

The Demographics of Innovation and Asset Returns*

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Abstract

We study asset-pricing implications of innovation in a general-equilibrium overlapping-generations economy. Innovation increases the competitive pressure on existing firms and workers, reducing the profits of existing firms and eroding the human capital of older workers. Thus, innovation creates a risk factor, which we call the “displacement risk.” Displacement risk is a systematic risk factor due to the lack of inter-generational risk sharing. We show that the standard aggregate consumption-based asset-pricing model must be modified to account for inter-cohort consumption differences generated by the displacement-risk factor. This new risk factor helps explain several empirical patterns in asset returns, including the existence of the growth-value factor in returns and the value premium, and the high equity premium. Our calibration results suggest that the proposed mechanism is quantitatively consistent with the data.

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1 Introduction

In this paper we explore asset-pricing implications of innovation. We concentrate on two effects of innovation. First, while innovation expands the productive capacity of the economy, it increases competitive pressure on existing firms and workers, reducing profits of existing firms and eroding the human capital of older workers. Thus, innovation creates a risk factor, which we call “displacement risk.” Second, since economic rents from innovation are captured largely by the future cohorts of inventors through the firms they create, older workers cannot use financial markets to avoid the negative effects of displacement.

We study an overlapping-generations general-equilibrium production economy with multiple intermediate goods, which are used to produce a single consumption good. Innovation creates a stochastically expanding variety of intermediate goods. Intermediate goods are partial substitutes, therefore growth in their variety intensifies competition between their producers and leads to displacement of the established firms by the new entrants. In addition, older workers are not as well adapted to the new technologies as the new cohorts of agents, which implies that innovation diminishes older worker’s human capital. Thus, there are two sides to innovation. The bright side is the increased productivity that it brings, which raises aggregate output, consumption, and wages. The dark side is the reduced wage bill and consumption share of the older agents.

The displacement risk faced by older agents is a systematic risk factor. Individual Euler equations in our model cannot be aggregated into a pricing model based solely on aggregate consumption because of the wedge between the future consumption of all agents *present currently* and the future *aggregate* consumption: the latter includes the consumption of future cohorts, but the former does not. This wedge is stochastic and driven by innovation shocks. Thus, the standard aggregate consumption-based pricing model must be augmented with the displacement-risk factor. This argument helps explain several important empirical patterns in asset returns.

First, the displacement risk factor is connected to cross-sectional differences in stock returns. We assume that existing firms participate in innovation, but some firms are more

likely to innovate than others. The more innovative firms derive a larger fraction of their value from future inventions and earn higher valuation ratios, which makes them “growth firms.” Because of their relatively high exposure to the innovation shocks, growth firms offer a hedge against displacement risk and, in equilibrium, earn lower average returns than less innovative “value firms.” Thus, heterogeneous exposure to displacement risk helps explain the positive average return premium earned by value stocks relative to growth stocks, called the value premium. Moreover, innovation shocks generate co-movement among value stocks and among growth stocks, giving rise to a value-growth factor in stock returns. We test this implication empirically. We identify innovation shocks through their effect on consumption of individual cohorts of agents and show that inter-generational differences in consumption correlate with the realized return differences between value and growth stocks.

Second, the aggregate equity premium in our model is boosted by the stock-market exposure to the displacement-risk factor. Large innovation shocks simultaneously lower the value of existing firms through increased competition, and reduce consumption of existing agents through the erosion of their human and financial wealths. As a result, agents require a higher premium to hold stocks than could be inferred from the aggregate consumption series using standard pricing models.

Third, the equilibrium interest rate in our model is lower than suggested by the aggregate consumption process and agents’ preferences. This is because individual agents’ consumption growth is lower on average and riskier than that of aggregate consumption. This property of overlapping-generation economies is noted in the seminal paper of Blanchard (1985) and emphasized recently by Gârleanu and Panageas (2007).¹

In addition to the empirical tests, we calibrate our model and verify that our empirical results are quantitatively consistent with the model’s predictions.

Our paper relates to several strands of the literature. Our model of innovation is based on Romer (1990), who studies endogenous sources of growth. We treat growth as exogenous

¹The size of this effect is magnified if we allow for some degree of catching up with the Joneses, as in Abel (1990).

and instead focus on the impact of innovation on financial asset returns.

Several papers use an overlapping-generations framework to study asset pricing phenomena, e.g., Abel (2003), Constantinides et al. (2002), Gârleanu and Panageas (2007), Gomes and Michaelides (2007), or Storesletten et al. (2007). None of these papers, however, considers the displacement risk, which is critical for our results. DeMarzo et al. (2004, 2008), discuss the importance of pecuniary externalities and complementarities for the behavior of existing agents in a setup where future endowments are non-tradable. Even though the non-tradability of future endowments is important for both these papers and ours, our model does not derive its implications from complementarities in the behavior of existing agents, but rather from limited intergenerational risk sharing coupled with increased rivalry due to the ideas introduced by new generations.

Our paper also relates to the literature that studies the cross-section of stock returns in an equilibrium framework. We contribute to this active literature, which includes Berk et al. (1999), Gomes et al. (2003), Carlson et al. (2004, 2006), Papanikolaou (2007), and Zhang (2005) among many, by providing a new approach to the value-premium puzzle. Many of the earlier papers consider single-factor models and rely on time-varying conditional betas to produce a value premium. Instead, we propose a novel source of systematic risk that generates return differences between value and growth stocks.

We also contribute to the vast literature on the equity-premium puzzle, (e.g., Mehra and Prescott (1985), Campbell and Cochrane (1999)). The displacement-risk factor helps reconcile a high equity premium with a smooth time series of aggregate consumption. We argue that the displacement risk can reconcile the historical moments of stock and bond returns with the fundamentals.

The rest of the paper is organized as follows. In Section 2, we formulate and in Section 3 we solve our model. Section 4 analyzes qualitative properties of the model, and Section 5 contains a quantitative evaluation, including empirical tests. We collect technical results and proofs in the Appendix.

2 Model

2.1 Agents' Preferences and Demographics

We consider a model with discrete and infinite time: $t \in \{\dots, 0, 1, 2, \dots\}$. The size of the population is normalized to 1. At each date a mass λ of agents, chosen randomly, die, and a mass λ of agents are born, so that the population remains constant. An agent born at time s has preferences of the form

$$E_s \sum_{t=s}^{s+\tau} \beta^{(t-s)} \frac{\left(c_{t,s}^\psi \left(\frac{c_{t,s}}{C_t} \right)^{1-\psi} \right)^{1-\gamma}}{1-\gamma}, \quad (1)$$

where τ is the (geometrically distributed) time of death, $c_{t,s}$ is the agent's consumption at time t , C_t is aggregate consumption at time t , $0 < \beta < 1$ is a subjective discount factor, $\gamma > 0$ is the agent's relative risk aversion, and ψ is a constant between 0 and 1. Preferences of the form (1) were originally proposed by Abel (1990), and are commonly referred to as “keeping-up-with-the-Joneses” preferences. These preferences capture the idea that agents derive utility from both their own consumption and their consumption relative to per capita consumption. When $\psi = 1$, these preferences specialize to the standard constant-relative-risk-aversion preferences. In general, for $\psi \in [0, 1]$ agents place a weight ψ on their own consumption (irrespective of what others are consuming) and a weight $1 - \psi$ on their consumption relative to average consumption in the population. Our qualitative results hold independently of the keeping-up-with-the-Joneses feature, which only helps at the calibration stage, by reducing the value of the interest rate.

A standard argument allows us to integrate over the distribution of the stochastic times of death and re-write preferences of the form (1) as

$$E_s \sum_{t=s}^{\infty} [(1-\lambda)\beta]^{(t-s)} \frac{\left(c_{t,s}^\psi \left(\frac{c_{t,s}}{C_t} \right)^{1-\psi} \right)^{1-\gamma}}{1-\gamma}. \quad (2)$$

2.2 Technology

Final-Good Firms

There is a representative (competitive) final-good producing firm that produces the single final good using two categories of inputs: a) labor and b) a continuum of intermediate goods. Specifically, the production function of a final good producing firm is

$$Y_t = Z_t (L_t^F)^{1-\alpha} \left[\int_0^{A_t} \omega_{j,t} (x_{j,t})^\alpha dj \right]. \quad (3)$$

In equation (3) Z_t denotes a stochastic productivity process, L_t^F captures the efficiency units of labor that enter into the production of the final good, A_t is the number of intermediate goods available at time t , and $x_{j,t}$ captures the quantity of intermediate good j that is used in the production of the final good. The constant $\alpha \in [0, 1]$ controls the relative weight of labor and intermediate goods in the production of the final good, while $\omega_{j,t}$ captures the relative importance placed on the various intermediate goods. We specify $\omega_{j,t}$ as

$$\omega_{j,t} = \left(\frac{j}{A_t} \right)^{\chi(1-\alpha)}, \quad \chi \geq 0 \quad (4)$$

For $\chi = 0$, the production function (3) is identical to the one introduced by the seminal Romer (1990) paper in the context of endogenous growth theory. Our version is slightly more general, since the factor weights $\omega_{j,t}$, which are increasing functions of the intermediate good index j , allow the production function to exhibit a “preference” for more recent intermediate goods. As we show below, this feature confers an additional degree of control over the individual-firm profit variability. Even though our aim here is not to explain the sources of growth in the economy, the production function (3) is useful for our purposes for several reasons: a) Innovation, i.e., an increase in the variety of intermediate goods (A_t) helps increase aggregate output; b) There is rivalry between existing and newly arriving intermediate goods, since increases in A_t strengthen the competition among intermediate goods producers, and c) Heterogeneity in intermediate, rather than final, goods is technically convenient, since we can keep one unit of the final good as numeraire throughout. An

exact illustration of the first two properties is provided in the next section, where we solve the model.

The productivity process Z_t follows a random walk (in logs) with drift μ and volatility σ_ε :

$$\log(Z_{t+1}) = \log(Z_t) + \mu + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2). \quad (5)$$

At each point in time t , the representative final-good firm chooses L_t^F and $x_{j,t}$ (where $j \in [0, A_t]$) so as to maximize its profits

$$\pi_t^F = \max_{L_t^F, x_{j,t}} \left\{ Y_t - \int_0^{A_t} p_{j,t} x_{j,t} dj - w_t L_t^F \right\}, \quad (6)$$

where $p_{j,t}$ is the price of intermediate good j and w_t is the prevailing wage (per efficiency unit of labor).

