{4}$ | -0.626 | -2.167 | 0.628 | 1.351 |
| $h_{5}$ | -0.264 | -1.463 | -0.001 | -0.012 |
| $h_{6}$ | -0.720 | -2.437 | -0.149 | -0.691 |
| $h_{7}$ | -0.426 | -2.140 | -0.018 | -0.210 |
| $h_{8}$ | 0.665 | 3.890 | -0.418 | -2.122 |
| $h_{1}^{3}$ | 0.115 | 3.767 | 0.049 | 1.733 |
| cons | 0.900 | 2.131 | 0.764 | 1.518 |
| $\bar{R}^{2}$ | 0.261 |  | 0.104 |  |

Notice that we could also have replaced $\widehat{h}_{t}$ in the above regression with $\widehat{H}_{t}$, where $\widehat{H}_{t}$ is comprised of predictors selected by either the in- or the out-of-sample BIC. However, $\widehat{H} 8_{t}$ is
a factor-based predictor that is arguable less vulnerable to the effects of data mining because it is simple a linear combination of all the estimated factors.

Tables 4 to 7 report results for maturities of $2,3,4$, and 5 years. The first four columns of each table are based on the static factors (ie. $\widehat{H}_{t}=\widehat{F}_{t}$ ), while columns 5 to 8 are based on the dynamic factors (ie. $\widehat{H}_{t}=\widehat{G}_{t}$ ). Of these, columns $1,2,5$, and 6 include the CP variable, while columns $3,4,7$, and 8 do not include the CP. Columns 9 and 10 report results using $\widehat{F} 8$ with and without CP and columns 11 and 12 do the same with $\widehat{G} 8$ in place. Our benchmark is a regression that has the CP variable as the sole predictor. This is reported in last column, ie. column 13 .

### 4.1 Two Year Returns

As can be seen from Table 4, the CP alone explains .309 of the variance in the two year excess bound returns. The variable $\widehat{F}_{8}$ alone explains 0.279 (column 10), while $\widehat{G}_{8}$ alone explains only .153 of the variation (column 12). Adding $\widehat{F} 8$ to the regression with the CP factor (column 9) increases $\bar{R}^{2}$ to .419 , and adding $\widehat{G} 8$ (column 11) to CP yields an $\bar{R}^{2}$ of .401. The macroeconomic factors thus have non trivial predictive power above and beyond the CP factor.

We next turn to regressions when both the factors and CP are included. In Ludvigson and $\mathrm{Ng}(2007)$, the static factors $\widehat{f}_{1 t}, \widehat{f}_{2 t}, \widehat{f}_{3 t}, \widehat{f}_{4 t}, \widehat{f}_{8 t}$ and CP are found to have the best predictive power for excess returns. The in-sample BIC still finds the same predictors to be important, but adds $\widehat{f}_{6 t}$ and $\widehat{f}_{5 t}^{2}$ to the predictor list. It is however noteworthy that some variables selected by the BIC have individual $t$ statistics that are not significant. The resulting model has an $\bar{R}^{2}$ of 0.460 (column 1). The out-of-sample BIC selects smaller models and finds $\widehat{f_{1}}, \widehat{f_{8}}, \widehat{f}_{5}^{2}, \widehat{f}_{1}^{3}$ and the CP to be important regressors (column 2).

Amongst the dynamic factors, $\widehat{g}_{2}$ (labor market), $\widehat{g}_{8}$ (stock market), $\widehat{g}_{6}^{2}$ (bonds and foreign exchange) along with CP are selected by both BIC procedures as predictors (columns 5 and 6). Interestingly, the output factor $\widehat{g}_{1}$ is not significant when the CP is included. The out-of-sample BIC has an $\bar{R}^{2}$ of 0.407 , showing that there is a substantial amount of variation in the two-year excess bond returns that can be predicted by macroeconomic factors. The in-sample BIC additionally selects $\widehat{g}_{3 t}, \widehat{g}_{6 t}$ and some higher order terms with an $\bar{R}^{2}$ of 0.477 . Thus, predictive regressions using $\widehat{f}_{t}$ and $\widehat{g}_{t}$ both find a factor relating to real activity ( $\widehat{f}_{1 t}$ or $\widehat{g}_{1 t}$ ) and one relating to the stock market ( $\widehat{f}_{8 t}$ or $\widehat{g}_{18}$ ) to have significant predictive power for two-year excess bond returns.

Results when the regressions do not include the CP variable are in columns 3, 4, 7, and
8. Evidently, $\widehat{f}_{2}$ is now important according to both the in- and out-of-sample BIC, showing that the main effect of CP is to render $\widehat{f}_{2}$ redundant. Furthermore, the out-of-sample BIC now selects a model that is only marginally more parsimonious than that selected by the in-sample BIC. The regressions with $\widehat{F}$ alone have an $\bar{R}^{2}$ of 0.283 and 0.258 respectively, slightly less than what is obtained with CP as the only regressor.

Regressions based on the dynamic factors are qualitatively similar. The factors $\widehat{g}_{1}, \widehat{g}_{3}$, and $\widehat{g}_{4}$, found not to be important when CP is included are now selected as relevant predictors when CP is dropped. Without CP, the dynamic factors selected by the in-sample BIC explain 0.2 of the one-year-ahead variation in excess bond returns, while the more parsimonious model selected by the out-of-sample BIC has an $\bar{R}^{2}$ of 0.192 These numbers are lower than what we obtain in columns (3) and (4) using $\widehat{F}_{t}$ as predictors.

It is important to stress that we consider the two sets of factor estimates not to perform a horse race of whether the PCA or the Bayesian estimator is better. The purpose instead is to show that macroeconomic factors have predictive power for excess bond returns irrespective of the way we estimate the factors. Although the precise degree of predictability depends on how the factors are estimated, a clear picture emerges. At least twenty percent of the variation in excess bound returns can be predicted by macroeconomic factors even in the presence of the CP factor.

### 4.2 Longer Maturity Returns and Overview

Table 5 to 7 report results for returns with maturity of three, four, and five years. Most of the static factors found to be useful in predicting $r x_{t+1}^{(2)}$ by the in-sample BIC remain useful in predicting the longer maturity returns. These predictors include $\widehat{f}_{1 t}, \widehat{f}_{4 t}, \widehat{f}_{6 t}, \widehat{f}_{7 t}, \widehat{f}_{8 t}, \widehat{f}_{1 t}^{3}$, and CP. Of these, $\widehat{f}_{1 t}, \widehat{f}_{8 t}$, and CP are also selected by the out-of-sample BIC procedure. The non-linear term $\widehat{f}_{1 t}^{3}$ is an important predictor in equations for all maturity returns except the five year. The factors add at least ten basis points to the $\bar{R}^{2}$ with CP as the sole predictor.

The dynamic factors found important in explaining two year excess return are generally also relevant in regressions for longer maturity excess returns. The in-sample BIC finds $\widehat{g}_{2 t}$, $\widehat{g}_{3 t}, \widehat{g}_{8 t}, \widehat{g}_{4 t}^{2}, \widehat{g}_{6 t}^{2}$ along with the CP to be important in regressions of all maturities. The output factor is again not significant in regressions with three and four year maturities. It is marginally significant in the five year maturity, but has the wrong sign. While $\widehat{g}_{8}$ was relevant in the two year regression, it is not an important predictor in the regressions for longer maturity returns. The out-of-sample BIC finds dynamic factors from the labor market $\left(\widehat{g}_{2 t}\right)$, the bond and foreign exchange markets $\left(\widehat{g}_{6 t}\right)$. Together, these factors have incremental
predictive power for excess bond returns over CP, improving the $\bar{R}^{2}$ by slightly less than 10 basis points.

The relevance of macroeconomic variables in explaining excess bond returns is reinforced by the results in columns 10 and 12 , which show that a simple linear combination of the eight factors still adds substantial predictive power beyond the CP factor. This result is robust across all four maturities considered, noting that the coefficient estimate on $\widehat{H} 8$ increases with the holding period without changing the statistical significance of the coefficient.

To see if the predictability varies over the sample, we also consider rolling regressions. Starting with the first regression that spans the sample 1964:1-1974:12, we add twelve monthly observations each time and record the $\bar{R}^{2}$. Figure 9 shows the $\bar{R}^{2}$ for regressions with CP included. Apart from a notable drop around the 1983 recession, $\bar{R}^{2}$ is fairly constant. Figure 10 depicts the $\bar{R}^{2}$ for regressions without CP. Notice that the $\bar{R}^{2}$ that corresponds to $\widehat{F} 8_{t}$ tends to be 15 basis points higher than $\widehat{G} 8_{t}$. As noted earlier, each of the eight $\widehat{f}_{t}$ is itself a combination of the current and lags of the eight $\widehat{g}_{t}$. This underscores the point that imposing a structure on the data to facilitate interpretation of the factors comes at the cost of not letting the data find the best predictive combination possible.

The results reveal that the estimated factors consistently have stronger predictive power for one- and multi-year ahead excess bond returns. The most parsimonious specification has just two variables - $\widehat{H} 8$ and $C P_{t}$ - explaining over forty percent of the variation in $r x_{t+1}^{n}$ of every maturity. A closer look reveals that the real activity factor $\widehat{f}_{1 t}$ is the strongest factor predictor, both numerically and statistically. As $\widehat{g}_{1 t}$ tends not to be selected as predictor, this suggests that the part of $\widehat{f}_{1 t}$ that has predictive power for excess bond returns is derived from real activity other than output. However, the dynamic factors $\widehat{g}_{2 t}$ (labor market) and $\widehat{g}_{3 t}$ (housing) have strong predictive power. Indeed, $\widehat{f}_{1 t}$ is highly correlated with $\widehat{g}_{2 t}$ and the coefficients for these predictors tend to be negative. This means that excess bond returns of every maturity are counter-cyclical, especially with the labor market. This result is in accord with the models of Campbell (1999) and Wachter (2006), which posit that forecasts of excess returns should be counter-cyclical because risk aversion is low in good times and high in recessions. We will subsequently show that yield risk premia, which are based on forecasts of excess returns, are also counter-cyclical.

## 5 Inference Issues

The results thus far assume that $N$ and $T$ are large and that $\sqrt{T} / N$ tends to zero. In this section, we first consider the implication for factor augmented regressions when $\sqrt{T} / N$ may
not be small as is assumed. We then examine the finite sample inference issues.

### 5.1 Asymptotic Bias

If excess bond returns truly depend on macroeconomic factors, then consistent estimates of the factors should be better predictors than the observed variables because these are contaminated measures of real activity. ${ }^{6}$ An appealing feature of PCA is that if $\sqrt{T} / N \rightarrow 0$ as $N, T \rightarrow \infty$, then $\widehat{F}_{t}$ can be treated in the predictive regression as though it were $F_{t}$. To see why this is the case, consider again the infeasible predictive regression, dropping the observed predictors $W_{t}$ for simplicity. We have

$$
\begin{aligned}
r x_{t+1}^{n} & =\alpha_{F}^{+\prime} F_{t}+\epsilon_{t+1} \\
& =\alpha_{F}^{\prime} \widehat{F}_{t}+\alpha_{F}^{\prime}\left(H F_{t}-\widehat{F}_{t}\right)+\epsilon_{t+1}
\end{aligned}
$$

where $\alpha_{F}=\alpha H^{-1}$, and $H$ is a $r \times r$ matrix defined in Bai and Ng (2006a). Let $S_{\widehat{F} \widehat{F}}=$ $T^{-1} \sum_{t=1}^{T} \widehat{F}_{t} \widehat{F}_{t}^{\prime}$. Then

$$
\begin{equation*}
\sqrt{T}\left(\widehat{\alpha}_{F}-\alpha_{F}\right)=\widehat{S}_{\widehat{F} \widehat{F}}^{-1}\left(\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \widehat{F}_{t} \epsilon_{t+1}\right)+S_{\widehat{F} \widehat{F}}^{-1}\left(\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \widehat{F}_{t}\left(H F_{t}-\widehat{F}_{t}\right)\right) \alpha_{F} . \tag{11}
\end{equation*}
$$

But $T^{-1} \widehat{F}^{\prime}\left(F H^{\prime}-\widehat{F}\right)=O_{p}\left(\min [N, T]^{-1}\right)$, a result that follows from Bai (2003). Thus if $\sqrt{T} / N \rightarrow 0$, the second term is negligible. It follows that

$$
\sqrt{T}\left(\widehat{\alpha}_{F}-\alpha_{F}\right) \xrightarrow{d} N\left(0, A \operatorname{var}\left(\widehat{\alpha}_{F}\right)\right)
$$

where

$$
\operatorname{Avar}\left(\widehat{\alpha}_{F}\right)=\operatorname{plim} S_{\widehat{F} \widehat{F}}^{-1} \widehat{\operatorname{Avar}}\left(g_{t}\right) S_{\widehat{F} \widehat{F}}^{-1},
$$

$\widehat{\operatorname{Avar}}\left(g_{t}\right)$ is an estimate of the asymptotic variance of $g_{t+1}=\widehat{\epsilon}_{t+1} \widehat{F}_{t}$.
Consider now the case when $\sqrt{T}$ is comparable to $N$. Although the first term on the right hand size of (11) is mean zero, the second term is a $O_{p}(1)$ random variable that may not be mean zero. This generates a bias in the asymptotic distribution for $\widehat{\alpha}_{F}$.

