ME
7/9/08
3:05 pm

Insights from an Estimated Search-Based Monetary Model with Nominal Rigidities*

S. Borağan Aruoba
University of Maryland

Frank Schorfheide
University of Pennsylvania

June 19, 2008

Abstract

We develop a search-based monetary dynamic stochastic equilibrium (DSGE) model with nominal rigidities by introducing the fundamental frictions that generate money demand in an otherwise standard New-Keynesian DSGE model. We use Bayesian methods to estimate two versions of the model based on post 1983 quarterly U.S. data and compare it to a money-in-the-utility (MIU) specification. While the decentralized market mechanism of the search-based models creates a stronger linkage between technology shocks and fluctuations in the stock of money, this linkage comes at a cost in terms of overall time series fit. On the other hand, if one uses the steady state relationships of the estimated DSGE models to predict velocity in periods of high target inflation rates as observed in the 1970s the search-based models deliver much more realistic predictions than the MIU model. In terms of welfare implications the estimated MIU model behaves very much like a New Keynesian DSGE model and a near-zero inflation rate is optimal. According to the search-based model, which also has embodied the same New Keynesian feature, the Friedman motive for keeping the nominal interest rate near zero dominates and negative inflation rates are optimal.

JEL CLASSIFICATION: C5, E4, E5

KEY WORDS: Bayesian Estimation, Monetary Models, Welfare Costs of Inflation.

---

*Correspondence: B. Aruoba: Department of Economics, University of Maryland, College Park, MD 20742. Email: aruoba@econ.umd.edu. F. Schorfheide: Department of Economics, 3718 Locust Walk, University of Pennsylvania, Philadelphia, PA 19104. Email: schorf@ssc.upenn.edu.

We thank Randy Wright for valuable discussions and Peter Ireland, Chris Otrok, Alex Wolman, as well as seminar participants at various institutions and conferences for helpful comments. Maxym Kryshko provided excellent research assistance. Schorfheide gratefully acknowledges financial support from the Alfred P. Sloan Foundation and the National Science Foundation under Grant SES 0617803. Gauss Programs that implement the empirical analysis will be made available at *http://www.ssc.upenn.edu/~schorf*.

1 Introduction

Dynamic stochastic general equilibrium (DSGE) models with nominal rigidities and a monetary policy represented by interest rate feedback rules are emerging as the workhorse of applied policy analysis in many central banks. Much of the empirical work with DSGE models, e.g. Smets and Wouters (2003), Christiano, Eichenbaum, and Evans (2005), Del Negro, Schorfheide, Smets, and Wouters (2007), Levin, Onatski, Williams, and Williams (2005), as well as the theoretical work summarized in Woodford (2003) is based on models in which real money balances directly enter the households' utility function. Such money-in-the-utility-function (MIU) specifications are informally motivated by the insight that money balances reduce transaction costs and therefore increase utility. Once monetary policy is represented by an interest-rate-feedback rule and real money balances enter the utility function in an additively separable fashion, the model becomes block triangular and aggregate outcomes are not affected by the money stock. In fact, it has become common practice to consider cash-less models, which are obtained by letting the weight on real money balances in the utility function converge to zero (see Woodford, 2003). Econometric work typically excludes a measure of the money stock from the list of observables and ignores the model implied money demand equation. While the cashless approach appears reasonable if the estimated model is used to study the propagation of structural shocks other than money demand shocks, it is not innocuous for welfare analysis.

To the extent that real money balances indeed affect households' utility, they are relevant for assessing the welfare consequences of changes in monetary policy, in particular at low levels of the nominal interest rate. In fact, when one includes a money demand motive in an otherwise standard New Keynesian model, the welfare consequences are not clear-cut. For example, in various contexts the standard result optimality of price stability emerge (e.g. Levin, Onatski, Williams, and Williams, 2005 in an MIU environment) in others Friedman rule of zero nominal net nominal interest rate is optimal (e.g. King and Wolman, 1996 in a shopping-time environment) and yet in some others the optimal inflation rate is somewhere in between these two extremems (e.g. Schmitt-Grohe and Uribe, 2007 in a transaction-cost environment).

The contribution of our paper is threefold. First, as an alternative to the commonly used MIU model, we develop an estimable DSGE model where money demand arises due to the micro-foundations laid out in the search-based monetary theory stemming from the work of Kiyotaki and Wright (1989). In our model, following the basic structure of in Lagos and Wright (2005, henceforth LW) and Aruoba, Waller, and Wright (2007, henceforth AWW), in every period economic activity takes place in two markets. In a decentralized market (DM), households engage in bilateral trade with a fraction of households producing and a fraction of households consuming. The centralized market (CM) resembles a standard DSGE model with nominal rigidities where production is carried out by firms. Physical capital is a factor of production in both markets. Demand for money arises because the transactions in the decentralized markets are facilitated by a medium of exchange. Our specification adds nominal rigidities in the centralized market, represents monetary policy by an interest rate feedback rule, and introduces stochastic disturbances to technology, preferences, government spending, and monetary policy to make the model amenable to econometric estimation methods. While the structure of our model to a large extent resembles that of a canonical New Keynesian model with capital, the presence of the decentralized market provides a micro-founded motive for holding money and creates a non-separability between consumption and the value of real money balances. Hence, our model differs from an MIU specification both in terms of the resulting money demand equation as well as its welfare implications.

Second, using post 1983 U.S. data on output, inflation, interest rates, and the money stock we use Bayesian techniques surveyed in An and Schorfheide (2007) to estimate our search-based DSGE model. We also fit a standard MIU model with nominal rigidities to the same set of observations. While most of the work on search-based monetary model has been theoretical, our analysis produces formal estimates of the taste and technology parameters that determine the exchange in the decentralized market. We compare the fit of the money demand equations obtained from the two estimated models. We also discuss dynamics by looking at variance decompositions and impulse-response functions.

Finally, we compare the effects of changes in the central bank's target inflation rate on steady state welfare using the two estimated DSGE models. Our choice of estimation objective function requires the models to fit both the post 1983 average velocity in U.S. data as well as the fluctuations in M2. We find that in the MIU model, New-Keynesian forces dominate the frictions created the opportunity cost of holding money and the optimal inflation target is near 0% inflation. In contrast, the optimal policy in the search-based model is to follow the Friedman rule of zero net nominal interest rate.

The remainder of the paper is organized as follows. We provide a detailed derivation and discussion of the search-based DSGE model in Section 2. A canonical MIU model with nominal rigidities and capital can be obtained by shutting down the decentralized market in the search-based model and adding a real-money-balance term to the households' utility function. This MIU model is described in Section 3. The Bayesian estimation results are presented in Section 4 and the welfare analysis is summarized in Section 5. Finally, Section 6 concludes. Detailed derivations for the two DSGE models are provided in the Appendix.

2 The Search-Based Model

The model is an extension of the two-sector model developed in LW. In every period, there is economic activity in two markets, which we label the decentralized market (DM) and the centralized market (CM). In the DM, households engage in decentralized bilateral trade with other households with one party producing and the other consuming, while the CM resembles a standard macro model where production is carried out by firms.

We extend the LW model in two dimensions. First, we include physical capital as a factor of production, following AWW. The only deviation we have from AWW in this regard is that we introduce an adjustment cost for investment to improve the empirical fit. Second, we replace the neoclassical structure on the firm side with a New Keynesian one. Intermediate goods producing firms sell their differentiated output to final good producers. The intermediate good producers face a downward sloping demand curve for their product and choose prices to maximize their profits. However, in any period only a fraction of these firms is able to re-optimize their prices. The remaining firms either adjust their prices by the lagged inflation rate or not at all. This mechanism of generating nominal rigidity is due to Calvo (1983) and widely used in the literature on New Keynesian DSGE models. Unlike in more elaborate empirical version in Smets and Wouters (2003) and Christiano, Eichenbaum, and Evans (2005), we exclude habit formation, wage stickiness, and variable capital utilization from our model specification. In turn we will describe the households' decision problems in both the centralized and the decentralized market (Section 2.1) and the firms' problem in the centralized market (Section 2.2). We then characterize the behavior of fiscal policy (Section 2.3), derive an aggregate resource constraint (Section 2.4) and characterize monetary policy (Section 2.5). Our model economy is subject to aggregate disturbances as we show in Section 2.6. A summary of all the equilibrium conditions is provided in the Appendix.

2.1 Households

There is a continuum of ex-ante identical households in the economy. These households derive utility from their activities in the two markets. A household that consumes q_t units of consumption good in the DM gets utility $\chi_t u(q_t)$ while it gets utility $U(x_t)$ by consuming x_t units in the CM. The disutility of effort in the DM for a seller and disutility of labor for a worker in the CM is linear:[1]

$$\mathcal{U}_t = U(x) - Ah_t \begin{cases} +\chi_t u(q_t) & \text{if buyer in DM} \\ -e_t & \text{if seller in DM} \end{cases} \tag{1}$$

Instead of using the disutility of effort e_t in the DM, we express the disutility as a function of output produced by the seller. To see this, we assume the following structure. For a seller, the output q_t is obtained using the production function $q_t = Z_t f(e_t, k_t)$ where Z_t is a technology shock which is common across the two markets. This production function can be inverted to get $e_t = \xi(q_t, k_t, Z_t)$. Using the linear disutility in effort, we can

[1] This assumption, in particular the linearity of disutility of labor in the CM is a critical assumption that prevents a non-degenerate distribution of money holdings.

define $c(q_t, k_t, Z_t) = \xi(q_t, k_t, Z_t)$ as the utility cost of production for the sellers. We have $c_q > 0$, $c_k < 0$, and $c_Z < 0$.

In a given period, the households participate in the DM followed by the CM. To characterize the household's behavior in this economy, we start from the problem of the household in the CM, followed by the DM problem.

2.1.1 Household Activity in the Centralized Market

The households take as given the aggregate price level in the CM, P_t, the nominal interest rate R_t, and the factor prices W_t and R_t^k. Using $W_t(\hat{m}_t, k_t, i_{t-1}, b_t, S_t)$ and $V_t(m_t, k_t, i_{t-1}, b_t, S_t)$ to denote the value functions in the CM and DM of period t where $\hat{m}_t$ is the money balances of the household entering the CM, the CM problem[2] is

$$\begin{aligned} & W_t(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) \\ = & \max_{x_t, h_t, m_{t+1}, i_t, k_{t+1}, b_{t+1}} \{U(x_t) - Ah_t + \beta E_t[V_{t+1}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1})]\} \\ \text{s.t.} \quad & P_t x_t + P_t i_t + b_{t+1} + m_{t+1} \leq P_t W_t h_t + P_t R_t^k k_t + \Pi_t + R_{t-1} b_t + \hat{m}_t - T_t \qquad (2) \\ & k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] i_t \qquad (3) \end{aligned}$$

given the laws of motion for the aggregate shocks, S_t. Here A is the disutility of one unit of labor, R_{t-1} is the gross nominal return on a government bond purchased in period $t-1$, T_t is a nominal lump-sum tax and Π_t denotes the total profits the household receives from intermediate good producers. (3) shows how capital is accumulated where the adjustment cost function $S(.)$ satisfies properties $S(1) = 0$, $S'(1) = 0$ and $S''(1) > 0$. Using Υ_t to denote the Lagrange multiplier for (3) and after eliminating h using (2), the FOC are

$$x_t \;:\; U'(x_t) = \frac{A}{W_t} \qquad (4)$$

$$m_{t+1} \;:\; \frac{U'(x_t)}{P_t} = \beta E V_{t+1,m}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \qquad (5)$$

$$i_t \;:\; U'(x_t) = \Upsilon_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) + \frac{i_t}{i_{t-1}} S'\left(\frac{i_t}{i_{t-1}}\right)\right] + \beta E V_{t+1,i}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \qquad (6)$$

$$k_{t+1} \;:\; \Upsilon_t = \beta E V_{t+1,k}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \qquad (7)$$

$$b_{t+1} \;:\; \frac{U'(x_t)}{P_t} = \beta E V_{t+1,b}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \qquad (8)$$

assuming that an interior solution exists. This leads to two key results. First, since the individual state variables, $(\hat{m}_t, k_t, i_{t-1}, b_t)$ do not appear in (5)-(8), household's decisions in the CM do not depend on its

[2]We could index households with j, but we will see that the assumption of complete markets implies that the index will drop out of most of these variables. In equilibrium households will make the same choice of consumption, money demand, and investment. So, we drop this index from the outset.

state variables. More specifically, for any distribution of assets $(\hat{m}_t, k_t, b_t)$ across agents entering the CM, the distribution of $(m_{t+1}, k_{t+1}, b_{t+1})$ is degenerate.[3] Second, we have the following envelope conditions,

$$\begin{aligned}
W_{t,m}(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) &= \frac{A}{P_t W_t} \\
W_{t,k}(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) &= \frac{A R_t^k}{W_t} + (1-\delta)\Upsilon_t \\
W_{t,i}(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) &= \Upsilon_t \left(\frac{i_t}{i_{t-1}}\right)^2 S'\left(\frac{i_t}{i_{t-1}}\right) \\
W_{t,b}(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) &= \frac{A R_{t-1}}{W_t}
\end{aligned}$$

which show that $W_t(.)$ is linear in $\hat{m}_t$ which will be important in the DM problem below. Finally, the Lagrange multiplier associated with the households' nominal budget constraint (2) is $U'(x_t)/P_t$. Under the assumption that households have access to a complete set of state-contingent claims we obtain that

$$\Xi^p_{t+1|t} = \frac{U'(x_{t+1})/P_{t+1}}{U'(x_t)/P_t}. \tag{9}$$

which the firms use to discount the future. We need to specify the details of the DM to characterize the equilibrium next. Specifically we will find V_m, V_k, V_i and V_b to obtain the equilibrium conditions.

2.1.2 Household Activity in the Decentralized Market

As we said, the centralized market in this model resembles a standard New Keynesian DSGE model. It is important to recognize that transactions in the CM take place without requiring a medium of exchange. Unlike a standard monetary model where money demand is generated by constructs such as cash-in-advance, money-in-the-utility-function or transaction costs, we follow a search-based approach. The DM is critical in generating the money demand. All trades take place in bilateral meetings. The agents are anonymous in the DM which means no household would accept an IOU from another household and any trade must be *quid pro quo*. Following AWW, at the start of each DM a measure σ of households receive a taste shock that make them buyers and another σ measure of households become sellers. Alternatively, we can consider the setup in LW where each household can produce a measure σ of goods out of a measure one of all possible goods, and they like consuming another σ measure of goods. When two households meet at random, with σ probability there is a single coincidence where one party likes the good the other party can produce but

[3]This result requires a small qualification for bond holdings. There are two parts of the argument that guarantees the degeneracy. The first part relies on the observation that $(\hat{m}_t, k_t, b_t)$ does not appear in (8). The second part relies on the strict concavity of $V(.)$ or, more specifically, the strict monotonicity of $V_b(.)$ which means the choice of b_{t+1} is unique. Both parts of the argument go through for money and capital in our environment, but only the first part goes through for bonds since $V_b(.)$ is constant as we show below. This means that in principle there could be multiple values of b_{t+1} that households choose, which can create a distribution of bond holdings. Fortunately, such a distribution of bonds holdings is not important for any of our results because bond-holdings will not affect the DM problem, as we show below.

not vice versa.[4] The literature started by Kiyotaki and Wright (1989) show that a medium of exchange will emerge in an environment where the agents are anonymous and there is a double-coincidence problem such as the one above. In a monetary equilibrium, in such single-coincidence meetings, the party who likes what the other party has (the buyer), uses money to purchase the good from the seller.[5] The possibility to consume in the DM generate a demand for money in this model.

The value of starting the DM for a household whose taste shock has not been realized yet is given by

$$\begin{aligned} V_t(m_t, k_t, i_{t-1}, b_t, S_t) &= \sigma V_t^b(m_t, k_t, i_{t-1}, b_t, S_t) + \sigma V_t^s(m_t, k_t, i_{t-1}, b_t, S_t) \\ &\quad +(1-2\sigma)W_t(m_t, k_t, i_{t-1}, b_t, S_t), \end{aligned} \tag{10}$$

where the values of being a buyer and a seller are

$$V_t^b(m_t, k_t, i_{t-1}, b_t, S_t) = \chi_t u(q_t^b) + W_t\left(m_t - d_t^b, k_t, i_{t-1}, b_t, S_t\right) \tag{11}$$

$$V_t^s(m_t, k_t, i_{t-1}, b_t, S_t) = -c(q_t^s, k_t, Z_t) + W_t\left(m_t + d_t^s, k_t, i_{t-1}, b_t, S_t\right) \tag{12}$$

with q_t^b and d_t^b (q_t^s and d_t^s) denoting output and money exchanged when buying (selling) which are determined via bilateral bargaining as describe below. We interpret χ_t as a money demand shock as it affects the utility from consuming in the DM and money serves as a medium of exchange. Using (9) we have

$$V_t(m_t, k_t, i_{t-1}, b_t, S_t) = W_t(m_t, k_t, i_{t-1}, b_t, S_t) + \sigma\left[\chi_t u(q_t^b) - \frac{d_t^b A}{P_t W_t}\right] + \sigma\left[\frac{d_t^s A}{P_t W_t} - c(q_t^s, k_t, Z_t)\right]. \tag{13}$$

To solve (5)-(8), we need:

$$\begin{aligned} V_{t,m}(m_t, k_t, i_{t-1}, b_t, S_t) &= \frac{A}{P_t W_t} + \sigma\left[\chi_t u'\left(q_t^b\right)\frac{\partial q_t^b}{\partial m_t} - \frac{A}{P_t W_t}\frac{\partial d_t^b}{\partial m_t}\right] \\ &\quad +\sigma\left[\frac{A}{P_t W_t}\frac{\partial d_t^s}{\partial m_t} - c_q(q_t^s, k_t, Z_t)\frac{\partial q_t^s}{\partial m_t}\right] \end{aligned} \tag{14}$$

$$\begin{aligned} V_{t,k}(m_t, k_t, i_{t-1}, b_t, S_t) &= \frac{A R_t^k}{W_t} + (1-\delta)\Upsilon_t + \sigma\left[\chi_t u'\left(q_t^b\right)\frac{\partial q_t^b}{\partial k_t} - \frac{A}{P_t W_t}\frac{\partial d_t^b}{\partial k_t}\right] \\ &\quad +\sigma\left[\frac{A}{P_t W_t}\frac{\partial d_t^s}{\partial k_t} - c_q(q_t^s, k_t, Z_t)\frac{\partial q_t^s}{\partial k_t} - c_k(q_t^s, k_t, Z_t)\right] \end{aligned} \tag{15}$$

$$V_{t,i}(m_t, k_t, i_{t-1}, b_t, S_t) = W_{t,i}(m_t, k_t, i_{t-1}, b_t, S_t) \tag{16}$$

$$V_{t,b}(m_t, k_t, i_{t-1}, b_t, S_t) = \frac{A}{P_t W_t} R_{t-1} \tag{17}$$

It remains to specify how the terms of trade (q, d) are determined, so that we can substitute for their derivatives in (14) and (15) which we turn to next. We consider two alternatives: bilateral bargaining via

[4] As AWW argue, the setup with idiosyncratic taste shocks and the setup with search leads to the same mathematical construct which we describe below.

[5] As with any deep model of money, there is a nonmonetary equilibrium in this model which is dominated by the monetary equilibrium in terms of welfare. We focus on the monetary equilibrium.

generalized Nash bargaining, which is one of the most common schemes in the search literature and price-taking (or Walrasian pricing) which is first considered by Rocheteau and Wright (2005). Apart from the mechanics, an important difference between these two schemes is the absence of the holdup problems in the price-taking version which are present in the bargaining version.

2.1.2.1 Bargaining in the Decentralized Market We drop the time subscripts since everything is period t. Our bargaining problem is

$$\max_{q,d}\left[\chi u(q)-\frac{Ad}{PW}\right]^{\theta}\left[\frac{Ad}{PW}-c(q,k_s,Z)\right]^{1-\theta}\quad \text{s.t. } d\leq m^b.$$

where θ is the bargaining power of the buyer, the first term is the buyer's surplus and the second term is the seller's surplus.

Using the insights of LW and AWW, in any monetary equilibrium $d=m^b$, that is the buyer spends all his money in exchange for some q that the seller produces using his capital and effort. Inserting $d=m^b$ and taking the FOC with respect to q, we get

$$\frac{m^b}{P}=\frac{g(q,k^s,\chi,Z)W}{A} \tag{18}$$

where

$$g(q,k,\chi,Z)\equiv\frac{\theta c(q,k,Z)\chi u'(q)+(1-\theta)\chi u(q)c_q(q,k,Z)}{\theta\chi u'(q)+(1-\theta)c_q(q,k,Z)}. \tag{19}$$

and the quantity produced will be $q=q(m^b,k^s,\chi,Z)$, where $q(\cdot)$ is given by solving (18) for q as a function of (m^b,k^s,χ,Z). Turning to the partial derivatives we need, we get

$$\frac{\partial d}{\partial m^b}=1,\quad \frac{\partial q}{\partial m^b}=\frac{A}{PW g_q(q,k,\chi,Z)}>0,\quad \text{and}\quad \frac{\partial q}{\partial k^s}=-\frac{g_k(q,k,\chi,Z)}{g_q(q,k,\chi,Z)}>0$$

while the other derivatives in (14) and (15) are 0.

Now reintroducing the time subscripts and inserting these results, (14) and (15) reduce to

$$V_{t,m}(m_t,k_t,i_{t-1},b_t,S_t) = \frac{(1-\sigma)A}{P_tW_t}+\frac{\sigma A\chi_t u'(q_t)}{P_tW_t g_q(q_t,k_t,\chi_t,Z_t)} \tag{20}$$

$$V_{t,k}(m_t,k_t,i_{t-1},b_t,S_t) = \frac{AR_t^k}{W_t}+(1-\delta)\Upsilon_t-\sigma\gamma(q_t,k_t,\chi_t,Z_t) \tag{21}$$

where

$$\gamma(q,k,\chi,Z)\equiv c_k+c_q\frac{\partial q}{\partial k}=\frac{c_k(q,k,Z)g_q(q,k,\chi,Z)-c_q\,(q,k,Z)\,g_k(q,k,\chi,Z)}{g_q(q,k,\chi,Z)}<0. \tag{22}$$

is the marginal return of having capital in the DM when the household is a seller. In particular, having more capital will reduce the seller's cost for a given quantity produced, which is captured by the c_k term. However, due to the non-competitive nature of DM, having more capital for the seller will also affect the terms of trade by increasing the output produced and this will increase his cost. This second term will be the source of one of the holdup problems we will discuss.

2.1.2.2 Price-Taking in the Decentralized Market With price taking, the DM value function has the same form as (10), but now

$$V_t^b(m_t, k_t, i_{t-1}, b_t, S_t) = \max_q \{\chi_t u(q) + W_t(m_t - \tilde{p}q, k_t, i_{t-1}, b_t, S_t)\} \text{ s.t. } \tilde{p}q \leq m \quad (23)$$

$$V_t^s(m_t, k_t, i_{t-1}, b_t, S_t) = \max_q \{-c(q_t^s, k_t, Z_t) + W_t(m_t + \tilde{p}q, k_t, i_{t-1}, b_t, S_t)\} \quad (24)$$

where $\tilde{p}$ is the DM price level taken as given by the household. Market clearing will guarantee that buyers and sellers choose the same q and buyers will choose to spend all of their money so that $q = m^b/\tilde{p}$ will hold. The FOC from (24) is

$$c_q(q, k^s, Z) = \tilde{p}W_m \quad (25)$$

Inserting $\tilde{p} = m^b/q$ and using (9) , we get the analog to (18) from the bargaining model

$$\frac{m^b}{P} = \frac{qc_q(q, k^s, Z)w}{A} \quad (26)$$

and the quantity produced will be $q = q(m^b, k^s, Z)$ using (26). The partial derivatives we need are

$$\frac{\partial d}{\partial m^b} = 1, \quad \frac{\partial q}{\partial m^b} = \frac{1}{\tilde{p}} = \frac{A}{PWc_q(q,k,Z)} > 0, \quad \frac{\partial q}{\partial k^s} = -\frac{c_{qk}(q,k,\chi,Z)}{c_{qq}(q,k,\chi,Z)} > 0 \quad \text{and} \quad \frac{\partial d}{\partial k^s} = \tilde{p}\frac{\partial q}{\partial k^s}$$

while the other derivatives in (14) and (15) are 0.

Finally, introducing time subscripts and using these results we get the envelope conditions

$$V_{t,m}(m_t, k_t, i_{t-1}, b_t, S_t) = \frac{(1-\sigma)A}{P_t W_t} + \frac{\sigma A \chi_t u'(q_t)}{P_t W_t q_t c_q(q_t, k_t, Z_t)} \quad (27)$$

$$V_{t,k}(m_t, k_t, i_{t-1}, b_t, S_t) = \frac{AR_t^k}{W_t} + (1-\delta)\Upsilon_t - \sigma c_k(q_t, k_t, Z_t) \quad (28)$$

It is useful to note that in (28), c_k is now the marginal return of having capital in the DM when the household is a seller and the extra terms in (22) do not appear due to the competitive nature of pricing. This will be key in understanding the (lack of) holdup problems with price-taking.