Intermediate-Goods Firms

The intermediate goods $x_{j,t}$ are produced by monopolistically competitive firms that own non-perishable blueprints to the production of intermediate good $x_{j,t}$. Each intermediate good is produced by a single firm, while a single firm may produce a countable number of intermediate goods. In analogy to Romer (1990), we assume that the production of the intermediate good $j \in [0, A_t]$ requires one unit of labor (measured in efficiency units) per unit of intermediate good produced, so that the total number of efficiency units of labor used in the intermediate-goods sector is

$$L_t^I = \int_0^{A_t} x_{j,t} dj. \quad (7)$$

The price $p_{j,t}$ of intermediate good j maximizes the profits of the intermediate good producer, taking the demand function of the representative final good firm $x_j(p_{j,t}; p_{j' \neq j}, w_t) \equiv \arg \max_{x_{j,t}} \pi_t^F$ as given. To simplify notation, we shall write $x_{j,t}(p_{j,t})$ instead of $x_{j,t}(p_{j,t}; p_{j' \neq j}, w_t)$. Production of the intermediate good j generates profits²

²Any firm produces a countable number (and hence zero-measure set) of intermediate goods. Hence,

$$\pi_t^I(j) = \max_{p_{j,t}} \{(p_{j,t} - w_t) x_{j,t}(p_{j,t})\}. \quad (8)$$

2.3 Arrival of New Intermediate Goods and New Agents

New Products

The number of intermediate goods A_t expands over time as a result of innovations. Given our focus on asset pricing, we assume that the innovation process is exogenous for simplicity. The number of intermediate goods in our economy follows a random walk (in logs):³

$$\log(A_{t+1}) = \log(A_t) + u_{t+1}. \quad (9)$$

The increment u_{t+1} is i.i.d. across time for simplicity. To ensure its positivity, we assume that u_{t+1} is Gamma distributed with parameters (k, ν) .

The intellectual property rights for the production of the $\Delta A_{t+1} = A_{t+1} - A_t$ new intermediate goods belong either to arriving agents or to existing firms. We assume that a fraction $\kappa \in [0, 1]$ of the total value of the new blueprints is allocated uniformly to the arriving agents, while the complementary fraction $1 - \kappa$ is introduced, also uniformly, by established firms and hence belongs indirectly to existing agents, who own these firms.

Value and Growth Firms

Agents who arrive endowed with ideas start a continuum of firms that produce the respective intermediate goods, and introduce them into the stock market.

it can ignore any feedback effects of the pricing of its intermediate good j on the demand for the other intermediate goods it produces. Hence, maximization of the overall firm's profits amounts to maximizing the profits from each intermediate good separately.

³We choose a random-walk specification in order to ensure that aggregate consumption is a random walk. The assumption of a random walk implies that — for a given u_{t+1} — the increase in production is proportional to the current level of production. This assumption is routinely used in the literature and is sometimes referred to as “standing on the shoulders of giants”. See e.g. Jones (1997).

Fractions of new blueprint value accruing to:

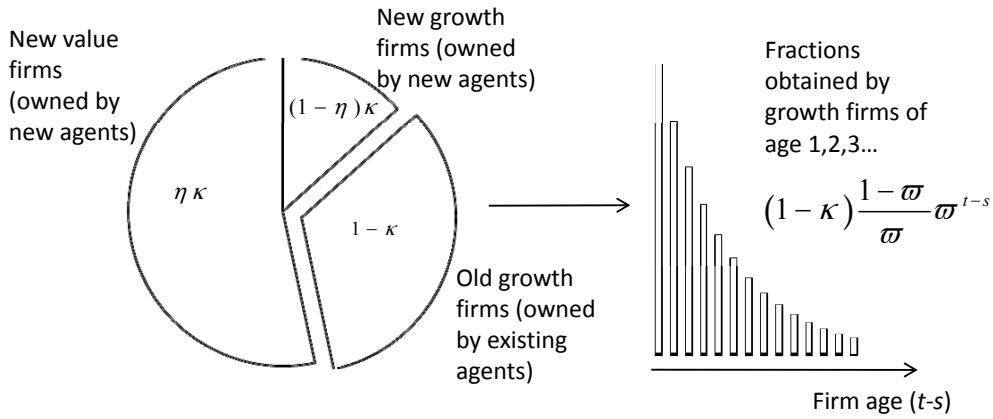


FIGURE 1: Illustration of the allocation of new blueprint value

These new firms come in two types, depending on whether they are capable of obtaining blueprints for new intermediate goods in the future or not. The first type are “value firms,” which cannot obtain any blueprints in future periods. They are only entitled to a fraction $\eta\kappa$ of the value of blueprints introduced at time $t + 1$, where $\eta \in (0, 1]$. The other kind of firms are “growth” firms. They are entitled to a fraction $(1 - \eta)\kappa$ of the value of new blueprints for intermediate goods, but they also receive a fraction of the new blueprints in future periods. Specifically, in period n , growth firms born at $s \in (-\infty, n - 1]$ obtain a fraction

$$(1 - \kappa) \left(\frac{1 - \varpi}{\varpi} \right) \varpi^{n-s} \quad (10)$$

of the value of the ΔA_n new blueprints. To simplify matters, we assume that there are no intra-cohort differences between growth firms and any two growth firms of the same cohort obtain the same value of blueprints in any given period. The geometric decay in the fraction of new blueprints that accrues to a given growth firm as a function of its age ensures that asymptotically the market capitalization of any firm goes to zero as a fraction of the aggregate market capitalization.

Workers

New business owners make up a fraction $\phi \in (0, 1)$ of new agents. The rest of the new agents are workers. Workers start life with a constant endowment of hours \bar{h} , which they supply inelastically. The ratio of efficiency units of labor to hours is affected by two factors: a) age and experience, and b) skill obsolescence. To capture the first notion, we assume that the ratio of labor efficiency units to hours changes geometrically with age at the rate δ , so that in the absence of skill obsolescence, the ratio of a worker's endowment of efficiency units at time $t > s$ to the respective endowment at the time of birth s is given by $(1 + \delta)^{t-s}$. The stylized assumption of geometric change is not crucial for our results, and we show how to relax it, when we calibrate the model.

In the real world younger workers are likely to be more productive in the presence of increased technological complexity than older workers. One potential reason is that their education gives them the appropriate skills for understanding the technological frontier. By contrast, older workers are likely to be challenged by technological advancements. In Appendix B we present a simple vintage model of the labor market that introduces imperfect substitution across labor supplied by agents born at different times. To expedite the presentation of the main results, in this section we assume that labor is a homogenous good and that workers' endowment of efficiency units depreciates in a way that replicates the outcome of the more elaborate model in Appendix B.

Specifically, we assume that a worker's total supply of efficiency units of labor is given by $\bar{h}(1 + \delta)^{t-s} q_{t,s}$ with

$$\log(q_{t+1,s}) = \log(q_{t,s}) - \rho u_{t+1}, \tag{11}$$

and $\rho \geq 0$. This specification captures the idea that advancements of the technological frontier act as depreciation shocks to the productivity of old agents. Such shocks generate cohort effects in individual consumption and income, which are present in historical data. We present empirical evidence in Section 5.1.

We normalize the initial endowment of efficiency units so that the aggregate number of

efficiency units in the economy is constant. In particular, we set

$$q_{s,s} = 1 - (1 - \lambda)(1 + \delta)e^{-\rho u_s} \quad (12)$$

and $\bar{h} = \frac{1}{\lambda}$. We assume that $(1 - \lambda)(1 + \delta) \leq 1$. as a result, the number of per-worker efficiency units, $L_t / (1 - \phi)$, is always equal to 1, and hence $\bar{h}(1 + \delta)^{t-s} q_{t,s}$ can be interpreted as the fraction of total wages that accrues to workers born at time s .⁴

2.4 Asset Markets

Agents face complete markets. At each point in time they are able to trade in zero net supply Arrow-Debreu securities contingent on the realization of future shocks $\varepsilon_{t+\tau}$ and $u_{t+\tau}$, $\forall \tau > 0$. This assumption implies the existence of a stochastic discount factor ξ_t , so that the time- s value of a claim paying D_t at time t is given by $E_s \frac{\xi_t}{\xi_s} D_t$.

Finally, agents have access to annuity markets as in Blanchard (1985). (We refer the reader to that paper for details). The joint assumptions of complete markets *for existing agents* and frictionless annuity markets simplifies the analysis considerably, since in a complete market with annuities an agent's feasible consumption choices are constrained by a single intertemporal budget constraint. For a worker, that intertemporal budget constraint is given by

$$E_s \sum_{t=s}^{\infty} (1 - \lambda)^{t-s} \left(\frac{\xi_t}{\xi_s} \right) c_{t,s}^w = E_s \sum_{t=s}^{\infty} (1 - \lambda)^{t-s} \left(\frac{\xi_t}{\xi_s} \right) w_t q_{t,s} \bar{h} (1 + \delta)^{t-s}, \quad (13)$$

⁴Note that our assumptions imply

$$\frac{L_t}{(1 - \phi)} = \lambda \sum_{s=-\infty \dots t} [(1 - \lambda)(1 + \delta)]^{t-s} q_{t,s},$$

so that iterating forward once to obtain $\frac{L_{t+1}}{(1 - \phi)}$ and using (11) and (12) yields

$$\begin{aligned} \frac{L_{t+1}}{(1 - \phi)} &= \lambda \sum_{s=-\infty \dots t+1} (1 - \lambda)(1 + \delta)^{t+1-s} q_{t+1,s} \\ &= (1 - \lambda)(1 + \delta) \left(\frac{L_t}{1 - \phi} \right) e^{-\rho u_{t+1}} + \lambda \bar{h} q_{t+1,t+1}. \end{aligned}$$

Setting $q_{t+1,t+1}$ as in (12) implies $\frac{L_{t+1}}{(1 - \phi)} = \frac{L_t}{(1 - \phi)} = 1$.

where $c_{t,s}^w$ denotes the time- t consumption of a representative worker who was born at time s . The left-hand side of (13) represents the present value of a worker's consumption, while the right hand side represents the present value of her income. Similarly, letting $c_{t,s}^e$ denote the time- t consumption of a representative inventor who was born at time s , her intertemporal budget constraint is

$$E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s} \right) c_{t,s}^e = \frac{1}{\lambda\phi} V_{s,s}, \quad (14)$$

where $V_{s,s}$ is the time- s total market capitalization of new firms created at time s . The left-hand side of Equation (14) is the present value of a representative inventor's consumption, while the right-hand side is the value of all new firms divided by the mass of new business owners, $\lambda\phi$. To determine the total market value of firms created at time s , let $\Pi_{j,s}$ be the present of profits from production of intermediate good j :

$$\Pi_{j,s} = \left[E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s} \right) \pi_{j,t}^I \right]. \quad (15)$$

The total market capitalization of all new firms can then be written as

$$V_{s,s} = \kappa \int_{A_{s-1}}^{A_s} \Pi_{j,s} dj + \left(\frac{1-\varpi}{\varpi} \right) E_s \sum_{t=s+1}^{\infty} \left(\frac{\xi_t}{\xi_s} \right) (1-\kappa) \varpi^{t-s} \int_{A_{t-1}}^{A_t} \Pi_{j,t} dj. \quad (16)$$

The first term in equation (16) is the value of the blueprints for the production of new intermediate goods that are introduced by new firms (both "growth" and "value" firms) at time s . Similarly, the latter term captures the value of "growth opportunities," namely, the value of blueprints to be received by growth firms in future periods.