Proposition 1 Suppose Assumptions $A-E$ of Bai and $N g$ (2006a) hold and let $\widehat{F}_{t} \subset \widehat{f_{t}}$, where $\widehat{f}_{t}$ are the principal component estimates of $f_{t}, x_{i t}=\lambda_{i t}^{\prime} f_{t}+e_{i t}$. Let $\widehat{\alpha}_{F}$ be obtained

[^5]from least squares estimation of the FAR $y_{t+h}=\alpha_{F}^{\prime} \widehat{F}_{t}+e_{t+h}$. An estimate of the bias in $\widehat{\alpha}_{F}$ is
$$
\widehat{B}_{1} \approx-S_{\widehat{F} \widehat{F}}^{-1}\left(\frac{1}{N T} \sum_{t=1}^{T} \widehat{\operatorname{Avar}_{t}\left(\widehat{F}_{t}\right)}\right) \widehat{\alpha}_{F}
$$
where $\operatorname{Avar}_{t}\left(F_{t}\right)=V^{-1} \Gamma_{t} V^{-1}$, V is a $r \times r$ diagonal matrix of the eigenvalues of $(N \cdot T)^{-1} x x^{\prime}$, and $\Gamma_{t}=\sum_{N \rightarrow \infty} N^{-1} \sum_{i=1}^{N} \sum_{j=1}^{N} E\left(\lambda_{i} \lambda_{j}^{\prime} e_{i t} e_{j t}\right)$. Let $\widehat{\alpha}_{F}^{B}=\widehat{\alpha}_{F}-\widehat{B}_{1}$ be the biased corrected estimate. Then
$$
\sqrt{T}\left(\widehat{\alpha}_{F}^{B}-\alpha_{F}\right) \xrightarrow{d} N\left(0, \operatorname{Avar}\left(\widehat{\alpha}_{F}\right)\right) .
$$

The asymptotic variance for the bias corrected estimator is the same as $\widehat{\alpha}_{F}$.
Proposition 1 makes use of the fact that

$$
\begin{aligned}
\frac{1}{T} \sum_{t=1}^{T} \widehat{F}_{t}\left(H F_{t}-\widehat{F}_{t}\right)^{\prime} & \left.=\frac{1}{T} \sum_{t=1}^{T}\left(\widehat{F}_{t}-H F_{t}\right)\left(H F_{t}-\widehat{F}_{t}\right)^{\prime}+H F_{t}\left(H F_{t}\right)-\widehat{F}_{t}\right) \\
& =-E\left[\frac{1}{T} \sum_{t=1}^{T}\left(\widehat{F}_{t}-H F_{t}\right)\left(H F_{t}-\widehat{F}_{t}\right)^{\prime}\right]+o_{p}(1) \\
& =-\frac{1}{N T} \sum_{t=1}^{T} \operatorname{Avar}\left(\widehat{F}_{t}\right)+o_{p}(1) .
\end{aligned}
$$

The estimation of $\operatorname{Avar}_{t}\left(\widehat{F}_{t}\right)$ was discussed at in Bai and $\operatorname{Ng}$ (2006a). If $E\left(e_{i t}^{2}\right)=\sigma^{2}$ for all $i$ and $t, \operatorname{Avar}_{t}\left(F_{t}\right)$ is the same for all $t$. Although $\Gamma_{t}$ will depend on $t$ if $e_{i t}$ is heteroskedastic, a consistent estimate of $\Gamma_{t}$ can be obtained for each $t$ when the errors are not cross-sectionally correlated, ie. $E\left(e_{i t} e_{j t}\right)=0$. Alternatively, if $E\left(e_{i t} e_{j t}\right)=\sigma_{i j} \neq 0$ for some or all $t$, panel data permit an estimate of $\operatorname{Avar}\left(\widehat{F}_{t}\right)$ that does not depend on $t$ even when the $e_{i t}$ are crosssectionally correlated. This estimator of $\Gamma_{t}$, referred to as CS-HAC in Bai and Ng (2006a), will be used below.

As this result on bias is new, we consider a small Monte Carlo experiment to gauge the magnitude of the bias as $N$ and $T$ changes. We consider a model with $r=1$ and 2 factors. We assume $\lambda_{i} \sim N(0,1)$ and $F_{t} \sim N(0,1)$. These are only simulated once. Samples of $x_{i t}=\lambda_{i} F_{t}+e_{i t}$ and $y_{t}=\alpha^{\prime} F_{t}+\epsilon_{t}$ are obtained by simulating $e_{i t} \sim \sigma N(0,1)$ and $\epsilon_{t} \sim N(0,1)$ for $i=1, \ldots N, t=1, \ldots T$. We let $\alpha=1$ when $r=1$ and $\alpha=(1,2)$ when $r=2$. We consider three values of $\sigma$. The smaller $\sigma$ is, the more informative are the data for the factors. The results are as follows:

Estimated Bias for $\widehat{\alpha}_{1}$

| DGP: $y_{t}=F_{t}^{\prime} \alpha+\epsilon_{t}, \quad x_{i t}=\lambda_{i} F_{t}+e_{i t}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\sigma=1$ |  |  |  |  |  |  |  |
|  | $\mathrm{~T}=50$ | 100 | 200 | 500 | 50 | 100 | 200 | 500 |
| $r=1$ |  |  |  |  |  |  |  |  |
| $\mathrm{~N}=50$ | -0.025 | -0.020 | -0.022 | -0.019 | -0.171 | -0.156 | -0.210 | -0.242 |
| 100 | -0.009 | -0.009 | -0.009 | -0.012 | -0.107 | -0.107 | -0.115 | -0.138 |
| 200 | -0.004 | -0.004 | -0.005 | -0.004 | -0.058 | -0.058 | -0.068 | -0.071 |
| 500 | -0.002 | -0.002 | -0.002 | -0.002 | -0.024 | -0.030 | -0.031 | -0.034 |
| $r=2$ |  |  |  |  |  |  |  |  |
| 50 | 0.014 | -0.035 | 0.026 | 0.017 | 0.002 | -0.244 | -0.077 | -0.124 |
| 100 | -0.020 | 0.003 | -0.018 | -0.020 | 0.116 | -0.170 | -0.056 | -0.158 |
| 200 | -0.010 | 0.001 | 0.007 | -0.009 | -0.104 | -0.036 | 0.077 | -0.092 |
| 500 | -0.005 | 0.002 | -0.004 | 0.001 | -0.047 | -0.043 | 0.028 | 0.031 |

As the true value of $\alpha$ is one, the entries can also be interpreted as percent bias. For large $N$ and $T$, the bias is quite small and ignoring the sampling error in $\widehat{F}_{t}$ should be inconsequential. Bias is smaller when $T / N=c$ than when $N / T=c$ for the same $c>$ 1 , confirming that the factors are more precisely estimated when there are more crosssection units to wash out the idiosyncratic noise. However, when $\sigma$ is large and the data are uninformative about the factors, the bias can be well over $10 \%$ and as large as $20 \%$. In such cases, the bias is also increasing in the number of estimated factors.

### 5.1.1 Bias When the Predictors are Functions of $\widehat{f_{t}}$

Our predictive regression has two additional complications. First, some of our predictors are powers of the estimated factors. Second, $\widehat{F} 8_{t}$ is a linear combination of a subset of $\widehat{f_{t}}$ and $\widehat{f_{1 t}^{3}}$, which is a non-linear function of $\widehat{f}_{1 t}$. To see how to handle the first problem, consider the case of the scalar predictor, $\widehat{m}_{t}=m\left(\widehat{f}_{1 t}\right)$ and let $m_{t}=m\left(H f_{1 t}\right)$ where $m$ takes its argument to the power $b$. The factor augmented regression becomes

$$
y_{t}=\alpha_{F}^{\prime} \widehat{m}_{t}+\alpha_{F}^{\prime}\left(m_{t}-\widehat{m}_{t}\right)+\epsilon_{t}
$$

where $\alpha_{F}=\alpha H^{-b}$. The required bias correction is now of the form

$$
B_{2}=S_{\widehat{m} \widehat{m}^{\prime}}^{-1}\left(\frac{1}{T} \sum_{t=1}^{T} \widehat{m}\left(m_{t}-\widehat{m}_{t}\right)^{\prime} \alpha_{F}\right)
$$

But since $m$ is continuous in $\widehat{f}_{1 t}$,

$$
m\left(\widehat{f}_{1 t}\right)=m\left(f_{t}\right)+m_{\widehat{f}_{1}, t}\left(\widehat{f}_{1 t}-H f_{1 t}\right)
$$

where $m_{\widehat{f}, t}=\left.\frac{\partial \widehat{m}_{t}}{\partial \widehat{f}_{1 t}}\right|_{\widehat{f}_{1 t}=H f_{1 t}}$. We have

$$
\widehat{m}_{t}-m_{t}=b\left(H f_{1 t}\right)^{b-1}\left(\widehat{f}_{1 t}-H f_{1 t}\right)=O_{p}\left(\min [N, T]^{-1}\right) .
$$

Given the foregoing result, it is then straightforward to show that

$$
T^{-1} \sum_{t=1}^{T} \widehat{m}_{t}\left(m_{t}-\widehat{m}_{t}\right)^{\prime}=\left[T^{-1} \sum_{t=1}^{T} m_{\widehat{f}_{1}, t} \operatorname{Avar}\left(\widehat{f}_{1 t}\right) m_{\hat{f}_{1}, t}^{\prime}\right]+o_{p}(1)
$$

Extending the argument to the case when $m_{t}$ is a vector leads to the bias correction

$$
\widehat{B}_{2}=-S_{\widehat{m} \widehat{m}^{\prime}}^{-1}\left(T^{-1} \sum_{t=1}^{T} m_{\widehat{F}, t} \operatorname{Avar}\left(\widehat{F}_{1 t}\right) m_{\widehat{F}, t}^{\prime}\right) \alpha_{F} .
$$

Finally, consider the predictive regression

$$
y_{t}=\alpha_{F}^{\prime} \widehat{M}_{t}+\epsilon_{t}
$$

where $\widehat{M}_{t}=\widehat{\gamma}_{0}+\widehat{\gamma}^{\prime} \widehat{m}_{t}$. The bias can be estimated by

$$
\widehat{B}_{3}=\widehat{\gamma}^{\prime} \widehat{B}_{2} \widehat{\gamma}
$$

In our application, $\widehat{\gamma}$ is obtained from estimation of (10).
While in theory, these bias corrections are required only when $\sqrt{T} / N$ does not tend to zero, in finite samples, the bias correction might be desirable even when $\sqrt{T} / N$ is small. We calculate the biased corrected estimates for two specifications of the predictive regressions. The first is when the predictors are selected by the in-sample BIC (column 1 of Tables 4-7). As this tends to lead to a larger model, the bias is likely more important. The second is when $\widehat{F} 8_{t}$ is used as predictor (column 9 of Table 4-7), which is the most parsimonious of our specifications. Note that the observed predictor CP is not associated with first step estimation error. As such, this predictor does not contribute to bias.

Reported in Table 8 are results using the CS-HAC, which allows the idiosyncratic errors to be cross-sectionally correlated. Results when the errors are heteroskedastic but crosssectionally uncorrelated are similar. The results indicate that the bias is quite small. For the present application, the effect of the bias correction is to increase the absolute magnitude of the coefficient estimates in the predictive regressions. The $t$ statistics (not reported) are correspondingly larger. The finding that the macroeconomic factors have predictive power for excess bond returns is not sensitive to the assumption underlying the asymptotically validity of the FAR estimates.

### 5.2 Bootstrap Inference

According to asymptotic theory, heteroskedasticity and autocorrelation consistent standard errors that are asymptotically $N(0,1)$ can be used to obtain robust $t$-statistics for the insample regressions. Moreover, provided $\sqrt{T} / N$ goes to zero as the sample increases, the $\widehat{F}_{t}$ can be treated as observed regressors, and the usual $t$-statistics are valid (Bai and Ng (2006a)). To guard against inadequacy of the asymptotic approximation in finite samples, we consider bootstrap inference in this section.

To proceed with a bootstrap analysis, we need to generate bootstrap samples of $r x_{t+1}^{(n)}$, and thus the exogenous predictors $Z_{t}$ (here just $C P_{t}$ ), as well as of the estimated factors $\widehat{F}_{t}$. Bootstrap samples of $r x_{t+1}^{(n)}$ are obtained in two ways: first by imposing the null hypothesis of no predictability, and second, under the alternative that excess returns are forecastable by the factors and conditioning variables studied above. The use of monthly bond price data to construct continuously compounded annual returns induces an MA(12) error structure in the annual log returns. Thus, under the null hypothesis that the expectations hypothesis is true, annual compound returns are forecastable up to an $\mathrm{MA}(12)$ error structure, but are not forecastable by other predictor variables or additional moving average terms.

Bootstrap sampling that captures the serial dependence of the data is straightforward when, as in this case, there is a parametric model for the dependence under the null hypothesis. In this event, the bootstrap may be accomplished by drawing random samples from the empirical distribution of the residuals of a $\sqrt{T}$ consistent, asymptotically normal estimator of the parametric model, in our application a twelfth-order moving average process. We use this approach to form bootstrap samples of excess returns under the null. Under the alternative, excess returns still have the MA(12) error structure induced by the use of overlapping data, but estimated factors $\widehat{F}_{t}$ are presumed to contain additional predictive power for excess returns above and beyond that implied by the moving average error structure.