2.1.3 Household's Optimality Conditions

We obtain the optimality conditions for the household under bargaining by simply substituting (16), (17), (20) and (21) in to the household's FOC to get the optimality conditions for the household. We also define $\mu_t \equiv \Upsilon_t / U'(x_t)$. Formally, taking as given $\{P_t, R_t, W_t, R_t^k, \Pi_t, T_t\}_{t=0}^{\infty}$ and exogenous aggregate states

$\{Z_t, \chi_t\}_{t=0}^{\infty}$, the household solves for $\{q_t, x_t, m_{t+1}, k_{t+1}, i_t, b_{t+1}, \mu_t\}_{t=0}^{\infty}$ using the following equations:

$$W_t = \frac{A}{U'(x_t)} \tag{29}$$

$$1 = \beta E_t \left[\frac{U'(x_{t+1})}{U'(x_t)} \frac{R_t}{\pi_{t+1}} \right] \tag{30}$$

$$1 = \mu_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) + \frac{i_t}{i_{t-1}} S'\left(\frac{i_t}{i_{t-1}}\right) \right] + \beta E_t \left\{ \mu_{t+1} \frac{U'(x_{t+1})}{U'(x_t)} \left(\frac{i_{t+1}}{i_t}\right)^2 S'\left(\frac{i_{t+1}}{i_t}\right) \right\} \tag{31}$$

$$k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) \right] i_t \tag{32}$$

$$\mu_t = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \left[R^k_{t+1} + (1-\delta)\mu_{t+1} \right] - \frac{\sigma}{U'(x_t)} \gamma(q_{t+1}, k_{t+1}, \chi_{t+1}, Z_{t+1}) \right\} \tag{33}$$

$$\frac{m_t}{P_t} = \frac{g(q_t, k_t, \chi_t, Z_t) W_t}{A} \tag{34}$$

$$1 = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)\pi_{t+1}} \left[\frac{\sigma \chi_{t+1} u'(q_{t+1})}{g_q(q_{t+1}, k_{t+1}, \chi_{t+1}, Z_{t+1})} + (1-\sigma) \right] \right\} \tag{35}$$

where we used $\pi_{t+1} \equiv P_{t+1}/P_t$. Equations (29) to (32) resemble the optimality conditions that arise in a standard DSGE model with capital. (29) is a labor supply equation that relates the wage to the marginal rate of substitution between consumption and labor, (30) is the Euler equation for Bond holdings. (31) describes the evolution of the shadow price of installed capital, μ_t, and (32) is the capital accumulation equation. Equations (33), (34) and (35) reflect the presence of the decentralized market. (33) is the Euler equation for capital stock holdings. The return to capital has two components, namely the return from renting capital to intermediate good producing firms in the centralized market, R^k_t, net of capital depreciation, and the return to capital when producing in the decentralized market which we discussed above. (34) defines the output produced in the DM. Finally, (35) is the Euler equation for holding money where the term in square brackets reflects the additional consumption provided in the DM by holding money. Note that combining (29), (34) and (35) we obtain the following equation that define money demand in this environment.

$$\frac{m_{t+1}}{P_t} = \frac{\beta}{U'(x_t)} E_t \left\{ g(q_{t+1}, k_{t+1}, \chi_{t+1}, Z_{t+1}) \left[\frac{\sigma \chi_{t+1} u'(q_{t+1})}{g_q(q_{t+1}, k_{t+1}, \chi_{t+1}, Z_{t+1})} + (1-\sigma) \right] \right\} \tag{36}$$

Turning to the price-taking version, we need to replace (33), (34), (35) and (36) by

$$\mu_t = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \left[R^k_{t+1} + (1-\delta)\mu_{t+1} \right] - \frac{\sigma}{U'(x_t)} c_k(q_{t+1}, k_{t+1}, Z_{t+1}) \right\} \tag{37}$$

$$\frac{m_t}{P_t} = \frac{q_t c(q_t, k_t, Z_t) W_t}{A} \tag{38}$$

$$1 = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)\pi_{t+1}} \left[\frac{\sigma \chi_{t+1} u'(q_{t+1})}{q_{t+1} c_q(q_{t+1}, k_{t+1}, Z_{t+1})} + (1-\sigma) \right] \right\} \tag{39}$$

$$\frac{m_{t+1}}{P_t} = \frac{\beta}{U'(x_t)} E_t \left\{ q_{t+1} c(q_{t+1}, k_{t+1}, Z_{t+1}) \left[\frac{\sigma \chi_{t+1} u'(q_{t+1})}{c_q(q_{t+1}, k_{t+1},, Z_{t+1})} + (1-\sigma) \right] \right\} \tag{40}$$

The set of equations above determines the path of money balances, given m_0 which is identical across all households assuming an interior solution. As all households start period t with the same money balances,

$m_t = M_t$ where M_t is the aggregate money stock, the buyers in the DM enter the CM with $\hat{m} = 0$, the sellers with $\hat{m} = 2M$ while the remaining $1 - 2\sigma$ households carry $\hat{m} = M$. Looking at (2), this means that individual labor supply depends on the status of the agent in the previous DM as the money holdings . In particular, we have

$$h_t = \begin{cases} H_t + \dfrac{(M_t - 0)}{P_t W_t} & \text{buyers} \\ H_t + \dfrac{(M_t - 2M_t)}{P_t W_t} & \text{sellers} \\ H_t & \text{others} \end{cases} \tag{41}$$

where H_t is aggregate hours which we define below. This shows buyers in the DM work more than others since they have to make up for the money they have spent and sellers work less than others. We only care about total hours H_t in equilibrium and will not track individual h_t.

2.2 Firms in the Centralized Market

The setup of the centralized market resembles that of a New Keynesian DSGE model. Production is carried out by two types of firms in the CM: final good producers combine differentiated intermediate goods. Intermediate goods producing firms hire labor and capital services from the households to produce the inputs for the final good producers. To introduce nominal rigidity we follow Calvo (1983) by assuming that only a constant fraction of the intermediate goods producers is able to re-optimize prices.

2.2.1 Final Good Producers

The final good Y_t in the CM is a composite made of a continuum of intermediate goods $Y_t(i)$:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda}} di \right]^{1+\lambda}. \tag{42}$$

Note that the elasticity is $(1 + \lambda)/\lambda$. $\lambda = 0$ corresponds to the linear case and $\lambda \to \infty$ corresponds to the Cobb-Douglas case. We will constrain $\lambda \in (0, \infty)$. The final good producers buy the intermediate goods on the market, package them into Y_t units of the composite good, and resell them to consumers. These firms maximize profits in a perfectly competitive environment. Their problem is:

$$\max_{Y_t, Y_t(i)} \; P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad \text{s.t.} \quad (42) \tag{43}$$

taking $P_t(i)$ as given. The first-order condition is:

$$P_t(i) = P_t Y_t^{\frac{\lambda}{1+\lambda}} Y_t(i)^{-\frac{\lambda}{1+\lambda}}. \tag{44}$$

Therefore,

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} Y_t. \tag{45}$$

Combining this condition with the zero profit condition one obtains an expression for the price of the composite good:

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda}} di\right]^{-\lambda}. \tag{46}$$

2.2.2 Intermediate Goods Producers

Intermediate goods producers, indexed by i, use the following technology:

$$Y_t(i) = \max\left\{Z_t K_t(i)^\alpha H_t(i)^{1-\alpha} - \mathcal{F}, 0\right\}. \tag{47}$$

Firm i's profit is given by:

$$\Pi_t(i) = P_t(i)Y_t(i) - P_t W_t H_t(i) - P_t R_t^k K_t(i). \tag{48}$$

All firms take factor prices W_t and R_t^k, as well as the prices of the other firms and the aggregate price level as given. We distinguish two types of firms: (i) firms are allowed to re-optimize their price $P_t(i)$ and (ii) firms that are not able to re-optimize their price. Firms that are not allowed to choose $P_t(i)$ optimally, satisfy the demand for their differentiated good (45) and choose capital and labor inputs to minimize costs. Firms that are able to change their price in an optimal fashion maximize future expected profits. The profit maximization problem can be solved in two steps. First, given a desired level of output $Y_t(i)$ we determine the cost-minimizing choice of factor inputs. Second, we determine the profit maximizing price $P_t(i)$ and quantity $Y_t(i)$ that satisfies (45).

Cost minimization subject to (47) yields the conditions:

$$P_t W_t = \mu_t(i) P_t(i)(1-\alpha) Z_t K_t(i)^\alpha H_t(i)^{-\alpha} \tag{49}$$

$$P_t R_t^k = \mu_t(i) P_t(i) \alpha Z_t K_t(i)^{\alpha-1} H_t(i)^{1-\alpha}, \tag{50}$$

where $\mu_t(i)$ is the Lagrange multiplier associated with (47). In turn, these conditions imply:

$$K_t(i) = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k} H_t(i).$$

If we integrate both sides of the equation with respect to di and define $K_t = \int K_t(i)di$ and $H_t = \int H_t(i)di$ we obtain a relationship between aggregate labor and capital:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k} H_t. \tag{51}$$

Thus, the aggregate capital labor ratio is a linear function of the ratio of factor prices.

Total variable cost (VC_t) is given by

$$VC_t(i) = \left(W_t + R_t^k \frac{K_t(i)}{H_t(i)}\right) H_t(i) = \left(W_t + R_t^k \frac{K_t(i)}{H_t(i)}\right) Z_t^{-1} \left(\frac{K_t(i)}{H_t(i)}\right)^{-\alpha} Y_t^v(i),$$

where $Y^v_t(i) = Z_t K_t(i)^\alpha H_t(i)^{1-\alpha}$ is the "variable" part of output $Y_t(i)$. The real marginal cost MC_t is the same for all firms and equal to:

$$\begin{aligned} MC_t &= \left(W_t + R^k_t \frac{K_t(i)}{H_t(i)}\right) Z_t^{-1} \left(\frac{K_t(i)}{H_t(i)}\right)^{-\alpha} \\ &= \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} W_t^{1-\alpha} (R^k_t)^\alpha Z_t^{-1}. \end{aligned} \tag{52}$$

Conditional on the optimal choice of factor inputs, nominal profits as a function of output $Y_t(i)$ and prices $P_t(i)$ can then be expressed as

$$\Pi_t(i) = \left[P_t(i) - P_t MC_t\right] Y_t(i) - P_t MC_t \mathcal{F}. \tag{53}$$

Since the last part of this expression does not depend on the firm's decision, it can be safely ignored subsequently.

We assume that prices are sticky as in Calvo (1983). Specifically, each firm can re-adjust prices with probability $1-\zeta$ in each period. We depart from the Calvo setup in assuming that for those firms that cannot adjust prices, $P_t(i)$ will increase at the geometric weighted average of the fixed rate π_{**} and of last period's inflation π_{t-1} with weights $1-\iota$ and ι, respectively. We define the price adjustment factor

$$\pi^{adj}_{t+s|t} = \prod_{l=1}^{s} \pi^{\iota}_{t+l-1} \pi^{1-\iota}_{**}$$

and adopt the convention that $\pi^{adj}_{t|t} = 1$. Firms that are unable to re-optimize their prices simply satisfy the demand for their product according to (45). For those firms that are allowed to re-optimize prices, the problem is to choose a price level $P^o_t(i)$ that maximizes the expected present discounted value of profits in all states of nature where the firm is stuck with that price in the future:

$$\begin{aligned} \max_{P^o_t(i)} \quad & I\!\!E_t \left[\sum_{s=0}^{\infty} \zeta^s \beta^s \Xi^p_{t+s|t} \left[P^o_t(i)\pi^{adj}_{t+s|t} - P_{t+s} MC_{t+s}\right] Y_{t+s}(i)\right] \\ \text{s.t.} \quad & Y_{t+s}(i) = \left[\frac{P^o_t(i)\pi^{adj}_{t+s|t}}{P_{t+s}}\right]^{-\frac{1+\lambda}{\lambda}} Y_{t+s}, \end{aligned} \tag{54}$$

where $\beta^s \Xi^p_{t+s|t}$ is the time t value of a dollar in period $t+s$ for the consumers. We assume that markets are complete so that $\beta^s \Xi^p_{t+s|t}$ is the same for all consumers. It is shown in the Appendix that the first-order conditions can be reduced to the following set of equations:

$$\mathcal{F}^{(1)}_t = (p^o_t)^{-\frac{1+\lambda}{\lambda}} Y_t + \zeta\beta \left(\pi^{\iota}_t \pi^{(1-\iota)}_{**}\right)^{-1/\lambda} I\!\!E_t \left[\left(\frac{p^o_t}{\pi_{t+1} p^o_{t+1}}\right)^{-\frac{1+\lambda}{\lambda}} \Xi^p_{t+1|t} \mathcal{F}^{(1)}_{t+1}\right] \tag{55}$$

$$\mathcal{F}^{(2)}_t = (p^o_t)^{-\frac{1+\lambda}{\lambda}-1} Y_t MC_t + \zeta\beta \left(\pi^{\iota}_t \pi^{(1-\iota)}_{**}\right)^{-\frac{1+\lambda}{\lambda}} I\!\!E_t \left[\left(\frac{p^o_t}{\pi_{t+1} p^o_{t+1}}\right)^{-\frac{1+\lambda}{\lambda}-1} \Xi^p_{t+1|t} \mathcal{F}^{(2)}_{t+1}\right] \tag{56}$$

$$\mathcal{F}^{(1)}_t = (1+\lambda)\mathcal{F}^{(2)}_t \tag{57}$$

Here we are considering only the symmetric equilibrium in which all firms that can readjust prices will choose the same $P_t^o(i)$. In the above formula, we dropped the i index and used the definitions $p_t^o = P_t^o/P_t$ and $\pi_t = P_t/P_{t-1}$. Equations (55) to (57) essentially determine the optimal price p_t^o as a function of marginal costs.

2.2.3 Aggregate Price Dynamics in the CM

From (46) it follows that:

$$P_t = \left[(1-\zeta)(P_t^o)^{-\frac{1}{\lambda}} + \zeta(\pi_{t-1}^{\iota}\pi_{**}^{1-\iota}P_{t-1})^{-\frac{1}{\lambda}}\right]^{-\lambda}. \tag{58}$$

Hence,

$$\pi_t = \left[(1-\zeta)\left(\pi_t p_t^o\right)^{-\frac{1}{\lambda}} + \zeta(\pi_{t-1}^{\iota}\pi_{**}^{1-\iota})^{-\frac{1}{\lambda}}\right]^{-\lambda}. \tag{59}$$

The system of equations (55) - (57) and (59) links inflation to real marginal costs and output and hence defines a so-called New Keynesian Phillips curve.

2.3 Goverment Spending and Fiscal Policy

In period t, the government in this model collects a nominal lump-sum tax T_t, spends G_t on goods from the centralized market, issues one-period nominal bonds B_{t+1} that pay R_t gross interest tomorrow and supplies the money to maintain the interest rate rule. It satisfies the following budget constraint every period

$$P_t G_t + R_{t-1}B_t + M_t = T_t + B_{t+1} + M_{t+1}. \tag{60}$$

We assume that government spending G_t evolves exogenously and will provide further details below.

2.4 Aggregate Resource Constraint and National Accounting

We begin by adding the households' CM budget constraints (remember that there are three types of households as they enter the CM depending on their status in the previous DM) and the government budget constraint to obtain

$$P_t X_t + P_t I_t + P_t G_t = P_t W_t H_t + P_t R_t^k K_t + \Pi_t. \tag{61}$$

Now consider firms' profits in the CM:

$$\begin{aligned}
\Pi_t &= \int P_t(i)Y_t(i)di - P_t W_t \int H_t(i)di - P_t R_t^k \int K_t(i)di \\
&= \int P_t(i)Y_t(i)di - P_t W_t H_t - P_t R_t^k K_t \\
&= P_t Y_t - P_t W_t H_t - P_t R_t^k K_t.
\end{aligned}$$

where the last equality follows from the zero profit conditions for the final goods producers. Combining the expression for profits with (61) we get

$$X_t + I_t + G_t = Y_t, \tag{62}$$

which is the resource constraint in the CM. Since there is no savings in the DM (and goods are perishable), there is a trivial resource constraint that sets consumption equal to output. The relationship between output and the aggregate labor and capital inputs in the CM is given by

$$\bar{Y}_t = Z_t \int K_t^\alpha(i) H_t^{1-\alpha}(i) di - \mathcal{F} = Z_t K_t^\alpha H_t^{(1-\alpha)} - \mathcal{F}.$$

where $\bar{Y}$ is the output of the intermediate good producers and the second equality follows from the fact that the optimal capital labor ratio $K_t(i)/H_t(i)$ only depends on relative factor prices which are common to all firms. The relationship between $\bar{Y}_t$ and Y_t is given by

$$\bar{Y}_t = Y_t \int \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} di. \tag{63}$$

using (45) or

$$Y_t D_t = Z_t K_t^\alpha L_t^{(1-\alpha)} - \mathcal{F}. \tag{64}$$

where

$$D_t \equiv \int \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} di \tag{65}$$

measures the extent of price dispersion across firms. Unless $P_t(i) = P_t$ for all firms, D_t will be greater than unity, which in turn implies the economy will produce inside its production possibilities frontier. We will refer to this as the price-dispersion distortion in our welfare analysis.

In order to understand the evolution of D_t, we need to determine the distribution of prices $P_t(i)$ in the CM. A fraction $1-\zeta$ of firms was allowed to re-optimize their prices in period t. For these firms $P_t(i) = P_t^o$. A fraction $\zeta(1-\zeta)$ of firms re-set their prices in period $t-1$. Hence, for these firms $P_t(i) = \pi_{t-1}^\iota \pi_{**}^{1-\iota} P_{t-1}^o$. Overall, we obtain

$$D_t = (1-\zeta) \sum_{j=0}^{\infty} \zeta^j \left(\frac{(\pi_{t-1}\pi_{t-2}\cdots\pi_{t-j})^\iota \pi_{**}^{j(1-\iota)}}{\pi_t \pi_{t-1} \cdots \pi_{t-j+1}} \frac{P_{t-j}^o}{P_{t-j}} \right)^{-\frac{1+\lambda}{\lambda}}. \tag{66}$$

We verify in the Appendix that D_t follows the law of motion:

$$D_t = \zeta \left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^\iota \left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)} \right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1-\zeta) \left[\frac{P_t^o}{P_t}\right]^{-\frac{1+\lambda}{\lambda}}. \tag{67}$$

To summarize, aggregate prices and quantities in the CM are given by P_t and Y_t. As is standard in New-Keynesian DSGE models, if we add an interest rate feedback rule to the system, we will only determine period to period changes in the aggregate price level, that is, inflation π_t, but not the level P_t.

Real and nominal output in the DM are given by σq_t and σM_t, respectively. Hence, we can define the price level in the DM as

$$P_t^{DM} = M_t/q_t. \tag{68}$$

Total nominal output in our model economy is given by

$$\mathcal{Y}_t^{(n)} = Y_t P_t + \sigma M_t. \tag{69}$$

Using the final good produced in the CM as numeraire, we can express real output as

$$\mathcal{Y}_t = Y_t + \sigma M_t/P_t = Y_t + \sigma \mathcal{M}_t/\pi_t, \tag{70}$$

where $\mathcal{M}_t = M_t/P_{t-1}$ are real money balances, in terms of the CM output.

To take the model to the data we will now construct a GDP deflator and a measure of real output that is consistent with this GDP deflator. Following NIPA conventions, we use a Fisher price index. However, to simplify the analysis we replace time-varying nominal shares by steady state shares. The DM share of nominal output in the steady state is

$$s_* = \frac{\sigma \mathcal{M}_*}{Y_* \pi_* + \sigma \mathcal{M}_*}. \tag{71}$$

Define $\pi_t^{DM} = P_t^{DM}/P_{t-1}^{DM}$ and let

$$\pi_t^{GDP} = \ln \frac{P_t^{GDP}}{P_{t-1}^{GDP}} = (1 - s_*) \ln \pi_t + s_* \ln \pi_t^{DM}. \tag{72}$$

Thus,

$$P_t^{GDP} = P_0^{GDP} \prod_{\tau=1}^{t} \pi_\tau^{1-s_*} (\pi_\tau^{DM})^{s_*}. \tag{73}$$

We now define real GDP as

$$\mathcal{Y}_t^{GDP} = \frac{\mathcal{Y}_t^{(n)}}{P_t^{GDP}} = \mathcal{Y}_t \frac{P_t}{P_t^{GDP}}. \tag{74}$$

It can be verified that up to a first-order approximation changes in real GDP evolve according to a Fisher quantity index with fixed (steady state) weights. Let X_* denote the steady state of a variable X_t and $\tilde{X}_t = \ln X_t/X_*$. Log-linearizing and differencing our expression for real output in terms of the CM good yields

$$\Delta \tilde{\mathcal{Y}}_t = (1 - s_*) \Delta \tilde{Y}_t + s_* [\Delta \tilde{M}_t - \Delta \tilde{\pi}_t].$$

Here Δ denotes the temporal difference operator. According to the definition of prices in the DM

$$\tilde{\pi}_t^{DM} = \Delta \tilde{M}_t - \Delta \tilde{q}_t.$$

Combining the two previous equations leads to:

$$\Delta \tilde{\mathcal{Y}}_t = (1 - s_*) \Delta \tilde{Y}_t + s_* [\Delta \tilde{q}_t + \tilde{\pi}_t^{DM} - \tilde{\pi}_t].$$

Thus,

$$\Delta\tilde{\mathcal{Y}}_t^{GDP} = \Delta\tilde{\mathcal{Y}}_t + \tilde{\pi}_t - (1-s_*)\tilde{\pi}_t - s_*\tilde{\pi}_t^{DM} = (1-s_*)\Delta\tilde{Y}_t + s_*\Delta\tilde{q}_t. \tag{75}$$

Hence, the level of GDP in period t is given by

$$\tilde{\mathcal{Y}}_t^{GDP} = (1-s_*)\tilde{Y}_t + s_*\tilde{q}_t + [\tilde{\mathcal{Y}}_0^{GDP} - (1-s_*)\tilde{Y}_0 - s_*\tilde{q}_0].$$

Under the normalizations $P_0^{GDP} = 1$ and $P_0 = 1$ we obtain

$$\tilde{\mathcal{Y}}_0^{GDP} = (1-s_*)\tilde{Y}_0 + s_*(\mathcal{M}_0 - \pi_0).$$

We can therefore further simplify our expression for GDP to

$$\tilde{\mathcal{Y}}_t^{GDP} = (1-s_*)\tilde{Y}_t + s_*\tilde{q}_t + s_*(\tilde{\mathcal{M}}_0 - \tilde{\pi}_0 - \tilde{q}_0). \tag{76}$$

2.5 Monetary Policy

Monetary policy is represented by an interest-rate feedback rule

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*}\right)^{\rho_R}\left[\left(\frac{\pi_t^{GDP}}{\pi_*}\right)^{\psi_1}\left(\frac{\mathcal{Y}_t}{\mathcal{Y}_*}\right)^{\psi_2}\right]^{1-\rho_R}\exp\left(\sigma_r\varepsilon_t^r\right), \tag{77}$$

where R_* is the gross steady state nominal interest rate, $\mathcal{Y}_*$ is the steady state of real GDP (in terms of the CM good), and π^* is the steady state (or target) inflation. It can be verified that steady state DM, CM, and GDP inflation are equal. Finally, ε_t^r is a monetary policy shock.

2.6 Aggregate Shocks

Government expenditures as a fraction of real GDP (both the DM and CM output, appropriately aggregated) $\mathcal{Y}_t$, denoted by g_t are assumed to be exogenous:

We consider four aggregate disturbances in our model economy. Z_t is the random productivity term that effects production in both markets and g_t is a shock that shifts government spending according to

$$G_t = (1 - 1/g_t)\,\mathcal{Y}_t. \tag{78}$$

We assume that although government consumption goods are purchased in the centralized market, the overall amount is a stochastic fraction of total GDP. The shock ε_t^r captures unanticipated deviations from the systematic part of the monetary policy rule. Finally, we have a money demand shock, χ_t, which we model as a taste shock in the DM. We define

$$\tilde{Z}_t = \ln\left(Z_t/Z_*\right), \quad \tilde{\chi}_t = \ln\left(\chi_t/\chi_*\right), \quad \tilde{g}_t = \ln\left(g_t/g_*\right),$$

where Z_*, χ_* and g_* are steady state values / means of the respective random variable. We assume that the exogenous disturbances evolve according to AR(1) processes:

$$\tilde{Z}_t = \rho_z \tilde{Z}_{t-1} + \sigma_z \varepsilon_t^z, \quad \tilde{\chi}_t = \rho_\chi \tilde{\chi}_{t-1} + \sigma_\chi \varepsilon_t^\chi, \quad \tilde{g}_t = \rho_g \tilde{g}_{t-1} + \sigma_g \varepsilon_t^g$$

and we collect all innovations in $\varepsilon = [\varepsilon_t^z, \varepsilon_t^\chi, \varepsilon_t^g, \varepsilon_t^r]$ which follows a multi-variate standard normal distribution.

The law of motion for the exogenous processes completes the specification of the search-based DSGE model. The equilibrium conditions are summarized in the Appendix. We derive the deterministic steady state for this model and use a log-linear approximation to its dynamics to form a state-space representation that is used for the Bayesian estimation.