2.5 Equilibrium

The definition of equilibrium is standard. To simplify notation, we let ϕ^e and ϕ^w denote the fraction of entrepreneurs and workers in the population, respectively, so that

$$\phi^i = \begin{cases} \phi & \text{if } i = e \\ 1 - \phi & \text{if } i = w \end{cases}. \quad (17)$$

An equilibrium is defined as follows.

Definition 1 *An equilibrium is a collection of adapted stochastic processes $\{x_{j,t}, L_t^F, c_{t,s}^w, c_{t,s}^e, \xi_t, p_{j,t}, w_t\}$ with $j \in [0, A_t]$ and $t \geq s$ such that*

1. *(Consumer optimality): Given ξ_t , the process $c_{t,s}^w$ (respectively, $c_{t,s}^e$) solves the optimization problem (2) subject to the constraint (13) (respectively, constraint (14)).*
2. *(Profit maximization) The prices $p_{j,t}$ solve the optimization problem (8) given L_t^F , $x_{j' \neq j,t}$, w_t and $L_t^F, x_{j,t}$ solve the optimization problem (6) given $p_{j,t}, w_t$.*
3. *(Market clearing). Labor and goods markets clear*

$$L_t^F + L_t^I = (1 - \phi) \quad (18)$$

$$\lambda \sum_{s=-\infty}^t \sum_{i \in \{w,e\}} (1 - \lambda)^{t-s} \phi^i c_{t,s}^i = Y_t. \quad (19)$$

Conditions 1 and 2 are the usual optimality conditions. Condition 3 requires that the total labor demand $L_t^F + L_t^I$ equals total labor supply $1 - \phi$. Finally, the last condition requires that aggregate consumption be equal to aggregate output.

3 Solution

3.1 Equilibrium Output, Profit, and Wages

Consider the intermediate-goods markets. We first derive the demand curve of the final-good firm for the intermediate input j at time t . Maximizing (6) with respect to $x_{j,t}$, we obtain

$$x_{j,t} = L_t^F \left[\frac{p_{j,t}}{\omega_{j,t} Z_t \alpha} \right]^{\frac{1}{\alpha-1}}. \quad (20)$$

Substituting this expression into (8) and maximizing over $p_{j,t}$ leads to

$$p_{j,t} = \frac{w_t}{\alpha}, \quad (21)$$

while combining (20) and (21) yields

$$x_{j,t} = L_t^F \left[\frac{w_t}{\omega_{j,t} Z_t \alpha^2} \right]^{\frac{1}{\alpha-1}}. \quad (22)$$

Next, consider the labor markets. Maximizing (6) with respect to L_t^F gives the first order condition

$$w_t L_t^F = (1 - \alpha) Y_t. \quad (23)$$

Substituting (22) into (3) and then into (23) and simplifying yields

$$w_t = (\alpha^2)^\alpha \left(\frac{1 - \alpha}{1 + \chi} \right)^{1-\alpha} Z_t A_t^{1-\alpha}. \quad (24)$$

We substitute equation (24) into (22) and then into (18) to obtain

$$x_{j,t} = \frac{1 + \chi}{A_t} \left(\frac{j}{A_t} \right)^\chi \frac{(1 - \phi) \alpha^2}{\alpha^2 + 1 - \alpha} \quad (25)$$

$$L_t^F = \frac{1 - \alpha}{\alpha^2 + 1 - \alpha} (1 - \phi). \quad (26)$$

Finally, aggregate output is given by

$$Y_t = \frac{(\alpha^2)^\alpha \left(\frac{1-\alpha}{1+\chi} \right)^{1-\alpha}}{\alpha^2 + 1 - \alpha} (1 - \phi) Z_t A_t^{1-\alpha}, \quad (27)$$

which we derive by combining (25) and (26) inside (3). The number of intermediate inputs (A_t) in equation (27) is raised to the power $1-\alpha$. This means that aggregate output is increasing in the number of intermediate inputs. However, the sensitivity of output to the number of inputs depends on the elasticity of substitution between different varieties of intermediate goods. For instance, as α approaches 1, intermediate goods become perfect substitutes, and larger variety of intermediate goods leads to more competition among firms and lower profits for existing intermediate-good producers without changing the overall productive capacity of the economy.

We now compute the income share of labor and the profits of firms. Total payments to labor $w_t (1 - \phi)$ are simply equal to a fraction $(\alpha^2 + 1 - \alpha)$ of output Y_t , which follows from (24) and (27). Because of constant returns to scale in the production of final goods, the

profits of the representative final goods firm are $\pi_t^F = 0$. The profits from the production of intermediate good j are

$$\pi_{j,t}^I = (1 + \chi) \left(\frac{j}{A_t} \right)^x \frac{Y_t}{A_t} \alpha (1 - \alpha), \quad (28)$$

which is obtained by combining (25) and (21) with (8) and (24). Note that the time series of profits $\pi_{j,t}^I$ is not cointegrated with aggregate output Y_t , since the variety of intermediate goods A_t grows over time and hence asymptotically $\pi_{j,t}^I/A_t \rightarrow 0$. As a result, dividends of an *individual firm* are not co-integrated with aggregate output, which is intuitive because of the constant arrival of competing firms. In comparison, aggregate profits are a constant fraction of total output, as $\int_0^{A_t} \pi_{j,t}^I dj = \alpha (1 - \alpha) Y_t$. This equality follows immediately from the fact that, in a general-equilibrium framework, total income shares must add up to aggregate output: $w_t (1 - \phi) + \pi_t^F + \int_0^{A_t} \pi_{j,t}^I dj = Y_t$.

3.2 The Stochastic Discount Factor

To determine the stochastic discount factor ξ_t , we recall that, since agents face complete markets *after their birth*, a consumer's lifetime consumption profile can be obtained by maximizing (2) subject to a single intertemporal budget constraint (constraint [13] if the agent is a worker and constraint [14] if the agent is an inventor). Attaching a Lagrange multiplier to the intertemporal budget constraint, maximizing with respect to $c_{t,s}^i$, and relating the consumption at time t to the consumption at time s for a consumer whose birth date is s gives

$$c_{t,s}^i = c_{s,s}^i \left(\frac{C_t^{(1-\psi)(1-\gamma)}}{C_s^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s} \right)^{-\frac{1}{\gamma}} \quad \text{for } i \in \{e, w\}. \quad (29)$$

From this equation, the aggregate consumption at any point in time is

$$C_t = \lambda \sum_{s=-\infty}^t \sum_{i \in \{w, e\}} (1 - \lambda)^{t-s} \phi^i c_{s,s}^i \left(\frac{C_t^{(1-\psi)(1-\gamma)}}{C_s^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s} \right)^{-\frac{1}{\gamma}}, \quad (30)$$

where ϕ^i was defined in (17). Iterating forward once to obtain C_{t+1} and then using (30) gives

$$C_{t+1} = (1 - \lambda) C_t \left(\beta^{-1} \frac{C_{t+1}^{(1-\psi)(1-\gamma)}}{C_t^{(1-\psi)(1-\gamma)}} \frac{\xi_{t+1}}{\xi_t} \right)^{-\frac{1}{\gamma}} + \lambda \sum_{i \in \{w, e\}} \phi^i c_{t+1, t+1}^i. \quad (31)$$

Dividing both sides of (31) by C_t , solving for $\frac{\xi_{t+1}}{\xi_t}$ and noting that $C_t = Y_t$ in equilibrium leads to

$$\frac{\xi_{t+1}}{\xi_t} = \beta \left(\frac{Y_{t+1}}{Y_t} \right)^{-1+\psi(1-\gamma)} \left[\frac{1}{1-\lambda} \left(1 - \lambda \sum_{i \in \{w, e\}} \phi^i \frac{c_{t+1, t+1}^i}{Y_{t+1}} \right) \right]^{-\gamma}. \quad (32)$$

To obtain an intuitive understanding of equation (32) it is easiest to focus on the case $\psi = 1$, so that agents have standard CRRA preferences. In this case, the stochastic discount factor consists of two parts. The first part is $\beta \left(\frac{Y_{t+1}}{Y_t} \right)^{-\gamma}$, which is the standard expression for the stochastic discount factor in an (infinitely-lived) representative-agent economy. The second part — the term contained inside square brackets in equation (32) — gives the proportion of output at time $t+1$ that accrues to agents already alive at time t . To see this, note that only a proportion $1 - \lambda$ of existing agents survive between t and $t + 1$, and that the arriving generation claims a proportion $1 - \lambda \sum_{i \in \{w, e\}} \phi^i \frac{c_{t+1, t+1}^i}{Y_{t+1}}$ of aggregate output. The combination of the two parts yields the consumption growth between t and $t + 1$ of the surviving agents.

Equation (32) states an intuitive point: since (ignoring consumption externalities) only agents alive at time t are relevant for asset pricing, it is exclusively their consumption growth that determines the stochastic discount factor, not the aggregate consumption growth. We elaborate on this point further in the next section.

To conclude the computation of equilibrium, we need to obtain an expression for the term inside square brackets on the right-hand side of (32). This can be done by using the intertemporal budget constraints (14) and (13). Proposition 1 in the Appendix shows that

$$1 - \lambda \sum_{i \in \{w, e\}} \phi^i \frac{c_{t+1, t+1}^i}{Y_{t+1}} = v(u_{t+1}; \theta^e, \theta^w, \theta^b),$$

with

$$v(u_{t+1}; \theta^e, \theta^w, \theta^b) \equiv 1 - \theta^e \alpha (1 - \alpha) \left(\kappa (1 - e^{-(1+\chi)u_{t+1}}) + \left(\frac{1 - \varpi}{\varpi} \right) \theta^b \right) - \theta^w (\alpha^2 + 1 - \alpha) (1 - (1 - \lambda)(1 + \delta) e^{-\rho u_{t+1}}), \quad (33)$$

and $\theta^e, \theta^b, \theta^w$ are three appropriate constants obtained from the solution of a system of three nonlinear equations in three unknowns. Given the interpretation of $v(u_{t+1}; \theta^e, \theta^w, \theta^b)$ as the fraction of consumption that accrues to new agents, we shall refer to it as the “displacement factor”.