To create bootstrapped samples of the factors, we re-sample the $T \times N$ panel of data, $x_{i t}$. For each $i$, we assume that the idiosyncratic errors $e_{i t}$ and the errors $u_{t}$ in the factor process are $\mathrm{AR}(1)$ processes. Least squares estimation of $\widehat{e}_{i t}=\rho_{i} \widehat{e}_{i t-1}+v_{i t}$ yields the estimates $\widehat{\rho}_{i}$ and $\widehat{v}_{i t}, t=2, \ldots T$, recalling that $\widehat{e}_{i t}=x_{i t}-\widehat{\lambda}_{i}^{\prime} \widehat{f}_{t}$. These errors are then re-centered. To generate a new panel of data, for each $i, \widehat{v}_{i t}$ is re-sampled (while preserving the cross-section correlation structure) to yield bootstrap samples of $\widehat{e}_{i t}$. In turn, bootstrap values of $x_{i t}$ are constructed by adding the bootstrap estimates of the idiosyncratic errors, $\widehat{e}_{i t}$, to $\widehat{\lambda}_{i}^{\prime} \widehat{F}_{t}$. Applying the method of principal components to the bootstrapped data yields a new set of estimated factors. Together with bootstrap samples of $C P_{t}$ created under the assumption that it is an
$\operatorname{AR}(1)$, we have a complete set of bootstrap regressors in the predictive regression.
Each regression using the bootstrapped data gives new estimates of the regression coefficients. This is repeated $B$ times. Bootstrap confidence intervals for the parameter estimates and $\bar{R}^{2}$ statistics are calculated from $B=10,000$ replications. We compute 90 th and 95 th percentiles of $\widehat{\beta}_{F}$ and $\widehat{\alpha}_{F}$, as well as the bootstrap estimate of the bias. This also allows us to compare the adequacy of our calculations for asymptotic bias considered in the previous subsection. The exercise is repeated for two-, three-, four- and five-year excess bond returns.

To conserve space, results are only reported for the largest model (corresponding to column 1 of Tables 4 to 7 ). The results based on bootstrap inference are consistent with asymptotic inference. In particular, the magnitude of predictability found in the historical data is too large to be accounted for by sampling error of the size we currently have. The coefficients on the predictors and factors are statistically different from zero at the $95 \%$ level and are well outside the $95 \%$ confidence interval under the null of no predictability. The bootstrap estimate of the bias on coefficients associated with the estimated factors are small, and the $\bar{R}^{2}$ are similar in magnitude to what was reported in Tables 4 to 7 .

### 5.3 Posterior Inference

In Tables 4 to 7 , we have used the posterior mean of $G_{t}$ in the predictive regression computed from 1000 draws (taken from a total of 25000 draws) from the posterior distribution of $G_{t}$. The $\widehat{\alpha}$ do not reflect sampling uncertainty about $G_{t}$. To have a complete account of sampling variability, we estimate the predictive regressions for each of the 1000 draws of $G_{t}$. This gives us the posterior distribution for $\alpha$ as well as the corresponding $t$ statistic.

Reported in Table 10 are the posterior mean of $\alpha_{G}$ along with the 5 and $95 \%$ percentage points of the $t$ statistic. The point estimates reported in Tables 4 to 7 are very close to the posterior means. Sampling variability from having to estimate the dynamic factors has little effect on the estimates of the factor augmented regressions.

So far we find that macroeconomic factors have non-trivial predictive power for bond excess returns and that the sampling error induced by $\widehat{F}_{t}$ or $\widehat{G}_{t}$ in the predictive regressions are numerically small. Multiple factors contribute to the predictability of excess returns, so it is not possible to infer the cyclicality of return risk premia by observing the signs of the individual coefficients on factors in forecasting regressions of excess returns. But Tables 4-7 provide a summary measure of how the factors are related to future excess returns by showing that excess bond returns are high when the linear combinations of all factors, $\widehat{F} 8_{t}$ and $\widehat{G} 8_{t}$, are high. Figures 11 and 12 show that $\widehat{F} 8_{t}$ and $\widehat{G} 8_{t}$ are in turn high when real activity (as
measured by industrial production growth) is low. The results therefore imply that excess returns are forecast to be high when economic activity is slow or contracting. That is, return risk premia are counter-cyclical. This is confirmed by the top panels of Figures 13 and 14, which plot return risk premia along with industrial production growth. The bottom panels of these figures show that the factors contribute significantly to the countercyclicality of riskpremia. Indeed, when factors are excluded (but $C P_{t}$ is included), risk-premia are a-cyclical. Of economic interest is whether yield risk-preimia are also counter-cyclical. We now turn to such an analysis.

## 6 Counter-cyclical Yield Risk Premia

The yield risk premium or term premium, should not be confused with the term spread, which is simply the difference in yields between the $n$-period bond and the one-period bond. Instead, the yield risk premium is a component of the the $n$-period yield:

$$
\begin{equation*}
y_{t}^{(n)}=\underbrace{\frac{1}{n} E_{t}\left(y_{t}^{(1)}+y_{t+1}^{(1)}+\cdots+y_{t+n-1}^{(1)}\right)}_{\text {expectations component }}+\underbrace{\varkappa_{t}^{(n)}}_{\text {yield risk premium }} . \tag{12}
\end{equation*}
$$

Under the expectations hypothesis, the yield risk premium, $\varkappa_{t}^{(n)}$, is assumed constant.
It is straightforward to show that the yield risk premium is identically equal to the average of expected future return risk premia of declining maturity:

$$
\begin{equation*}
\varkappa_{t}^{(n)}=\frac{1}{n}\left[E_{t}\left(r x_{t+1}^{(n)}\right)+E_{t}\left(r x_{t+2}^{(n-1)}\right)+\cdots+E_{t}\left(r x_{t+n-1}^{(2)}\right)\right] . \tag{13}
\end{equation*}
$$

To form an estimate of the risk premium component in yields, $\varkappa_{t}^{(n)}$, we need estimates of the multi-step ahead forecasts that appear on the right-hand-side of (13),. Denote estimated variables with "hats." Then

$$
\begin{equation*}
\widehat{\varkappa}_{t}^{(n)}=\frac{1}{n}\left[\widehat{E}_{t}\left(r x_{t+1}^{(n)}\right)+\widehat{E}_{t}\left(r x_{t+2}^{(n-1)}\right)+\cdots+\widehat{E}_{t}\left(r x_{t+n-1}^{(2)}\right)\right], \tag{14}
\end{equation*}
$$

where $\widehat{E}_{t}(\cdot)$ denotes an estimate of the conditional expectation $E_{t}(\cdot)$ formed by a linear projection. As estimates of the conditional expectations are simply linear forecasts of excess returns, multiple steps ahead our earlier results for the FAR have direct implications for risk premia in yields.

To generate multi-step ahead forecasts we estimate a monthly $p$ th-order vectorautoregression (VAR). The idea behind the VAR is that multi-step ahead forecasts may be obtained
by iterating one-step ahead linear projections from the VAR. The VAR vector contains observations on excess returns, the Cochrane-Piazzesi factor, $C P_{t}$ and $\widehat{H}_{t}$, where $\widehat{H}_{t}$ are the estimated factors (or a linear combination of them). Let

$$
Z_{t}^{U} \equiv\left[r x_{t}^{(5)}, r x_{t}^{(4)}, \ldots, r x_{t}^{(2)}, C P_{t}, \widehat{H} 8_{t}\right]^{\prime}
$$

where $\widehat{H} 8$ is either $\widehat{F} 8$ or $\widehat{G} 8$. For comparison, we will also form bond forecasts with a restricted VAR that excludes the estimated factors, but still includes $C P_{t}$ as a predictor variable:

$$
Z_{t}^{R} \equiv\left[r x_{t}^{(5)}, r x_{t}^{(4)}, \ldots, r x_{t}^{(2)}, C P_{t}\right]^{\prime}
$$

We use a monthly VAR with $p=12$ lags, where, for notational convenience, we write the VAR in terms of mean deviations: ${ }^{7}$

$$
\begin{equation*}
Z_{t+1 / 12}-\boldsymbol{\mu}=\mathbf{\Phi}_{1}\left(Z_{t}-\boldsymbol{\mu}\right)+\mathbf{\Phi}_{2}\left(Z_{t-1 / 12}-\boldsymbol{\mu}\right)+\cdots+\mathbf{\Phi}_{p}\left(Z_{t-11 / 12}-\boldsymbol{\mu}\right)+\boldsymbol{\varepsilon}_{t+1 / 12} \tag{15}
\end{equation*}
$$

Let $k$ denote the number of variables in $Z_{t}$. Then (15) can be can be expressed as a $V A R(1)$ :

$$
\begin{equation*}
\boldsymbol{\xi}_{t+1 / 12}=\mathbf{A} \boldsymbol{\xi}_{t}+\mathbf{v}_{t+1 / 12} \tag{16}
\end{equation*}
$$

where,

$$
\begin{aligned}
& \underset{\substack{(k p \times 1)}}{\boldsymbol{\xi}_{t+1 / 12}} \equiv\left[\begin{array}{c}
Z_{t}-\boldsymbol{\mu} \\
Z_{t-1 / 12}-\boldsymbol{\mu} \\
\cdot \\
\cdot \\
\cdot \\
Z_{t-11 / 12}-\boldsymbol{\mu}
\end{array}\right] \underset{(k p \times 1)}{\mathbf{v}_{t}} \equiv\left[\begin{array}{c}
\boldsymbol{\varepsilon}_{t+1 / 12} \\
\mathbf{0} \\
\cdot \\
\cdot \\
\cdot \\
\mathbf{0}
\end{array}\right] \\
& \underset{(k p \times k p)}{\mathbf{A}}=\left[\begin{array}{ccccccc}
\boldsymbol{\Phi}_{1} & \boldsymbol{\Phi}_{2} & \boldsymbol{\Phi}_{3} & . & . & \boldsymbol{\Phi}_{p-1} & \boldsymbol{\Phi}_{p} \\
\mathbf{I}_{n} & \mathbf{0} & \mathbf{0} & . & . & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{I}_{n} & \mathbf{0} & . & . & \mathbf{0} & \mathbf{0} \\
. & . & . & . & . & . & . \\
. & . & . & . & . & . & . \\
. & . & . & . & . & . & . \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & . & . & \mathbf{I}_{n} & \mathbf{0}
\end{array}\right] .
\end{aligned}
$$

Multi-step ahead forecasts are straightforward to compute using the first-order VAR:

$$
E_{t} \boldsymbol{\xi}_{t+j / 12}=\mathbf{A}^{j} \boldsymbol{\xi}_{t}
$$

[^6]When $j=12$, the monthly VAR produces forecasts of one-year ahead variables, $E_{t} \boldsymbol{\xi}_{t+1}=$ $\mathbf{A}^{12} \boldsymbol{\xi}_{t}$; when $j=24$, it computes two-year ahead forecasts, and so on. Define a vector $e j$ that picks out the $j$-th element of $\boldsymbol{\xi}_{t}$, i.e., $e 1^{\prime} \boldsymbol{\xi}_{t} \equiv r x_{t}^{(5)}$. In the notation above, we have $e 1_{(k p \times 1)}=[1,0,0, \ldots 0]^{\prime}, e 2_{(k p \times 1)}=[0,1,0, \ldots 0]^{\prime}$, analogously for $e 3$ and $e 4$. Thus, given estimates of the VAR parameters A, we may form estimates of the conditional expectations on the right-hand-side of (14) using the VAR forecasts of return risk premia. For example, the estimate of the expectation of the five-year bond, one year ahead, is given by $\widehat{E}_{t}\left(r x_{t+1}^{(5)}\right)=$ $e 1^{\prime} \mathbf{A}^{12} \boldsymbol{\xi}_{t}$; the estimate of the expectation of the four-year bond, two years ahead, is given by $\widehat{E}_{t}\left(r x_{t+2}^{(4)}\right)=e 2^{\prime} \mathbf{A}^{24} \boldsymbol{\xi}_{t}$, and so on.

Letting $\widehat{H}_{t}=\widehat{F} 5_{t}$ where $\widehat{F} 5_{t}$ is a linear combination of $\widehat{f}_{1 t}, \widehat{f}_{1 t}^{3}, \widehat{f}_{3 t}, \widehat{f}_{4 t}$ and $\widehat{f}_{8 t}$. we showed in Ludvigson and Ng (2007) that both yield and return risk premia are more countercyclical and reach greater values in recessions than in the absence of $\widehat{H}_{t}$. Here, we verify that this result holds up for different choices of $\widehat{H}_{t}$. To this end, we let $\widehat{H}_{t}$ be the static and dynamic factors selected by the out-of-sample BIC. These two predictor sets embody information in fewer factors than the ones implied by the in-sample BIC, $\widehat{H} 8$, or $F 5_{t}$ used in Ludvigson and Ng (2007). The point is to show that a few macroeconomic factors are enough to generate an important difference in the properties of risk premia. Specifically, without $\widehat{F}_{t}$ in $Z_{t}^{U}$, the correlation between the estimated return risk premium and IP growth is -0.014 . With $\widehat{F}_{t}$ in $Z_{t}^{U}$, the correlation is -0.223 . These correlations are -0.045 and -0.376 for yield risk premia. With $\widehat{G}_{t}$ in $Z_{t}^{U}$, the correlation of IP growth with return and yield risk premium are -0.218 and -0.286 respectively. Return and yield risk premia are thus more countercyclical when the factors are used to forecast excess returns.