3 A Money-in-the-Utility-Function Model

The specification of the MIU model closely resembles search-theoretic model described in the previous section. The production side of the MIU economy is identical to the production sector in the centralized market. Moreover, fiscal and monetary policy are identical as well and the economy is subject to the same set of stochastic shocks. We only discuss the modifications to the household's problem.

Since there is no decentralized market, households' consumption is restricted to x_t. The instantaneous utility function is of the form

$$\mathcal{U}_t = U(x_t) - Ah_t + \frac{\chi_t}{1-\nu_m}\left(\frac{m_t}{P_t}\frac{A}{Z_*^{1/1-\alpha}}\right)^{1-\nu_m}, \tag{79}$$

The third term on the right-hand-side of (79) captures the value of holding real money balances. The scaling by $A/Z_*^{1/(1-\alpha)}$ can be interpreted as a re-parameterization of χ_t, which has the effect that steady state velocity stays constant as we change A and Z. Here m_t are the (pre-determined) money holdings at the beginning of the period, and P_t is the price at which the final good is sold in period t. Using again $W_t(m_t, k_t, b_t, S_t)$ to denote the value function of the household in the centralized market, the household's problem is given by

$$\begin{aligned}
&W_t(m_t, k_t, i_{t-1}, b_t, S_t) \\
&\quad = \max_{x_t, h_t, m_{t+1}, k_{t+1}, i_t, b_{t+1}} \left\{ U(x_t) - Ah_t + \frac{\chi_t}{1-\nu_m}\left(\frac{m_t}{P_t}\frac{A}{Z_t^{1/1-\alpha}}\right)^{1-\nu_m} + \beta E W_{t+1}(m_{+1}, k_{+1}, i_t, b_{+1}, S_{+1}) \right\} \\
&\text{s.t.} \quad P_t x_t + P_t i_t + b_{t+1} + m_{t+1} \leq P_t W_t h_t + P_t R_t^k k_t + \Pi_t + R_{t-1} b_t + \hat{m}_t - T_t \qquad (80) \\
&\qquad\; k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] i_t \qquad (81)
\end{aligned}$$

To a large extent, the optimality conditions for the households resemble the ones derived for the centralized market in the search-based model. In fact, the labor supply function, the Euler equation for Bond holdings, the evolution of the shadow price of installed capital, and the capital accumulation equation are identical to Equations (29) to (32). The Euler equation for capital stock holdings is given by

$$\mu_t = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \left[R^k_{t+1} + (1-\delta)\mu_{t+1} \right] \right\}, \tag{82}$$

which is identical to (33) except that the term related to the DM does not appear. Similarly, the Euler equation for money in (34) is replaced by

$$\frac{U'(x_t)}{P_t} = \beta E_t \left[\frac{U'(x_{t+1})}{P_{t+1}} + \frac{\chi_{t+1}}{P_{t+1}} \left(\frac{A}{Z_{t+1}^{1/1-\alpha}} \right)^{1-\nu_m} \left(\frac{m_{t+1}}{P_{t+1}} \right)^{-\nu_m} \right] \tag{83}$$

which implies the money demand equation

$$\left(\frac{m_{t+1}}{P_t} \right)^{\nu_m} = \frac{\beta R_t}{U'(x_t)(R_t - 1)} E_t \left[\left(\frac{A}{Z_*^{1/1-\alpha}} \right)^{1-\nu_m} \frac{\chi_{t+1}}{\pi_{t+1}^{1-\nu_m}} \right]. \tag{84}$$

Equations (82) and (84) replace the optimality conditions (33) and (35) in the search-based model. Notice that m_{t+1} has been chosen in period t based on the realization of time t shocks. Hence, we detrend it by P_t and define $\mathcal{M}_{t+1} = m_{t+1}/P_t$ with the understanding that $\mathcal{M}_{t+1}$ only depends on realizations of shocks dated t and earlier. Since $\mathcal{M}_{t+1}$ does neither enter in the firms' problems nor is it included in the interest-rate feedback rule of the central bank, the model has a block-diagonal structure: the determination of output, inflation, and interest rates does not depend on the money stock. Since our MIU model is a one-sector model without a decentralized market, aggregate output and prices are simply $\mathcal{Y}_t = Y_t$ and $\mathcal{P}_t = P_t$.

The equilibrium conditions for the MIU model are summarized in the Appendix. As we did for the search-based model, we derive the deterministic steady state for the MIU model and use a log-linear approximation to its dynamics to form a state-space representation that is used for the Bayesian estimation.

4 Empirical Analysis

We now turn to the estimation of the search-based and the MIU model. We use a Bayesian approach discussed in detail in An and Schorfheide (2007). We begin by describing the data set (Section 4.1). We then proceed by specifying the functional forms we use (Section 4.2) and the specification of the prior distributions used for the parameters of the two DSGE models (Section 4.3). Next, we present the parameter estimates as well as implied steady states, (Section 4.4) and discuss dynamics via variance decompositions, and impulse responses (Section 4.5). Finally we discuss the implications of both models for money demand in section Section 4.6.

4.1 Data

Our empirical analysis focuses on quarterly U.S. postwar data on aggregate output, inflation, interest rates, and (inverse) velocity of money.[6] Unless otherwise noted, the data are obtained from the FRED2 database maintained by the Federal Reserve Bank of St. Louis. Our measure of per capita output is defined as real GDP (GDPC96) divided by civilian noninstitutional population (CNP16OV). The population series is provided at a monthly frequency and converted to quarterly frequency by simple averaging. Since the quarterly flow statistics reported in the National Income and Product Accounts are annualized, we divide real GDP by 4. The models presented in Sections 2 and 3 are specified to capture stationary fluctuations around a deterministic steady state. Hence, we take the natural log of per capita output and extract a deterministic trend by an OLS regression over the sample period 1959:I to 2006:IV. We then scale the deviations from the linear trend by 100 to convert them into percentages and relate them to log deviations from the steady state in the models. Inflation is defined as the log difference of the GDP deflator (GDPDEF) and multiplied by 400 to obtain annualized percentages. Our measure of nominal interest rates corresponds to the Federal Funds Rate (FEDFUNDS). The Fed Funds Rate is also provided at monthly frequency and we use simple averaging to convert it to quarterly frequency.

We incorporate money as an observable by using inverse velocity. As a measure of money we use the sweep-adjusted M2S series provided by Cynamon, Dutkowsky and Jones (2006). This series is provided at monthly frequency without seasonal adjustment. We apply the EVIEWS default version of the X12 filter to perform the seasonal adjustment and then use the observation for the last month of every quarter. We divide the M2S series by quarterly nominal output to obtain inverse velocity. We take natural log of inverse velocity, scale it by 100 to relate to the log deviations from $100 * \ln(\mathcal{M}^*/\mathcal{Y}^*)$ in the models. We plot M1 and M2 inverse velocity (not scaled by 100, measured per quarter) from 1960 to 2005 in Figure 1. Notice that M1 is falling between 1960 and 1980 and increasing afterwards. The trends in M1 are presumably due to factors such as payment technologies, which do not appear in our theoretical models. Trends in M2 inverse velocity, on the other hand, are less pronounced. While we believe that the ideal measure of money supply for our search-based models is M1,[7] we opt for M2 velocity for our empirical analysis because, at least after 1984, the series seems roughly consistent with a model that implies that velocity evolves like a stationary, albeit fairly persistent, process around a constant mean. For the subsequent estimation we restrict the sample period to 1984:I to 2005:IV.

[6] Instead of using real money balances directly, we use inverse velocity, i.e. nominal output divided by nominal money balances as one of our observables.

[7] While not explicitly present in our model, checking accounts could be introduced.

4.2 Functional Forms

We use the following functional forms in our estimation:

$$u(q) = \ln(q+\kappa) - \ln(\kappa), \quad U(x) = B\ln(x_t), \quad f(e,k) = e^{1-\alpha}k^{\alpha}$$

where $\kappa > 0$ is a small constant to make sure $q_t = 0$ can be handled[8] and B determines the relative weight of the utility from consuming the CM and DM goods. We use a natural logarithm for both utility functions and use the same Cobb-Douglas production function as the function used by the intermediate good producers in the CM as these are necessary conditions for balanced-growth in this model. The functional form assumption for $f(e,k)$ implies that

$$c(q,k,Z) = \frac{1}{Z^{1/(1-\alpha)}} q^{1/(1-\alpha)} k^{-\alpha/(1-\alpha)}.$$

4.3 Restricted Parameters and Prior Distributions

A goal of our empirical analysis is to compare the propagation of shocks and the steady state welfare implications of the search-based DSGE model and the MIU model. Hence, it is desirable to restrict a subset of the model parameters prior to estimation. These restrictions, which apply to both models, are reported in Table 1. We fix π_* at the average inflation rate in our sample. Moreover, we let r_A be equal to the difference of the average Federal Funds Rate and the average inflation rate between 1984 and 2005 and let $\beta = 1/(1 + r_A/400)$. Using these parameter values for both DSGE models implies that the steady state inflation and nominal interest rates are equal to the post-1983 sample averages. We fix the depreciation rate δ at 0.014. This value is obtained as the average ratio of fixed asset depreciation and the stock of fixed assets between 1959 and 2005.[9] We set $g_* = 1.2$, which is computed from the average ratio of government consumption plus investment and GDP. The deviations of output from a linear trend in our post-1983 sample are highly persistent. To capture this persistence we let $\rho_z = 0.98$, a value that is consistent with the stationarity assumption embodied in our theoretical models as well as the observed persistence in the data.

We also impose that the estimated models agree on the conduct of post-1983 monetary policy. To this end, we conduct a preliminary estimation of the MIU model with capital without the money series. The mechanics of this estimation are identical to those of the subsequent estimation of the full model. This preliminary analysis yields $\hat{\psi}_1 = 1.82$, $\hat{\psi}_2 = 0.18$, and $\hat{\rho}_R = 0.78$. We fix the policy rule coefficients at these

[8] We use $\kappa = 0.0001$ in our empirical implementation.

[9] We use NIPA-FAT11 (current cost net stock) and NIPA-FAT13 (current cost depreciation) for fixed assets and consumer durables).

estimates for the subsequent analysis with both models.[10] Moreover, we let $\mathcal{F} = 0$ (no fixed costs) and $\pi_{**} = 1$, meaning that there is no static indexation for the firms that cannot change their prices.

In order to have a fair comparison between the two models, especially in terms of welfare, we normalize three parameters – two arbitrarily at unity and one at the value given by the data. We normalize steady state real GDP in both models to be unity and we normalize the mean of the money demand shocks in both models to be unity. Average inverse labor productivity in our sample is 0.03 (a worker produces about \$33 of real GDP in one hour) and we use this value in both our models for H_*/Y_*. Given the two restrictions that we impose on labor productivity, real GDP and the fact that we use inverse velocity in our estimation means we endogenously determine the values of our model parameters A, B and Z_*.

The marginal prior distributions for the remaining parameters of the two models are summarized in Table 2. The priors for parameters common to both models are identical except for λ as we explain below. The prior for inverse velocity is based on pre-sample information. Similarly, the prior for α is chosen so that the implied prior for the labor share is consistent with pre-sample evidence. We use a uniform prior on the indexation parameter ι and our prior for ζ is broadly consistent with micro-evidence on the frequency of price changes. The prior distributions for ρ_g and ρ_χ reflect the belief that the government spending (demand) disturbance and the money demand shock are fairly persistent. The remaining priors were loosely chosen such that the implied distribution of the variability of the endogenous variables is broadly in line with the pre-sample variability of the observed series. We assume that all the parameters listed in Table 2 are *a priori* independent. Since we fix the policy rule parameters at values that are far away from the boundary of the determinacy region, no further adjustment of the prior is needed.

There are two additional parameters in the search-based model, probability of single-coincidence meeting σ and in the bargaining version, the bargaining power of the buyer θ.[11] While it is difficult to fully disentangle the effect of these parameters, it is instructive to rewrite the steady state share of the decentralized market as follows:

$$s_* = \sigma \frac{1}{\pi_*} \frac{\mathcal{M}_*}{\mathcal{Y}_*}.$$

Since σ relates velocity to the share of the decentralized market, a prior for σ is linked to prior beliefs about the DM share. The parameter θ affects the bargaining power of the seller in the decentralized market and hence the markup and we use a uniform prior over the full range $(0, 1]$.

In addition to the normalizations we explained above, we want the two models to be similar in terms of two more steady state implications: the investment-output ratio (or equivalently capital-output ratio)

[10] Ex ante, we do not want the parameters for the policy rule to be different across the two models as we want to be able to compare them, holding monetary policy fixed. It turns out when we free policy rule parameters to be estimated in both models, the parameter estimates we get are very similar.

[11] Since according to our model $\sigma \in [0, 1/2]$ we introduce the transformed parameter $\tilde{\sigma}$, which lives on the unit interval.

and the average economy-wide markup. To achieve this we follow the approach proposed in Del Negro and Schorfheide (2008). Specifically, we combine the marginal prior distributions reported in Table 2 and denoted as $\tilde{p}(\vartheta)$ with a function $f(\vartheta)$ that reflects beliefs about the investment output ratio and the economy-wide markup. Here the vector ϑ stacks the parameters of the DSGE model. The overall prior is given by

$$p(\vartheta) \propto \tilde{p}(\vartheta) \exp\left\{-\frac{1}{2}\left[\frac{(I_*(\vartheta)/\mathcal{Y}_*(\vartheta) - 0.16)^2}{0.005^2} - \frac{(mu(\vartheta) - 0.15)^2}{0.01^2}\right]\right\} \mathcal{I}\{\vartheta \in \Theta_D\}, \tag{85}$$

where $\propto$ signifies proportionality, I_* and $\mathcal{Y}_*$ denote the steady states of investment and output (as a function of ϑ), $mu(\vartheta)$ is the economy-wide mark-up, and $\mathcal{I}\{\vartheta \in \Theta_D\}$ is an indicator function that is one if ϑ falls in the region of the parameter space in which the linearized search-based model has a unique stable rational expectations solution.[12] The adjustment function down-weights parameter combinations for which the investment output ratio deviates from 0.16 and the economy-wide mark-up deviates from 15%. We use $p(\vartheta)$ as the prior for both models.[13]

Turning to why we use different priors for λ across the two models, note that our goal is to use priors in both models that imply that the economy-wide mark-up is 15%. We use the function $f(\cdot)$ to induce this prior. In the MIU model the economy-wide mark-up equals λ. Hence, the $f(\cdot)$ function in conjunction with a prior density for λ that is equal to one on the unit interval implements our *a priori* belief. In the search-based model the economy-wide markup is a weighted average of the mark-ups in the centralized and decentralized market. This weighted average enters the $f(\cdot)$ function. To incorporate the belief that the mark-up in the centralized market is not too different from 15%, we combine $f(\cdot)$ with a Gamma density function for λ with mean 15% and standard deviation of 0.5%.

4.4 Parameter Estimates and the Implied Steady States

We report prior and posterior means and 90% credible intervals for the parameters of the three models in Tables 3, 4 and 5. The posterior is obtained by combining the prior distribution described in the previous subsection with the likelihood function derived from the state-space representations of the linearized DSGE models. We then use a random-walk Metropolis algorithm to generate draws from the posterior distribution of the parameters. To make inference about steady states, impulse responses, and variance decompositions, we convert the parameter draws into the statistics of interest. Further technical details are described in An and Schorfheide (2007).

The parameters ζ and ι determine the shape of the Phillips curve. From the search-based model we obtain a posterior mean estimate of the Calvo parameter, $\hat{\zeta} = 0.695$ for the bargaining version and $\hat{\zeta} = 0.801$

[12]For the MIU model, $mu(\vartheta)$ is simply equal to λ while in the search-based model it is a weighted average between λ and the markup in the DM.

[13]Draws from this prior can be generated using the random-walk Metropolis algorithm described in An and Schorfheide (2007). The normalization constant can be computed using Geweke's (1999) harmonic mean estimator.

for the price-taking version. In both versions there is some dynamic indexation, $\hat{\iota} = 0.268$ in the bargaining version and $\hat{\iota} = 0.117$ in the price-taking version. Because of the nonzero markup in the DM in the bargaining version, the CM markup needs to be smaller than 0.15 at $\hat{\lambda} = 0.094$ in order for the economy-wide markup to be close to our target of 0.15 while since there is no markup in the DM for the price-taking version, λ is substantially higher at $\hat{\lambda} = 0.238$. Finally, the two parameters specific to the search-based model are in line with our priors and independent calibration results in AWW.[14]

Comparing some of the common parameters of interest across the MIU and search-based models, we see that price rigidities are strongest in the price-taking version, followed by the MIU model and the bargaining version but all of them are in a reasonable range of estimates from past studies.[15] The implied average duration of price stickiness are 3.3 quarters, 5 quarters and 4.1 quarters for the search-based model with bargaining and price-taking and the MIU model, respectively. Moreover, the dynamic indexation in the MIU model is considerably lower than the search-based models at $\hat{\iota} = 0.05$. In all models, we obtain a similar estimate of the adjustment cost parameter which is quite large, reducing the volatility of the return to capital and dampening its effect on the marginal costs that enter the Phillips curve relationship. $\hat{\alpha}$ is estimated to be significantly larger in the bargaining version of the search-based model due to the lower value of investment-output ratio that results in this model at the more conventional $\alpha = 0.28$ due to the holdup problems related to investment. In order to reduce the penalty coming from our target of investment-output ratio, α needs to be higher.

The implied posterior distribution of the steady states is reported in Table 6. As we explained in Section 4.3, one of our goals in the estimation of our models was to have them display similar long-run characteristics, which are in line with the U.S. post-war experience. The first panel of the table reports the marginal distributions of important steady-state variables for the two models. All of these distributions are centered very close to our targets and they are fairly tight. This will enable us to make welfare analysis using these models which will be comparable across models. The second panel of the table reports the distributions of some other endogenous objects of interest. We see that about a third of economic activity takes place in the DM and the average markup in the DM for the bargaining version is about 0.27, roughly three times the estimated markup in the CM.

Finally, we turn to the issue of overall time series fit. As we report in Tables 3, 4 and 5, the marginal log-likelihood values for our models are -441.1 for the MIU model, -501.3 for the search-based model with price-taking, and -503.0 for the search-based model with bargaining. Thus, someone placing equal prior probabilities on the models, would conclude that the log odds of the MIU model versus the search-based

[14]In AWW, θ was calibrated to be around 0.75. This somewhat lower value, which is needed for a higher markup in the DM can be attributed to the lower share of the DM in AWW due to using M1 as the measure of money and the competitive pricing in the CM which puts all the burden of matching the aggregate markup to the DM.

[15]For example, Christiano, Eichenbaum and Evans (2005) use $\zeta = 0.6$.

model with bargaining are exp[61.9].[16] This result is not surprising in light of Ireland (2004)'s finding that the data overwhelmingly rejects non-separabilities in the MIU model when money is included as an observable, as the search-based model is essentially a non-separable model. Our MIU has a block-recursive structure, which insulates output, inflation, and interest rates from money demand shocks. Thus, as long as the correlation between real money balances and the remaining variables is fairly weak in the data, the separable utility function of the MIU model allows the money demand shock to capture the fluctuations in real money balances without compromising the fit for any other variable. A comparison of in-sample root-mean-squared-errors suggests that the MIU model is more successful in tracking velocity than the search-based model. The goodness of the in-sample predictions of the other three series is very similar across MIU and search-based models.

We will revisit this issue in the next section.

4.5 Dynamics

We now turn to exploring the dynamics of the two models. The unconditional variance decompositions for output, inflation, Federal Funds Rate and real money balances associated with the two estimated models are reported in Table 7. Most of the fluctuations of output, inflation, and the Federal Funds Rate are driven by the highly persistent technology shock. Neither in the MIU model nor in the search-based model, monetary policy shocks play an important role for the overall variability of output, inflation, and the Federal Funds Rate. Due to the block-triangular structure of the MIU model, money demand shocks have no effect on output, inflation, and the interest rate. However, money demand shocks in the MIU model explain almost 80% of the variance of the monetary aggregate. We interpret this as a disconnect between fluctuations in money and the shocks that drive output and inflation.

The estimated search-based model produces a tighter link between technology shocks and monetary aggregates: about 70% of the variance of real money balances is due to technology shocks in both versions. Money demand shocks, modelled as shocks to the taste for goods produced in the decentralized market, play a much smaller role for the fluctuations of real money balances. They explain only about 23% of the variation.

Due to the highly persistent technology shock much of the unconditional variance of the endogenous variables is generated at low frequencies and therefore the technology shock dominates the variance decompositions. If we restrict our attention to business-cycle frequencies the technology shock is somewhat less important. Variance decompositions at business cycle frequencies are reported in Table 8. We again see

[16]The overall fit of the bargaining and the price taking model is very similar, albeit the log odds favor the price taking model by a slight margin.

that the money demand shock explains over 70% of the fluctuations in real money balances while for the search-based model this number is about 50% and the technology shock still plays a more significant role in the latter. It is also noteworthy that even in the business-cycle frequencies, the money demand shock plays a somewhat important role (over 10%) in capturing fluctuations in output and inflation.

Revisiting the issue of fit from the previous section, it is clear from these two tables that the money demand shock has no impact on any variable other than real money balances where it explains about 75% of the fluctuations. Thus we conclude that its block-triangular structure allows the MIU model to capture the fluctuations in real money balances with the money demand shock at not cost in terms of loss of fit in the three other variables. This is not the case for the search-based model where the money demand shock effects all other variables and apparently fluctuations in the money demand shock which would enable the search-based model match the fluctuations in real money balances comes at a cost of reduced fit in other variables.

To emphasize the link between the money demand and technology shocks and some of the key variables in our model, we compute the impulse-response dynamics for the two models, which are depicted in Figure 2. Focusing first on an impulse to the technology shock, for both models we see an initial jump and sustained increase in real GDP resulting from increased investment in capital (now shown). Some of the increase in the search-based model is a result of increased DM production. An increase in the technology shock reduced current and future marginal costs, and since inflation can be represented as the expected sum of discounted future marginal costs, this in turn lowers inflation. The increase in output above steady state and reduction in inflation below the target inflation rate reduces the Fed Funds rate through the interest-rate feedback rule where apparently the effect of the latter dominates the effect of the former. As a result of these easing of monetary policy, real money demand increases, counteracting the effect of lower inflation. So far, qualitatively both models yield similar responses but because the technology shock effects the desirability of the DM good (by making it cheaper to produce) its demand increases and this gives an additional boost to real money demand since households use money to purchase the DM good. This explains the extra 0.4% increase in real money balances and the increased importance of the technology shock for explaining fluctuations in real money balances in the search-based model.

Turning to the money demand shock, in the MIU model clearly it has no effect on any of the variables except for real money balances. In the search-based model, on the other hand, a shock to money demand increase real GDP through increased consumption in the DM which also reduces inflation. The net effect of these changes on the Fed Funds rate is a decrease and this increases money demand. Money demand is almost twice as responsive on impact in the search-based model compared to the MIU model. Thus, we see that capturing fluctuations in real money through the money demand shock comes at a cost of creating fluctuations in other variables in the search-based model.

4.6 Money Demand

We now turn to investigating various aspects of money demand across the two models as it is the key difference between them. We begin by simulating the models conditional on the posterior mean parameter estimates and creating scatter plots of log inverse velocity and nominal interest rates, depicted in Figure 3. Lucas (2000) argues that such plots trace out money demand functions under the assumption that the income elasticity of money demand is equal to one. The slope of a hypothetical regression line fitted to the observations would correspond to the semi-elasticity of money balances to interest rates. In the context of our models there are two major caveats to such an interpretation: the money demand functions have a more complicated formand the interest rate feedback rule makes money supply endogenous. Nonetheless, one can ask the question whether velocity and interest rate data generated by the estimated model have similar features as the actual data.

In the two panels of Figure 3 we overlay actual data (red circles) and 500 model generated observations (green dots). For the search-based model (bargaining) the artificial observations inherit the negative correlation between inverse velocity and interest rates that is apparent in the U.S. data. The estimated MIU model, on the other hand, generates a slight positive correlation, which is at odds with the actual data.

The variation in the model generated data is due to four structural shocks: technology, government spending, money demand, and monetary policy. We proceed by simulating the DSGE models using one shock at a time. The resulting velocity-interest rate pairs are plotted in Figures 4 and 5. The lower-right panels of the two figures make clear that according to our estimates most of the variability in the interest rate is generated by the endogenous response to technology shocks. This is consistent with the variance decompositions reported in Table 7. The impulse responses in depicted Figure 2 show that in the search-based models real money balances increase by more than real GDP in response to a technology shock, while interest rates fall. This creates the negative correlation between inverse velocity and interest rates apparent from the scatter plot in Figure 4. To the contrary, in the MIU model real money balances increase by less than real output in response to a technology shock, which creates a positive correlation between inverse velocity and interest rate, as can be seen in Figure 5. The other three shocks create negative correlations of different degrees in both the search-based and the MIU model. Due to the block-triangular structure of the MIU model, the money demand shock has no effect on nominal interest rates.