4 Qualitative Properties of the Equilibrium

The consumption externality aside, the stochastic discount factor in our model depends only on the consumption of existing agents, as we show above using equation (32). Thus, the standard version of the consumption CAPM (CCAPM), which relies on the aggregate consumption series, does not hold. This is intuitive, since the total consumption of existing agents at a future date is not equal to the *aggregate* consumption at that date, which includes consumption of the agents born in the interim. In this section, we explore qualitative implications of our model for asset returns.

4.1 Equity Premium without Aggregate Risk

To highlight the departure from the standard CCAPM in our model, we consider a limiting case of the model with no aggregate consumption risk. Specifically, suppose that $\sigma = 0$, $\rho > 0$, and let α approach 1. Equation (27) implies that the volatility of aggregate output, and aggregate consumption, approaches zero. Then, according to the standard CCAPM, risk premia must vanish in the limit. This is not the case in our model, as shown in the following lemma.

Lemma 1 *Assume that $\sigma = 0$, $\rho > 0$, $\kappa = 1$, and*

$$1 > \beta (1 - \lambda)^\gamma e^{\mu\psi(1-\gamma) + \frac{1}{2}\sigma^2\psi^2(1-\gamma)^2} \quad (34)$$

$$1 \leq \beta (1 + \delta)^{-\gamma} e^{\mu\psi(1-\gamma) + \frac{1}{2}\sigma^2\psi^2(1-\gamma)^2} E[e^{\rho\gamma u_{t+1}}]. \quad (35)$$

Then an equilibrium exists. Letting R_t be the return of any stock,

$$\lim_{\alpha \rightarrow 1} \text{Var}(\Delta Y_{t+1}) = 0 \quad (36)$$

$$\lim_{\alpha \rightarrow 1} \frac{\partial (\xi_{t+1}/\xi_t)}{\partial u_{t+1}} > 0 \quad (37)$$

$$\lim_{\alpha \rightarrow 1} \{E(R_t) - (1 + r^f)\} > 0. \quad (38)$$

Inequalities (34)–(35) define the set of admissible parameters and distribution functions. These conditions are necessary and sufficient for the existence of an equilibrium.

The intuition behind Lemma 1 is straightforward. While the volatility of aggregate consumption vanishes as α approaches 1, the volatility of existing agents' consumption does not. As α approaches 1, intermediate inputs behave more and more like perfect substitutes. This implies that new innovations have a vanishing effect on aggregate output, and instead lead to redistribution from old to young firms and from old to young agents (since $\rho > 0$). Thus, innovation shocks (u_t) are systematic consumption shocks from the perspective of existing agents, but they are not aggregate shocks in the traditional sense. Since profits of existing firms are exposed to the innovation shocks (u_t), stock returns of existing companies are correlated with consumption growth of existing agents and therefore command a risk premium.

While the limiting case $\alpha = 1$ is a special case of the model,⁵ the results in Lemma 1 illustrate why the standard aggregate CCAPM relationship can understate the risks associated with investing in stocks.

⁵A caveat behind Lemma 1 is that in the limit $\alpha = 1$ the profits of intermediate goods firms disappear. Hence, even though the rate of return on a stock is well defined in the limit (because rates of return are not affected by the level of dividends and prices), the limiting case $\alpha = 1$ is of limited practical relevance. However, it has theoretical interest, because it illustrates in a simple way the asset pricing implications of the wedge between aggregate consumption and existing agents' consumption.

4.2 The Cross-Section of Stock Returns

The distinction between consumption of current agents and aggregate consumption is important for understanding the cross-sectional patterns in stock returns, particularly the value premium. Assume henceforth that $\kappa < 1$, so that “growth firms” receive a fraction of new blueprints over time. The price-to-earnings ratio (P/E) for a typical value firm is $\Phi \equiv \frac{\Pi_{j,t}^I}{\pi_{j,t}^I}$, where j is the index of any intermediate good in $[0, A_t]$, and Π_t^I is given by (15). The ratio does not depend on the index j , and thus all value firms have the same P/E, regardless of which intermediate goods they produce. Since the increments to the (log) stochastic discount factor and the increments to (log) profits of value firms are independent across time, Φ is a constant.⁶ For any admissible set of parameter values, the stock price of a typical growth firm is given by the following lemma.

Lemma 2 *The (end of) period- t value of the representative “growth firm” created at time s is given by*

$$P_{t,s} = \alpha(1 - \alpha)Y_t \left[(\Phi - 1)N_{t,s} + (1 - \varpi)\varpi^{t-s}\frac{\theta^b}{\varpi}\Phi \right], \quad (39)$$

where

$$N_{t,s} = (1 - \eta)\kappa \left(\frac{A_s}{A_t} \right)^{1+\chi} (1 - e^{-(1+\chi)u_s}) + \sum_{n=s+1}^t (1 - \varpi)(1 - \kappa)\varpi^{n-(s+1)} \left(\frac{A_n}{A_t} \right)^{1+\chi} (1 - e^{-(1+\chi)u_n}).$$

The first term inside the square brackets in equation (39) is the value of all the blueprints that the growth firm has received since its creation, or its *assets in place*, while the second term is the value of the future blueprints it will receive, or the *present value of its growth opportunities*. Thus, the return on a typical growth stock is a weighted average of the return on assets in place and the return on growth opportunities. Specifically, based on (39), the gross return on the representative growth firm R_{t+1}^g at time $t+1$ is given by the dividends from

⁶We show this formally as part of the proof of Proposition 1.

all the blueprints collected by the firm up to and including period $t + 1$, $\alpha(1 - \alpha)Y_{t+1}N_{t+1,s}$, and the end-of-period price $P_{t+1,s}$, all divided by the beginning-of-period price $P_{t,s}$:

$$R_{t+1,s}^g \equiv \frac{\alpha(1 - \alpha)Y_{t+1}N_{t+1,s} + P_{t+1,s}}{P_{t,s}}.$$

Define the gross return of assets in place, which is the same as the return of a value firm in our model, R_{t+1}^a , and the gross return of growth opportunities, R_{t+1}^o , as

$$R_{t+1}^a = \left(\frac{\Phi}{\Phi - 1} \right) \left(\frac{\pi_{j,t+1}^I}{\pi_{j,t}^I} \right), \quad \forall j \in [0, A_t], \quad (40)$$

$$R_{t+1}^o = \varpi \left(\frac{Y_{t+1}}{Y_t} \right) \frac{(1 - \kappa)(1 - e^{-(1+\chi)u_{t+1}}) + \theta^b}{\theta^b}. \quad (41)$$

Then, the rate of return on any growth firm can be expressed as

$$R_{t+1,s}^g = (1 - w_{t,s}^o) R_{t+1}^a + w_{t,s}^o R_{t+1}^o, \quad (42)$$

where $w_{t,s}^o$ is the relative weight of growth opportunities in the value of the firm, and is obtained from Lemma 2 as

$$w_{t,s}^o = \frac{(1 - \varpi) \varpi^{t-s} \frac{\theta^b}{\varpi} \Phi}{(\Phi - 1) N_{t,s} + (1 - \varpi) \varpi^{t-s} \frac{\theta^b}{\varpi} \Phi}.$$

The value of assets in place and the value of growth opportunities of a firm have identical exposures to the productivity shocks ϵ_t , but different exposures to the innovation shocks u_t . Specifically, combining (28) and (27) with (40) and (41), we obtain that the return on assets in place has a negative loading on the innovation shock, $\frac{\partial R_{t+1}^a}{\partial u_{t+1}} < 0$, while the return on the growth-opportunity component of the firm value has a positive loading, $\frac{\partial R_{t+1}^o}{\partial u_{t+1}} > 0$. Thus, we conclude that there is a “value factor” in our model, i.e., a diversified portfolio with long positions in value stocks and short positions in growth stocks is exposed to a source of systematic risk, namely, the innovation shocks.

If consumption of existing agents falls in response to innovation, then their marginal utility loads positively on innovation shocks, and hence $\frac{\partial(\xi_{t+1}/\xi_t)}{\partial u_{t+1}} > 0$. Then, in equilibrium, value stocks earn a higher average rate of return than growth stocks — in other words,

our model produces a value premium.⁷ Intuitively, this is because the growth opportunities embedded in growth stocks act as a hedge against innovation shocks and hence drive down the expected return on growth stocks. Thus, the rationale behind the value premium in our model is quite different from the explanations proposed previously.⁸ While in most of these models the conditional CAPM holds, and value stocks earn higher average return because of their higher exposure to the aggregate market risk factor, in our model there exists a distinct risk factor affecting the return differential between value and growth stocks. We explore the empirical implication that this risk factor is driven by innovation shocks in Section 5.

4.3 Relationship to CAPM

We now consider the relationship between the equilibrium stochastic discount factor in our model and the CAPM. Without the consumption externality in preferences ($\psi = 1$), the CAPM relationship holds in our model with respect to the *total* wealth of existing agents, which includes both their stock holdings and their human capital. The reason is that the ratio of consumption to total wealth is constant in our model, and hence a reasoning similar to the one in Section 3.2 implies that when $\psi = 1$, the stochastic discount factor can be expressed in terms of the growth rate of total wealth of existing agents.⁹ Since the two components of total wealth (financial wealth and human capital) are not perfectly correlated, the stock market cannot be used as a proxy for total wealth. This well-known critique of the empirical implementations of the CAPM applies, at a theoretical level, within our model.

⁷Equation (37) of Lemma 1 shows that, if α is sufficiently close enough to 1, then value stocks earn higher average returns than growth stocks. While Lemma 1 assumes that $\sigma = 0$, equation (37) continues to hold when $\sigma > 0$, with a few minor modifications of the technical conditions of the lemma.

⁸A representative sample of papers employing time varying-single factor conditional beta models includes Berk et al. (1999), Gomes et al. (2003), Carlson et al. (2004, 2006), and Zhang (2005)

⁹More generally, when $\psi < 1$, the stochastic discount factor is an exponential affine function of the aggregate consumption growth and the wealth growth of existing agents.

5 Quantitative Evaluation

In this section we explore an original implication of our model that individual consumption exhibits cohort effects that are related to the cross-section of stock returns. We derive this result within the model and perform empirical tests using microeconomic data and cohort analysis.