Figure 15 shows the twelve month moving-average of risk-premium component of the five-year bond yield. As we can see, yield risk premia were particularly high in the 1982-83 recession, as well as shortly after the 2001 recession. Figure 16 shows the yield risk premia estimated with and without using $\widehat{F}_{t}$ to forecast excess returns, while Figure 17 shows a similar picture with and without $\widehat{G}_{t}$. The difference between the risk premia estimated with and without the factors is largest around recessions. For example, the yield risk premium on the five-year bond estimated using the information contained in $\widehat{F}_{t}$ or $\widehat{G}_{t}$ was over $2 \%$ in the 2001 recession, but it was slightly below $1 \%$ without $\widehat{G}_{t}$. The return risk premia (not reported) show a similar pattern.

When the economy is contracting, the countercyclical nature of the risk factors contributes to a steepening of the yield curve even as future short term rates fall. Conversely, when the economy is expanding, the factors contribute to a flattening of the yield curve even
as expectations of future short-term rates rise. This implies that information in the factors is ignored. Too much variation in the long term yields is attributed to the expectations component in recessions. Information in the macro factors are thus important in accurate decomposition of risk premia, especially in recessions.

## 7 Conclusion

There is a good deal of evidence that excess bond returns are predictable by financial variables. Yet, macroeconomic theory postulates that it is real variables relating to macroeconomic activity that should forecast bond returns. This paper presents robust evidence in support of the theory. Macroeconomic factors, especially the real activity factor, has strong predictive power for excess bond returns even in the presence of financial predictors. Our analysis consists of estimating two sets of factors and a comprehensive specification search. We also account for sampling uncertainty that might arise from estimation of the factors. While the estimated risk premia without using the macro factors to forecast excess returns are acyclical, both bond returns and yield risk premia are counter-cyclical when the factors are used. The evidence indicate that investors seek compensation for macroeconomic risks associated with recessions.

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Table 1: First Order Autocorrelation Coefficients

|  | $\hat{f}_{t}$ | $t$ | $\widehat{g}_{t}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.767 | 20.589 | -0.361 | -6.298 |
| 2 | 0.748 | 18.085 | 0.823 | 22.157 |
| 3 | -0.239 | -2.852 | 0.877 | 32.267 |
| 4 | 0.456 | 7.594 | 0.660 | 14.385 |
| 5 | 0.362 | 6.819 | -0.344 | -1.635 |
| 6 | 0.422 | 4.232 | 0.448 | 4.552 |
| 7 | -0.112 | -0.672 | 0.050 | 0.609 |
| 8 | 0.225 | 4.526 | 0.157 | 2.794 |

Table 2: Correlation between $\widehat{f_{t}}$ and $g_{t}$

|  | $\widehat{g}_{1}$ | $\widehat{g}_{2}$ | $\widehat{g}_{3}$ | $\widehat{g}_{4}$ | $\widehat{g}_{5}$ | $\widehat{g}_{6}$ | $\widehat{g}_{7}$ | $\widehat{g}_{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | output | labor | housing | mfg | money | finance | prices | stocks |
| $\widehat{f}_{1}$ | 0.601 | 0.903 | 0.551 | 0.766 | -0.067 | 0.489 | 0.126 | -0.092 |
| $\widehat{f}_{2}$ | 0.181 | -0.120 | 0.376 | 0.269 | 0.095 | -0.462 | -0.227 | 0.449 |
| $\widehat{f}_{3}$ | 0.037 | 0.027 | -0.150 | -0.010 | -0.148 | 0.144 | -0.800 | -0.067 |
| $\widehat{f}_{4}$ | -0.303 | 0.118 | 0.253 | -0.128 | 0.185 | -0.417 | -0.194 | 0.092 |
| $\widehat{f}_{5}$ | 0.306 | 0.179 | -0.365 | 0.026 | 0.046 | -0.474 | -0.009 | 0.183 |
| $\widehat{f}_{6}$ | 0.103 | -0.140 | 0.321 | 0.179 | -0.398 | 0.008 | 0.050 | 0.177 |
| $\widehat{f}_{7}$ | 0.064 | -0.023 | 0.125 | 0.004 | 0.743 | 0.088 | -0.078 | 0.100 |
| $\widehat{f}_{8}$ | -0.241 | 0.073 | -0.023 | 0.111 | -0.057 | 0.119 | -0.052 | 0.689 |

Table 3: Long run correlation between $\widehat{f}_{t}$ and $\widehat{g}_{t}$.

|  | $\widehat{g}_{1}$ | $\widehat{g}_{2}$ | $\widehat{g}_{3}$ | $\widehat{g}_{4}$ | $\widehat{g}_{5}$ | $\widehat{g}_{6}$ | $\widehat{g}_{7}$ | $\widehat{g}_{8}$ | $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | output | labor | housing | mfg | money | finance | prices | stocks |  |
| $\widehat{f}_{1}$ | 0.447 | 0.536 | 0.215 | 0.066 | -0.008 | 0.140 | -0.002 | -0.038 | 0.953 |
| $\widehat{f}_{2}$ | 0.548 | -0.466 | 0.296 | 0.299 | 0.031 | -0.536 | -0.135 | 0.266 | 0.689 |
| $\widehat{f}_{3}$ | 0.100 | 0.026 | -0.152 | -0.036 | -0.007 | 0.211 | -0.390 | -0.026 | 0.935 |
| $\widehat{f}_{4}$ | -0.925 | 0.699 | 0.491 | -0.242 | 0.004 | -0.444 | -0.077 | -0.064 | 0.723 |
| $\widehat{f}_{5}$ | 0.682 | 0.417 | -0.624 | -0.135 | -0.000 | -0.488 | 0.018 | 0.146 | 0.790 |
| $\widehat{f}_{6}$ | 0.070 | -0.357 | 0.467 | -0.098 | -0.294 | 0.144 | 0.061 | 0.100 | 0.490 |
| $\widehat{f}_{7}$ | 0.226 | -0.252 | 0.136 | -0.095 | 0.540 | 0.325 | -0.080 | 0.180 | 0.692 |
| $\widehat{f}_{8}$ | -0.986 | 0.447 | -0.224 | 0.167 | 0.025 | 0.313 | -0.049 | 0.905 | 0.797 |

Reported are estimates of $A_{r .0}$, obtained from the regression: $\widehat{f}_{r t}=A_{r .0} \widehat{g}_{t}+\sum_{i=1}^{p-1} A_{r . i} \Delta g_{t-i}+e_{t}$ with $p=4$.

Table 4: Regressions $r x_{t+1}^{(2)}=a+\alpha^{\prime} \widehat{H}_{t}+\beta^{\prime} C P_{t}+\epsilon_{t+1}$

| $\widehat{H}$ | $\widehat{H}=\widehat{F}$ |  |  |  | $\widehat{H}=\widehat{G}$ |  |  |  | $\widehat{H}=\widehat{F}$ |  | $\widehat{H}=\widehat{G}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | in | out | in | out | in | out | in | out |  |  |  |  |  |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| $\widehat{H}_{1}$ | -0.761 | -0.793 | -0.935 | -0.931 | - | - | 0.147 | 0.170 | - | - | - | - | - |
| tstat | -5.387 | -4.848 | -5.748 | -5.449 | - | - | 2.947 | 2.623 | - | - | - | - | - |
| $\widehat{H}_{2}$ | - | - | 0.325 | 0.326 | -0.494 | -0.627 | -0.699 | -0.646 | - | - | - | - | - |
| tstat | - | - | 2.663 | 2.520 | -3.151 | -3.623 | -2.905 | -3.062 | - | - | - | - | - |
| $\widehat{H}_{3}$ | - | - | - | - | -0.492 | - | -0.532 | -0.487 | - | - | - | - | - |
| tstat | - | - | - | - | -4.813 | - | -2.889 | -3.012 | - | - | - | - | - |
| $\widehat{H}_{4}$ | -0.291 | - | -0.399 | -0.399 | - | - | 0.186 | - | - | - | - | - | - |
| tstat | -2.716 | - | -3.103 | -2.974 | - | - | 1.039 | - | - | - | - | - | - |
| $\widehat{H}_{6}$ | -0.151 | - | -0.281 | -0.280 | 0.137 | - | -0.163 | - | - | - | - | - | - |
| tstat | -1.322 | - | -1.949 | -1.795 | 1.679 | - | -1.594 | - | - | - | - | - | - |
| $\widehat{H}_{7}$ | -0.128 | - | -0.143 | -0.144 | - | - | - | - | - | - | - | - | - |
| tstat | -1.577 | - | -1.517 | -1.365 | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{8}$ | 0.240 | 0.241 | 0.302 | - | -0.136 | - | -0.164 | - | - | - | - | - | - |
| tstat | 2.981 | 3.297 | 3.575 | - | -1.562 | - | -1.997 | - | - | - | - | - | - |
| $\widehat{H}_{2}^{2}$ | - | - | - | - | - | -0.100 | - | - | - | - | - | - | - |
| tstat | - | - | - | - | - | -2.147 | - | - | - | - | - | - | - |
| $\widehat{H}_{4}^{2}$ | - | - | - | - | -0.074 | - | -0.121 | -0.118 | - | - | - | - | - |
| tstat | - | - | - | - | -3.165 | - | -3.167 | -3.076 | - | - | - | - | - |
| $\widehat{H}_{5}^{2}$ | -0.080 | -0.110 | - | - | - | - | - | - | - | - | - | - | - |
| tstat | -2.468 | -2.925 | - | - | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{6}{ }^{2}$ | - | - | - | - | -0.086 | -0.083 | -0.084 | -0.080 | - | - | - | - | - |
| tstat | - | - | - | - | -6.245 | -6.804 | -3.642 | -3.176 | - | - | - | - | - |
| $\widehat{H}_{1}^{3}$ | 0.044 | 0.047 | 0.057 | 0.056 | 0.019 | - | - | - | - | - | - | - | - |
| tstat | 2.912 | 2.887 | 3.081 | 3.338 | 2.254 | - | - | - | - | - | - | - | - |
| CP | 0.385 | 0.411 | - | - | 0.452 | 0.433 | - | - | 0.336 | - | 0.413 | - | 0.455 |
| tstat | 5.647 | 6.981 | - | - | 7.488 | 7.738 | - | - | 4.437 | - | 6.434 | - | 8.836 |
| $\widehat{H} 8$ | - | - | - | - | - | - | - | - | 0.332 | 0.482 | 0.427 | 0.544 | - |
| tstat | - | - | - | - | - | - | - | - | 4.336 | 7.212 | 3.880 | 3.493 | - |
| $\bar{R}^{2}$ | 0.460 | 0.430 | 0.283 | 0.258 | 0.477 | 0.407 | 0.200 | 0.192 | 0.419 | 0.279 | 0.401 | 0.153 | 0.309 |

Notes: The table reports estimates from OLS regressions of excess bond returns on the lagged variables named in column 1. The dependent variable $r x_{t+1}^{n}$ is the excess $\log$ return on the $n$ year Treasury bond. $\widehat{H}_{t}$ denotes a set of regressors formed from consisting of functions of $\widehat{f_{t}}$ or $\widehat{g_{t}}$ where $\widehat{f_{t}}$ is a set of eight factors estimated by the method of principal components, and $\widehat{g_{t}}$ is a vector of eight dynamic factors estimated by Bayesian factors. The panel of data used in estimation consists of 131 individual series over the period 1964:1-2007:12. $\widehat{H} 8_{t}$ is the single factor constructed as a linear combination of the eight estimated factors and $\widehat{f}_{1}^{3} . C P_{t}$ is the Cochrane and Piazzesi (2005) factor that is a linear combination of five forward spreads. Newey and West (1987) corrected $t$-statistics have lag order 18 months and are reported in parentheses. A constant is always included in the regression even though its estimate is not reported in the Table.