So far, we have only explored contemporaneous correlations between inverse velocity and interest rates. In Figure 6 we consider correlations at different leads and lags. Moreover, we are taking uncertainty about the DSGE model parameters into account. Formally, the bands in the figure correspond to pointwise 90% intervals of the posterior predictive distribution. Draws from this distribution are obtained by repeatedly sampling DSGE model parameters from their posterior distribution, simulating sample paths conditional on these parameter draws, and computing the correlations of interest. The solid line in the figure represents correlations calculated from post 1983 U.S. data. Actual sample correlations that fall into the far tails of the posterior predictive distribution can be interpreted as evidence of model misspecification. The left-hand-side panels of Figure 6 are computed based on raw data, whereas the right-hand-side panels are based on HP-filtered observations that isolate variation at business-cycle frequencies.

As was apparent from Figure 3, the contemporaneous correlation between inverse velocity and interest rates is slightly negative in our sample, about -0.2. This negative correlation is reproduced by the search-based models but not by the MIU model. In fact, the observed correlations fall onto the boundary of the posterior predictive interval associated with the MIU model and in the center of the predictive interval associated with the search model. If we isolate the business-cycle frequency variation the bands of the predictive distributions become narrower. Again the search model does a better job in capturing the negative contemporaneous correlation of inverse velocity and interest rates. All models have unfortunately difficulties reproducing the negative correlations at leads and lags. Figure 7 depicts posterior predictive intervals for the correlation of inverse-velocity and output. The contemporaneous correlation in the data is near zero. The search-based models tend to predict a positive correlation, whereas the MIU model predicts a negative correlation. If we remove the low frequency variation from the data, the contemporaneous correlation between inverse velocity and output becomes negative, about 0.5. Overall, the estimated MIU model does a slightly better job in reproducing the observed money-output correlations at business cycle frequencies.

We now proceed by considering predictive distributions for coefficient estimates in money demand regressions. We consider one of the specifications used by Stock and Watson (1993) and Ball (2001). Using our notation and timing convention, the regression is of the form

$$\ln(M_{t+1}/P_t) = \beta_0 + \beta_1 \ln Y_t + \beta_2 \tilde{R}_t + \beta_3 \Delta \ln Y_{t-1} + \beta_3 \Delta \ln \tilde{R}_{t-1} + u_t. \tag{86}$$

For consistency with the DSGE model estimation, $\ln Y_t$ corresponds to linearly detrended real GDP (scaled by 100), $\tilde{R}_t$ is the net interest rate measured in annualized percentages, and $\ln(M_{t+1}/P_t)$ is constructed as the sum of log inverse velocity and detrended log real output (both scaled by 100). Thus, we are essentially removing a common deterministic trend component from both output and real money balances prior to the estimation of the money demand function. The coefficient β_1 is the income elasticity and β_2 is the semi-elasticity of money demand with respect to interest rates. Results of the predictive check are summarized in Table 9. Based on quarterly U.S. data from 1984 to 2005 the estimated income elasticity is 0.6 and the interest rate semi-elasticity is about -0.3. The model-implied posterior predictive distributions for the coefficient estimates are quite diffuse because the persistence of the technology shock makes it difficult to estimate the money demand function coefficients precisely. Strictly speaking, the estimates obtained from actual U.S. data are outside of the 90% bands of the predictive distribution generated by the search-based model and inside the bands of the predictive distribution associated with the MIU model. If we take the probability limit (plim) of the model implied money demand estimates conditional on the posterior mean parameter estimates we obtain plim $\hat{\beta}_1 = 0.3$ and plim $\hat{\beta}_2 = -1.3$ for the search-based model. The estimated MIU model, on the other hand, implies a negative income elasticity of -0.1 and a smaller interest rate semi-elasticity of -0.6.

Finally, we consider the steady state relationship between inverse velocity and the nominal interest rate as a function of the target inflation rate. Lucas (2000) argues that there is empirical evidence that the income elasticity of money demand is unity. To the extent that the variation in U.S. interest rates is due to exogenous shifts in monetary policy, the correlation between inverse velocity and interest rates provides a measure of the interest elasticity of money demand. In turn, the area under the money demand function measures the welfare costs of inflation.

Our analysis differs from Lucas' in the following dimensions. We agree with the observation that velocity has been fairly stable over time, while real output has been growing. We attribute output growth to a deterministic trend in total factor productivity that we remove prior to estimation. Both the specification of the search-based models and the MIU model are such that along the growth path velocity exhibits no trend. We restricted our estimation to a fairly short sample in which the target inflation rate has been quite low and monetary policy has been fairly stable. Monetary policy is largely endogenous in our analysis as it reacts to the state of the economy through an interest rate feedback rule. Based on our model estimates we concluded that most of the post 1984 variation in the nominal interest rate is due to technology shocks which shift both money demand as well as money supply.

The only exogenous variation in money supply is generated by the monetary policy shock. Arguably, an approximation of the money demand relationship embodied in the DSGE models that is relevant for the subsequent welfare analysis could be obtained simulating data in which the only source of variation is the monetary policy shock. If we estimate (86) based on large samples of such model generated data, we obtain estimates of -0.05 and -2.0 for β_1 and β_2 under the search-based model, and estimates of -0.2 and -0.94 for the MIU model.

In the subsequent welfare analysis we will use the DSGE model estimates and evaluate the effects of changes in the target inflation rate using the steady state relationships. Figure 8 traces out steady state velocity as a function of the nominal interest rate – and hence target inflation – for the search-based models and the MIU model. We overlay the actual observations. The circles correspond to velocity-interest rate pairs that have been used in the estimation, whereas the crosses correspond to observations prior to 1984. For the estimation we fixed the steady state interest rate at 5.34%. Differences in velocity given the historical steady state interest rate are due to the slightly different estimates of $\ln(\mathcal{M}_*/\mathcal{Y}_*)$ across model specifications. For interest rates between 2 and 6% the velocity predictions of the three DSGE models are quite similar.

Below 2% and above 6% the predictions of the MIU model start to diverge from the predictions of the search-based model. In particular the MIU model implies for interest rates greater than 8% that velocity is increasing, whereas the search-based models predict a decrease in velocity. Which of these predictions is more reasonable? If one is willing to interpret the 1970s as a period in which the target inflation rate was substantially higher than post 1983, then Figure 8 can be interpreted as a pseudo-out-of-sample check. The search-based models correctly predict the observed drop in inverse velocity, whereas the MIU model incorrectly predicts a strong increase of inverse velocity. This is an difference between the search-based models and the MIU model which casts some doubt on the results of the welfare experiments using the MIU model in the next section, especially those that involve large values of inflation.

5 Welfare Experiments

In this section, we consider experiments where we change the inflation target of the central bank, π_*, which is the inflation rate that prevails at the steady state. For the calculations in this section, we fix the parameter values at the posterior mean estimates reported in Section 4 (see Tables 1, 3, 4 and 5).

There are five sources of changes in welfare in these experiments that are present in the search-based model, three of which are also shared by the MIU model. First, inflation is a tax on money holdings in both models and as inflation rises (and the nominal interest rate increases) welfare will be reduced. This logic underlies Friedman's prescription of a zero percent net nominal interest rate to eliminate the opportunity cost of holding money which has come to be known as the Friedman rule. We will label this channel of welfare loss the Friedman channel. This channel will reveal itself as a reduction in real money balances and hence lower utility from the MIU part in the MIU model while we will see a lower q in the search-based model, which will lower utility in the DM. Second, both models display some level of price rigidity given by positive ζ and the fact that some firms cannot optimally change their prices create a relative price distortion.[17] This distortion is captured by the deviation of D_t in (64) from unity which would move the economy inside the production possibilities frontier by effectively destroying some of the outputs of the intermediate good firms. The distortion becomes more severe as the steady state inflation rate is away from 0% (in both directions), reducing welfare. We will label this channel the relative-price distortion channel. Third, the monopolistic competition among intermediate good producers create an additional distortion in both models. This distortion is captured by a positive markup, given by λ and by moving the real wage rate away from the marginal product of labor, it can be thought of creating a wedge similar to a labor income tax. We will label this channel the markup channel and will refer to the this and the previous channel collectively

[17] See Wolman (2001) for a more in-depth discussion of this and the next channel.

as the New-Keynesian channel. The three channels discussed so far will be the only channels in the MIU model and the price-taking version of the search-based model.

There are two more distortions in the bargaining version of the search-based model which are explained in detail in AWW. To summarize, the bilateral nature of trade and the fact that the surplus in a meeting is split by the two parties in the DM create two holdup problems: the buyers do not bring in the optimal amount of money (a money demand holdup problem) and the sellers do not bring in the optimal amount of capital (an investment holdup problem). These holdup problems are aggravated as inflation increases as this further reduces the payoffs in the DM by reducing q. We will collectively refer to these two sources of welfare loss as the holdup problem channel. An important reason for including the price-taking version of the search-based model is to investigate the importance of the holdup problem channels and as we show below, the qualitative results are unchanged in their absence.

Putting the holdup problem channel aside, the Friedman channel and the New-Keynesian channel has different implications for welfare. The welfare loss of inflation from the Friedman channel is minimized (in fact eliminated) at the Friedman rule of zero percent net nominal interest rate (or an inflation target for the central bank equal to the minus the rate of time preference). On the other hand, the loss due to the New-Keynesian channel is minimized around zero percent inflation target.[18] When both of these channels are present, the inflation rate that minimizes the overall distortions may be at either of the two extremes or somewhere in between.[19] There are three key considerations that determine which of the two channels dominate: the level of price stickiness given by ζ, the markup for the intermediate good producer given by λ and the importance of the opportunity cost of holding money. Our estimates of ζ across the two models are fairly similar while estimated of λ vary depending on the availability of some other source of markup: in the bargaining version we only need about 9% markup in the CM while in the price-taking version we need about 24%. A traditional way to think about the opportunity cost of holding money is the welfare triangle (the area under the money demand curve), which has first been discussed by Bailey (1956) and subsequently by Lucas (2000). It can be shown that in the absence of any other distortion, the welfare cost of inflation can be very well approximated by (in fact for some models exactly equal to) the area under the money demand curve between the two rates being compared.[20] As such, the shape of the steady state relationship between inverse velocity and interest rate depicted in Figure 8 is key for determining the strength of the Friedman channel.

[18] As Wolman (2001) notes the relative-price distortion is minimized at exactly zero percent inflation but the markup distortion may be minimized at a slightly positive inflation rate.

[19] As Schmitt-Grohe and Uribe (2007) note, in a medium-scale New Keynesian model with some money-demand motive, the welfare-maximizing inflation target may be about -2% or close to 0% depending on the parameterization.

[20] Craig and Rocheteau (2008) show that in the basic Lagos-Wright model, in the absence of any holdup problems, the area under the demand curve very closely approximate the consumption-equivalent welfare measure we will also use.

Before we turn to the results, a brief discussion about how we compute the welfare loss is in order. In the MIU model, the steady state welfare up to a constant is given by

$$W(\pi_*) = U(x_*) - Ah_* + \frac{\chi_*}{1-\nu_m}\left(\frac{A\mathcal{M}_*}{\pi_* Z_*^{1/1-\alpha}}\right)^{1-\nu_m} \tag{87}$$

and we solve for the percentage change required in x_* to make the households indifferent between two economies with different steady state inflation rates. In the search-based model, the reduced-form steady state welfare up to a constant is given by

$$V(\pi_*) = \sigma\left[u(q_*) - c(q_*, k_*, Z_*)\right] + U(x_*) - Ah_* \tag{88}$$

and we solve for the percentage change required in x_* and consumption in the DM (the q_* in $u(q_*)$) to make the households indifferent between two economies with different steady state inflation rates.[21] Finally, as a technical point, we replace $\left(1-\frac{1}{g_*}\right)\mathcal{Y}_*$ with simply a constant G_* obtained from the estimations to prevent any welfare effects coming through this term.

We focus on two questions related to welfare. First, we want to find the inflation target that minimizes the distortions and maximizes welfare. This requires us to study the behavior of welfare between the Friedman rule and zero percent inflation. Second, we want to investigate the welfare loss of moderate inflation, around 10% per year, similar to the rates that have been observed in the late 1970s. Figure 9 depicts the answers to both of these questions where we plot the welfare loss of deviating from 0% annual inflation. First focusing on the left side of this graph, it is clear that for both versions of the search-based model, the optimal inflation rate is the Friedman rule, as evidenced by the monotonically increasing curves. In contrast, for the MIU model the optimal inflation is around 0%.[22] Evidently, the Friedman channel dominates the New-Keynesian channel in the search-based model while the reverse is true in the MIU model. Turning to the second question, a 10% annual inflation costs about 8% of consumption according to both versions of the search-based model while the MIU model indicates a loss of more than twice this number. This shows that the New Keynesian channel becomes increasingly costly in the MIU model, relative to the search-based model. In the rest of this section, we dissect these results to understand them better.

Holding household-related parameters constant, as they are related to the Friedman channel, the relative importance of the two channels are determined by the degree of price stickiness, ζ, and the degree of dynamic indexation, ι. Intuitively, the former will be important in determining the optimal inflation target and the

[21] Note that we will not change the q_* term inside the cost function as it is a part of production.

[22] As a side remark, note that welfare at the Friedman rule is not defined for the MIU model as unless we put an artificial bound on money holdings, there is no solution to the household's problem. Money is costless to hold and utility is increasing in money balances so the households would like to hold arbitrarily large amounts. This is not the case in the search-based model as households would never want to hold more money than what they need to purchase q^*, the first-best quantity. As a result welfare is well-defined at the Friedman rule. For the figures, we omit the Friedman rule for the MIU model.

latter will be affect the welfare loss of moderate inflation rates since the more firms who cannot optimally choose their price can dynamically index their price, the lower the effect of price stickiness on welfare will be. To establish this point, and also to investigate the robustness of our results, in Figure 10 and Figure 11 we vary ζ and ι. Zooming in on inflation rates between the Friedman rule and 1% annual inflation, Figure 10 shows the tension between the Friedman channel and the New-Keynesian channel. In the search-based model, with the estimated degree of price stickiness, $\zeta = 0.70$ and $\zeta = 0.80$, respectively, as we argued above the Friedman channel is dominant. However, with an increase in the degree of price stickiness, to $\zeta = 0.90$ in the bargaining version and $\zeta = 0.85$ in the price-taking version, the relative-price distortion becomes more sever and the optimal policy is no longer the Friedman rule as evidenced from the U-shaped welfare loss curve. Similarly, the MIU model, when we lower the degree of price stickiness to $\zeta = 0.5$, the New-Keynesian channels loses its dominance and the Friedman rule becomes optimal. In short, in the empirically plausible range of the price-stickiness parameter, both models have a robust prediction about the optimal inflation target. Turning to Figure 11, if we reduce the dynamic indexation parameter in the bargaining version of the search-based model to the estimated value in the MIU model, $\iota = 0.05$, the welfare cost of 10% annual inflation increases to 16% of consumption and similarly in the MIU model, increasing the level of dynamic indexation to the level in the bargaining version of the search-based model reduces the welfare cost to just above 6% of consumption. This finding shows that most of the differences in welfare for moderate levels of inflation can be related to dynamic indexation, or lack thereof. In the price-taking version of the search-based model, which has an intermediate level of dynamic indexation at $\iota = 0.12$, moving ι has the same qualitative effect, albeit smaller quantitatively.

Finally, in order to see the effect of the different channels on welfare, Figure 12 shows the welfare loss of deviating from 0% inflation for different version of each model. In the search-based model, if we shut down the DM, which would amount to setting $\chi_* = 0$, we see the U-shaped welfare loss, reflecting the New-Keynesian channel. As soon as we turn on money demand, going to the curve labelled benchmark, the Friedman channel dominates. Approaching the benchmark from the other direction, the curve labelled perfect competition, shuts down the New-Keynesian channel completely and adding them makes the Friedman rule slightly less desirable but it remains optimal. It is also useful to note that if we shut down the holdup problems by considering price-taking (at the estimated parameters of the bargaining version) despite some quantitative changes, Friedman rule remains optimal. Turning to the MIU model, shutting down the New-Keynesian channel, we get a mild welfare gain – about an order of magnitude smaller compared to the search-based model – of going to the Friedman rule, which shows that the Friedman channel is not very strong in the MIU model. Similarly, moving from the cashless version where money demand is not modelled to the benchmark version there is only a slight move away from 0% inflation.

6 Conclusion

As an alternative to the commonly used MIU model, we have developed an estimable DSGE model in which the presence of a decentralized market creates an incentive for households to hold money, because money is needed as a medium of exchange. The model specification is closely tied to the theoretical literature that is developing microfounded models of monetary exchange. In particular, we base our model on recent work by Lagos and Wright (2005), and Aruoba, Waller, and Wright (2007).

Using post-1983 U.S. on output, inflation, interest rates, and real money balances, we estimate two versions of our search-based DSGE model along with a standard New Keynesian model in which real money balances enter the utility function. We obtain parameter estimates for the taste and technology parameters that determine the exchange in the decentralized market of the search-based models. These parameter estimates are potentially useful for the theoretically-oriented literature on microfounded monetary models.

We compare the dynamics of the estimated search-based model and the MIU model. While the decentralized market mechanism of the search-based models creates a stronger linkage between technology shocks and fluctuations in the stock of money, this linkage comes at a cost in terms of overall time series fit – at least for post 1983 data. On the other hand, if one uses the steady state relationships of the estimated DSGE models to predict velocity in periods of high target inflation rates as observed in the 1970s the search-based models deliver much more realistic predictions than the MIU model.

Finally, we explore the steady state welfare implications of the two models. The estimated MIU model behaves very much like a New Keynesian DSGE model and a near-zero inflation rate is optimal. According to the search-based model, which also has embodied some New Keynesian feature, the Friedman motive for keeping the nominal interest rate near zero dominates and negative inflation rates are optimal. This paper is part of a research agenda that tries to link the literatures on microfounded monetary models and estimable New Keynesian DSGE models that are popular at central banks. Many interesting questions are left unanswered and will hopefully be addressed in future research.

References

Aruoba, S. Boragan, Christopher Waller, and Randall Wright (2007): "Money and Capital," *Manuscript*, University of Maryland, University of Notre Dame, and University of Pennsylvania.

An, Sungbae and Frank Schorfheide (2007): "Bayesian Analysis of DSGE Models," *Econometric Reviews*, **26**(2-4), 113-172.

Bailey, Martin (1956): "The Welfare Cost of Inflationary Finance," *Journal of Political Economy*, **64**, 93-110.

Ball, Lawrence (2001): "Another Look at Long-Run Money Demand," *Journal of Monetary Economics*, *47*, 31-44.

Calvo, Guillermo (1983): " Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, **12**, 383-398.

Christiano, Lawrence , Martin Eichenbaum, and Charles Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, **113**, 1-45.

Craig, Ben and Guillaume Rocheteau (2008): "Inflation and Welfare: A Search Approach," *Journal of Money, Credit, and Banking*, **40**, 89-120.

Cynamon, Barry, Donald Dutkowsky, and Barry Jones (2006): "Redefining the Monetary Aggregates: A Clean Sweep," *Eastern Economic Journal*, **32**, 661-672.

Del Negro, Marco and Frank Schorfheide (2008): "Forming Priors for DSGE Models (and How it Affects the Assessment of Nominal Rigidities)," *NBER Working Paper No. 13741.*

Del Negro, Marco, Frank Schorfheide, Frank Smets, and Rafael Wouters (2007): "On the Fit of New Keynesian Models," *Journal of Business and Economic Statistics*, **25**, 123-162.

Geweke, John (1999): "Using Simulation Methods for Bayesian Econometric Models: Inference, Development and Communication," *Econometric Reviews*, **18**, 1-126.

Ireland, Peter (2004): "Money's Role in the Monetary Business Cycle," *Journal of Money, Credit, and Banking*, **36**, 969-983.

King, Robert, and Alexander Wolman (1996): "Inflation Targeting in a St. Louis Model of the 21st Century," *FRB St. Louis Review*, May-June.

Kiyotaki, Nobu and Randall Wright (1989): "On Money as a Medium of Exchange," *Journal of Political Economy*, **97**, 927-954.

Lagos, Ricardo and Randall Wright (2005): "A Unified Framework for Monetary Theory and Policy Evaluation," *Journal of Political Economy*, **113**, 463-484.

Levin, Andrew, Alexei Onatski, John Williams, and Noah Williams (2006): "Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models" in Mark Gertler and Kenneth Rogoff (eds.) *NBER Macroeconomics Annual 2005*, MIT Press, Cambridge, MA.

Lucas, Robert (2000): "Inflation and Welfare," *Econometrica*, **68**, 247-274.

Rocheteau, Guillaume and Randall Wright (2005): "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica*, **73**, 175-202.

Schmitt-Grohe, Stephanie, and Martin Uribe (2007): "Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model," in *Monetary Policy Under Inflation Targeting* , edited by Klaus Schmidt-Hebbel and Rick Mishkin, Central Bank of Chile, Santiago, Chile, p. 125-186.

Smets, Frank and Raf Wouters (2003): "An Estimated Stochastic Dynamic General Equilibrium Model for the Euro Area," *Journal of the European Economic Association*, **1**, 1123-1175.

Stock, James and Mark Watson (1993): "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems," *Econometrica*, **61**, 783-820.

Wolman, Alexander (2001): "A Primer on Optimal Monetary Policy with Staggered Price Setting," *FRB Richmond Quarterly Review*, **87**-4, 27-52.

Woodford, Michael (2003): *Interest and Prices*, Princeton University Press, Princeton.

Table 1: Parameters Fixed During Estimation

Name	MIU Model	Search-Based Model
Depreciation Rate δ	0.014	0.014
Persistence of TFP ρ_z	0.980	0.980
Fixed Costs $\mathcal{F}$	0.000	0.000
Indexation π_{**}	1.000	1.000
Steady State GDP $\mathcal{Y}_*$	1.000	1.000
Steady State $\ln(H_*/Y_*)$	-3.50	-3.50
Preference Parameter χ_*	1.000	1.000
Preference Parameter κ	N/A	.0001
Steady State Real Rate r_A	2.840	2.840
Steady State Inflation Rate π_A	2.500	2.500
Policy Rule ψ_1	1.820	1.820
Policy Rule ψ_2	0.180	0.180
Policy Rule ρ_R	0.780	0.780
Share of Government Spending g_*	1.200	1.200

Notes: We use the following transformations: $\beta = 1/(1 + r_A/400)$, $\pi_* = 1 + \pi_A/400$.

Table 2: Prior Distributions

			SBM - Bargaining		SBM - Price-Taking		MIU Model	
Name	Domain	Density	Para (1)	Para (2)	Para (1)	Para (2)	Para (1)	Para (2)
			Household					
$\ln(\mathcal{M}_*/\mathcal{Y}_*)$	$I\!R$	Normal	0.75	0.50	0.75	0.50	0.75	0.50
ν_m	$I\!R^+$	Gamma	-	-	-	-	20.00	5.00
θ	$[0,1)$	Uniform	0.00	1.00	-	-	-	-
$\tilde{\sigma}$	$[0,1)$	Beta	0.20	0.10	0.20	0.10	-	-
			Firms					
α	$[0,1)$	Beta	0.30	0.03	0.30	0.03	0.30	0.03
λ	$I\!R^+$	Gamma	0.15	0.05	0.15	0.05	-	-
λ	$[0,1]$	Uniform	-	-	-	-	0.00	1.00
ζ	$[0,1)$	Beta	0.60	0.15	0.60	0.15	0.60	0.15
ι	$[0,1)$	Beta	0.50	0.25	0.50	0.25	0.50	0.25
S''	$I\!R^+$	Gamma	5.00	2.50	5.00	2.50	5.00	2.50
			Shocks					
ρ_g	$[0,1)$	Beta	0.80	0.10	0.80	0.10	0.80	0.10
ρ_χ	$[0,1)$	Beta	0.80	0.10	0.80	0.10	0.80	0.10
σ_g	$I\!R^+$	InvGamma	1.00	4.00	1.00	4.00	1.00	4.00
σ_χ	$I\!R^+$	InvGamma	1.00	4.00	1.00	4.00	1.00	4.00
σ_R	$I\!R^+$	InvGamma	0.50	4.00	0.50	4.00	0.50	4.00
σ_Z	$I\!R^+$	InvGamma	1.00	4.00	1.00	4.00	1.00	4.00

Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu,s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. For the SBM, we multiply the product of the marginal densities reported in the table with the function $f(\cdot) = -0.5(I_*/\mathcal{Y}_* - 0.16)^2/0.005^2 - 0.5(mu - 0.15)^2/0.01^2$ where mu is the economy-wide markup and for the MIU model with $f(\cdot) = -0.5(I_*/\mathcal{Y}_* - 0.16)^2/0.005^2 - 0.5(\lambda - 0.15)^2/0.01^2$ and truncate the effective prior at the boundary of the determinacy region.