5.1 Cohort Effects and Asset Returns

Econometric specification

According to the model, an individual agent's consumption can be decomposed into cohort effects (a_s), time effects (b_t), and individual-specific effects (ε^i , $i \in \{e, w\}$):

$$\log c_{t,s}^i = a_s + b_t + \varepsilon^i, \quad (43)$$

where, according to (29),

$$a_s = \sum_{j \in \{e, w\}} \phi^j \log c_{s,s}^j + \frac{1}{\gamma} \log (C_s^{(1-\psi)(1-\gamma)} \beta^{-s} \xi_s), \quad (44)$$

$$b_t = -\frac{1}{\gamma} \log (\xi_t \beta^{-t} C_t^{(1-\psi)(1-\gamma)}), \quad (45)$$

$$\varepsilon_s^i = \log c_{s,s}^i - \sum_{j \in \{e, w\}} \phi^j \log c_{s,s}^j. \quad (46)$$

Equation (43) provides a basic testable implication of our model. In a model with unrestricted transfers between altruistically linked generations, every agent's consumption is a constant proportion of the aggregate consumption regardless of her birth date.¹⁰ Consequently, cohort effects are zero in such a model. By contrast, in our model consumption exhibits non-zero cohort effects. The following lemma derives the dynamic behavior of these cohort effects according to the model.

¹⁰Such a model is isomorphic to the standard infinitely-lived representative-agent model.

Lemma 3 For any $T \geq 1$, the cohort effect in individual consumption satisfies

$$a_{s+T} - a_s = - \sum_{i=1}^T \log \left(\frac{v(u_{s+i})}{1-\lambda} \right) + z_{s+T} - z_s, \quad (47)$$

where z_s is a series of independently and identically distributed random variables defined by

$$z_s = (1 - \phi) \log \left(\frac{c_{s,s}^w}{Y_s} \right) + \phi \log \left\{ \frac{c_{s,s}^e}{Y_s} \right\}.$$

Equation (47) implies that cohort effects should be non-stationary according to our model, since the first term on the right hand side of (47) is a random walk with drift. The permanent shocks to the consumption cohort effects (increments to the random walk component in (47)) reveal the unobserved displacement factor $v(u_s)$. One can estimate the variance of these permanent shocks, namely, the variance of $\log \left(\frac{v(u_s)}{1-\lambda} \right)$, by forming an estimate of the long-run variance of first differences of the cohort effects Δa_s — for instance, using a Newey-West heteroscedasticity- and autocorrelation-consistent variance estimator.

Furthermore, our model implies a testable relationship between consumption cohort effects and stock returns. According to the model, the innovation shocks u_t affect both the displacement factor $v(u_t)$ and the return spread between growth and value stocks. Equations (40) and (41), together with (28) and (27), imply that the return spread between growth opportunities and assets in place, R_{t+1}^o/R_{t+1}^a , is an increasing function of u_{t+1} . Hence, for any growth firm whose value contains a fraction w_t^o of growth opportunities, we obtain $R_{t+1}^g/R_{t+1}^a = 1 + w_t^o \left(\frac{R_{t+1}^o}{R_{t+1}^a} - 1 \right) = f(u_{t+1})$, for an increasing function $f(\cdot)$. Finally, letting $l(x)$ be defined as $l(x) \equiv -\log(v(f^{-1}(x)))$, the consumption cohort effects can be related to the cross-sectional differences in stock returns as

$$a_{s+T} - a_s = \sum_{i=1}^T l \left(\log \frac{R_{s+i}^g}{R_{s+i}^a} \right) + T \log(1-\lambda) + z_{s+T} - z_s. \quad (48)$$

(we use equation (47) to establish this relationship). Since the function $v(u_t)$ is decreasing in u_t and the function $f^{-1}(x)$ is increasing in x , $l(x)$ is an increasing function. Consequently, the long-run covariance of Δa_s and $\log(R_s^g/R_s^a)$ should be non-negative.

Empirical results

We use CEX data from 1981-2003 provided on the NBER website¹¹ to estimate consumption cohort effects. We obtain a measurement of the consumption of each household over four quarters, the year corresponding to the first observation quarter, and the age of the head of household at that time. We define the cohort of households who were “born” in a given year as all those households whose head was twenty years old in that year. Given these measurements, it is possible to estimate equation (43).

Equation (43) takes the model’s predictions literally and omits some important effects present in the data. Consumption in the model only features time and cohort effects, but no age effects. One would expect the age effects to be present in the data either because of borrowing constraints early in life, or because of changes in the consumption patterns of households over the life cycle. For these reasons, we estimated (43) allowing for age effects. We also included a control for (log) household size.¹²

Linear trends in age, cohort, and time effects cannot be identified separately if age effects are included in (43).¹³ However, the exact identification of the linear trends is immaterial for our purposes. As has been shown in the literature, it is possible to estimate differences in differences of cohort effects ($a_{s+1} - a_s - (a_s - a_{s-1})$) without any normalizing assumptions

¹¹The CEX data on the NBER website were compiled by Ed Harris and John Sabelhaus. See http://www.nber.org/ces_cbo/Cexfam.pdf for a description of the data. We provide more details on the data in the appendix.

¹²As a robustness check, we also adjust for family size by dividing by the average family equivalence scales reported in Fernandez-Villaverde and Krueger (2007). Our empirical findings remain unchanged.

¹³Some of the literature addresses this problem by following Deaton and Paxson (1994) and making the normalizing assumption that the time effects add up to zero and are orthogonal to the time trend. In our model, the time effects b_t follow a random walk and hence such an assumption is not appropriate.

and even after including a full set of age dummies.¹⁴ This statement implies that there exists a function of time a_s^* such that, for any normalizing assumptions, the estimated cohort effects \widehat{a}_s are given by $\widehat{a}_s = \beta_0 + \beta_1 s + a_s^*$. (The coefficients β_0 and β_1 are not identified and their magnitude depends on the normalizing assumptions.)

According to our model, the de-trended cohort effects, a_s^* , are non-zero. Hence, the first hypothesis we test is that $a_s^* = 0$. The three columns of Table 1 report the results of estimating equation (43) including 1) no age effects, 2) parametric age effects, and 3) a full set of age dummies. The model with parametric age effects is fitted by assuming that age effects are given by a function $h(\text{age})$ which we parameterize with a cubic spline having knots at ages 33, 45, and 61. The first row reports the results from a Wald test that the de-trended cohort effects are zero, namely, that $\widehat{a}_{s+1} - \widehat{a}_s - (\widehat{a}_s - \widehat{a}_{s-1}) = 0$ for all s . The second row reports the associated p -values. As can be seen, the data strongly reject the hypothesis that cohort effects are either non-existent or given by a deterministic linear trend, and instead offers evidence of imperfect inter-generational risk sharing.

We document the magnitude of variation of cohort effects in consumption and their co-movement with asset returns in Tables 2 and 3. These estimates help evaluate the economic significance of cross-cohort variation in consumption, and we use them to calibrate the model in Section 5.2.

The first two rows of Table 2 present estimates of the volatility of consumption cohort effects under different ways of controlling for age effects. The first row contains estimates of the standard deviation¹⁵ of the first differences in cohort effects ($\widehat{a}_{s+1} - \widehat{a}_s$). As we discussed

¹⁴See McKenzie (2006) for a proof. The easiest way to see why, is to allow for age effects in equation (43), consider the resulting equation $\log c_{t,s}^i = a_s + b_t + \gamma_{t-s} + \varepsilon^i$, and observe that

$$E \log c_{t+1,s+1}^i - E \log c_{t+1,s}^i - (E \log c_{t,s}^i - E \log c_{t,s-1}^i) = a_{s+1} - a_s - (a_s - a_{s-1}),$$

where $E()$ denotes the time t mean log consumption of an agent who belongs to cohort s . Replacing expectations with the respective cross sectional averages shows how the differences in differences of cohort effects ($a_{s+1} - a_s - (a_s - a_{s-1})$) can be estimated in the data.

¹⁵Since cohort effects are estimated rather than observed, we use only cohorts from 1927-1995 for the calculations in Table 2, because cohorts prior to 1927 and after 1995 are not sufficiently populated. With this

	No age effects	Parametric Age Effects	Age Dummies
Wald Test $\Delta a_{s+1} - \Delta a_s = 0$	31.3	4.21	4.25
p-value	0.000	0.000	0.000
Observations	52245	52245	52245
R-squared	0.373	0.382	0.384

TABLE 1: Results from regression of log consumption expenditure on time dummies (one dummy for each quarter), a control for $\log(\text{fam.size})$ and various specifications of cohort and age effects. Cohort effects are included via cohort dummies. In the first specification the regression does not contain age effects, while the second specification allows age effects parameterized via a cubic spline. The third specification allows for a full set of age dummies. The Wald test refers to the test that $a_{s+1} - a_s - (a_s - a_{s-1}) = 0$ for all s . Standard errors are computed using a robust covariance matrix clustered by cohort and quarter. The CEX data are from 1980-2003 and include observations on cohorts as far back as 1911.

above, *permanent shocks* to consumption cohort effects are closely related to the displacement shocks in our model. The second row reports estimates of the standard deviation of permanent shocks to consumption cohort effects. Here we fit an ARIMA (1,1,1) model to the estimated cohort effects \hat{a}_s and use the methods of Beveridge and Nelson (1981) to estimate the standard deviation of the permanent component of \hat{a}_s . We report two additional estimates of the standard deviation of permanent shocks. The third row contains Newey-West estimates of the long-run variance of the first differences in \hat{a}_s using 10-year autocovariance lags. In the fourth row, we report the standard deviation of differences between consumption

choice of sample, the minimal cohort has 199 observations, the first quartile of cohorts has 521 observations and the median cohort has 657 observations. Accordingly, cohorts are sufficiently well populated so that our variance estimate of the first differences of cohorts is unlikely to be materially affected by measurement error. We would also like to point out that our estimates of the variance of *permanent* components of cohort effects are less affected by (i.i.d) measurement error than the variance of first differences, because heteroscedasticity- and autocovariance-consistent variance estimators control for the moving-average error structure introduced by the noisy measurement of first differences.

	Param. Age Effects	Age Dummies
Std. Dev. (Cohort–Lagged Cohort)	0.030	0.030
Std. Dev. (Perm. Component (Newey West))	0.020	0.020
Std. Dev. (Perm. Component (Beveridge Nelson))	0.021	0.023
Std. Dev. (Perm. Component $\sqrt{10}$ (10-year Aveg))	0.019	0.018
$\frac{\text{Cov}(\text{coh.diffs}; \text{FF})}{\text{Var}(\text{coh.diffs})}$ (Newey West)	3.39	3.92
$\frac{\text{Cov}(10\text{-year coh.diffs}; 10\text{-year FF})}{\text{Var}(10\text{-year coh.diffs})}$	2.93	3.43
Observations	68	68

TABLE 2: Various moments of the permanent components of the estimated cohort effects.

cohorts evaluated at ten-year intervals, normalized by $\sqrt{10}$. All three alternative estimates of the volatility of the permanent component of the cohort effects are similar. Consumption cohorts exhibit substantial variability, which is comparable to the volatility of aggregate consumption growth over the same period. This shows that displacement risk should have economically significant impact on pricing of risk in the economy.