Table 5: Regressions $r x_{t+1}^{(3)}=a+\alpha^{\prime} \widehat{H}_{t}+\beta^{\prime} C P_{t}+\epsilon_{t+1}$

| $\widehat{H}$ | $\widehat{H}=\widehat{F}$ |  |  |  | $\widehat{H}=\widehat{G}$ |  |  |  | $\widehat{H}=\widehat{F}$ |  | $\widehat{H}=\widehat{G}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | in | out | in | out | in | out | in | out |  |  |  |  |  |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| $\widehat{H}_{1}$ | -1.232 | -1.280 | -1.624 | -1.592 | - | - | - | - | - | - | - | - | - |
| tstat | -5.079 | -4.581 | -5.553 | -5.479 | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{2}$ | -0.028 | - | 0.694 | 0.703 | -0.782 | -1.094 | -1.259 | -1.056 | - | - | - | - | - |
| tstat | -0.147 | - | 2.851 | 2.982 | -2.805 | -3.773 | -2.983 | -3.092 | - | - | - | - | - |
| $\widehat{H}_{3}$ | - | - | - | - | -0.807 | - | -0.843 | -0.734 | - | - | - | - | - |
| tstat | - | - | - | - | -4.297 | - | -2.667 | -2.548 | - | - | - | - | - |
| $\widehat{H}_{4}$ | -0.423 | - | -0.588 | -0.592 | - | - | 0.421 | - | - | - | - | - | - |
| tstat | -2.193 | - | -2.518 | -2.496 | - | - | 1.225 | - | - | - | - | - | - |
| $\widehat{H}_{6}$ | -0.433 | - | -0.598 | -0.590 | - | - | -0.356 | - | - | - | - | - | - |
| tstat | -1.890 | - | -2.294 | -2.269 | - | - | -2.006 | - | - | - | - | - | - |
| $\widehat{H}_{7}$ | -0.338 | - | -0.360 | -0.342 | - | - | - | - | - | - | - | - | - |
| tstat | -2.138 | - | -2.109 | -1.989 | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{8}$ | 0.389 | 0.428 | 0.550 | 0.553 | -0.308 | - | -0.329 | - | - | - | - | - | - |
| tstat | 2.593 | 3.190 | 3.718 | 3.738 | -2.018 | - | -2.143 | - | - | - | - | - | - |
| $\widehat{H}_{1}^{2}$ | - | - | 0.156 | - | - | - | - | - | - | - | - | - | - |
| tstat | - | - | 0.854 | - | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{2}^{2}$ | - | - | - | - | - | -0.208 | - | - | - | - | - | - | - |
| tstat | - | - | - | - | - | -2.668 | - | - | - | - | - | - | - |
| $\widehat{H}_{3}^{2}$ | 0.111 | - | - | - | - | - | - | - | - | - | - | - | - |
| tstat | 1.999 | - | - | - | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{4}^{2}$ | - | - | - | - | -0.190 | - | -0.250 | -0.275 | - | - | - | - | - |
| tstat | - | - | - | - | -3.925 | - | -3.005 | -3.622 | - | - | - | - | - |
| $\widehat{H}_{5}^{2}$ | - | -0.161 | - | - | - | - | - | - | - | - | - | - | - |
| tstat | - | -2.179 | - | - | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{6}^{2}$ | - | - | - | - | -0.152 | -0.147 | -0.140 | -0.127 | - | - | - | - | - |
| tstat | - | - | - | - | -7.130 | -6.883 | -3.307 | -2.551 | - | - | - | - | - |
| $\widehat{H}_{7}^{2}$ | - | - | - | - | 0.089 | - | - | - | - | - | - | - | - |
| tstat | - | - | - | - | 2.687 | - | - | - | - | - | - | - | - |
| $\widehat{H}_{1}^{3}$ | 0.095 | 0.086 | 0.141 | 0.106 | 0.032 | - | 0.031 | - | - | - | - | - | - |
| tstat | 3.235 | 3.204 | 2.922 | 3.445 | 2.233 | - | 1.942 | - | - | - | - | - | - |
| $C P$ | 0.760 | 0.784 | - | - | 0.847 | 0.821 | - | - | 0.644 | - | 0.786 | - | 0.856 |
| tstat | 5.329 | 6.885 | - | - | 7.516 | 7.770 | - | - | 4.661 | - | 6.381 | - | 8.301 |
| SH | - | - | - | - | - | - | - | - | 0.588 | 0.877 | 0.710 | 0.931 | - |
| tstat | - | - | - | - | - | - | - | - | 4.494 | 7.133 | 3.624 | 3.256 | - |
| $\bar{R}^{2}$ | 0.455 | 0.424 | 0.268 | 0.267 | 0.475 | 0.418 | 0.182 | 0.167 | 0.432 | 0.277 | 0.404 | 0.135 | 0.328 |

Table 6: Regressions $r x_{t+1}^{(4)}=a+\alpha^{\prime} \hat{H}_{t}+\beta^{\prime} C P_{t}+\epsilon_{t+1}$

| $\widehat{H}$ | $\widehat{H}=\widehat{F}$ |  |  |  | $\widehat{H}=\widehat{G}$ |  |  |  | $\widehat{H}=\widehat{F}$ |  | $\widehat{H}=\widehat{G}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | in | out | in | out | in | out | in | out |  |  |  |  |  |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| $\widehat{H}_{1}$ | -1.521 | -1.521 | -2.011 | -2.050 | - | - | - | - | - | - | - | - | - |
| tstat | -5.138 | -4.149 | -5.013 | -5.290 | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{2}$ | - | - | 1.069 | 1.069 | -0.952 | -1.342 | -1.619 | -1.601 | - | - | - | - | - |
| tstat | - | - | 3.028 | 3.095 | -2.680 | -3.754 | -2.812 | -2.848 | - | - | - | - | - |
| $\widehat{H}_{3}$ | - | - | - | - | -1.036 | - | -1.080 | -1.078 | - | - | - | - | - |
| tstat | - | - | - | - | -4.127 | - | -2.486 | -2.401 | - | - | - | - | - |
| $\widehat{H}_{4}$ | -0.436 | - | -0.689 | -0.681 | - | - | 0.590 | 0.452 | - | - | - | - | - |
| tstat | -1.595 | - | -1.957 | -1.978 | - | - | 1.221 | 0.927 | - | - | - | - | - |
| $\widehat{H}_{5}$ | - | - | -0.321 | - | - | - | - | - | - | - | - | - | - |
| tstat | - | - | -1.475 | - | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{6}$ | -0.668 | - | -0.889 | -0.889 | - | - | -0.605 | - | - | - | - | - | - |
| tstat | -2.160 | - | -2.522 | -2.449 | - | - | -2.333 | - | - | - | - | - | - |
| $\widehat{H}_{7}$ | -0.534 | - | -0.535 | -0.541 | - | - | - | - | - | - | - | - | - |
| tstat | -2.401 | - | -2.222 | -2.209 | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{8}$ | 0.578 | 0.636 | 0.820 | 0.822 | -0.474 | - | -0.521 | - | - | - | - | - | - |
| tstat | 2.820 | 3.365 | 3.935 | 3.914 | -2.344 | - | -2.277 | - | - | - | - | - | - |
| $\widehat{H}_{1}^{2}$ | - | -0.146 | - | - | - | - | - | - | - | - | - | - | - |
| tstat | - | -0.770 | - | - | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{2}^{2}$ | - | - | - | - | - | -0.284 | - | - | - | - | - | - | - |
| tstat | - | - | - | - | - | -2.934 | - | - | - | - | - | - | - |
| $\widehat{H}_{3}^{2}$ | 0.177 | - | - | - | - | - | - | - | - | - | - | - | - |
| tstat | 2.527 | - | - | - | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{4}^{2}$ | - | - | - | - | -0.262 | - | -0.354 | -0.367 | - | - | - | - | - |
| tstat | - | - | - | - | -3.692 | - | -2.976 | -3.552 | - | - | - | - | - |
| $\widehat{H}_{5}^{2}$ | - | -0.228 | - | - | - | - | - | - | - | - | - | - | - |
| tstat | - | -2.309 | - | - | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{6}^{2}$ | - | - | - | - | -0.231 | -0.227 | -0.219 | -0.189 | - | - | - | - | - |
| tstat | - | - | - | - | -6.923 | -9.811 | -4.375 | -3.248 | - | - | - | - | - |
| $\widehat{H}_{7}^{2}$ | - | - | - | - | 0.148 | 0.104 | - | - | - | - | - | - | - |
| tstat | - | - | - | - | 3.258 | 2.233 | - | - | - | - | - | - | - |
| $\widehat{H}_{1}^{3}$ | 0.131 | 0.081 | 0.142 | 0.148 | 0.037 | - | 0.036 | - | - | - | - | - | - |
| tstat | 3.436 | 1.483 | 3.938 | 3.602 | 1.964 | - | 1.599 | - | - | - | - | - | - |
| CP | 1.115 | 1.158 | - | - | 1.238 | 1.219 | - | - | 0.955 | - | 1.150 | - | 1.235 |
| tstat | 6.077 | 7.028 | - | - | 7.821 | 8.197 | - | - | 4.765 | - | 6.417 | - | 8.224 |
| $\widehat{H} 8$ | - | - | - | - | - | - | - | - | 0.777 | 1.204 | 0.864 | 1.188 | - |
| tstat | - | - |  | - | - | - | - | - | 4.474 | 7.247 | 3.388 | 3.061 | - |
| $\bar{R}^{2}$ | 0.473 | 0.441 | 0.263 | 0.260 | 0.496 | 0.445 | 0.171 | 0.155 | 0.452 | 0.273 | 0.416 | 0.114 | 0.357 |

Table 7: Regressions $r x_{t+1}^{(5)}=a+\alpha^{\prime} \widehat{H}_{t}+\beta^{\prime} C P_{t}+\epsilon_{t+1}$

| $\widehat{H}$ | $\widehat{H}=\widehat{F}$ |  |  |  | $\widehat{H}=\widehat{G}$ |  |  |  | $\widehat{H}=\widehat{F}$ |  | $\widehat{H}=\widehat{G}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | in | out | in | out | in | out | in | out |  |  |  |  |  |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| $\widehat{H}_{1}$ | -1.653 | -1.373 | -2.214 | -2.277 | 0.308 | - | 0.326 | - | - | - | - | - | - |
| tstat | -4.723 | -3.686 | -4.503 | -4.819 | 1.701 | - | 2.049 | - | - | - | - | - | - |
| $\widehat{H}_{2}$ | - | - | 1.355 | 1.355 | -1.145 | -1.573 | -1.928 | -1.609 | - | - | - | - | - |
| tstat | - | - | 3.111 | 3.195 | -2.653 | -3.691 | -2.760 | -2.994 | - | - | - | - | - |
| $\widehat{H}_{3}$ | - | - | - | - | -1.161 | - | -1.199 | -1.003 | - | - | - | - | - |
| tstat | - | - | - | - | -3.615 | - | -2.224 | -2.021 | - | - | - | - | - |
| $\widehat{H}_{4}$ | -0.516 | - | -0.818 | -0.805 | - | - | 0.654 | - | - | - | - | - | - |
| tstat | -1.478 | - | -1.861 | -1.881 | - | - | 1.128 | - | - | - | - | - | - |
| $\widehat{H}_{5}$ | - | - | -0.523 | - | - | - | - | - | - | - | - | - | - |
| tstat | - | - | -1.969 | - | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{6}$ | -0.856 | - | -1.120 | -1.120 | - | - | -0.678 | - | - | - | - | - | - |
| tstat | -2.150 | - | -2.566 | -2.462 | - | - | -2.049 | - | - | - | - | - | - |
| $\widehat{H}_{7}$ | -0.686 | - | -0.685 | -0.694 | - | - | - | - | - | - | - | - | - |
| tstat | -2.479 | - | -2.321 | -2.299 | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{8}$ | 0.702 | 0.725 | 0.985 | 0.988 | -0.563 | - | -0.608 | - | - | - | - | - | - |
| tstat | 2.756 | 3.292 | 3.956 | 3.907 | -2.217 | - | -2.156 | - | - | - | - | - | - |
| $\widehat{H}_{1}^{2}$ | - | -0.563 | - | - | - | - | - | - | - | - | - | - | - |
| tstat | - | -3.037 | - | - | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{2}^{2}$ | - | - | - | - | - | -0.339 | - | - | - | - | - | - | - |
| tstat | - | - | - | - | - | -2.955 | - | - | - | - | - | - | - |
| $\widehat{H}_{3}^{2}$ | 0.204 | - | - | - | - | - | - | - | - | - | - | - | - |
| tstat | 2.327 | - | - | - | - | - | - | - | - | - | - | - | - |
| $\widehat{H}_{4}^{2}$ | - | - | - | - | -0.357 | - | -0.465 | -0.466 | - | - | - | - | - |
| tstat | - | - | - | - | -4.429 | - | -3.497 | -3.684 | - | - | - | - | - |
| $\widehat{H}_{6}^{2}$ | - | - | - | - | -0.269 | -0.279 | -0.253 | -0.234 | - | - | - | - | - |
| tstat | - | - | - | - | -6.235 | -9.685 | -4.407 | -3.596 | - | - | - | - | - |
| $\widehat{H}_{7}^{2}$ | - | - | - | - | 0.179 | - | - | - | - | - | - | - | - |
| tstat | - | - | - | - | 3.221 | - | - | - | - | - | - | - | - |
| $\widehat{H}_{1}^{3}$ | 0.150 | - | 0.160 | 0.170 | - | - | - | - | - | - | - | - | - |
| tstat | 3.310 | - | 3.893 | 3.440 | - | - | - | - | - | - | - | - | - |
| $C P$ | 1.316 | 1.394 | - | - | 1.457 | 1.413 | - | - | 1.115 | - | 1.359 | - | 1.453 |
| tstat | 5.603 | 6.985 | - | - | 7.237 | 7.409 | - | - | 4.370 | - | 5.969 | - | 7.576 |
| SH | - | - | - | - | - | - | - | - | 0.938 | 1.437 | 0.955 | 1.338 | - |
| tstat | - | - | - | - | - | - | - | - | 4.542 | 7.281 | 3.078 | 2.854 | - |
| $\bar{R}^{2}$ | 0.435 | 0.392 | 0.251 | 0.245 | 0.453 | 0.408 | 0.152 | 0.135 | 0.422 | 0.259 | 0.377 | 0.097 | 0.330 |