Table 3: Prior and Posterior Moments for SBM - Bargaining

Name	Prior		Posterior	
	Mean	90% Intv	Mean	90% Intv
$\ln(\mathcal{M}_*/\mathcal{Y}_*)$	0.427	[-0.524, 1.157]	0.768	[0.742, 0.793]
θ	0.938	[0.897, 0.998]	0.939	[0.931, 0.948]
$\tilde{\sigma}$	0.141	[0.019, 0.284]	0.305	[0.271, 0.339]
Firms				
α	0.305	[0.275, 0.334]	0.372	[0.352, 0.390]
λ	0.136	[0.086, 0.189]	0.094	[0.049, 0.127]
ζ	0.629	[0.448, 0.810]	0.695	[0.643, 0.753]
ι	0.703	[0.426, 0.995]	0.268	[0.123, 0.426]
S''	4.447	[1.209, 6.849]	11.921	[9.231, 15.24]
Shocks				
ρ_g	0.808	[0.677, 0.962]	0.859	[0.827, 0.896]
ρ_χ	0.829	[0.692, 0.956]	0.933	[0.914, 0.953]
σ_g	1.344	[0.657, 2.008]	0.936	[0.816, 1.048]
σ_χ	1.168	[0.572, 1.787]	1.619	[1.400, 1.815]
σ_R	1.027	[0.287, 1.890]	0.246	[0.215, 0.278]
σ_Z	1.162	[0.551, 1.880]	0.393	[0.327, 0.455]

Notes: The log marginal likelihood for this specification is -503.0.

Table 4: Prior and Posterior Moments for SBM - Price-Taking

Name	Prior		Posterior	
	Mean	90% Intv	Mean	90% Intv
$\ln(\mathcal{M}_*/\mathcal{Y}_*)$	0.524	[-0.085, 1.143]	0.769	[0.748, 0.790]
$\tilde{\sigma}$	0.162	[0.035, 0.277]	0.358	[0.310, 0.404]
Firms				
α	0.278	[0.265, 0.293]	0.277	[0.264, 0.291]
λ	0.175	[0.145, 0.204]	0.238	[0.205, 0.268]
ζ	0.589	[0.360, 0.821]	0.801	[0.764, 0.840]
ι	0.515	[0.117, 0.916]	0.117	[0.003, 0.231]
S''	5.046	[1.281, 8.640]	14.246	[8.782, 19.513]
Shocks				
ρ_g	0.799	[0.647, 0.957]	0.858	[0.823, 0.895]
ρ_χ	0.803	[0.659, 0.963]	0.928	[0.909, 0.947]
σ_g	1.247	[0.533, 1.992]	0.914	[0.791, 1.030]
σ_χ	1.229	[0.551, 1.957]	1.540	[1.338, 1.735]
σ_R	0.623	[0.265, 0.997]	0.280	[0.239, 0.320]
σ_Z	1.366	[0.503, 2.222]	0.390	[0.326, 0.448]

Notes: The log marginal likelihood for this specification is -501.3.

Table 5: Prior and Posterior Moments for MIU Model

Name	Prior		Posterior	
	Mean	90% Intv	Mean	90% Intv
		Household		
$\ln(\mathcal{M}_*/\mathcal{Y}_*)$	0.743	[-0.098, 1.547]	0.779	[0.729, 0.827]
ν_m	19.900	[12.018, 28.046]	25.943	[19.581, 31.647]
		Firms		
α	0.280	[0.266, 0.294]	0.282	[0.269, 0.296]
λ	0.151	[0.134, 0.167]	0.150	[0.133, 0.166]
ζ	0.595	[0.362, 0.833]	0.759	[0.709, 0.809]
ι	0.509	[0.094, 0.897]	0.050	[0.000, 0.101]
S''	5.060	[1.084, 8.726]	11.079	[6.299, 15.683]
		Shocks		
ρ_g	0.799	[0.646, 0.953]	0.886	[0.850, 0.920]
ρ_χ	0.800	[0.652, 0.960]	0.974	[0.958, 0.992]
σ_g	1.267	[0.521, 2.065]	1.227	[1.062, 1.388]
σ_χ	1.202	[0.546, 1.871]	0.865	[0.757, 0.972]
σ_R	0.583	[0.273, 0.897]	0.199	[0.175, 0.223]
σ_Z	1.194	[0.555, 1.874]	0.557	[0.471, 0.639]

Notes: The log marginal likelihood for this specification is -441.1.

Table 6: Posterior Steady States

Shock	Data	SBM - Bargaining		SBM - Price-Taking		MIU Model	
		Mean	90% Intv	Mean	90% Intv	Mean	90% Intv
$\mathcal{Y}_*$	-	1.000		1.000		1.000	
$I_*/\mathcal{Y}_*$	0.16	0.154	[0.147, 0.161]	0.162	[0.155,0.170]	0.163	[0.155, 0.171]
$K_*/\mathcal{Y}_*$	11.43	10.99	[10.47, 11.48]	11.60	[11.05,12.14]	11.64	[11.09, 12.19]
$\mathcal{M}_*/\mathcal{Y}_*$	2.12	2.155	[2.099, 2.207]	2.157	[2.111,2.202]	2.179	[2.072, 2.289]
H_*/Y_*	0.03	0.030		0.030		0.030	
W_*H_*/Y_*	0.70	0.576	[0.546, 0.604]	0.584	[0.565,0.605]	0.625	[0.608, 0.642]
Overall Markup	0.15	0.149	[0.134, 0.167]	0.146	[0.130,0.162]	0.150	[0.133, 0.166]
DM Share	-	0.326	[0.289, 0.359]	0.384	[0.335,0.435]	N/A	N/A
DM Markup	-	0.267	[0.172, 0.356]	0		N/A	N/A
A	-	5.129	[4.707, 5.544]	8.396	[8.066,8.716]	7.065	[6.334, 7.822]
B	-	0.095	[0.085, 0.105]	0.125	[0.103,0.146]	0.229	[0.206, 0.252]
Z_*	-	3.209	[2.837, 3.648]	5.572	[5.002,6.140]	6.189	[5.579, 6.761]

Notes: The data column, where available, refer to the relevant number in our larger sample.

Table 7: Posterior Variance Decomposition (Unconditional)

Shock	SBM - Bargaining		SBM - Price-Taking		MIU Model	
	Mean	90% Intv	Mean	90% Intv	Mean	90% Intv
			Output			
Gov Spending	0.046	[0.032, 0.063]	0.050	[0.032, 0.068]	0.099	[0.065, 0.136]
Money Demand	0.025	[0.013, 0.037]	0.052	[0.023, 0.079]		
Monetary Policy	0.008	[0.005, 0.011]	0.014	[0.006, 0.022]	0.012	[0.007, 0.017]
Technology	0.921	[0.899, 0.945]	0.883	[0.843, 0.926]	0.889	[0.850, 0.923]
			Inflation			
Gov Spending	0.003	[0.002, 0.005]	0.004	[0.001, 0.006]	0.020	[0.007, 0.034]
Money Demand	0.017	[0.007, 0.026]	0.032	[0.011, 0.054]		
Monetary Policy	0.021	[0.011, 0.030]	0.011	[0.005, 0.017]	0.009	[0.003, 0.016]
Technology	0.959	[0.944, 0.977]	0.952	[0.929, 0.977]	0.971	[0.956, 0.988]
			Federal Funds Rate			
Gov Spending	0.003	[0.002, 0.005]	0.002	[0.001, 0.003]	0.001	[0.000, 0.003]
Money Demand	0.004	[0.002, 0.007]	0.013	[0.002, 0.024]		
Monetary Policy	0.022	[0.013, 0.030]	0.032	[0.015, 0.047]	0.040	[0.022, 0.055]
Technology	0.970	[0.961, 0.981]	0.954	[0.932, 0.976]	0.959	[0.943, 0.977]
			Real Money Balances			
Gov Spending	0.023	[0.015, 0.034]	0.019	[0.007, 0.030]	0.002	[0.001, 0.004]
Money Demand	0.223	[0.142, 0.297]	0.244	[0.149, 0.346]	0.767	[0.595, 0.923]
Monetary Policy	0.030	[0.019, 0.040]	0.054	[0.023, 0.083]	0.021	[0.008, 0.036]
Technology	0.724	[0.654, 0.818]	0.683	[0.569, 0.808]	0.209	[0.052, 0.358]

Notes: Real money balances are measured in terms of the CM Good.

Table 8: Posterior Variance Decomposition (Business Cycle Freq)

Shock	SBM - Bargaining		SBM - Price-Taking		MIU Model	
	Mean	90% Intv	Mean	90% Intv	Mean	90% Intv
			Output			
Gov Spending	0.502	[0.421, 0.586]	0.432	[0.360, 0.512]	0.620	[0.539, 0.704]
Money Demand	0.105	[0.070, 0.139]	0.153	[0.098, 0.195]		
Monetary Policy	0.079	[0.049, 0.107]	0.105	[0.055, 0.143]	0.101	[0.065, 0.138]
Technology	0.314	[0.233, 0.382]	0.309	[0.233, 0.379]	0.280	[0.214, 0.357]
			Inflation			
Gov Spending	0.021	[0.012, 0.029]	0.023	[0.011, 0.037]	0.118	[0.062, 0.175]
Money Demand	0.103	[0.058, 0.140]	0.122	[0.062, 0.173]		
Monetary Policy	0.190	[0.126, 0.251]	0.096	[0.055, 0.143]	0.076	[0.025, 0.128]
Technology	0.686	[0.606, 0.774]	0.758	[0.678, 0.834]	0.806	[0.719, 0.887]
			Federal Funds Rate			
Gov Spending	0.013	[0.006, 0.021]	0.013	[0.005, 0.020]	0.004	[0.000, 0.009]
Money Demand	0.008	[0.002, 0.015]	0.028	[0.006, 0.050]		
Monetary Policy	0.213	[0.153, 0.285]	0.263	[0.167, 0.346]	0.333	[0.254, 0.425]
Technology	0.766	[0.701, 0.839]	0.696	[0.607, 0.788]	0.664	[0.580, 0.751]
			Real Money Balances			
Gov Spending	0.083	[0.056, 0.114]	0.051	[0.031, 0.069]	0.008	[0.002, 0.012]
Money Demand	0.522	[0.447, 0.592]	0.470	[0.379, 0.545]	0.725	[0.614, 0.836]
Monetary Policy	0.177	[0.114, 0.225]	0.266	[0.171, 0.357]	0.144	[0.086, 0.197]
Technology	0.217	[0.155, 0.287]	0.213	[0.136, 0.285]	0.124	[0.059, 0.193]

*Notes:*Real money balances are measured in terms of the CM good.

Table 9: Posterior Predictive Check: Money Demand Coefficients

Regressor	Data	Search Model		MIU Model	
		Mean	90% Intv	Mean	90% Intv
Intcpt	78.483	84.186	[78.766, 88.300]	81.083	[74.521, 87.749]
$\ln Y_t$	0.596	0.067	[-0.544, 0.579]	-0.152	[-0.684, 0.650]
R_t	-0.337	-1.324	[-2.006, -0.564]	-0.678	[-1.605, 0.038]
$\Delta \ln Y_{t-1}$	-1.661	0.022	[-0.345, 0.320]	0.032	[-0.423, 0.485]
ΔR_{t-1}	-1.330	0.035	[-0.315, 0.315]	0.075	[-0.347, 0.468]

Notes: The column "Data" contains least squares estimates of the regression

$$\ln(M_{t+1}/P_t) = \beta_0 + \beta_1 \ln Y_t + \beta_2 \tilde{R}_t + \beta_3 \Delta \ln Y_{t-1} + \beta_3 \Delta \tilde{R}_{t-1} + u_t.$$

The remaining columns summarize the posterior predictive distribution for these least squares estimates based on the Search and MIU models. Draws from the posterior predictive distributions are obtained by simulating sample paths from the DSGE model conditional on the posterior parameter draws and estimating the money demand equation from the simulated sample paths.

Figure 1: M1 and M2 Log Inverse Velocity

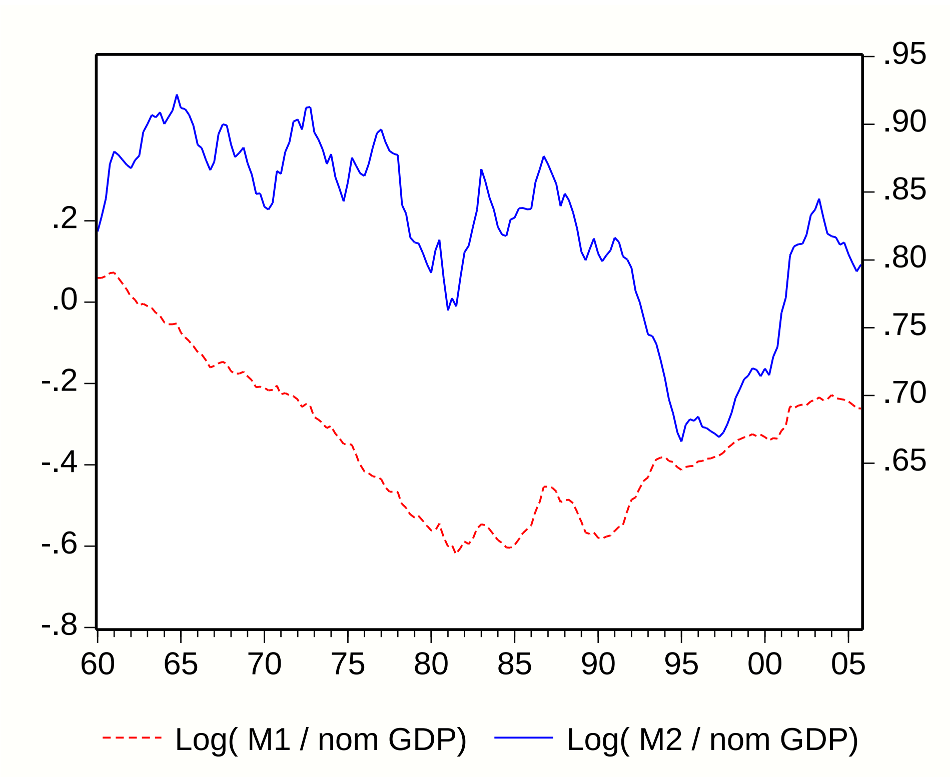
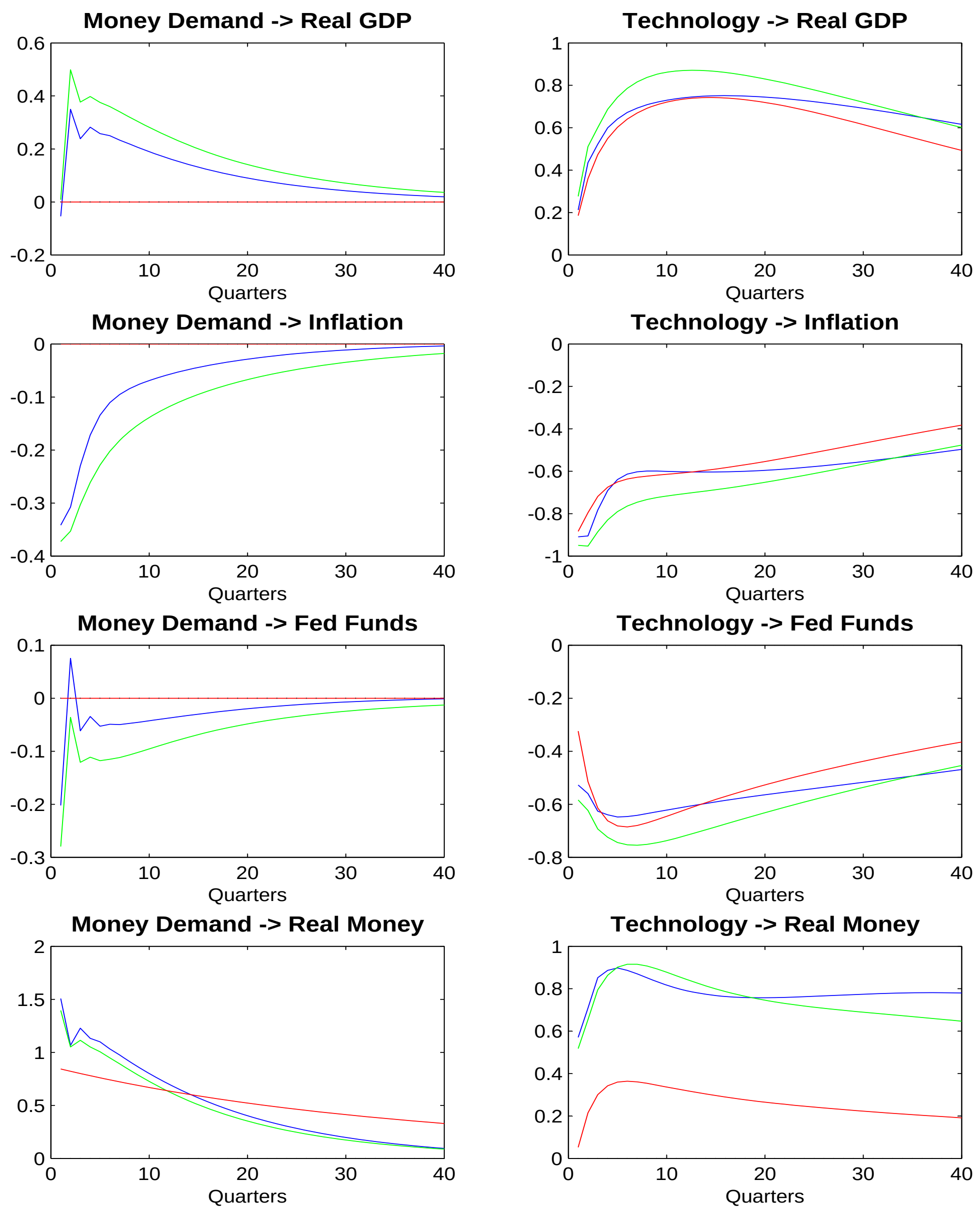
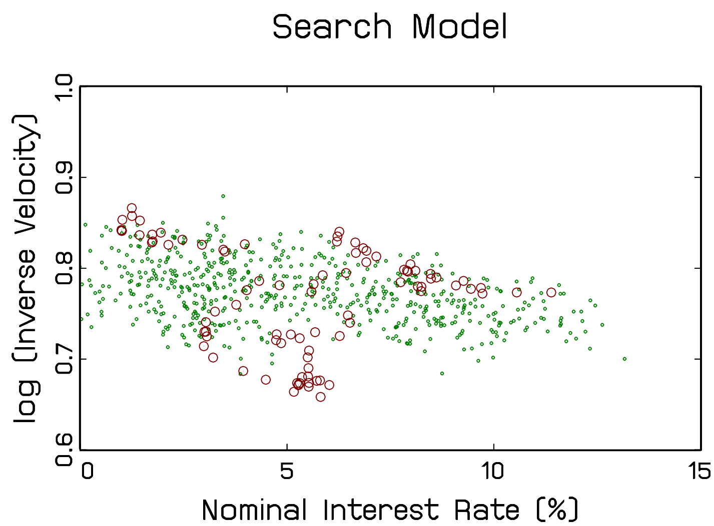


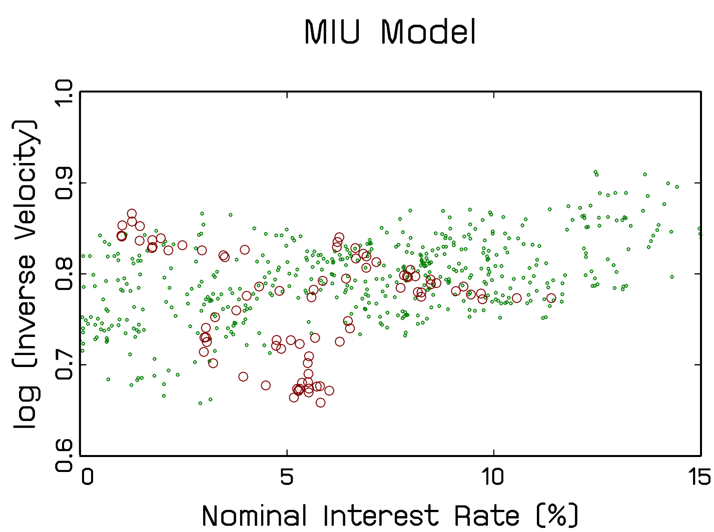
Figure 2: Impulse Responses to Money Demand and Technology Shocks



Notes: Figure depicts pointwise posterior mean of impulse response functions for the MIU model (red), search-based model with bargaining (blue) and with price-taking (green) to the money demand and technology shocks. Responses of inflation and Fed Funds rate are measured in percentage points and responses of real GDP and real money are measured in percentage deviations from the steady state.

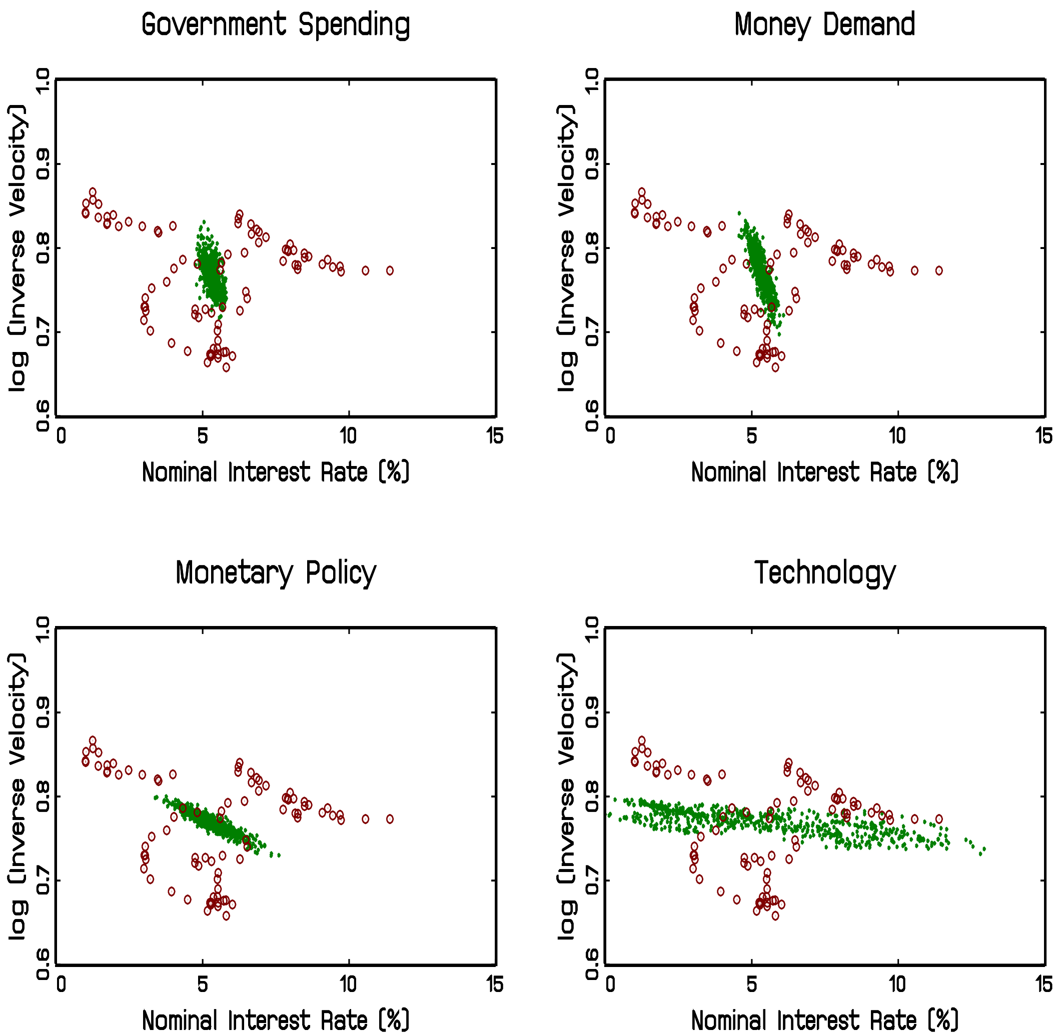
Figure 3: Log Inverse Velocity vs. Nominal Interest Rate