We now relate increments in consumption cohort effects to cross-sectional differences in stock returns. The solid line in Figure 2 depicts the estimated cohort effects. As can be seen from the figure, these cohort effects are persistent, in line with the results reported in the first two rows of Table 2. The dashed line depicts $\sum_{i=1}^T \log(R_{t+i}^g/R_{t+i}^a)$, where we have used the negative of the logarithmic gross return associated with the HML factor of Fama and French (1992) as a measure of $\log(R_{t+i}^g/R_{t+i}^a)$ and have removed a linear trend¹⁶. According to equation (48), the consumption cohort effects should be co-trending with the sum of an appropriate non-linear increasing function $l(\log(R_{t+1}^g/R_{t+1}^a))$. Assuming that $l(\cdot)$ is reasonably well approximated by an affine first order Taylor expansion, cohort effects and cumulative (log) returns on a growth-value portfolio should be co-trending, as the picture suggests.

¹⁶We report the cohort effects from 1927 onward, since data on the Fama-French HML factor are available from 1927 onward. We also report results up to 1995 because of the sparsity of data on cohorts post 1995.

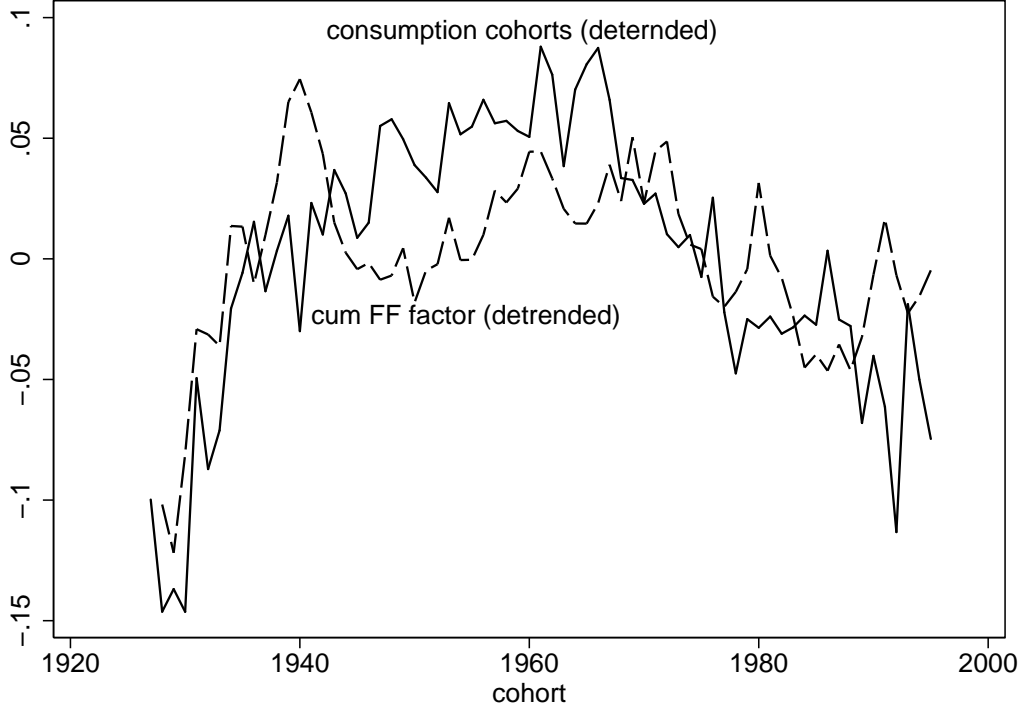


FIGURE 2: Consumption cohort effects and cumulative returns on a growth-value portfolio (negative of the HML factor) after removing a constant time trend from both series and multiplying the latter series by a scalar to fit in one scale. A full set of age dummies was used in estimating the consumption cohort effects.

To quantify the displacement-risk exposure of stocks, we estimate the covariance between the innovations to the permanent components of consumption cohorts, a_s , and the permanent components of the stock-return differential between growth and value stocks, $\sum_{i=1}^s \log(R_{t+i}^g/R_{t+i}^a)$, to which we refer as the long-run covariance. According to Equation (48), this long-run covariance is a measure of $\text{cov}(\log(R_t^g/R_t^a), \log v(u_t))$, i.e., it is a measure of the displacement-risk exposure of the HML factor. We estimate the long-run covariance using a multivariate Newey-West estimate¹⁷ of the covariance between the growth-value returns, defined as the negative of the logarithmic gross return on the HML factor, and the first

¹⁷We use 10 lags in the computation.

differences of the estimated cohort effects. The fifth row of Table 2 reports this covariance, normalized by the long-run variance of the consumption cohort effects as obtained in the second row of the table. As a robustness check, we also report in the final row of the table the results from computing the covariance of 10-year consumption cohort differences and 10-year cumulative returns on the growth-value returns, normalized by the variance of 10-year consumption-cohort differences. To interpret these numbers, note that since the standard deviation of the permanent component of cohort effects is about 0.02 and the respective number for the HML factor is about 0.12, the implied correlation between the permanent components of the two series is $3.92 \times 0.02/0.12 = 0.65$. Therefore, the HML factor has significant displacement-risk exposure.

Table 3 reports further results on the relationship between consumption cohort effects and stock returns. This table shows that the displacement-risk exposure can help explain differences in average stock returns across the book-to-market deciles. Specifically, the first row in Table 3 reports the average difference between the (log) gross return on the first nine book-to-market decile portfolios and the respective return on the 10th-decile portfolio. The second row of Table 3 reports the long-run covariance between these return differences and the increments of the permanent component of (log) consumption cohort effects, normalized by the long-run variance of the latter. This is the same measure of displacement-risk exposure as reported for the HML factor in the fifth row in Table 2. Stocks in low book-to-market deciles (growth stocks) have lower average returns than stocks in high book-to-market deciles (value stocks), which is a well-known value premium. The last row of the table shows that displacement-risk exposure decreases across the book-to-market deciles, which means that growth stocks offer a hedge against the displacement risk.

In addition to testing the empirical implications of the model for consumption and stock returns, we also report empirical evidence that relates to the model's mechanism. In particular, since the displacement risk in the model is generated by shocks to agent's human capital, we estimate cohort effects in individual income and relate them to the stock market returns, in the same manner as we do with consumption cohort effects. Applying a similar

1 -10	2-10	3-10	4-10	5-10	6-10	7-10	8-10	9-10
-0.033	-0.020	-0.023	-0.028	-0.015	-0.011	-0.016	0.005	0.009
6.293	7.162	5.609	4.995	5.125	4.184	3.090	3.474	2.636

TABLE 3: Average return differences between book-to-market decile portfolios sorted along their long-run covariances with consumption cohort innovations. The first row reports average annual return differences. Label $n-10$ denotes the average difference between the (log) gross return on the n^{th} decile portfolio and the 10^{th} decile portfolio. The second row reports the long-run covariance between the return differences and innovations in the permanent component of (log) consumption cohort effects, normalized by the long-run variance of the latter. Covariances and variances are computed using the Newey-West approach with 10 lags. The data on return differentials are from the website of K. French (annual 1927-2007).

argument to the one that led to equation (43), equations (11) and (12) imply the presence of cohort effects in income data that should be correlated with the cumulative return of a growth-value portfolio. Figure 3 is analogous to Figure 2, with consumption replaced by agents' disposable income net of dividends, rents, and interest. This picture confirms that, in agreement with the model, the qualitative properties that hold for consumption cohort effects also hold for earned income cohort effects.

5.2 Calibration

Our empirical results suggest that the key predictions of the model are qualitatively consistent with the data. In this section we assess whether the model can account quantitatively for the empirical relationships between asset returns, aggregate consumption growth, and individual consumption cohort effects.

Our parameter choices are summarized in Table 4. The values of μ and σ are chosen to match the moments of aggregate consumption growth. The parameter α controls the share of profits in aggregate income in the model, according to equation (21). We set $\alpha = 0.8$,

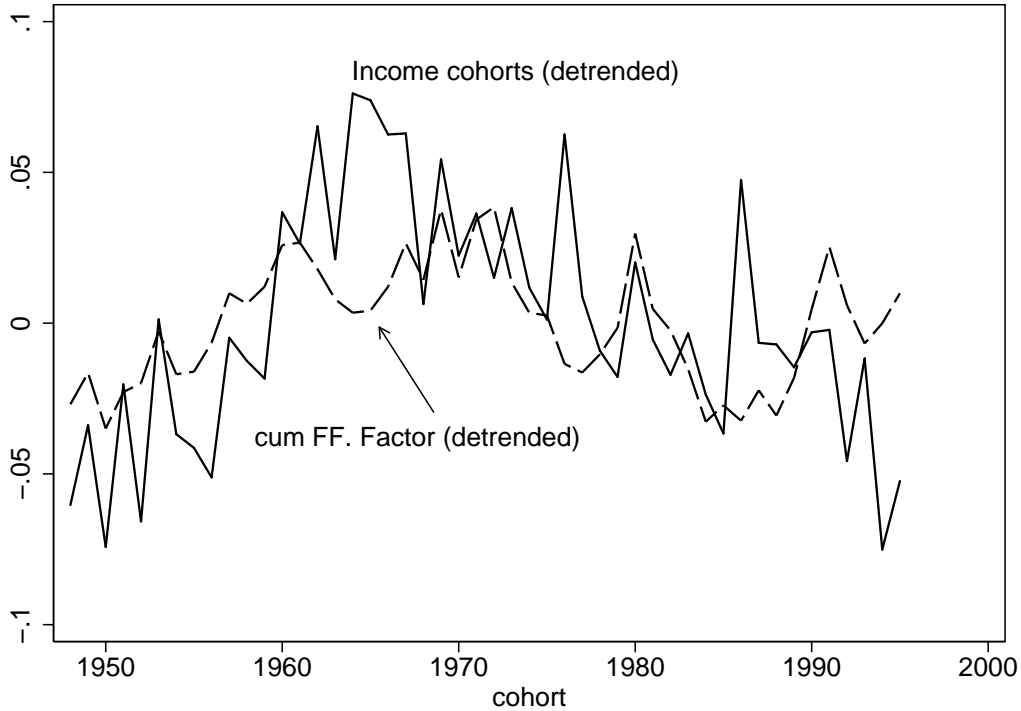


FIGURE 3: Earned (log) income cohort effects and cumulative returns on a growth-value portfolio (negative of the HML factor) after removing a constant time trend from both series and multiplying the latter series by a scalar to fit on one scale. A full set of age dummies was used in estimating the income cohort effects. The regression was estimated with post-war cohorts (1947 onward) to avoid discontinuities in earned income that arise at the retirement age of 65.