Table 8: Biased Corrected Estimates: $r x_{t+1}^{(n)}=a+\alpha^{\prime} \widehat{F}_{t}+\beta^{\prime} C P_{t}+\epsilon_{t+1}$

| $\widehat{F}$ | $n=2$ |  | $n=3$ |  | $n=4$ |  | $n=5$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\widehat{H}_{1}$ | -0.761 | - | -1.232 | - | -1.521 | - | -1.653 | - |
| $\widetilde{\alpha}$ | -0.785 | - | -1.277 | - | -1.576 | - | -1.724 | - |
| bias | 0.024 | - | 0.045 | - | 0.054 | - | 0.072 | - |
| $\widehat{H}_{2}$ | - | - | -0.028 | - | - | - | - | - |
| $\widetilde{\alpha}$ | - | - | -0.059 | - | - | - | - | - |
| bias | - | - | 0.032 | - | - | - | - | - |
| $\widehat{H}_{4}$ | -0.291 | - | -0.423 | - | -0.436 | - | -0.516 | - |
| $\widetilde{\alpha}$ | -0.307 | - | -0.454 | - | -0.472 | - | -0.564 | - |
| bias | 0.016 | - | 0.031 | - | 0.036 | - | 0.048 | - |
| $\widehat{H}_{6}$ | -0.151 | - | -0.433 | - | -0.668 | - | -0.856 | - |
| $\widetilde{\alpha}$ | -0.168 | - | -0.468 | - | -0.710 | - | -0.912 | - |
| bias | 0.018 | - | 0.035 | - | 0.042 | - | 0.055 | - |
| $\widehat{H}_{7}$ | -0.128 | - | -0.338 | - | -0.534 | - | -0.686 | - |
| $\widetilde{\alpha}$ | -0.145 | - | -0.372 | - | -0.573 | - | -0.737 | - |
| bias | 0.017 | - | 0.034 | - | 0.039 | - | 0.051 | - |
| $\widehat{H}_{8}$ | 0.240 | - | 0.389 | - | 0.578 | - | 0.702 | - |
| $\widetilde{\alpha}$ | 0.225 | - | 0.355 | - | 0.542 | - | 0.654 | - |
| bias | 0.016 | - | 0.033 | - | 0.036 | - | 0.048 | - |
| $\widehat{H}_{3}^{2}$ | - | - | 0.111 | - | 0.177 | - | 0.204 | - |
| $\widetilde{\alpha}$ | - | - | 0.114 | - | 0.181 | - | 0.209 | - |
| bias | - | - | -0.004 | - | -0.004 | - | -0.006 | - |
| $\widehat{H}_{5}^{2}$ | -0.080 | - | - | - | - | - | - | - |
| $\widetilde{\alpha}$ | -0.078 | - | - | - | - | - | - | - |
| bias | -0.003 | - | - | - | - | - | - | - |
| $\widehat{H}_{1}^{3}$ | 0.044 | - | 0.095 | - | 0.131 | - | 0.150 | - |
| $\widetilde{\alpha}$ | 0.045 | - | 0.096 | - | 0.133 | - | 0.153 | - |
| bias | -0.001 | - | -0.002 | - | -0.002 | - | -0.003 | - |
| $C P$ | 0.385 | 0.336 | 0.760 | 0.644 | 1.115 | 0.955 | 1.316 | 1.115 |
| $\widetilde{\alpha}$ | 0.381 | 0.343 | 0.760 | 0.660 | 1.108 | 0.980 | 1.306 | 1.147 |
| bias | 0.004 | -0.007 | -- | -0.016 | 0.007 | -0.026 | 0.010 | -0.032 |
| $\widehat{H} 8$ | - | 0.332 | - | 0.588 | - | 0.777 | - | 0.938 |
| $\widetilde{\alpha}$ | - | 0.342 | - | 0.607 | - | 0.802 | - | 0.972 |
| bias | - | -0.010 | - | -0.019 | - | -0.025 | - | -0.035 |

Note: The bias unadjusted estimates are reported in Columns 1 and 9 of Tables 4 to 7, respectively.

Table 9: Bootstrap Estimates when $\widehat{H}_{t}=\widehat{F}_{t}$ : Regression $r x_{t+1}^{(n)}=\alpha^{\prime} \widehat{F}_{t}+\beta^{\prime} C P_{t}+\epsilon_{t+1}$

|  |  |  | Bootstrap |  | Bootstrap under the Null |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\widehat{\alpha}$ | bias | 95\% CI | 99\% CI | 95\% CI | 99\% CI |
|  | $n=2$ |  |  |  |  |  |
| $\widehat{H}_{1}$ | -0.761 | 0.012 | ( -1.143-0.343) | ( -1.071-0.399) | (-0.021-0.015) | ( -0.021-0.016) |
| $\widehat{H}_{4}$ | -0.291 | -0.006 | ( -0.554-0.031) | ( $-0.508-0.073$ ) | ( -0.003 0.003) | (-0.002 0.003) |
| $\widehat{H}_{6}$ | -0.151 | -0.002 | ( -0.467 0.166) | ( -0.408 0.100) | ( -0.015 0.016) | ( -0.015 0.016) |
| $\widehat{H}_{7}$ | -0.128 | -0.004 | ( -0.285 0.027) | ( -0.258-0.010) | ( -0.008 0.011) | (-0.007 0.009) |
| $\widehat{H}_{8}$ | 0.240 | 0.004 | ( 0.0540 .425 ) | ( 0.088 0.404) | ( -0.011 0.010) | ( -0.010 0.008) |
| $\widehat{H}_{5}^{2}-0.080$ | 0.003 | (-0.187 0.040) | ( -0.170 0.015) | ( -0.010-0.003) | ( -0.009-0.003) |  |
| $\widehat{H}_{1}^{3} 0.044$ | -0.001 | ( 0.0100 .076 ) | ( 0.0160 .071 ) | ( -0.000 0.000) | ( -0.000 0.000) |  |
| CP 0.385 | -0.003 | ( 0.2620 .516 ) | ( 0.2760 .490 ) | ( 0.003 0.009) | ( 0.0030 .008 ) |  |
| $\mathrm{R}^{2}$ | 0.460 |  | ( 0.2370 .523 ) | ( 0.2610 .500 ) | ( 0.0190 .045 ) | ( 0.0210 .042 ) |
|  | $n=3$ |  |  |  |  |  |
| $\widehat{H}_{1}$ | -1.232 | 0.027 | ( -1.914-0.506) | ( -1.797-0.655) | ( -0.021-0.015) | ( -0.021-0.016) |
| $\widehat{H}_{2}$ | -0.028 | -0.017 | ( -0.574 0.505) | ( -0.486 0.426) | ( -0.001 0.005) | ( -0.000 0.005) |
| $\widehat{H}_{4}$ | -0.423 | -0.004 | ( -0.881 0.030) | ( -0.811-0.050) | ( -0.003 0.003) | (-0.003 0.003) |
| $\widehat{H}_{6}$ | -0.433 | 0.012 | ( -0.969 0.093) | ( -0.870 0.024) | ( -0.014 0.015) | ( -0.013 0.014) |
| $\widehat{H}_{7}$ | -0.338 | -0.002 | ( -0.585-0.094) | ( -0.549-0.140) | ( -0.009 0.010) | ( -0.007 0.009) |
| $\widehat{H}_{8}$ | 0.389 | -0.002 | ( 0.0820 .669 ) | ( 0.140 0.632) | ( -0.009 0.008) | ( -0.008 0.007) |
| $\widehat{H}_{3}^{2} 0.111$ | -0.003 | ( -0.046 0.250) | ( -0.006 0.221) | ( 0.0000 .002 ) | ( 0.0000 .002 ) |  |
| $\widehat{H}_{1}^{3} 0.095$ | -0.002 | ( 0.0340 .145 ) | ( 0.0460 .136 ) | ( 0.0000 .001 ) | ( 0.0000 .001 ) |  |
| $C P$ | 0.760 | -0.001 | ( 0.5460 .980 ) | ( 0.5820 .935 ) | ( 0.0030 .009 ) | ( 0.0030 .008 ) |
| $\bar{R}^{2}$ | 0.455 |  | ( 0.2800 .559 ) | ( 0.3030 .533 ) | ( 0.0130 .035 ) | ( 0.0140 .032 ) |
|  | $n=4$ |  |  |  |  |  |
| $\widehat{H}_{1}$ | -1.521 | 0.047 | (-2.488-0.480) | ( -2.323-0.617) | ( -0.021-0.015) | ( -0.021-0.016) |
| $\widehat{H}_{4}$ | -0.436 | 0.001 | ( -1.048 0.178) | ( -0.958 0.090) | ( -0.004 0.003) | ( -0.003 0.003) |
| $\widehat{H}_{6}$ | -0.668 | -0.002 | ( -1.410 0.131) | (-1.297 0.002) | ( -0.014 0.015) | ( -0.013 0.014) |
| $\widehat{H}_{7}$ | -0.534 | 0.004 | ( -0.942-0.178) | ( -0.849-0.230) | ( -0.009 0.010) | ( -0.007 0.008) |
| $\widehat{H}_{8}$ | 0.578 | 0.004 | ( 0.119 1.022) | ( 0.2060 .957 ) | ( -0.010 0.009) | ( -0.009 0.007) |
| $\widehat{H}_{3}^{2}$ | 0.177 | -0.001 | ( -0.031 0.375) | ( 0.0020 .339 ) | ( 0.0000 .002 ) | ( 0.000 0.002) |
| $\widehat{H}_{1}^{3}$ | 0.131 | -0.003 | ( 0.0550 .206 ) | ( 0.068 0.189) | ( 0.000 0.001) | ( 0.000 0.001) |
| $C P$ | 1.115 | -0.006 | ( 0.820 1.401) | ( 0.8611 .348 ) | ( 0.0030 .009 ) | ( 0.0030 .009 ) |
| $R^{2}$ | 0.473 |  | ( 0.2770 .567 ) | ( 0.3030 .545 ) | ( 0.0140 .036 ) | ( 0.0160 .034 ) |
|  | $n=5$ |  |  |  |  |  |
| $\widehat{H}_{1}$ | -1.653 | 0.026 | (-2.832-0.429) | ( -2.648-0.606) | ( -0.021-0.015) | ( -0.021-0.016) |
| $\widehat{H}_{4}$ | -0.516 | -0.004 | ( -1.306 0.321) | ( -1.190 0.169) | ( -0.003 0.003) | (-0.003 0.003) |
| $\widehat{H}_{6}$ | -0.856 | 0.011 | ( -1.870 0.190) | ( -1.666 0.012) | ( -0.014 0.014) | ( -0.013 0.014) |
| $\widehat{H}_{7}$ | -0.686 | 0.012 | ( -1.182-0.119) | ( -1.071-0.244) | ( -0.007 0.010) | ( -0.006 0.009) |
| $\widehat{H}_{8}$ | 0.702 | -0.004 | ( 0.1391 .286 ) | ( 0.224 1.160) | ( -0.009 0.008) | ( -0.009 0.007) |
| $\widehat{H}_{3}{ }^{2}$ | 0.204 | 0.000 | ( -0.059 0.491) | ( -0.017 0.419) | ( 0.0000 .002 ) | ( 0.0010 .002 ) |
| $\widehat{H}_{1}^{3}$ | 0.150 | -0.001 | ( 0.0510 .242 ) | ( 0.069 0.232) | ( 0.0000 .001 ) | ( 0.000 0.001) |
| $C P$ | 1.316 | -0.009 | ( 0.896 1.723) | ( 0.9451 .663 ) | ( 0.0030 .009 ) | ( 0.0030 .008 ) |
| $\bar{R}^{2}$ | 0.435 |  | ( 0.2250 .518 ) | ( 0.2510 .488 ) | ( 0.0150 .036 ) | ( 0.0160 .033 ) |