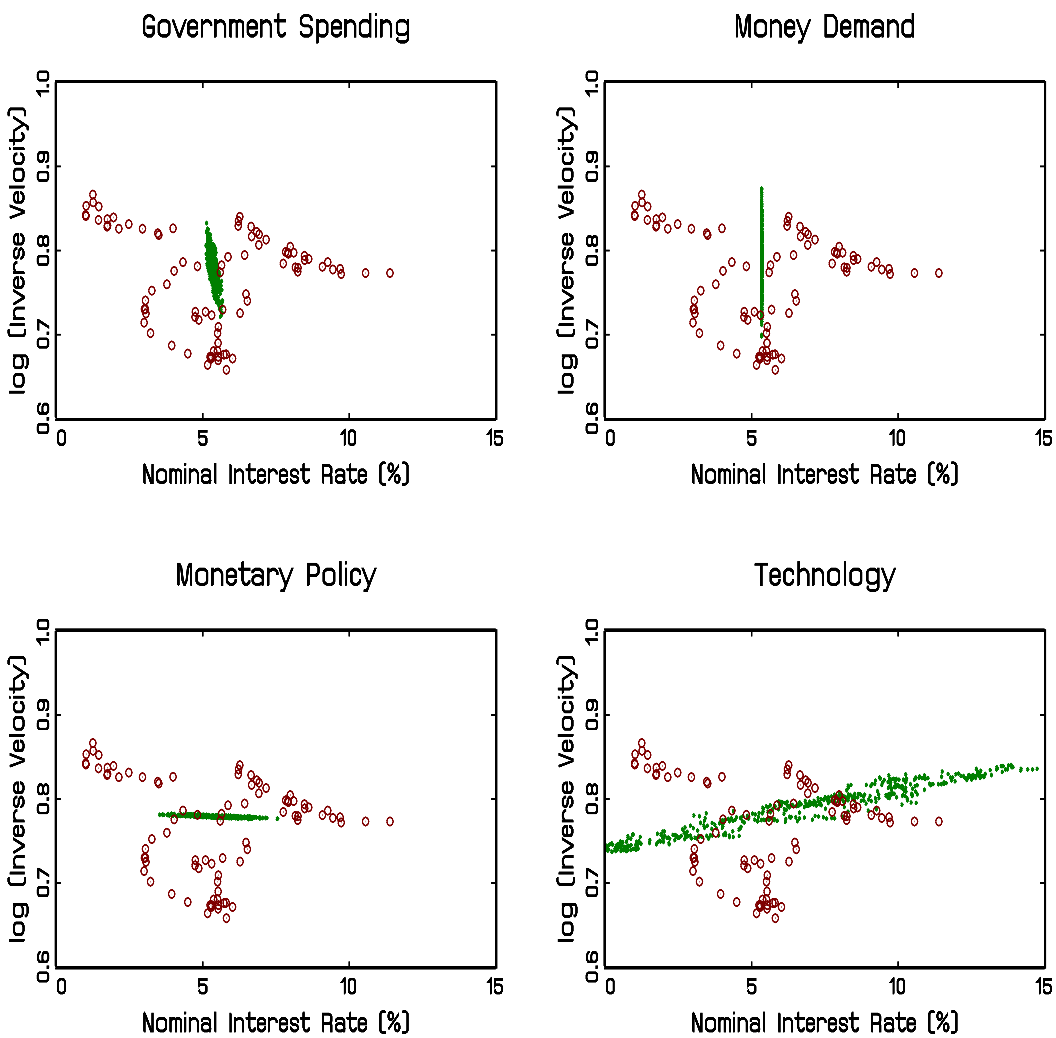
Notes: Conditional on the posterior mean estimates we depict log inverse velocity and interest rates for a simulated sample of 500 observations (green dots) as well as actual U.S. data (red circles)

Figure 4: LOG INVERSE VELOCITY VS. NOMINAL INTEREST RATE - SBM (BARGAINING)



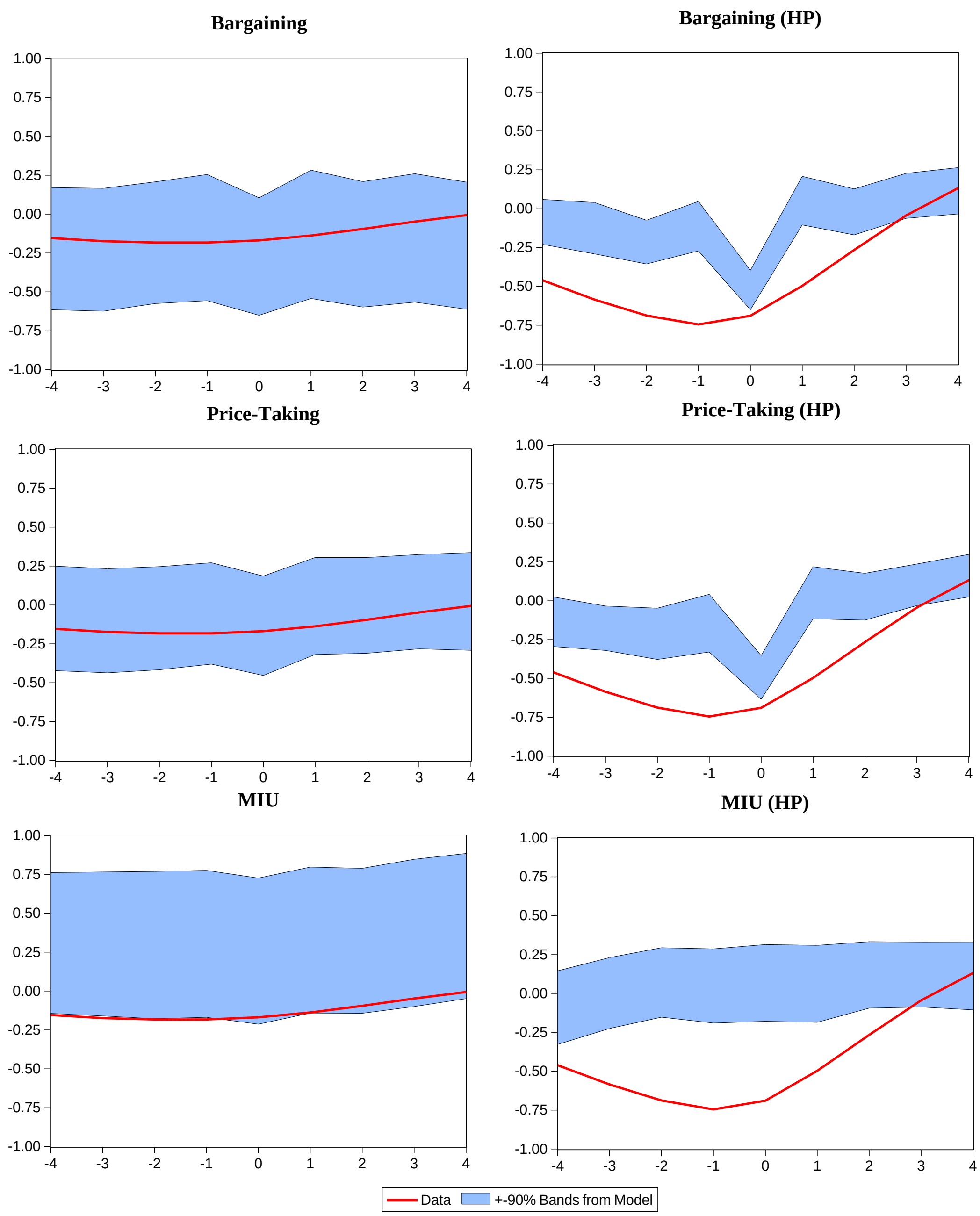
Notes: Conditional on the posterior mean estimates we depict log inverse velocity and interest rates for a simulated sample of 500 observations (green dots) as well as actual U.S. data (red circles) Each panel shows the results when only the respective shock is active.

Figure 5: Log Inverse Velocity vs. Nominal Interest Rate - MIU Model



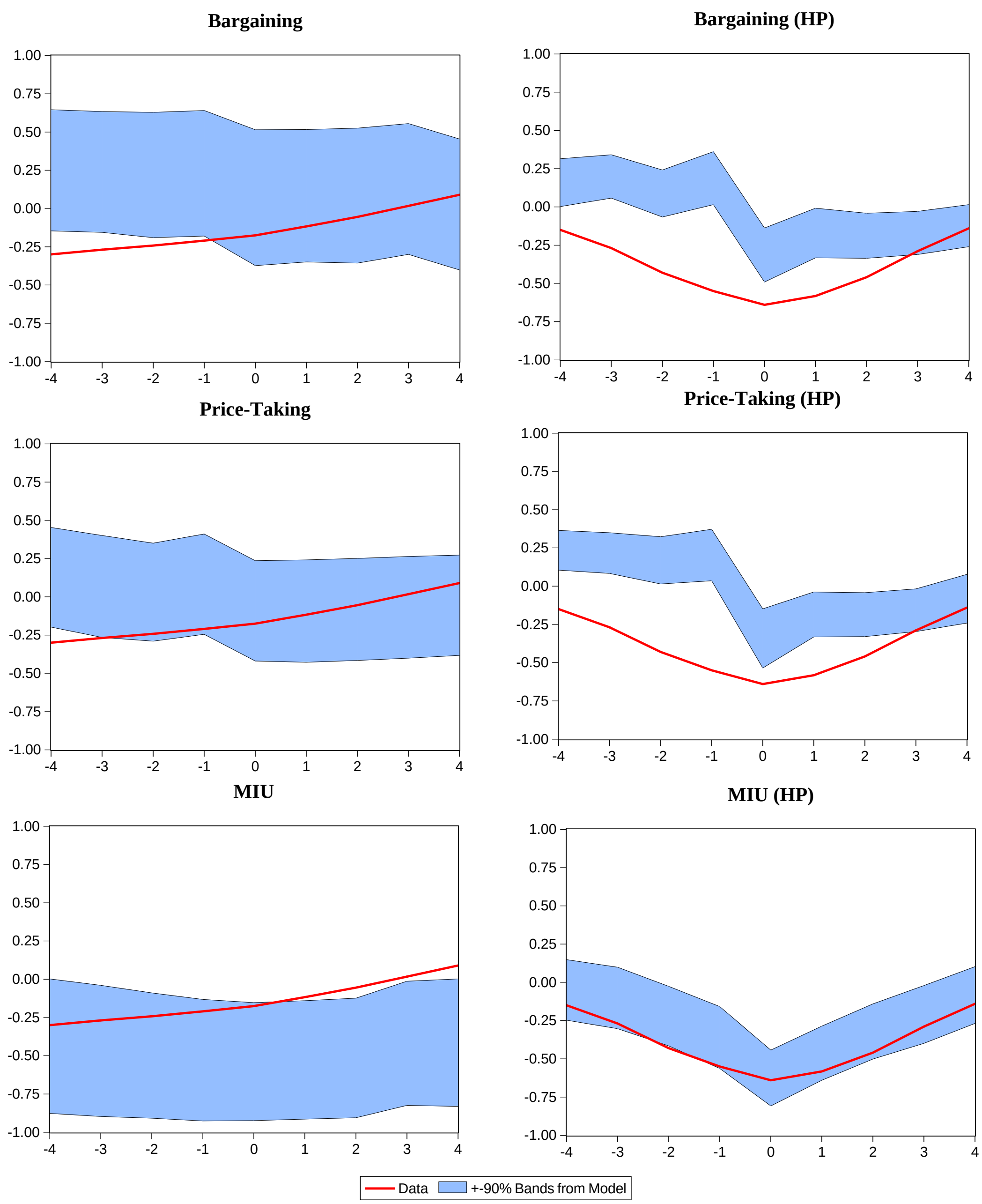
Notes: See Figure 4.

Figure 6: Correlations of Log Inverse Velocity at $t + h$ with Nominal Interest Rate at t



Notes: The lines labelled "Data" contain sample correlations. For model-based bands, draws from the posterior predictive distributions are obtained by simulating sample paths from the DSGE model conditional on posterior parameter draws and calculating moments from the simulated sample paths. The panels on the left show the results with raw data and the panels on the right show results with HP-filtered data.

Figure 7: Correlations of Log Inverse Velocity at $t+h$ with Real GDP at t



Notes: See Figure 6.

Figure 8: Steady State Relationship Between Log Inverse Velocity and Nominal Interest Rate

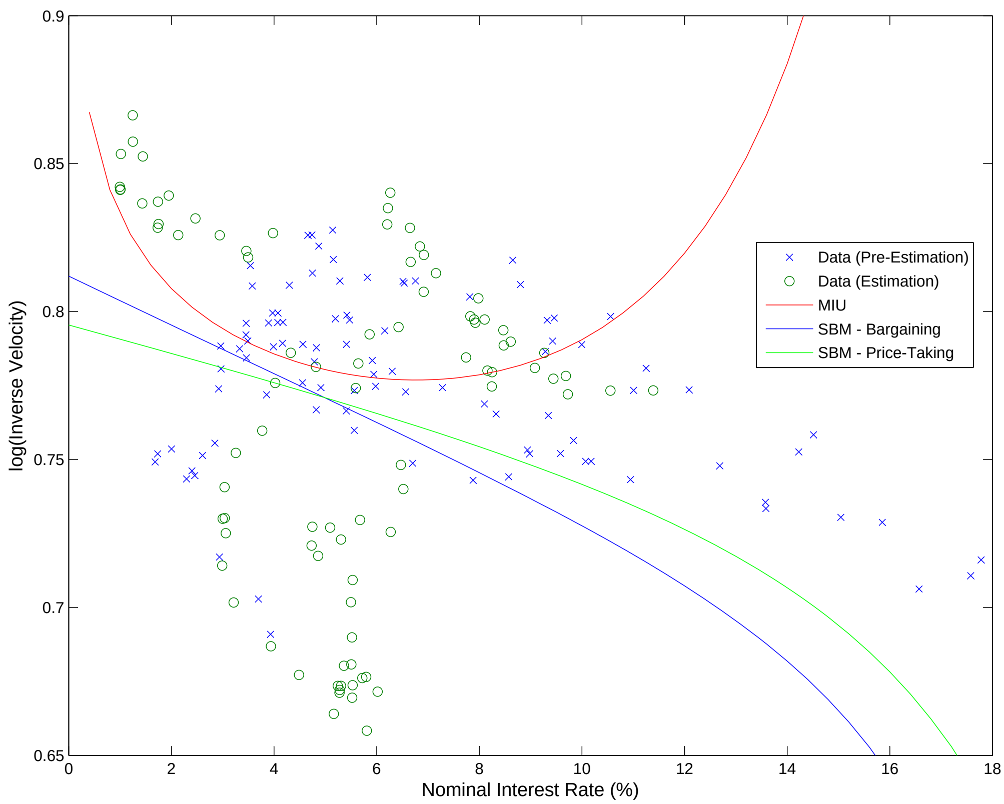


Figure 9: Welfare Loss of Deviating from 0% Inflation

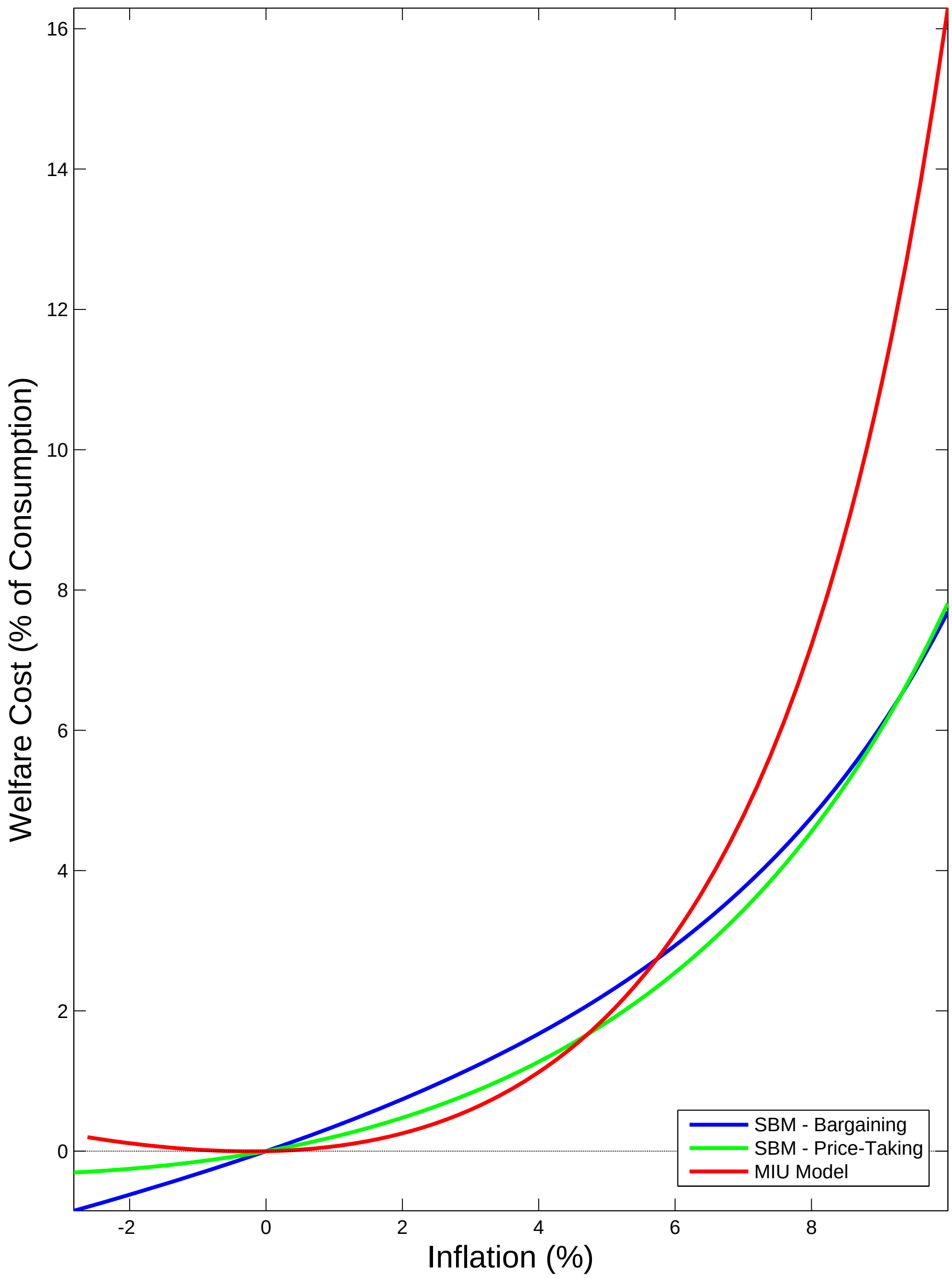


Figure 10: Welfare Loss of Deviating from 0% Inflation - Sensitivity to Price Stickiness

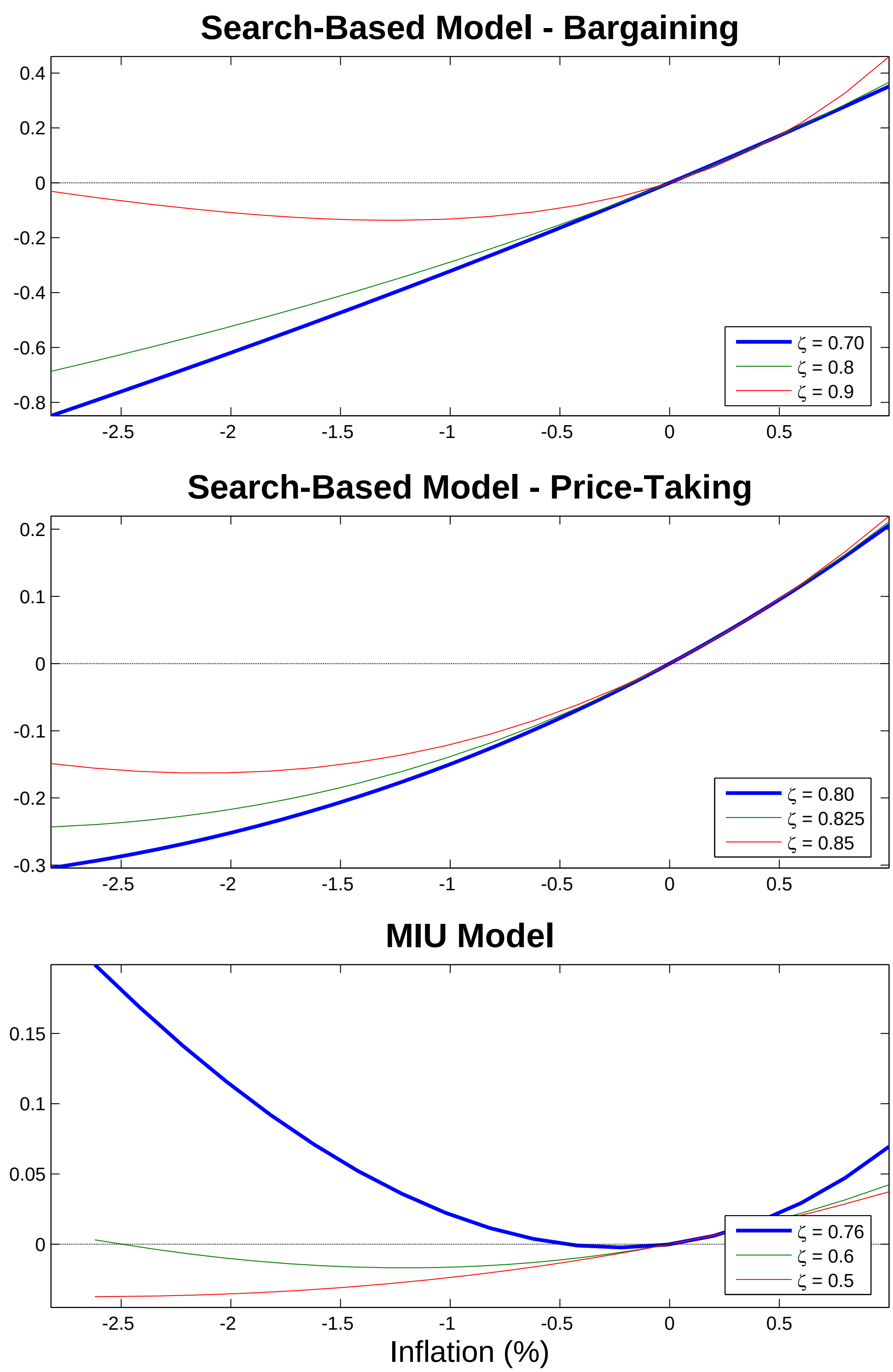


Figure 11: Welfare Loss of Deviating from 0% Inflation - Sensitivity to Dynamic Indexation

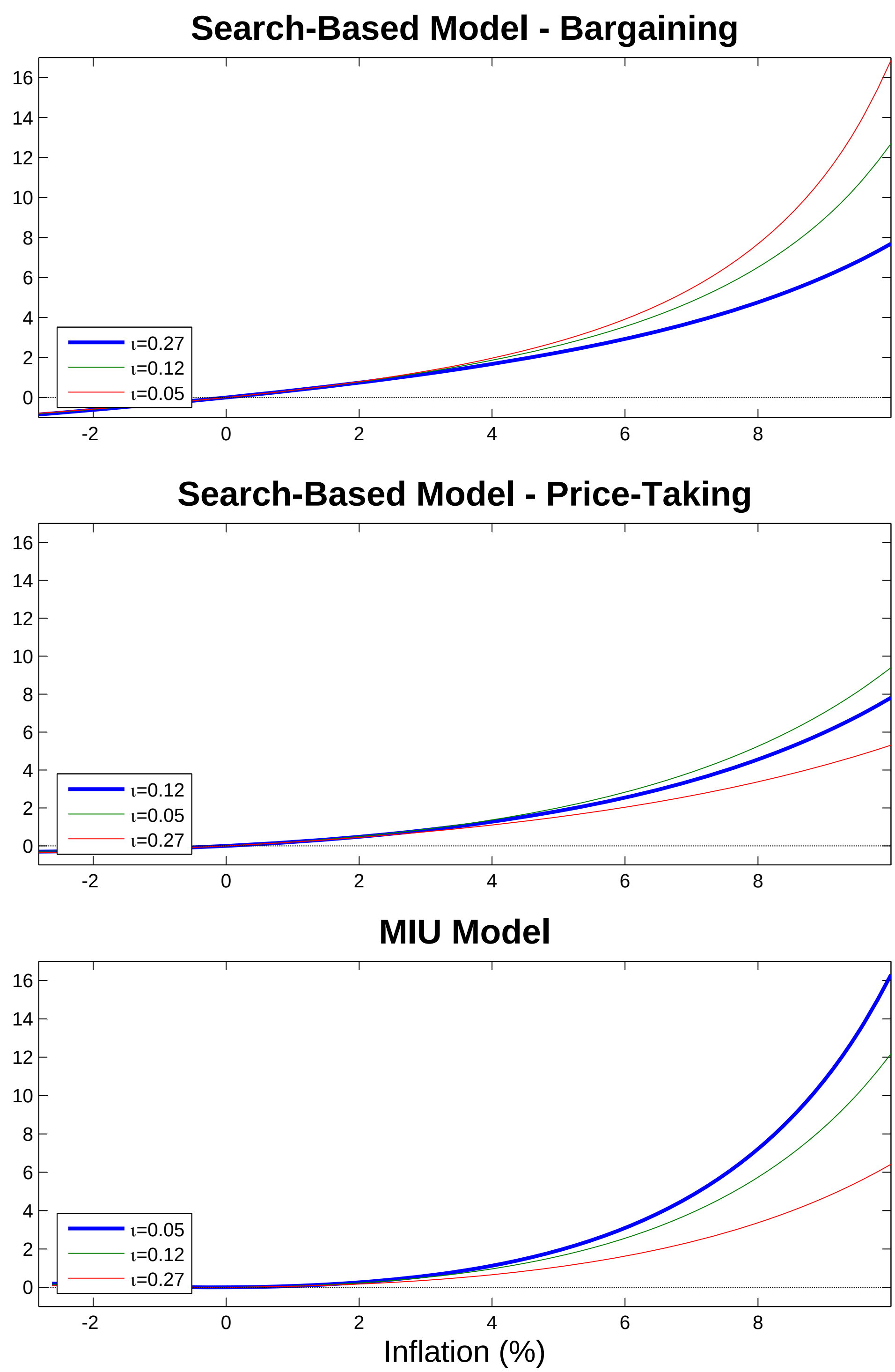
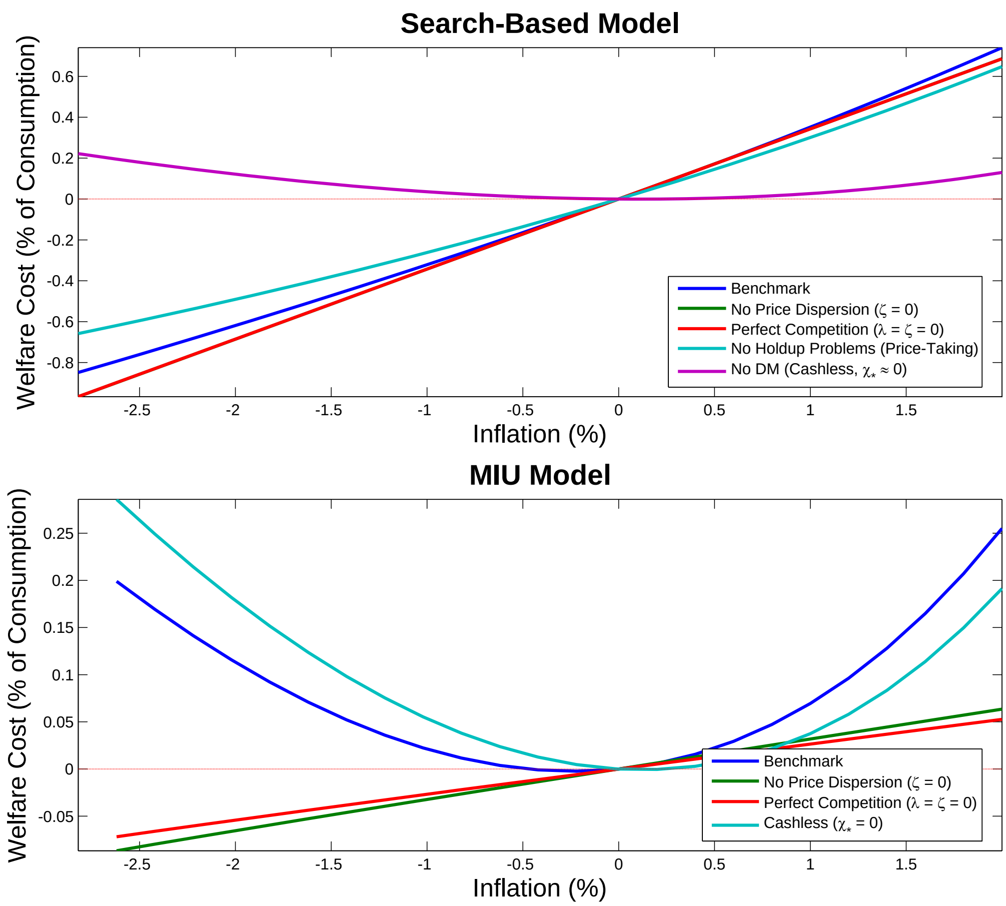


Figure 12: Channels of Welfare Loss



A The Search-Based Model

We use a slightly more general specification of the utility and production functions in the subsequent exposition:

$$U(x) = B\frac{x^{1-\gamma}}{1-\gamma}, \quad u(q) = \frac{(q+\kappa)^{1-\eta} - \kappa^{1-\eta}}{1-\eta}.$$

Moreover, we let $f(e,k) = e^{\Phi}k^{1-\Phi}$.