which implies a profit share of 16%. In yearly NIPA data for the U.S. since 1929, the average share of (after depreciation) profits and interest payments is about 15% of national income, or 18% if one imputes that 1/3 of proprietor's income is due to profits.¹⁸ The parameter λ

¹⁸Since in our model there is no financial leverage, it seems appropriate to combine dividend and interest payments. Moreover, it also seems appropriate to deduct depreciation from profits, because otherwise the relative wealths of agents e and w would be unduly affected by a quantity that should not be counted as income of either. We note here that our choice of a profit share of 16 percent is consistent with the real business cycle literature, which assumes a capital share (i.e. profits prior to depreciation) of 1/3 and

is chosen to capture the arrival of new agents. In post-war data, the average birth rate is about 0.016. Immigration rates are estimated to be between 0.002 – 0.004, which implies an overall arrival rate of new agents between 0.018 and 0.02. We take the time-discount factor to be close to 1, since in an overlapping generations model the presence of death makes the “effective” discount factor of agents equal to $\beta(1-\lambda)$. Given a choice of $\lambda = 0.02$, the effective discount rate is 0.98, which is a standard choice in the literature. The constant ψ influences the growth rate of agents’ marginal utilities, and hence is important for the determination of interest rates. We choose $\psi = 0.5$ in order to approximately match observed interest rates. On behavioral grounds, this assumption implies that an individual places equal weights on his own consumption and on his consumption relative to the aggregate.

In the real world, income is hump shaped as a function of age, whereas in the model age effects are assumed to follow a geometric trend. Fortunately, it is straightforward to account for arbitrary age-earnings profiles, after observing that any path of earnings over the life cycle matters for general-equilibrium outcomes only to the extent that it affects the present value of income at birth.¹⁹ For this reason we use the age-earnings profiles that we observe in the data and we calibrate δ so that the present value of income using the age-earnings profile in the data and the present value of income using a simple geometric trend with parameter δ coincide inside the model. Specifically, we use the estimated age-(log) earnings profiles of Hubbard et al. (1994) and determine δ so that

$$\begin{aligned} E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{w_t}{w_s}\right) (1+\delta)^{t-s} \left(\frac{A_t}{A_s}\right)^{-\rho} \\ = E_s \sum_{t=s}^{\infty} \Lambda_{t-s} G(t-s) \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{w_t}{w_s}\right) \left(\frac{A_t}{A_s}\right)^{-\rho}, \end{aligned}$$

deducts investment from gross profits to obtain dividends. Since in stochastic steady state, investment and depreciation are typically close to each other, the share of net output that accrues to equity holders is approximately equal to the number we assume here.

¹⁹This is due to the fact that agents are not subject to borrowing constraints, so that agents’ optimal consumption paths only depend on the present value of their initial endowments. Accordingly, the stochastic discount factor depends on assumptions of age-earnings profiles only to the extent that they affect the present value of agents’ endowments at birth.

β	0.999	k	0.25
ψ	0.5	ν	0.05
μ	0.015	ρ	0.9
σ	0.032	κ	0.9
α	0.8	χ	4
λ	0.018	ω	0.87
δ	0.012	η	0.9

TABLE 4: Baseline parameters used in the calibration.

where Λ_{t-s} is an agents' survival probability at age $t - s$ obtained from the National Center for Health Statistics and $G(t - s)$ is the age-(log) earnings profile, as estimated by Hubbard et al. (1994).

Innovation shocks u_t follow a Gamma distribution with parameters k and ν . Parameters ρ and χ control the exposures of labor and dividend income to the shock u_t . We choose k , ν , ρ , and χ jointly to match a) the volatility of the permanent component of consumption cohort effects as reported in Table 2, b) the volatility of the permanent component of income cohort effects,²⁰ c) the volatility of aggregate dividends, and d) the correlation between aggregate dividends and aggregate consumption.

The parameter κ controls the proportion of growth opportunities owned by existing firms, which are therefore tradeable, while ϖ controls the term structure of existing firms' growth opportunities (high ϖ means that growth opportunities are front-loaded). As a consequence, these two parameters jointly determine the aggregate price-to-earnings ratio, as well as the properties of growth firms. We therefore calibrate them to the aggregate price-to-earnings ratio and the long-run covariance between HML returns and changes in the permanent component of consumption cohort effects. We chose this covariance as a target in calibration

²⁰We obtain the permanent component of income cohort effects by using earned log income on the left-hand side of Equation (43), estimating the resulting cohort effects and isolating their permanent component, as we did for consumption.

because of its importance in determination of the value premium.

Parameter η affects only the relative weight of assets in place and growth opportunities in firms' values, and thus the cross-sectional dispersion of valuation ratios. We calibrate it to match the spread in price-to-earnings ratios between the top and the bottom price-to-earnings deciles of firms.

We treat the risk-aversion coefficient γ as a free parameter and examine the model's ability to match a number of moments of asset returns and fundamentals for a range of values of γ . Since we are attempting to match more moments than parameters, it is impossible to obtain an exact fit, but the model fits the empirical moments quite well. As can be seen in Table 5, with $\gamma = 10$ the model can match about 66% of the equity premium and about 80% of the value premium. As γ increases to 12, the model does well in almost all dimensions. In interpreting these results, it should be noted that the model has no financial leverage, which, as Barro (2006) argues, implies that the unlevered equity premium produced by the model should account for two thirds of the levered equity premium estimated empirically. Moreover, in the model, there is no time-variation in interest rates, stock return volatility, and conditional equity premium. Therefore, it is not surprising that the model needs relatively high levels of risk aversion to match the data. However, even in the absence of time-varying conditional moments of returns, levels of risk aversion around 10 explain a non-trivial fraction of asset-pricing data. Therefore, the evidence in Table 5 suggests that the model's mechanisms are quantitatively powerful enough to match the salient moments of asset returns and macroeconomic fundamentals.

5.3 Discussion

Inspecting the mechanism

In this section, we attribute the quantitative performance of our model to its key ingredients.

Our model produces large equity and value premia for three reasons. First, current agents' consumption growth is more volatile than aggregate consumption growth because of the displacement risk. Second, current firms' dividends are more sensitive to the displacement

	Data	$\gamma = 10$	$\gamma = 12$	$\gamma = 15$
Aggregate (log) Consumption Growth rate	0.017	0.017	0.017	0.017
Aggregate (log) Consumption Volatility	0.033	0.032	0.032	0.032
Riskless Rate	0.010	0.022	0.015	0.014
Equity premium	0.061	0.040	0.051	0.062
Aggregate Earnings / Price	0.075	0.103	0.108	0.119
Dividend Volatility	0.112	0.10	0.101	0.101
Correl. (divid. growth, cons.growth)	0.2	0.189	0.189	0.189
Std ($\Delta\alpha_s^{\text{perm}}$)	0.023	0.024	0.024	0.023
$\frac{\text{cov}(\Delta\alpha_s^{\text{perm}}, \log R^g - \log R^a)}{\text{var}(\Delta\alpha_s^{\text{perm}})}$	3.92	4.226	4.378	4.598
Std (Δw_s^{perm})	0.022	0.023	0.023	0.023
Earnings / Price 90th Perc.	0.120	0.11	0.118	0.132
Earnings / Price 10th Perc.	0.04	0.041	0.039	0.041
Average Value premium	0.081	0.064	0.081	0.097
Std (Value Premium)	0.120	0.104	0.105	0.105
E ($\log R^g - \log R^a$)		0.102	0.121	0.141

TABLE 5: Data and model moments for different values of risk aversion γ . Data on consumption, the riskless rate, the equity premium, and dividends per share are from Campbell and Cochrane (1999). Data on the aggregate E/P ratio are from the long sample (1871-2005) on R. Shiller’s website. The E/P for value and growth firms are the respective E/P ratios of firms in the bottom and top book-to-market deciles from Fama and French (1992). The value premium is computed as the difference in value weighted returns of stocks in the top and bottom book-to-market deciles, available from the website of Kenneth French. Std ($\Delta\alpha_s^{\text{perm}}$) denotes the standard deviation of the permanent component of consumption cohort effects as estimated in Table 2. Std (Δw_s^{perm}) refers to the cohort effects of earned income. E ($\log R^g - \log R^a$) is the expected return difference between assets in place and growth opportunities.

risk factor than current agents' consumption. Third, there is co-skewness between current firms' dividend growth and current agents' consumption growth.

A simple back-of-the-envelope calculation helps illustrate the magnitude of each factor. Taking logarithms of the pricing kernel in equation (32), using (5), (27), and the definition of $v(u_{t+1})$ in equation (33) leads to

$$\Delta \log \xi_{t+1} + \text{const} = (-1 + \psi(1 - \gamma))\varepsilon_{t+1} - \gamma v(u_{t+1}) + (-1 + \psi(1 - \gamma))(1 - \alpha)u_{t+1}. \quad (49)$$

At the same time, the stochastic component of the aggregate dividend growth equals $\varepsilon_{t+1} + h(u_{t+1})$ for some function h .²¹ Given our choice of parameters $\psi = 0.5$, $\gamma = 10$ and $\sigma = 0.032$, the standard deviations of the first term in (49) is 0.18; the standard deviation of $v(u_{t+1})$ is 0.023, so that of the second term is 0.23; and the last term has a standard deviation of 0.03. Thus, the standard deviation of the sum of the last two terms is approximately 0.2. The standard deviation of dividend growth is approximately 0.1, and that of ε is 0.032, which means that $h(u_{t+1})$ has a standard deviation of $\sqrt{0.1^2 - 0.032^2} = 0.095$. Since the price-to-dividend ratio is constant, the volatility of market returns is the same as the volatility of dividend growth.

If ε_{t+1} and u_{t+1} (and therefore $v(u_{t+1})$, approximately) were jointly normally distributed, then the equity premium would equal approximately $0.18 \times 0.032 + 0.2 \times 0.095 = 0.025$. The difference between this number and the equity premium of 0.04 in our base-case calibration owes to the fact that shock u_{t+1} is not Gaussian, making the consumption growth of existing agents and the stock market returns co-skewed.