Table 10: Posterior Mean: $r x_{t+1}^{(n)}=a+\alpha^{\prime} \widehat{G}_{t}+\beta^{\prime} C P_{t}+\epsilon_{t+1}$

| $\widehat{F}$ | $n=2$ |  | $n=3$ |  | $n=4$ |  | $n=5$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\widehat{H}_{1}$ | - | - | - | - | - | - | 0.288 | - |
| $t_{\text {. }} 05$ | - | - | - | - | - | - | 1.275 | - |
| $t .95$ | - | - | - | - | - | - | 1.912 | - |
| $\widehat{H}_{2}$ | -0.506 | - | -0.801 | - | -0.976 | - | -1.159 | - |
| $t .05$ | -3.676 | - | -3.239 | - | -3.140 | - | -3.099 | - |
| $t .95$ | -2.942 | - | -2.622 | - | -2.477 | - | -2.397 | - |
| $\widehat{H}_{3}$ | -0.456 | - | -0.746 | - | -0.959 | - | -1.074 | - |
| $t .05$ | -5.335 | - | -4.749 | - | -4.616 | - | -3.302 | - |
| $t .95$ | -4.050 | - | -3.637 | - | -3.482 | - | -3.374 | - |
| $\widehat{H}_{6}$ | 0.139 | - | - | - | - | - | - | - |
| $t .05$ | 1.819 | - | - | - | - | - | - | - |
| $t .95$ | 1.712 | - | - | - | - | - | - | - |
| $\widehat{H}_{8}$ | -0.139 | - | -0.309 | - | -0.473 | - | -0.561 | - |
| $t .05$ | -1.872 | - | -2.366 | - | -2.622 | - | -2.523 | - |
| $t .95$ | -1.332 | - | -1.732 | - | -1.994 | - | -1.863 | - |
| $\widehat{H}_{4}^{2}$ | -0.070 | - | -0.183 | - | -0.253 | - | -0.348 | - |
| $t .05$ | -2.395 | - | -2.982 | - | -2.920 | - | -3.713 | - |
| $t .95$ | -2.787 | - | -3.319 | - | -3.089 | - | -3.681 | - |
| $\widehat{H}_{6}{ }^{2}$ | -0.086 | - | -0.154 | - | -0.235 | - | -0.274 | - |
| $t .05$ | -5.427 | - | -6.109 | - | -6.109 | - | -5.559 | - |
| $t .95$ | -6.629 | - | -7.223 | - | -6.838 | - | -6.138 | - |
| $\widehat{H}_{7}^{2}$ | - | - | 0.087 | - | 0.146 | - | 0.178 | - |
| $t .05$ | - | - | 2.408 | - | 2.866 | - | 2.852 | - |
| $t .95$ | - | - | 2.404 | - | 3.006 | - | 2.914 | - |
| $\widehat{H}_{1}^{3}$ | 0.019 | - | 0.032 | - | 0.037 | - | - | - |
| $t .05$ | 2.092 | - | 2.090 | - | 1.836 | - | - | - |
| $t .95$ | 2.346 | - | 2.357 | - | 2.095 | - | - | - |
| $C P$ | 0.452 | 0.416 | 0.845 | 0.790 | 1.236 | 1.155 | 1.456 | 1.365 |
| $t .05$ | 7.200 | 6.334 | 7.285 | 6.300 | 7.568 | 6.348 | 7.012 | 5.900 |
| $t .95$ | 7.566 | 6.919 | 7.641 | 6.770 | 7.926 | 6.760 | 7.331 | 6.262 |
| $\widehat{H} 8$ | - | 0.428 | - | 0.712 | - | 0.867 | - | 0.959 |
| $t .05$ | - | 3.330 | - | 3.096 | - | 2.888 | - | 2.610 |
| $t .95$ | - | 4.316 | - | 4.033 | - | 3.803 | - | 3.489 |
| $\bar{R}_{0.95}^{2}$ | 0.471 | 0.399 | 0.469 | 0.403 | 0.489 | 0.415 | 0.448 | 0.377 |
| $\bar{R}_{0.05}^{2}$ | 0.469 | 0.397 | 0.467 | 0.401 | 0.488 | 0.413 | 0.446 | 0.375 |

Note: Reported are the mean estimates when a predictive regression is run for each draw of $G_{t}$. Estimates when the regressors are the posterior mean of the $G_{t}$ are reported in Columns 5 and 10 of Tables 4 to 7, respectively.

Figure 1: Marginal R-squares for $F_{1}$


Notes: Chart shows the R-square from regressing the series number given on the $x$-axis onto the estimated factor named in the heading. See the appendix for a description of the numbered series. The factors are estimated using data from 1964:1-2007:12.

Figure 2: Marginal R-squares for $\mathrm{F}_{2}$


Notes: See Figure 1.

Figure 3: Marginal R-squares for $\mathrm{F}_{3}$


Figure 4: Marginal R-squares for $F_{4}$


Notes: See Figure 1.

Figure 5: Marginal R-squares for $\mathrm{F}_{5}$


Notes: See Figure 1.

Figure 6: Marginal R-squares for $\mathrm{F}_{6}$


Notes: See Figure 1.

Figure 7: Marginal R-squares for $\mathrm{F}_{7}$


Notes: See Figure 1.

Figure 8: Marginal R-squares for $\mathrm{F}_{8}$


Notes: See Figure 1.

Figure 9:


Fin and Gin are the $\bar{R}^{2}$ from rolling estimation of (8), with predictors selected by the in-sample BIC. Fout and Gout use predictors selected by the out-of-sample BIC. F8 and G8 use a linear combination of eight factors as predictors, where the weights are based on (10).

Figure 10:


Fin and Gin are the $\bar{R}^{2}$ from rolling estimation of (8), with predictors selected by the in-sample BIC. Fout and Gout use predictors selected by the out-of-sample BIC. F8 and G8 use a linear combination of eight factors as predictors, where the weights are based on (10).

Figure 11:


Figure 12:


Figure 13:



Figure 14:



Figure 15:


Figure 16:


Figure 17:


## 1 Data Appendix

This appendix lists the short name of each series, its mnemonic (the series label used in the source database), the transformation applied to the series, and a brief data description. All series are from the Global Insights Basic Economics Database, unless the source is listed (in parentheses) as TCB (The Conference Board's Indicators Database) or AC (author's calculation based on Global Insights or TCB data). In the transformation column, $\ln$ denotes logarithm, $\Delta \ln$ and $\Delta^{2} \ln$ denote the first and second difference of the logarithm, lv denotes the level of the series, and $\Delta$ lv denotes the first difference of the series. The data are available from 1959:01-1997:12.

## Group 1: Output and Income

| No. | Gp | Short Name | Mnemonic | Tran | Descripton |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | PI | ypr | $\Delta l n$ | Personal Income (AR, Bil. Chain 2000 \$) (TCB) |
| 6 | 1 | IP: total | ips10 | $\Delta l n$ | Industrial Production Index - Total Index |
| 7 | 1 | IP: products | ips11 | $\Delta l n$ | Industrial Production Index - Products, Total |
| 8 | 1 | IP: final prod | ips299 | $\Delta l n$ | Industrial Production Index - Final Products |
| 9 | 1 | IP: cons gds | ips12 | $\Delta l n$ | Industrial Production Index - Consumer Goods |
| 10 | 1 | IP: cons dble | ips13 | $\Delta l n$ | Industrial Production Index - Durable Consumer Goods |
| 11 | 1 | IP: cons nondble | ips18 | $\Delta l n$ | Industrial Production Index - Nondurable Consumer Goods |
| 12 | 1 | IP: bus eqpt | ips25 | $\Delta l n$ | Industrial Production Index - Business Equipment |
| 13 | 1 | IP: matls | ips32 | $\Delta l n$ | Industrial Production Index - Materials |
| 14 | 1 | IP: dble matls | ips34 | $\Delta l n$ | Industrial Production Index - Durable Goods Materials |
| 15 | 1 | IP: nondble matls | ips38 | $\Delta l n$ | Industrial Production Index - Nondurable Goods Materials |
| 16 | 1 | IP: mfg | ips43 | $\Delta l n$ | Industrial Production Index - Manufacturing (Sic) |
| 17 | 1 | IP: res util | ips307 | $\Delta l n$ | Industrial Production Index - Residential Utilities |
| 18 | 1 | IP: fuels | ips306 | $\Delta l n$ | Industrial Production Index - Fuels |
| 19 | 1 | NAPM prodn | pmp | lv | Napm Production Index (Percent) |
| 20 | 1 | Cap util | utl11 | $\Delta l v$ | Capacity Utilization (SIC-Mfg) (TCB) |

## Group 2: Labor Market

| No. | Gp | Short Name | Mnemonic | Tran | Descripton |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | 2 | Help wanted indx | lhel | $\Delta l v$ | Index Of Help-Wanted Advertising In Newspapers (1967=100;Sa) |
| 22 | 2 | Help wanted/emp | lhelx | $\Delta l v$ | Employment: Ratio; Help-Wanted Ads:No. Unemployed Clf |
| 23 | 2 | Emp CPS total | lhem | $\Delta l n$ | Civilian Labor Force: Employed, Total (Thous.,Sa) |
| 24 | 2 | Emp CPS nonag | lhnag | $\Delta l n$ | Civilian Labor Force: Employed, Nonagric.Industries (Thous.,Sa) |
| 25 | 2 | U: all | lhur | $\Delta l v$ |  |
| 26 | 2 | U : mean duration | lhu680 | $\Delta l v$ | Unemploy.By Duration: Average(Mean)Duration In Weeks (Sa) |
| 27 | 2 | $\mathrm{U}<5$ wks | lhu5 | $\Delta l n$ | Unemploy.By Duration: Persons Unempl.Less Than 5 Wks (Thous.,Sa) |
| 28 | 2 | U 5-14 wks | lhu14 | $\Delta l n$ | Unemploy.By Duration: Persons Unempl. 5 To 14 Wks (Thous.,Sa) |
| 29 | 2 | U $15+$ wks | lhu15 | $\Delta l n$ | Unemploy.By Duration: Persons Unempl. 15 Wks + (Thous.,Sa) |
| 30 | 2 | U 15-26 wks | lhu26 | $\Delta l n$ | Unemploy.By Duration: Persons Unempl. 15 To 26 Wks (Thous.,Sa) |
| 31 | 2 | U $27+$ wks | lhu27 | $\Delta l n$ | Unemploy.By Duration: Persons Unempl. 27 Wks + (Thous,Sa) |
| 32 | 2 | UI claims | claimuii | $\Delta l n$ | Average Weekly Initial Claims, Unemploy. Insurance (Thous.) (TCB) |
| 33 | 2 | Emp: total | ces002 | $\Delta l n$ | Employees On Nonfarm Payrolls: Total Private |
| 34 | 2 | Emp: gds prod | ces003 | $\Delta l n$ | Employees On Nonfarm Payrolls - Goods-Producing |
| 35 | 2 | Emp: mining | ces006 | $\Delta l n$ | Employees On Nonfarm Payrolls - Mining |
| 36 | 2 | Emp: const | ces011 | $\Delta l n$ | Employees On Nonfarm Payrolls - Construction |
| 37 | 2 | Emp: mfg | ces015 | $\Delta l n$ | Employees On Nonfarm Payrolls - Manufacturing |
| 38 | 2 | Emp: dble gds | ces017 | $\Delta l n$ | Employees On Nonfarm Payrolls - Durable Goods |
| No. | Gp | Short Name | Mnemonic | Tran | Descripton |
| 39 | 2 | Emp: nondbles | ces033 | $\Delta l n$ | Employees On Nonfarm Payrolls - Nondurable Goods |
| 40 | 2 | Emp: services | ces046 | $\Delta l n$ | Employees On Nonfarm Payrolls - Service-Providing |
| 41 | 2 | Emp: TTU | ces048 | $\Delta l n$ | Employees On Nonfarm Payrolls - Trade, Transportation, And Utilities |
| 42 | 2 | Emp: wholesale | ces049 | $\Delta l n$ | Employees On Nonfarm Payrolls - Wholesale Trade. |
| 43 | 2 | Emp: retail | ces053 | $\Delta l n$ | Employees On Nonfarm Payrolls - Retail Trade |
| 44 | 2 | Emp: FIRE | ces088 | $\Delta l n$ | Employees On Nonfarm Payrolls - Financial Activities |
| 45 | 2 | Emp: Govt | ces140 | $\Delta l n$ | Employees On Nonfarm Payrolls - Government |
| (46) | 2 | Emp-hrs nonag | a0m048 | $\Delta l n$ | Employee Hours In Nonag. Establishments (AR, Bil. Hours) (TCB) |
| 47 | 2 | Avg hrs | ces151 | lv | Avg Weekly Hrs of Prod or Nonsup Workers On Private Nonfarm Payrolls - Goods-Producing |
| 48 | 2 | Overtime: mfg | ces155 | $\Delta l v$ | Avg Weekly Hrs of Prod or Nonsup Workers On Private Nonfarm Payrolls - Mfg Overtime Hours |
| 49 | 2 | Avg hrs: mfg | aom001 | lv | Average Weekly Hours, Mfg. (Hours) (TCB) |
| 50 | 2 | NAPM empl | pmemp | lv | Napm Employment Index (Percent) |
| 129 | 2 | AHE: goods | ces275 | $\Delta^{2} l n$ | Avg Hourly Earnings of Prod or Nonsup Workers On Private Nonfarm Payrolls - Goods-Producing |
| 130 | 2 | AHE: const | ces277 | $\Delta^{2} l n$ | Avg Hourly Earnings of Prod or Nonsup Workers On Private Nonfarm Payrolls - Construction |
| 131 | 2 | AHE: mfg | $\operatorname{ces} 278$ | $\Delta^{2} l n$ | Avg Hourly Earnings of Prod or Nonsup Workers On Private Nonfarm Payrolls - Manufacturing |

## Group 3: Housing

| No. | Gp | Short Name | Mnemonic | Tran | Descripton |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 51 | 3 | Starts: nonfarm | hsfr | $\ln$ | Housing Starts:Nonfarm(1947-58);Total Farm\&Nonfarm(1959-)(Thous.,Saar) |
| 52 | 3 | Starts: NE | hsne | $\ln$ | Housing Starts:Northeast (Thous.U.)S.A. |
| 53 | 3 | Starts: MW | hsmw | $\ln$ | Housing Starts:Midwest(Thous.U.)S.A. |
| 54 | 3 | Starts: South | hssou | $\ln$ | Housing Starts:South (Thous.U.)S.A. |
| 55 | 3 | Starts: West | hswst | $\ln$ | Housing Starts:West (Thous.U.)S.A. |
| 56 | 3 | BP: total | hsbr | $\ln$ | Housing Authorized: Total New Priv Housing Units (Thous.,Saar) |
| 57 | 3 | BP: NE | hsbne* | $\ln$ | Houses Authorized By Build. Permits:Northeast(Thou.U.)S.A |
| 58 | 3 | BP: MW | hsbmw* | $\ln$ | Houses Authorized By Build. Permits:Midwest(Thou.U.)S.A. |
| 59 | 3 | BP: South | hsbsou* | $\ln$ | Houses Authorized By Build. Permits:South(Thou.U.)S.A. |
| 60 | 3 | BP: West | hsbwst* | $\ln$ | Houses Authorized By Build. Permits:West(Thou.U.)S.A. |