A.1 Further Details: Intermediate Good Producers

The first-order condition for a intermediate good producing firm is:

$$\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\zeta^s\beta^s\Xi^p_{t+s|t}\frac{1}{P^o_t(i)}\left(\frac{P^o_t(i)\pi^{adj}_{t+s|t}}{P_{t+s}}\right)^{-\frac{1+\lambda}{\lambda}}Y_{t+s}\left[P^o_t(i)\pi^{adj}_{t+s|t} - (1+\lambda)P_{t+s}MC_{t+s}\right]\right\} = 0. \tag{89}$$

Define and rewrite

$$\begin{aligned}
\mathcal{F}^{(1)}_t &= \mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta^s\beta^s\Xi^p_{t+s|t}\left(\frac{P^o_t(i)\pi^{adj}_{t+s|t}}{P_{t+s}}\right)^{-\frac{1+\lambda}{\lambda}}Y_{t+s}\pi^{adj}_{t+s|t}\right] \qquad (90)\\
&= \left(\frac{P^o_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}}Y_t + \zeta\beta\mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta^s\beta^s\Xi^p_{t+1+s|t}\left(\frac{P^o_t(i)\pi^{adj}_{t+1+s|t}}{P_{t+1+s}}\right)^{-\frac{1+\lambda}{\lambda}}Y_{t+1+s}\pi^{adj}_{t+1+s|t}\right]\\
&= \left(\frac{P^o_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}}Y_t + \zeta\beta\left(\pi^{\iota}_t\pi^{(1-\iota)}_{**}\right)^{-1/\lambda}\\
&\quad\times\mathbb{E}_t\left[\left(\frac{P^o_t(i)}{P^o_{t+1}(i)}\right)^{-\frac{1+\lambda}{\lambda}}\Xi^p_{t+1|t}\sum_{s=0}^{\infty}\zeta^s\beta^s\Xi^p_{t+1+s|t+1}\left(\frac{P^o_{t+1}(i)\pi^{adj}_{t+1+s|t+1}}{P_{t+1+s}}\right)^{-\frac{1+\lambda}{\lambda}}Y_{t+1+s}\pi^{adj}_{t+1+s|t+1}\right]\\
&= \left(\frac{P^o_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}}Y_t + \zeta\beta\left(\pi^{\iota}_t\pi^{(1-\iota)}_{**}\right)^{-1/\lambda}\mathbb{E}_t\left[\left(\frac{P^o_t(i)}{P^o_{t+1}(i)}\right)^{-\frac{1+\lambda}{\lambda}}\Xi^p_{t+1|t}\mathcal{F}^{(1)}_{t+1}\right].
\end{aligned}$$

Similarly,

$$\begin{aligned}
\mathcal{F}^{(2)}_t &= \mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta^s\beta^s\Xi_{t+s}\left(\frac{P^o_t(i)\pi^{adj}_{t+s|t}}{P_{t+s}}\right)^{-\frac{1+\lambda}{\lambda}}Y_{t+s}\frac{P_{t+s}MC_{t+s}}{P^o_t(i)}\right] \qquad (91)\\
&= \left(\frac{P^o_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}}Y_t\frac{P_tMC_t}{P^o_t(i)} + \zeta\beta\left(\pi^{\iota}_t\pi^{(1-\iota)}_{**}\right)^{-\frac{1+\lambda}{\lambda}}\mathbb{E}_t\left[\left(\frac{P^o_t(i)}{P^o_{t+1}(i)}\right)^{-\frac{1+\lambda}{\lambda}-1}\Xi^p_{t+1|t}\mathcal{F}^{(2)}_{t+1}\right].
\end{aligned}$$

and the first-order condition becomes

$$\mathcal{F}^{(1)}_t = (1+\lambda)\mathcal{F}^{(2)}_t. \tag{92}$$

A.2 Further Details: Price Dispersion

To capture the evolution of the price distribution we introduced a new variable D_t. Its law of motion can be derived as follows:

$$\begin{aligned} D_t &= (1-\zeta)\sum_{j=0}^{\infty}\zeta^j\left(\frac{(\pi_{t-1}\pi_{t-2}\cdots\pi_{t-j})^{\iota}\pi_{**}^{j(1-\iota)}}{\pi_t\pi_{t-1}\cdots\pi_{t-j+1}}\frac{P^o_{t-j}}{P_{t-j}}\right)^{-\frac{1+\lambda}{\lambda}} \\ &= (1-\zeta)\left[\frac{P^o_t}{P_t}\right]^{-\frac{1+\lambda}{\lambda}} \\ &\quad +(1-\zeta)\zeta\left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\frac{P^o_{t-1}}{P_{t-1}}\right]^{-\frac{1+\lambda}{\lambda}} \\ &\quad +(1-\zeta)\zeta^2\left[\left(\frac{\pi_{t-2}}{\pi_t}\right)^{\iota}\left(\frac{\pi^2_{**}}{\pi_t\pi_{t-1}}\right)^{(1-\iota)}\frac{P^o_{t-2}}{P_{t-2}}\right]^{-\frac{1+\lambda}{\lambda}}\cdots. \end{aligned}$$

Lagging D_t by one period yields

$$\begin{aligned} D_{t-1} &= (1-\zeta)\left[\frac{P^o_{t-1}}{P_{t-1}}\right]^{-\frac{1+\lambda}{\lambda}} \\ &\quad +(1-\zeta)\zeta\left[\left(\frac{\pi_{t-2}}{\pi_{t-1}}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_{t-1}}\right)^{(1-\iota)}\frac{P^o_{t-2}}{P_{t-2}}\right]^{-\frac{1+\lambda}{\lambda}} \\ &\quad +(1-\zeta)\zeta^2\left[\left(\frac{\pi_{t-3}}{\pi_{t-1}}\right)^{\iota}\left(\frac{\pi^2_{**}}{\pi_{t-1}\pi_{t-2}}\right)^{(1-\iota)}\frac{P^o_{t-3}}{P_{t-3}}\right]^{-\frac{1+\lambda}{\lambda}}\cdots. \end{aligned}$$

Therefore, we obtain the following law of motion for the price dispersion:

$$D_t = \zeta\left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1-\zeta)\left[\frac{P^o_t}{P_t}\right]^{-\frac{1+\lambda}{\lambda}}. \tag{93}$$

A.3 Equilibrium Conditions

We now summarize the equilibrium conditions for the search-based model. The timing is such that all t shocks are realized at the beginning of t and $\bar{S}_t = (Z_t, g_t, \chi_t)$ and R_t are observed. $\bar{S}_t$ summarizes the exogenous state variables. We define $S_t = (\bar{S}_t, R_t)$ which will be the aggregate state variables of the household's problem. In the following definitions, we do not track h_t (individual labor supply) and B_t (the bond supply of the government). We also do not track nominal money balances but instead track $\mathcal{M}_t = M_t/P_{t-1}$. Recall that M_t is determined based on $t-1$ information and so is $\mathcal{M}_t$. Finally, we use $\pi_t \equiv P_t/P_{t-1}$ and do not track the level of prices.

Given exogenous states $\{\bar{S}_t\}_{t=0}^{\infty}$, a monetary equilibrium is defined as allocations $\{q_t, X_t, H_t, K_t, I_t, \mu_t, Y_t, \mathcal{M}_t, \mathcal{Y}_t\}_{t=0}^{\infty}$, policy $\{R_t\}_{t=0}^{\infty}$ and prices $\{W_t, R^k_t, p^0_t, \pi_t, D_t\}_{t=0}^{\infty}$ such that :

Household's Problem: Given exogenous states, policy and prices, $\left\{q_t, X_t, H_t, K_t, I_t, \mu_t, \mathcal{M}_t, \Xi^p_{t+1|t}\right\}_{t=0}^{\infty}$ satisfy

$$
\begin{aligned}
W_t &= \frac{A}{U'(X_t)} && (94)\\
1 &= \beta E_t\left[\frac{U'(X_{t+1})}{U'(X_t)}\frac{R_t}{\pi_{t+1}}\right] && (95)\\
1 &= \mu_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}}S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta E_t\left\{\mu_{t+1}\frac{U'(X_{t+1})}{U'(X_t)}\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)\right\} && (96)\\
K_{t+1} &= (1-\delta)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t && (97)\\
\mu_t &= \beta E_t\left\{\frac{U'(X_{t+1})}{U'(x_t)}\left[R^k_{t+1} + (1-\delta)\mu_{t+1}\right] - \frac{\sigma}{U'(X_t)}\gamma(q_{t+1}, K_{t+1}, \chi_{t+1}, Z_{t+1})\right\} && (98)\\
\mathcal{M}_t &= \frac{g\left(q_t, K_t, \chi_t, Z_t\right)W_t\pi_t}{A} && (99)\\
U'(X_t) &= \beta E_t\left\{\frac{U'(X_{t+1})}{\pi_{t+1}}\left[\frac{\sigma\chi_{t+1}u'(q_{t+1})}{g_q(q_{t+1}, K_{t+1}, \chi_{t+1}, Z_{t+1})} + (1-\sigma)\right]\right\} && (100)\\
\Xi^p_{t+1|t} &= \frac{U'(X_{t+1})}{U'(X_t)\pi_{t+1}} && (101)
\end{aligned}
$$

In the price-taking version we replace (98), (99) and (100) with

$$
\begin{aligned}
\mu_t &= \beta E_t\left\{\frac{U'(X_{t+1})}{U'(x_t)}\left[R^k_{t+1} + (1-\delta)\mu_{t+1}\right] - \frac{\sigma}{U'(X_t)}c_k(q_{t+1}, K_{t+1}, Z_{t+1})\right\} && (102)\\
\mathcal{M}_t &= \frac{q_t c_q\left(q_t, K_t, Z_t\right)W_t\pi_t}{A} && (103)\\
U'(X_t) &= \beta E_t\left\{\frac{U'(X_{t+1})}{\pi_{t+1}}\left[\frac{\sigma\chi_{t+1}u'(q_{t+1})}{c_q(q_{t+1}, K_{t+1}, Z_{t+1})} + (1-\sigma)\right]\right\} && (104)
\end{aligned}
$$

Intermediate Goods Producing Firms' Problem: Intermediate goods firms choose their capital labor ratio as a function of the factor prices to minimize costs:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R^k_t}H_t. \quad (105)$$

Firms that are allowed to change prices are choosing a relative price $p^o_t(i)$ (relative to the aggregate price level) to maximize expected profits subject to the demand curve for their differentiated product, taking the aggregate price level P_t as well as the prices charged by other firms as given, which leads to

$$
\begin{aligned}
MC_t &= \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W_t^{1-\alpha}(R^k_t)^{\alpha}Z_t^{-1} && (106)\\
\mathcal{F}^{(1)}_t &= (p^o_t)^{-\frac{1+\lambda}{\lambda}}Y_t + \zeta\beta\left(\pi^{\iota}_t\pi^{(1-\iota)}_{**}\right)^{-1/\lambda}\mathbb{E}_t\left[\left(\frac{p^o_t}{\pi_{t+1}p^o_{t+1}}\right)^{-\frac{1+\lambda}{\lambda}}\Xi^p_{t+1|t}\mathcal{F}^{(1)}_{t+1}\right] && (107)\\
\mathcal{F}^{(2)}_t &= (p^o_t)^{-\frac{1+\lambda}{\lambda}-1}Y_t MC_t + \zeta\beta\left(\pi^{\iota}_t\pi^{(1-\iota)}_{**}\right)^{-\frac{1+\lambda}{\lambda}}\mathbb{E}_t\left[\left(\frac{p^o_t}{\pi_{t+1}p^o_{t+1}}\right)^{-\frac{1+\lambda}{\lambda}-1}\Xi^p_{t+1|t}\mathcal{F}^{(2)}_{t+1}\right] && (108)\\
\mathcal{F}^{(1)}_t &= (1+\lambda)\mathcal{F}^{(2)}_t && (109)
\end{aligned}
$$

Final Good Producing Firms' Problem: Final goods producers take factor prices and output prices as given and choose inputs $Y_t(i)$ and output Y_t to maximize profits. Free entry ensures that final good producers make zero profits and leads to

$$\pi_t = \left[(1-\zeta)\left(\pi_t p_t^o\right)^{-\frac{1}{\lambda}} + \zeta(\pi_{t-1}^{\iota}\pi_{**}^{1-\iota})^{-\frac{1}{\lambda}}\right]^{-\lambda} \tag{110}$$

Monetary Policy: The central bank supplies the quantity of money necessary to attain the nominal interest rate

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*}\right)^{\rho_R}\left[\left(\frac{\pi_t^{GDP}}{\pi_*}\right)^{\psi_1}\left(\frac{\mathcal{Y}_t}{\mathcal{Y}_*}\right)^{\psi_2}\right]^{1-\rho_R}\exp\left(\sigma_r \varepsilon_t^r\right) \tag{111}$$

Aggregate Resource Constraint for CM is given by

$$Y_t = D_t^{-1}(Z_t K_t^{\alpha} H_t^{(1-\alpha)} - \mathcal{F}), \tag{112}$$

where

$$D_t = \zeta\left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1-\zeta)\left(p_t^o\right)^{-\frac{1+\lambda}{\lambda}}. \tag{113}$$

Market Clearing: The goods market in the CM clears:

$$X_t + I_t + \left(1 - \frac{1}{g_t}\right)\mathcal{Y}_t = Y_t \tag{114}$$

GDP and GDP Deflator: Prices and inflation in the DM are given by

$$P_t^{DM} = \frac{\sigma \mathcal{M}_t P_{t-1}}{q_t}, \quad \pi_t^{DM} = \frac{P_t^{DM}}{P_{t-1}^{DM}} = \frac{\mathcal{M}_t q_{t-1}}{\mathcal{M}_{t-1} q_t}\pi_{t-1}. \tag{115}$$

According to our (approximate) Fisher index the GDP deflator evolves according to

$$\pi_t^{GDP} = (\pi_t)^{(1-s_*)}(\pi_t^{DM})^{s_*}. \tag{116}$$

Real output in terms of the CM good and GDP are

$$\mathcal{Y}_t = Y_t + \frac{\sigma \mathcal{M}_t}{\pi_t}, \quad \mathcal{Y}_t^{GDP} = \mathcal{Y}_t P_t / P_t^{GDP}. \tag{117}$$

Finally, measured real money balances and (inverse) velocity in the data are given by

$$\frac{M_{t+1}}{P_t^{GDP}} = \mathcal{M}_{t+1}\frac{P_t}{P_t^{GDP}}, \quad \frac{M_{t+1}}{P_t^{GDP} Y_t^{GDP}} = \frac{\mathcal{M}_{t+1}}{(P_t^{GDP}/P_t)\mathcal{Y}_t^{GDP}} = \frac{\mathcal{M}_{t+1}}{\mathcal{Y}_t}. \tag{118}$$

A.4 Steady States

For estimation purposes it is useful to parameterize the model in terms of $\mathcal{Y}_*$, H_*, and $\mathcal{M}_*$ and solve the steady state conditions for A, B, and Z_*. Suppose q_* and K_* are given then we can solve for the following steady states recursively:

$$\begin{aligned}
R_* &= \pi_*/\beta \\
p_*^o &= \left[\frac{1}{1-\zeta} - \frac{\zeta}{1-\zeta}\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1-\iota}{\lambda}}\right]^{-\lambda} \\
D_* &= \frac{(1-\zeta)(p_*^o)^{-\frac{1+\lambda}{\lambda}}}{1-\zeta\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}}} \\
Y_* &= \mathcal{Y}_* - \sigma\mathcal{M}_*/\pi_* \\
\bar{Y}_* &= Y_* D_* \\
Z_* &= (\bar{Y}_* + \mathcal{F})/(K_*^\alpha H_*^{1-\alpha}) \\
R_*^k &= \frac{\alpha Z_* p_*^o}{1+\lambda}\left[\frac{1-\zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}}{1-\zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}}\right]^{-1}\left(\frac{H_*}{K_*}\right)^{1-\alpha} \\
W_* &= \frac{1-\alpha}{\alpha}\frac{K_*}{H_*}R_*^k \\
I_* &= \delta K_* \\
X_* &= Y_* - I_* - (1-1/g_*)\mathcal{Y}_* \\
A &= \frac{g(q_*, K_*, \chi_*, Z_*)W_*\pi_*}{\mathcal{M}_*} \\
U_*' &= A/W_* \qquad (119) \\
B &= U_*' X_*^\gamma \\
\pi_*^{DM} &= \pi_*^{GDP} = \pi_*
\end{aligned}$$

To determine q_* and K_* we solve the following equations jointly:

$$R_* = 1 + \sigma\left[\frac{\chi_* u'(q_*)}{g_q(q_*, K_*, \chi_*, Z_*)} - 1\right] \qquad (120)$$

$$1 = \beta(1 + R_*^k - \delta) - \sigma\beta\frac{\gamma(q_*, K_*, \chi_*, Z_*)}{U_*'} \qquad (121)$$

In the price-taking version, we replace (119), (120) and (121) with

$$\begin{aligned}
A &= \frac{q_* c_q(q_*, K_*, \chi_*, Z_*)W_*\pi_*}{\mathcal{M}_*} \\
R_* &= 1 + \sigma\left[\frac{\chi_* u'(q_*)}{c_q(q_*, K_*, Z_*)} - 1\right] \\
1 &= \beta(1 + R_*^k - \delta) - \sigma\beta\frac{c_k(q_*, K_*, Z_*)}{U_*'}
\end{aligned}$$

Note that from the firm's problem we have

$$\begin{aligned}
\mathcal{F}_*^{(1)} &= \left(1-\zeta\beta\pi_*\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}\right)^{-1}(p_*^o)^{-\frac{1+\lambda}{\lambda}}Y_* \\
\mathcal{F}_*^{(2)} &= \left(1-\zeta\beta\pi_*\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}\right)^{-1}(p_*^o)^{-\frac{1+\lambda}{\lambda}-1}Y_*MC_* \\
\mathcal{F}_*^{(1)} &= (1+\lambda)\mathcal{F}_*^{(2)} \\
MC_* &= \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W_*^{1-\alpha}(R_*^k)^\alpha Z_*^{-1} \\
\pi_* &= \left[(1-\zeta)\left(\pi_* p_*^o\right)^{-\frac{1}{\lambda}}+\zeta\left(\pi_*^\iota\pi_{**}^{1-\iota}\right)^{-\frac{1}{\lambda}}\right]^{-\lambda}
\end{aligned}$$

which lead to the conditions for p_*^o above. The term D_* measures the steady state price dispersion. The larger π_*/π_{**}, that is, the faster the price of the non-adjusters is eroding in real terms, the bigger D_*. Finally, in steady state the DM share of nominal output and the DM markup are given by

$$\begin{aligned}
s_* &= \frac{\sigma\mathcal{M}_*}{\sigma\mathcal{M}_*+Y_*\pi_*} \\
\text{markup}(dm) &= \frac{g(q_*,K_*,\chi_*,Z_*)}{q_*c_q(q_*,K_*,Z_*)}-1.
\end{aligned}$$

A.5 Log-Linearizations

In the subsequent presentation of the log-linearized equations we adopt the convention that we abbreviate time t expectations of a $t+1$ variable simply by a time $t+1$ subscript, omitting the expectation operator.

Firms's Problem: Marginal costs evolve according to

$$\tilde{MC}_t = (1-\alpha)\tilde{w}_t + \alpha\tilde{R}_t^k - \tilde{Z}_t.. \tag{122}$$

Conditional on capital and factor prices, the labor demand is determined according to

$$\tilde{H}_t = \tilde{K}_t + \tilde{R}_t^k - \tilde{W}_t. \tag{123}$$

Since $\mathcal{F}_t^{(1)}$ and $\mathcal{F}_t^{(2)}$ are proportional, $\tilde{\mathcal{F}}_t^{(1)} = \tilde{\mathcal{F}}_t^{(2)} = \tilde{\mathcal{F}}_t$. The remaining optimality conditions can be written as follows.

$$\begin{aligned}
\tilde{\mathcal{F}}_t &= (1-\mathcal{A})\left[-\frac{1+\lambda}{\lambda}\tilde{p}_t^o+\tilde{\mathcal{Y}}_t\right] \qquad (124)\\
&\quad +\mathcal{A}\left[-\frac{\iota}{\lambda}\tilde{\pi}_t-\frac{1+\lambda}{\lambda}\tilde{p}_t^o+\frac{1+\lambda}{\lambda}\tilde{\pi}_{t+1}+\frac{1+\lambda}{\lambda}\tilde{p}_{t+1}^o+\tilde{\mathcal{F}}_{t+1}+\tilde{\Xi}_{t+1|t}^p\right] \\
\mathcal{A} &= \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}
\end{aligned}$$

and

$$\tilde{\mathcal{F}}_t = (1-\mathcal{A})\left[-\left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}^o_t + \tilde{\mathcal{Y}}_t + \tilde{MC}_t\right] \tag{125}$$
$$+\mathcal{A}\Bigg[-\frac{\iota(1+\lambda)}{\lambda}\tilde{\pi}_t - \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}^o_t + \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{\pi}_{t+1}$$
$$+\left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}^o_{t+1} + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}^p_{t+1|t}\Bigg]$$
$$\mathcal{A} = \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}.$$

The relationship between the optimal price charged by the adjusting firms and the inflation rate is given by

$$\tilde{p}^o_t = (\mathcal{A}-1)\tilde{\pi}_t - \mathcal{A}\iota\zeta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}\tilde{\pi}_{t-1} \tag{126}$$
$$\mathcal{A} = \frac{(p^o_*)^{1/\lambda}}{1-\zeta}$$

Equations (124) to (126) determine $\tilde{\pi}_t$, $\tilde{F}_t$, and $\tilde{\pi}^o_t$.

Household's Problem: The optimality conditions for the household can be expressed as

$$\tilde{W}_t = \gamma\tilde{X}_t \tag{127}$$
$$\tilde{X}_t = \tilde{X}_{t+1} - \frac{1}{\gamma}(\tilde{R}_t - \tilde{\pi}_{t+1}) \tag{128}$$
$$\tilde{i}_t = \frac{1}{1+\beta}\tilde{i}_{t-1} + \frac{\beta}{1+\beta}\tilde{i}_{t+1} + \frac{1}{(1+\beta)S''}\tilde{\mu}_t \tag{129}$$
$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \delta\tilde{i}_t \tag{130}$$
$$\tilde{\mu}_t - \gamma\tilde{X}_t = \beta(1-\delta)\tilde{\mu}_{t+1} - \gamma\beta(1-\delta+R^k_*)\tilde{X}_{t+1} + \beta R^k_*\tilde{R}^k_{t+1} \tag{131}$$
$$+(1-\beta(1-\delta+R^k_*))\tilde{\Gamma}_{t+1}$$
$$\tilde{\mathcal{M}}_t = \tilde{g}_t + \tilde{W}_t + \tilde{\pi}_t \tag{132}$$
$$\tilde{R}_t = \frac{R_*-1+\sigma}{R_*}[\tilde{\chi}_{t+1} - \tilde{g}_{q,t+1} - \eta\frac{q_*}{(q_*+\kappa)}\tilde{q}_{t+1}] \tag{133}$$
$$\tilde{\Xi}^p_{t|t-1} = -\gamma(\tilde{X}_t - \tilde{X}_{t-1}) - \tilde{\pi}_t \tag{134}$$

Equations (127) to (134) determine wages, CM consumption, investment, capital, the shadow price of installed capital, the rental rate of capital, real money balances, the stochastic discount factor used in the firms' problem, and DM consumption.

For the price-taking version, we replace (131), (132) and (133) with

$$\tilde{\mu}_t - \gamma\tilde{X}_t = \beta(1-\delta)\tilde{\mu}_{t+1} - \gamma\beta(1-\delta+R^k_*)\tilde{X}_{t+1} + \beta R^k_*\tilde{R}^k_{t+1} \tag{135}$$
$$+(1-\beta(1-\delta+R^k_*))\tilde{c}_{k,t+1}$$
$$\tilde{\mathcal{M}}_t = \tilde{q}_t + \tilde{c}_{q,t} + \tilde{W}_t + \tilde{\pi}_t \tag{136}$$
$$\tilde{R}_t = \frac{R_*-1+\sigma}{R_*}[\tilde{\chi}_{t+1} - \tilde{c}_{q,t+1} - \eta\frac{q_*}{(q_*+\kappa)}\tilde{q}_{t+1}] \tag{137}$$

Decentralized Market: We now determine the law of motion for $\tilde{g}_{q,t}$, $\tilde{\Gamma}_t$, and $\tilde{g}_t$. In addition, we are introducing some auxiliary variables. We begin with (omitting t subscripts),

$$
\begin{aligned}
u &= \frac{(q+\kappa)^{1-\eta} - \kappa^{1-\eta}}{1-\eta} \\
u' &= (q+\kappa)^{-\eta} \\
u'' &= -\eta(q+\kappa)^{-\eta-1} \\
c &= \exp\{-\tilde{Z}\}q^{\psi}k^{1-\psi} \\
c_q &= \psi \exp\{-\tilde{Z}\}q^{\psi-1}k^{1-\psi} \\
c_k &= (1-\psi)\exp\{-\tilde{Z}\}q^{\psi}k^{-\psi} \\
c_{qq} &= \psi(\psi-1)\exp\{-\tilde{Z}\}q^{\psi-2}k^{1-\psi} \\
c_{kk} &= \psi(\psi-1)\exp\{-\tilde{Z}\}q^{\psi}k^{-\psi-1} \\
c_{qk} &= \psi(1-\psi)\exp\{-\tilde{Z}\}q^{\psi-1}k^{-\psi}
\end{aligned}
$$

which can be log-linearized as follows

$$
\begin{aligned}
\tilde{u}u_* &= \frac{q_*}{(q_*+\kappa)^{\eta}}\tilde{q} \\
\tilde{u}' &= -\eta\frac{q_*}{q_*+\kappa}\tilde{q} \\
\tilde{u}'' &= -(\eta+1)\frac{q_*}{q_*+\kappa}\tilde{q} \\
\tilde{c} &= -\psi\tilde{Z} + \psi\tilde{q} + (1-\psi)\tilde{k} \\
\tilde{c}_q &= -\psi\tilde{Z} + (\psi-1)\tilde{q} + (1-\psi)\tilde{k} \\
\tilde{c}_k &= -\psi\tilde{Z} + \psi\tilde{q} - \psi\tilde{k} \\
\tilde{c}_{qq} &= -\psi\tilde{Z} + (\psi-2)\tilde{q} + (1-\psi)\tilde{k} \\
\tilde{c}_{kk} &= -\psi\tilde{Z} + \psi\tilde{q} - (1+\psi)\tilde{k} \\
\tilde{c}_{qk} &= -\psi\tilde{Z} + (\psi-1)\tilde{q} + -\psi\tilde{k}
\end{aligned}
$$

Recall that

$$\Gamma_t = \frac{c_{k,t}g_{q,t} - c_{q,t}g_{k,t}}{g_{q,t}}$$

which implies that $\tilde{\Gamma}_t$ evolves according to

$$\tilde{g}_{q,t} + \tilde{\Gamma}_t = \frac{c_{k*}g_{q*}}{c_{k*}g_{q*} - c_{q*}g_{k*}}[\tilde{c}_{k,t} + \tilde{g}_{q,t}] - \frac{c_{q*}g_{k*}}{c_{k*}g_{q*} - c_{q*}g_{k*}}[\tilde{c}_{q,t} + \tilde{g}_{k,t}]. \tag{138}$$

Now consider the equation

$$g_t(\theta\chi u_t' + (1-\theta)c_{q,t}) = \theta\chi c_t u_t' + (1-\theta)\chi c_{q,t}u_t,$$

which can be written in log-linear form as

$$\begin{aligned} & \left[\theta\chi_* u_*' + (1-\theta)\, c_{q*}\right] g_* \tilde{g}_t \\ = \; & \theta\chi_* u_*' \left(c_* - g_*\right) \tilde{u}_t' + (1-\theta)\, \chi_* c_{q*} u_* \tilde{u}_t + (1-\theta)\, c_{q*} \left(\chi_* u_* - g_*\right) \tilde{c}_{q,t} && (139) \\ & + \theta\chi_* c_* u_*' \tilde{c} + \left[-\theta\chi_* g_* u_*' + \theta\chi_* c_* u_*' + (1-\theta)\, \chi_* c_{q*} u_*\right] \tilde{\chi}_t && (140) \end{aligned}$$

and determines $\tilde{g}_t$.

Now consider

$$g_q = \frac{\chi u' c_q [\theta\chi u' + (1-\theta)c_q] + \theta(1-\theta)(\chi u - c)\left(\chi u' c_{qq} - c_q \chi u''\right)}{[\theta\chi u' + (1-\theta)c_q]^2}$$

In log-linear form, the equation can be rewritten as

$$\begin{aligned} & g_{q*} \left[\theta\chi_* u_*' + (1-\theta)\, c_{q*}\right]^2 \tilde{g}_{q,t} \\ = \; & -\eta g_{q*} \left[\theta\chi_* u_*' + (1-\theta)\, c_{q*}\right] \left[\theta\chi_* u_*' \left(\tilde{u}_t + \tilde{\chi}_t\right) + (1-\theta)\, c_{q*} \tilde{c}_{q,t}\right] && (141) \\ & + \chi_* u_*' c_{q*} \left[\theta\chi u_*' + (1-\theta)\, c_{q*}\right] \left(\tilde{u}_t' + \tilde{\chi}_t + \tilde{c}_{q,t}\right) \\ & + \theta \left(\chi_* u_*'\right)^2 c_{q*} \left(\tilde{u}_t' + \tilde{\chi}_t\right) + \chi_* (1-\theta)\, u_*' c_{q*}^2 \tilde{c}_{q,t} \\ & + \theta (1-\theta)\, \chi_* \left(u_*' c_{qq*} - c_{q*} u_*''\right) \left[\chi_* u_* \left(\tilde{u}_t + \tilde{\chi}_t\right) - c_* \tilde{c,} t\right] \\ & + \theta (1-\theta)\, \chi_* \left(\chi_* u_* - c_*\right) u_*' c_{qq*} \left(\tilde{u}_t' + \tilde{\chi}_t + \tilde{c}_{qq,tt}\right) \\ & - \theta (1-\theta)\, \chi_* \left(\chi_* u_* - c_*\right) u_*'' c_{q*} \left(\tilde{u}_t'' + \tilde{\chi}_t + \tilde{c}_{q,t}\right). \end{aligned}$$

Moreover,

$$g_k = \frac{\theta\chi u' c_k \left[\theta\chi u' + (1-\theta)c_q\right] + \theta(1-\theta)(\chi u - c)\chi u' c_{qk}}{\left[\theta\chi u' + (1-\theta)c_q\right]^2},$$

which leads to an equation for $\tilde{g}_{k,t}$:

$$\begin{aligned} & g_{k*}[\theta\chi u_*' + (1-\theta)c_{q*}]^2 \tilde{g}_{k,t} \\ = \; & -2 g_{k*} [\theta\chi_* u_*' + (1-\theta) c_{q*}] \Big(\theta\chi_* u_*' (\tilde{u}_t \tilde{\chi}_t) + (1-\theta) c_{q*} \tilde{c}_{q,t}\Big) && (142) \\ & + \theta\chi_* u_*' c_{k*} \left[\theta\chi_* u_*' + (1-\theta)\, c_{q*}\right] \left(\tilde{u}_t' + \tilde{\chi}_t + \tilde{c}_{k,t}\right) \\ & + \left(\theta\chi_* u_*'\right)^2 c_{k*} \left(\tilde{u}_t' + \tilde{\chi}_t\right) + \chi_* \theta (1-\theta)\, u_*' c_{k*} c_{q*} \tilde{c}_{q,t} \\ & + \theta (1-\theta)\, \chi_* \left(\chi_* u_* - c_*\right) u_*' c_{qk*} \left(\tilde{u}_t' + \tilde{\chi}_t + \tilde{c}_{qk,t}\right) \\ & + \theta (1-\theta)\, \chi_* u_*' c_{qk*} \left[\chi_* u_* \left(\tilde{u}_t + \tilde{\chi}_t\right) - c_* \tilde{c}_t\right]. \end{aligned}$$

To summarize, Equations (138) to (142) determine $\tilde{\Gamma}_t$, $\tilde{g}_t$, $\tilde{g}_{q,t}$, and $\tilde{g}_{k,t}$. The first three variables appear in the characterization of the households' problem above.

Resource Constraint, Market Clearing Conditions in the CM: Aggregate output across evolves according to

$$\tilde{\dot{Y}}_t = \tilde{Y}_t + \tilde{D}_t = (1 + \mathcal{F}/\dot{Y}_*)[\tilde{Z}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{H}_t]. \tag{143}$$

and the steady state price dispersion follows

$$\tilde{D}_t = \zeta \left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}} \left[\tilde{D}_{t-1} + \frac{(1+\lambda)}{\lambda}\tilde{\pi}_t - \frac{\iota(1+\lambda)}{\lambda}\tilde{\pi}_{t-1}\right] - \frac{p_*^o(1+\lambda)(1-\zeta)}{\lambda D_*}\tilde{p}_t^o \tag{144}$$

The goods market clearing condition is of the form

$$\tilde{Y}_t = \frac{X_*}{Y_*}\tilde{X}_t + \frac{I_*}{Y_*}\tilde{I}_t + \left(1 - \frac{1}{g_*}\right)\frac{\mathcal{Y}_*}{Y_*}\mathcal{Y}_t + \frac{\mathcal{Y}_*}{Y_* g_*}\tilde{g}_t \tag{145}$$

and determines investment.

Aggregate Output and Prices, Measured Real Money Balances

In log-linear terms, inflation in the DM evolves according to

$$\tilde{\pi}_t^{DM} = \tilde{M}_t - \tilde{M}_{t-1} - (\tilde{q}_t - \tilde{q}_{t-1}) + \tilde{\pi}_{t-1}. \tag{146}$$

Since all inflation rates share the same steady state, changes in the GDP deflator are given by

$$\tilde{\pi}_t^{GDP} = (1 - s_*)\tilde{\pi}_t + s_*\tilde{\pi}_t^{DM}. \tag{147}$$

Real output in terms of the CM final good evolves according to

$$\tilde{\mathcal{Y}}_t = (1 - s_*)\tilde{Y}_t + s_*(\tilde{M}_t - \tilde{\pi}_t). \tag{148}$$

As we showed in the main text, real GDP can be expressed as

$$\tilde{\mathcal{Y}}_t^{GDP} = (1 - s_*)\tilde{Y}_t + s_*\tilde{q}_t + s_*(\tilde{\mathcal{M}}_0 - \tilde{\pi}_0 - \tilde{q}_0). \tag{149}$$

Finally, inverse velocity evolves according to

$$\widetilde{\mathcal{M}_{t+1}/\mathcal{Y}_t} = \tilde{\mathcal{M}}_{t+1} - \tilde{\mathcal{Y}}_t. \tag{150}$$

Government Policies: The monetary policy rule can be written as

$$\tilde{R}_t = \rho_R\tilde{R}_{t-1} + (1 - \rho_R)[\psi_1\tilde{\pi}_t^{GDP} + \psi_2\tilde{\mathcal{Y}}_t] + \epsilon_{R,t}. \tag{151}$$

B The MIU Model

The subsequent exposition is based on a slightly more general utility function:

$$U(x) = B\frac{x^{1-\gamma}}{1-\gamma}.$$

B.1 Equilibrium Conditions

Household's Problem: Given exogenous states, policy and prices,

$$U'(x_t) = \frac{A}{W_t} \tag{152}$$

$$1 = \beta E_t\left[\frac{U'(x_{t+1})}{U'(x_t)}\frac{R_t}{\pi_{t+1}}\right] \tag{153}$$

$$1 = \mu_t\left[1 - S\left(\frac{i_t}{i_{t-1}}\right) + \frac{i_t}{i_{t-1}}S'\left(\frac{i_t}{i_{t-1}}\right)\right] + \beta E_t\left\{\mu_{t+1}\frac{U'(x_{t+1})}{U'(x_t)}\left(\frac{i_{t+1}}{i_t}\right)^2 S'\left(\frac{i_{t+1}}{i_t}\right)\right\} \tag{154}$$

$$k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] \tag{155}$$

$$\mu_t = \beta E_t\left\{\frac{U'(x_{t+1}}{U'(x_t)}\left[R^k_{t+1} + (1-\delta)\mu_{t+1}\right]\right\} \tag{156}$$

$$\frac{U'(x_t)}{P_t} = \beta E_t\left[\frac{U'(x_{t+1})}{P_{t+1}} + \frac{\chi_{t+1}}{P_{t+1}}\left(\frac{A}{Z_*^{1/1-\alpha}}\right)^{1-\nu_m}\left(\frac{m_{t+1}}{P_{t+1}}\right)^{-\nu_m}\right] \tag{157}$$

$$\Xi^p_{t+1|t} = \frac{U'(x_{t+1})}{U'(x_t)\pi_{t+1}} \tag{158}$$

Intermediate Goods Producing Firms' Problem: Intermediate goods firms choose their capital labor ratio as a function of the factor prices to minimize costs:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R^k_t}H_t. \tag{159}$$

Firms that are allowed to change prices are choosing a relative price $p^o_t(i)$ (relative to the aggregate price level) to maximize expected profits subject to the demand curve for their differentiated product, taking the aggregate price level P_t as well as the prices charged by other firms as given, which leads to

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W_t^{1-\alpha}(R^k_t)^\alpha Z_t^{-1} \tag{160}$$

$$\mathcal{F}^{(1)}_t = (p^o_t)^{-\frac{1+\lambda}{\lambda}}Y_t + \zeta\beta\left(\pi_t^\iota\pi_{**}^{(1-\iota)}\right)^{-1/\lambda}\mathbb{E}_t\left[\left(\frac{p^o_t}{\pi_{t+1}p^o_{t+1}}\right)^{-\frac{1+\lambda}{\lambda}}\Xi^p_{t+1|t}\mathcal{F}^{(1)}_{t+1}\right] \tag{161}$$

$$\mathcal{F}^{(2)}_t = (p^o_t)^{-\frac{1+\lambda}{\lambda}-1}Y_tMC_t + \zeta\beta\left(\pi_t^\iota\pi_{**}^{(1-\iota)}\right)^{-\frac{1+\lambda}{\lambda}}\mathbb{E}_t\left[\left(\frac{p^o_t}{\pi_{t+1}p^o_{t+1}}\right)^{-\frac{1+\lambda}{\lambda}-1}\Xi^p_{t+1|t}\mathcal{F}^{(2)}_{t+1}\right] \tag{162}$$

$$\mathcal{F}^{(1)}_t = (1+\lambda)\mathcal{F}^{(2)}_t \tag{163}$$

Final Good Producing Firms' Problem: Final goods producers take factor prices and output prices as given and choose inputs $Y_t(i)$ and output Y_t to maximize profits. Free entry ensures that final good producers make zero profits and leads to

$$\pi_t = \left[(1-\zeta)\left(\pi_t p_t^o\right)^{-\frac{1}{\lambda}} + \zeta(\pi_{t-1}^{\iota}\pi_{**}^{1-\iota})^{-\frac{1}{\lambda}}\right]^{-\lambda} \tag{164}$$

Monetary Policy: The central bank supplies the quantity of money necessary to attain the nominal interest rate

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1} \left(\frac{\mathcal{Y}_t}{\mathcal{Y}_*}\right)^{\psi_2}\right]^{1-\rho_R} \exp\left(\sigma_r \varepsilon_t^r\right) \tag{165}$$

Aggregate Resource Constraint: is given by

$$Y_t = D_t^{-1}(Z_t K_t^{\alpha} H_t^{(1-\alpha)} - \mathcal{F}), \tag{166}$$

where

$$D_t = \zeta\left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1-\zeta)\left(p_t^o\right)^{-\frac{1+\lambda}{\lambda}}. \tag{167}$$

The gross domestic product of this economy is given by

$$\mathcal{Y}_t = Y_t \tag{168}$$

Market Clearing: The goods market in the CM clears:

$$X_t + I_t + \left(1 - \frac{1}{g_t}\right)\mathcal{Y}_t = Y_t \tag{169}$$

B.2 Steady States

For estimation purposes it is useful to parameterize the model in terms of $\mathcal{Y}_* = Y_*$, H_*, and $\mathcal{M}_*$ and solve the steady state conditions for A, B, and Z_*.

$$
\begin{aligned}
R_* &= \pi_*/\beta \\
p_*^o &= \left[\frac{1}{1-\zeta} - \frac{\zeta}{1-\zeta}\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1-\iota}{\lambda}}\right]^{-\lambda} \\
R_*^k &= \frac{1}{\beta} + \delta - 1 \\
D_* &= \frac{(1-\zeta)(p_*^o)^{-\frac{1+\lambda}{\lambda}}}{1-\zeta\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}}} \\
\bar{Y}_* &= Y_* D_* \\
Z_* &= (\bar{Y}_* + \mathcal{F})/(K_*^\alpha H_*^{1-\alpha}) \\
K_* &= \frac{\alpha(\bar{Y}_* + \mathcal{F})p_*^o}{(1+\lambda)R_*^k}\left[\frac{1-\zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}}{1-\zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}}\right]^{-1} \\
W_* &= \frac{1-\alpha}{\alpha}\frac{K_*}{H_*}R_*^k \\
I_* &= \delta K_* \\
X_* &= Y_* - I_* - (1-1/g_*)Y_* \\
A &= \frac{1}{\mathcal{M}_*}\left[\frac{\chi_* \pi_*^{\nu_m} W_*}{(R_*-1)Z_*^{(1-\nu_m)/(1-\alpha)}}\right]^{1/\nu_m} \\
U_*' &= A/W_* \\
B &= U_*' X_*^\gamma
\end{aligned}
$$

B.3 Log-Linearizations

We will frequently use equation-specific constants, such as $\mathcal{A}$ and $\mathcal{B}$. Variables dated $t+1$ refer to time t conditional expectations.

Firms's Problem: Marginal costs evolve according to

$$\tilde{MC}_t = (1-\alpha)\tilde{w}_t + \alpha\tilde{R}_t^k - \tilde{Z}_t. \tag{170}$$

Conditional on capital, the labor demand is determined according to

$$\tilde{H}_t = \tilde{K}_t + \tilde{R}_t^k - \tilde{W}_t \tag{171}$$

Since $\mathcal{F}_t^{(1)}$ and $\mathcal{F}_t^{(2)}$ are proportional, $\tilde{\mathcal{F}}_t^{(1)} = \tilde{\mathcal{F}}_t^{(2)} = \tilde{\mathcal{F}}_t$. The remaining optimality conditions can be written as follows.

$$\begin{aligned}
\tilde{\mathcal{F}}_t &= (1-\mathcal{A})\left[-\frac{1+\lambda}{\lambda}\tilde{p}_t^o + \tilde{\mathcal{Y}}_t\right] \\
&\quad + \mathcal{A}\left[-\frac{\iota}{\lambda}\tilde{\pi}_t - \frac{1+\lambda}{\lambda}\tilde{p}_t^o + \frac{1+\lambda}{\lambda}\tilde{\pi}_{t+1} + \frac{1+\lambda}{\lambda}\tilde{p}_{t+1}^o + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}^p_{t+1|t}\right] \\
\mathcal{A}_1 &= \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}
\end{aligned} \tag{172}$$

and

$$\begin{aligned}
\tilde{\mathcal{F}}_t &= (1-\mathcal{A})\left[-\left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}_t^o + \tilde{\mathcal{Y}}_t + \tilde{MC}_t\right] \\
&\quad + \mathcal{A}\Bigg[-\frac{\iota(1+\lambda)}{\lambda}\tilde{\pi}_t - \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}_t^o + \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{\pi}_{t+1} \\
&\quad + \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}_{t+1}^o + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}^p_{t+1|t}\Bigg] \\
\mathcal{A}_2 &= \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}.
\end{aligned} \tag{173}$$

The relationship between the optimal price charged by the adjusting firms and the inflation rate is given by

$$\begin{aligned}
\tilde{p}_t^o &= (\mathcal{A}-1)\tilde{\pi}_t - \mathcal{A}\iota\zeta\left(\frac{\pi_{**}}{\pi_*}\right)^{-(1-\iota)/\lambda}\tilde{\pi}_{t-1} \\
\mathcal{A}_p &= \frac{(p_*^o)^{1/\lambda}}{1-\zeta}
\end{aligned} \tag{174}$$

Equations (172) to (174) determine $\tilde{\pi}_t$, $\tilde{\mathcal{F}}_t$, and $\tilde{p}_t^o$.

Household's Problem The optimality conditions for the household can be expressed as

$$\tilde{W}_t = \frac{1}{\gamma}\tilde{X}_t \tag{175}$$

$$-\gamma\tilde{X}_t = -\gamma\tilde{X}_{t+1} + (\tilde{R}_t - \tilde{\pi}_{t+1}) \tag{176}$$

$$\tilde{i}_t = \frac{1}{1+\beta}\tilde{i}_{t-1} + \frac{\beta}{1+\beta}\tilde{i}_{t+1} + \frac{1}{(1+\beta)S''}\tilde{\mu}_t \tag{177}$$

$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \delta\tilde{i}_t \tag{178}$$

$$\tilde{\mu}_t - \gamma\tilde{X}_t = \beta(1-\delta)\tilde{\mu}_{t+1} - \gamma\tilde{X}_{t+1} + \beta R_*^k\tilde{R}_{t+1}^k \tag{179}$$

$$\nu_m\tilde{\mathcal{M}}_{t+1} = \gamma\tilde{X}_t + \nu_m\tilde{\chi}_{t+1} - (1-\nu_m)\tilde{\pi}_{t+1} - \frac{1}{R_*-1}\tilde{R}_t \tag{180}$$

$$\tilde{\Xi}^p_{t|t-1} = -\gamma(\tilde{X}_t - \tilde{X}_{t-1}) - \tilde{\pi}_t. \tag{181}$$

Equations (175) to (181) determine wages, consumption, investment, capital, the shadow value of installed capital, the rental rate of capital, real money balances, and the stochastic discount factor.

Resource Constraint, Market Clearing Conditions Aggregate output across evolves according to

$$\tilde{\tilde{Y}}_t = \tilde{Y}_t + \tilde{D}_t = (1+\mathcal{F}/\bar{Y}_*)[\tilde{Z}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{H}_t]. \tag{182}$$

and the steady state price dispersion follows

$$\tilde{D}_t = \zeta \left(\frac{\pi_{**}}{\pi_*} \right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}} \left[\tilde{D}_{t-1} + \frac{(1+\lambda)}{\lambda} \tilde{\pi}_t - \frac{\iota(1+\lambda)}{\lambda} \tilde{\pi}_{t-1} \right] - \frac{p^o_*(1+\lambda)(1-\zeta)}{\lambda D_*} \tilde{p}^o_t \tag{183}$$

The goods market clearing condition is of the form

$$\tilde{\mathcal{Y}}_t = \frac{X_*}{X_* + I_*} \tilde{X}_t + \frac{I_*}{X_* + I_*} \tilde{I}_t + \tilde{g}_t. \tag{184}$$

Government Policies The monetary policy rule can be written as

$$\tilde{R}_t = \rho_R \tilde{R}_{t-1} + (1 - \rho_R)[\psi_1 \tilde{\pi}_t + \psi_2 \tilde{Y}_t] + \epsilon_{R,t}. \tag{185}$$