## Group 4: Consumption, Orders and Inventories

| 61 | 4 | PMI | pmi | lv | Purchasing Managers' Index (Sa) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 62 | 4 | NAPM new ordrs | pmno | lv | Napm New Orders Index (Percent) |
| 63 | 4 | NAPM vendor del | pmdel | lv | Napm Vendor Deliveries Index (Percent) |
| 64 | 4 | NAPM Invent | pmnv | lv | Napm Inventories Index (Percent) |
| 65 | 4 | Orders: cons gds | a 1 m 008 | $\Delta l n$ | Mfrs' New Orders, Consumer Goods And Materials (Mil. \$) (TCB) |
| 66 | 4 | Orders: dble gds | $\mathrm{a} 0 \mathrm{m007}$ | $\Delta l n$ | Mfrs' New Orders, Durable Goods Industries (Bil. Chain 2000 \$ ) (TCB) |
| 67 | 4 | Orders: cap gds | $\mathrm{a} 0 \mathrm{m027}$ | $\Delta l n$ | Mfrs' New Orders, Nondefense Capital Goods (Mil. Chain 1982 \$) (TCB) |
| 68 | 4 | Unf orders: dble | a 1 m 092 | $\Delta l n$ | Mfrs' Unfilled Orders, Durable Goods Indus. (Bil. Chain 2000 \$) (TCB) |
| 69 | 4 | M\&T invent | a 0 m 070 | $\Delta l n$ | Manufacturing And Trade Inventories (Bil. Chain 2000 \$) (TCB) |
| 70 | 4 | M\&T invent/sales | a 0 m 077 | $\Delta l v$ | Ratio, Mfg. And Trade Inventories To Sales (Based On Chain 2000 \$) (TCB) |
| 3 | 4 | Consumption | cons-r | $\Delta l n$ | Real Personal Consumption Expenditures (AC) (Bill \$) pi031 / gmdc |
| 4 | 4 | M\&T sales | mtq | $\Delta l n$ | Manufacturing And Trade Sales (Mil. Chain 1996 \$) (TCB) |
| 5 | 4 | Retail sales | $\mathrm{a} 0 \mathrm{m059}$ | $\Delta l n$ | Sales Of Retail Stores (Mil. Chain 2000 \$) (TCB) |
| 132 | 4 | Consumer expect | hhsntn | $\Delta l v$ | U. Of Mich. Index Of Consumer Expectations(Bcd-83) |

## Group 5: Money and Credit

| No. | Gp | Short Name | Mnemonic | Tran | Descripton |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 71 | 5 | M1 | fm1 | $\Delta^{2} l n$ | Money Stock: M1(Curr,Trav.Cks,Dem Dep,Other Ck'able Dep)(Bil\$,Sa) |
| 72 | 5 | M2 | fm2 | $\Delta^{2} l n$ | Money Stock:M2(M1+O'nite Rps,Euro\$,G/P\&B/D \& Mmmfs\&Sav\& Sm Time Dep(Bil\$,Sa) |
| 73 | 5 | Currency | fmscu | $\Delta^{2} l n$ | Money Stock: Currency held by the public (Bil\$,Sa) |
| 74 | 5 | M2 (real) | fm2-r | $\Delta l n$ | Money Supply: Real M2, fm2 / gmdc (AC) |
| 75 | 5 | MB | fmfba | $\Delta^{2} l n$ | Monetary Base, Adj For Reserve Requirement Changes(Mil\$,Sa) |
| 76 | 5 | Reserves tot | fmrra | $\Delta^{2} l n$ | Depository Inst Reserves:Total, Adj For Reserve Req Chgs(Mil\$,Sa) |
| 77 | 5 | Reserves nonbor | fmrnba | $\Delta^{2} l n$ | Depository Inst Reserves:Nonborrowed,Adj Res Req Chgs(Mil\$,Sa) |
| 78 | 5 | C\&I loans | fclnbw | $\Delta^{2} l n$ | Commercial \& Industrial Loans Outstanding + NonFin Comm. Paper (Mil\$, SA) (Bci) |
| 79 | 5 | C\&I loans | fclbmc | $l^{2}$ | Wkly Rp Lg Com'l Banks:Net Change Com'l \& Indus Loans(Bil\$,Saar) |
| 80 | 5 | Cons credit | ccinrv | $\Delta^{2} l n$ | Consumer Credit Outstanding - Nonrevolving(G19) |
| 81 | 5 | Inst cred/PI | ccipy | $\Delta l v$ | Ratio, Consumer Installment Credit To Personal Income (Pct.) (TCB) |

## Group 6: Bond and Exchange rates

| 86 | 6 | Fed Funds | fyff | $\Delta l v$ | Interest Rate: Federal Funds (Effective) (\% Per Annum,Nsa) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 87 | 6 | Comm paper | cp90 | $\Delta l v$ | Commercial Paper Rate |
| 88 | 6 | 3 mo T-bill | fygm3 | $\Delta l v$ | Interest Rate: U.S.Treasury Bills,Sec Mkt,3-Mo.(\% Per Ann,Nsa) |
| 89 | 6 | 6 mo T-bill | fygm6 | $\Delta l v$ | Interest Rate: U.S.Treasury Bills,Sec Mkt,6-Mo.(\% Per Ann,Nsa) |
| 90 | 6 | 1 yr T-bond | fygt1 | $\Delta l v$ | Interest Rate: U.S.Treasury Const Maturities,1-Yr.(\% Per Ann,Nsa) |
| 91 | 6 | 5 yr T-bond | fygt5 | $\Delta l v$ | Interest Rate: U.S.Treasury Const Maturities,5-Yr.(\% Per Ann,Nsa) |
| 92 | 6 | 10 yr T-bond | fygt10 | $\Delta l v$ | Interest Rate: U.S.Treasury Const Maturities,10-Yr.(\% Per Ann,Nsa) |
| 93 | 6 | Aaa bond | fyaaac | $\Delta l v$ | Bond Yield: Moody's Aaa Corporate (\% Per Annum) |
| 94 | 6 | Baa bond | fybaac | $\Delta l v$ | Bond Yield: Moody's Baa Corporate (\% Per Annum) |
| 95 | 6 | CP-FF spread | scp90F | $l v$ | cp90-fyff (AC) |
| 96 | 6 | 3 mo-FF spread | sfygm3 | $l v$ | fygm3-fyff (AC) |
| 97 | 6 | 6 mo-FF spread | sfygm6 | $l v$ | fygm6-fyff (AC) |
| 98 | 6 | 1 yr-FF spread | sfygt1 | $l v$ | fygt1-fyff (AC) |
| 99 | 6 | 5 yr-FF spread | sfygt5 | $l v$ | fygt5-fyff (AC) |
| 100 | 6 | 10 yr-FF spread | sfygt10 | $l v$ | fygt10-fyff (AC) |
| 101 | 6 | Aaa-FF spread | sfyaaac | $l v$ | fyaaac-fyff (AC) |
| 102 | 6 | Baa-FF spread | sfybaac | $l v$ | fybaac-fyff (AC) |
| 103 | 6 | Ex rate: avg | exrus | $\Delta l n$ | United States;Effective Exchange Rate(Merm)(Index No.) |
| 104 | 6 | Ex rate: Switz | exrsw | $\Delta l n$ | Foreign Exchange Rate: Switzerland (Swiss Franc Per U.S.S) |
| 105 | 6 | Ex rate: Japan | exrjan | $\Delta l n$ | Foreign Exchange Rate: Japan (Yen Per U.S.\$) |
| 106 | 6 | Ex rate: UK | exruk | $\Delta l n$ | Foreign Exchange Rate: United Kingdom (Cents Per Pound) |
| 107 | 6 | EX rate: Canada | exrcan | $\Delta l n$ | Foreign Exchange Rate: Canada (Canadian \$ Per U.S.\$) |

## Group 7: Prices

| 108 | 7 | PPI: fin gds | pwfsa | $\Delta^{2} l n$ | Producer Price Index: Finished Goods (82=100,Sa) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 109 | 7 | PPI: cons gds | pwfcsa | $\Delta^{2} l n$ | Producer Price Index: Finished Consumer Goods (82=100,Sa) |
| 110 | 7 | PPI: int materials | pwimsa | $\Delta^{2} l n$ | Producer Price Index:I ntermed Mat.Supplies \& Components( $82=100$, Sa) |
| 111 | 7 | PPI: crude materials | pwcmsa | $\Delta^{2} l n$ | Producer Price Index: Crude Materials (82=100,Sa) |
| 112 | 7 | Spot market price | psccom | $\Delta^{2} l n$ | Spot market price index: bls \& crb: all commodities(1967=100) |
| 113 | 7 | PPI: nonferrous materials | pw102 | $\Delta^{2} l n$ | Producer Price Index: Nonferrous Materials (1982=100, Nsa) |
| 114 | 7 | NAPM com price | pmcp | lv | Napm Commodity Prices Index (Percent) |
| 115 | 7 | CPI-U: all | punew | $\Delta^{2} l n$ | Cpi-U: All Items (82-84=100,Sa) |
| 116 | 7 | CPI-U: apparel | pu83 | $\Delta^{2} l n$ | Cpi-U: Apparel \& Upkeep (82-84=100,Sa) |
| 117 | 7 | CPI-U: transp | pu84 | $\Delta^{2} l n$ | Cpi-U: Transportation (82-84=100,Sa) |
| 118 | 7 | CPI-U: medical | pu85 | $\Delta^{2} l n$ | Cpi-U: Medical Care (82-84=100,Sa) |
| 119 | 7 | CPI-U: comm. | puc | $\Delta^{2} l n$ | Cpi-U: Commodities (82-84=100,Sa) |
| 120 | 7 | CPI-U: dbles | pucd | $\Delta^{2} l n$ | Cpi-U: Durables (82-84=100,Sa) |
| 121 | 7 | CPI-U: services | pus | $\Delta^{2} l n$ | Cpi-U: Services (82-84=100,Sa) |
| 122 | 7 | CPI-U: ex food | puxf | $\Delta^{2} l n$ | Cpi-U: All Items Less Food (82-84=100,Sa) |
| 123 | 7 | CPI-U: ex shelter | puxhs | $\Delta^{2} l n$ | Cpi-U: All Items Less Shelter (82-84=100,Sa) |
| 124 | 7 | CPI-U: ex med | puxm | $\Delta^{2} l n$ | Cpi-U: All Items Less Midical Care (82-84=100,Sa) |
| 125 | 7 | PCE defl | gmdc | $\Delta^{2} l n$ | Pce, Impl Pr Defl:Pce (2000=100) (AC) (BEA) |
| 126 | 7 | PCE defl: dlbes | gmdcd | $\Delta^{2} l n$ | Pce, Impl Pr Deff:Pce; Durables (2000=100) (AC) (BEA) |
| 127 | 7 | PCE defl: nondble | gmdcn | $\Delta^{2} l n$ | Pce, Impl Pr Defl:Pce; Nondurables (2000=100) (AC) (BEA) |
| 128 | 7 | PCE defl: service | gmdcs | $\Delta^{2} l n$ | Pce, Impl Pr Defl:Pce; Services (2000=100) (AC) (BEA) |

## Group 8: Stock Market

| No. | Gp | Short Name | Mnemonic | Tran | Descripton |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 82 | 8 | S\&P 500 | fspcom | $\Delta l n$ | S\&P's Common Stock Price Index: Composite (1941-43=10) |
| 83 | 8 | S\&P: indust | fspin | $\Delta l n$ | S\&P's Common Stock Price Index: \& Industrials (1941-43=10) |
| 84 | 8 | S\&P div yield | fsdxp | $\Delta l v$ | S\&P's Composite Common Stock: Dividend Yield (\% Per Annum) |
| 85 | 8 | S\&P PE ratio | fspxe | $\Delta l n$ | S\&P's Composite Common Stock: \&Price-Earnings Ratio (\%,Nsa) |


[^0]:    ${ }^{1}$ See Bai and Ng (2008) for a survey on this literature.

[^1]:    ${ }^{2}$ See, for example, Boivin and Ng (2005).

[^2]:    ${ }^{3}$ See, for example, Bai and $\operatorname{Ng}$ (2006b) and DeMol, Giannone, and Reichlin (2006).

[^3]:    ${ }^{4}$ It is useful to remark that the convergence rate established in Stock and Watson (2002a) is too slow to permit consistent estimation of the parameters in (5).

[^4]:    ${ }^{5}$ This is obtained by considering $C_{18, j}$ for $j=1, \ldots 8$, where $C_{n, k}$ denotes choosing $k$ out of $n$ potential predictors.

[^5]:    ${ }^{6}$ Moench (2008) finds that factors estimated from a large panel of macroeconomic data explain the short rate better than output and inflation.

[^6]:    ${ }^{7}$ This is only for notational convenience. The estimation will include the means.