This back-of-the-envelope calculation shows that the high equity premium in our model is *not* due to the fact that consumption of existing agents is excessively volatile. The total volatility of existing agents' consumption is approximately equal to 0.039, which is not far from the aggregate volatility of 0.032.

Our model generates high volatility of stock market returns despite the low volatility of

²¹According to equation (28), $h(x) = -(1 + \chi)x$ for the existing value firms. However, since the dividends of existing growth firms also include the dividends from the blueprints that are created within the period, h is generally different.

consumption growth. This is explained by the joint dynamics of dividends and consumption in our model. Specifically, future dividends of existing firms are not co-integrated with future aggregate consumption, or even existing agents' consumption. Because of this lack of co-integration, dividend growth can be much more volatile than consumption growth, with both driven by permanent shocks. Nevertheless, the long-run dynamics of dividends and consumption are mutually consistent. Dividends of existing firms become a negligible fraction of aggregate consumption over time, while the aggregate dividends paid by all firms at any point in time are a constant fraction of aggregate consumption.

Our model not only produces a sizeable equity premium, but also an interest rate that is constant and low. The reasons are that a) current agents's consumption has a slightly lower mean growth rate (1.5%) than aggregate consumption (1.7%) and is more volatile (3.9%) than aggregate consumption (3.2%), and b) external habit formation (captured by ψ) implies that agents' marginal utility of consumption declines more slowly than suggested solely by risk aversion and consumption growth.

More general endowment processes

We simplify some aspects of the model for tractability. One of the stylized assumptions is that innovating agents receive their blueprints "at birth." In reality, it takes time to start a new firm, and each cohort of agents does not innovate simultaneously. Moreover, innovation shocks u_t are more likely to follow a moving average process rather than being independent, as we assume. We provide a simple example to illustrate why such frictions and perturbations of the baseline model are unlikely to affect our conclusions about the long-run properties of the model-implied asset returns.

Suppose that all agents are born as workers with an initial endowment of labor efficiency units of $\bar{h}(1 - \phi)q_{s,s}$. Furthermore, suppose that a fraction ϕ of them become entrepreneurs in the second period of their lives and permanently drop out of the workforce, whereas the ones that remain workers have an endowment of labor efficiency units equal to the baseline

model from the second period of their lives onward, namely $\bar{h}(1+\delta)^{t-s}q_{t,s}$ for all $t \geq s+1$.²² Finally, assume that agents can only access financial markets in the second period of their lives, while in the first period they consume their wage income. These assumptions capture the idea that an agent’s “birth” cohort and the date at which that agent innovates may not coincide. Moreover, exclusion from markets captures in a stylized manner the idea that the agent cannot smooth consumption between the “birth” date and the innovation arrival date.

Repeating the argument of Section 3.2, the equilibrium stochastic discount factor in this modified setup is

$$\frac{\xi_{t+1}}{\xi_t} = \beta \left(\frac{Y_{t+1}}{Y_t} \right)^{-1+\psi(1-\gamma)} \hat{v}(u_{t+1}, u_t)^{-\gamma},$$

where

$$\hat{v}(u_{t+1}, u_t) = (1-\lambda)^{-1} \left(1 - \frac{\lambda y_t}{C_t} \right)^{-1} \left(1 - \lambda(1-\lambda) \sum_{i \in \{w,e\}} \phi^i \frac{C_{t+1,t}^i}{C_{t+1}} + \lambda \frac{y_{t+1}}{C_{t+1}} \right)$$

and y_t denotes an agent’s initial wage income. Furthermore, the same reasoning as in the proof of Lemma 2 implies that the variance of the permanent component of (log) consumption cohort effects equals $Var(\hat{v}(u_{t+1}, u_t))$.

This simple example illustrates the fact that the frictions affecting agents’ life-cycle of earnings change the transitory dynamics of cohort effects, returns, and the stochastic discount factor. Such frictions do not alter our main qualitative conclusion that the permanent component of cohort effects reflects the permanent component of the displacement factor, as reflected in the stochastic discount factor.

²²Note that since agents are born with $\bar{h}(1-\phi)q_{s,s}$ rather than $\bar{h}q_{s,s}$ efficiency units, the total supply of labor efficiency units remains equal to the baseline model.

Appendix

A Auxiliary Results and Proofs

Proposition 1 *Let ζ be defined as*

$$\zeta \equiv \beta (1 - \lambda)^\gamma e^{\mu\psi(1-\gamma) + \frac{\sigma^2}{2}\psi^2(1-\gamma)^2}$$

and consider the solution to the following system of three equations in three unknowns θ^e, θ^w , and θ^b

$$\theta^e = \frac{1 - \zeta E_t [e^{\psi(1-\alpha)(1-\gamma)u_{t+1}} v(u_{t+1}; \theta^e, \theta^w, \theta^b)^{1-\gamma}]}{1 - \zeta E_t [e^{((1-\alpha)\psi(1-\gamma) - (1+\chi))u_{t+1}} v(u_{t+1}; \theta^e, \theta^w, \theta^b)^{-\gamma}]} \quad (50)$$

$$\theta^w = \frac{1 - \zeta E_t [e^{\psi(1-\alpha)(1-\gamma)u_{t+1}} v(u_{t+1}; \theta^e, \theta^w, \theta^b)^{1-\gamma}]}{1 - \zeta (1 - \lambda) (1 + \delta) E_t [e^{((1-\alpha)\psi(1-\gamma) - \rho)u_{t+1}} v(u_{t+1}; \theta^e, \theta^w, \theta^b)^{-\gamma}]} \quad (51)$$

$$\theta^b = \frac{\varpi \zeta E_t [e^{(1-\alpha)\psi(1-\gamma)u_{t+1}} v(u_{t+1}; \theta^e, \theta^w, \theta^b)^{-\gamma} (1 - e^{-(1+\chi)u_{t+1}}) (1 - \kappa)]}{1 - \varpi \zeta E_s [e^{(1-\alpha)\psi(1-\gamma)u_{t+1}} v(u_{t+1}; \theta^e, \theta^w, \theta^b)^{-\gamma}]} \quad (52)$$

Here,

$$v(x; \theta^e, \theta^w, \theta^b) \equiv 1 - \theta^e \alpha (1 - \alpha) \left(\kappa (1 - e^{-(1+\chi)x}) + \left(\frac{1 - \varpi}{\varpi} \right) \theta^b \right) - \theta^w (\alpha^2 + 1 - \alpha) (1 - (1 - \lambda) (1 + \delta) e^{-\rho x}). \quad (53)$$

Assuming positivity of the numerators and denominators in (50) and (51) and positivity of the denominator in (52), there exists an equilibrium with stochastic discount factor

$$\frac{\xi_{t+1}}{\xi_t} = \beta \left(\frac{Y_{t+1}}{Y_t} \right)^{-1+\psi(1-\gamma)} \left[\frac{1}{1 - \lambda} v(u_{t+1}; \theta^e, \theta^w, \theta^b) \right]^{-\gamma}. \quad (54)$$

Proof of Proposition 1. To prove Proposition 1 we conjecture that the expression $\frac{c_{t+1,t+1}^i}{Y_{t+1}}$ is exclusively a function of u_{t+1} , and then confirm our conjecture based on the resulting expression for $\frac{\xi_{t+1}}{\xi_t}$. To start, we note that if $\frac{c_{t+1,t+1}^i}{Y_{t+1}}$ is exclusively a function of u_{t+1} , then an appropriate function $f(u_{t+1})$ exists such that the stochastic discount factor is given by $\frac{\xi_{t+1}}{\xi_t} = \beta \left(\frac{Y_{t+1}}{Y_t} \right)^{-1+\psi(1-\gamma)} \times f(u_{t+1})$.

To determine $c_{t+1,t+1}^i$ for a worker ($i = w$) under this conjecture for $\frac{\xi_{t+1}}{\xi_t}$ we use (29), (11), and the fact that $h_{t,s} = \bar{h}(1 + \delta)^{t-s}$ inside the intertemporal budget constraint (13) to obtain

$$c_{s,s}^w = h_{s,s} q_{s,s} w_s \left[\frac{E_s \sum_{t=s}^{\infty} (1 - \lambda)^{t-s} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{w_t}{w_s} \right) (1 + \delta)^{t-s} \left(\frac{A_t}{A_s} \right)^{-\rho}}{E_s \sum_{t=s}^{\infty} (1 - \lambda)^{t-s} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{Y_t^{(1-\psi)(1-\gamma)}}{Y_s^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s} \right)^{-\frac{1}{\gamma}}} \right]. \quad (55)$$

Under our conjecture the expression ξ_{t+1}/ξ_t is a deterministic function of ε_{t+1} and u_{t+1} , and it follows that the distribution of $\frac{\xi_t}{\xi_s}$ for $t \geq s$ depends *only* on $t - s$. The same is true for A_t/A_s and for w_t/w_s (by equation [24]). Therefore, the expectations in both the numerator and the denominator inside the square brackets of equation (55) are time-invariant constants. Hence, using (11) we can express (55) as

$$c_{s,s}^w = \bar{h} \left(1 - (1 - \lambda) (1 + \delta) e^{-\rho u_s} \right) w_s \theta^w, \quad (56)$$

where θ^w is defined as

$$\theta^w \equiv \frac{E_s \sum_{t=s}^{\infty} (1 - \lambda)^{t-s} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{w_t}{w_s} \right) (1 + \delta)^{t-s} \left(\frac{A_t}{A_s} \right)^{-\rho}}{E_s \sum_{t=s}^{\infty} (1 - \lambda)^{t-s} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{(Y_t/N_t)^{(1-\psi)(1-\gamma)}}{(Y_s/N_s)^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s} \right)^{-\frac{1}{\gamma}}}. \quad (57)$$

The initial consumption of new business owners, who are born at time s , can be computed in a similar fashion. To start, we observe that

$$\Pi_s = \pi_s^I \left[E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{\pi_t^I}{\pi_s^I} \right) \right], \quad (58)$$

Our conjecture on $\frac{\xi_{t+1}}{\xi_t}$, together with (28), implies that the expression inside square brackets in (58) is a constant. Observing that $A_s - A_{s-1} = A_s(1 - e^{-u_s})$ (from [9]) and that $\int_{A_{s-1}}^{A_s} \pi_{j,s}^I dj = (1 - e^{-u_s}) \alpha (1 - \alpha) Y_s$ (from [28]), and using (58) inside (16) gives

$$V_{s,s} = \alpha (1 - \alpha) Y_s \times \left\{ E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{\pi_t^I}{\pi_s^I} \right) \right\} \\ \times \left\{ \kappa (1 - e^{-(1+\chi)u_s}) + \left(\frac{1 - \varpi}{\varpi} \right) E_s \sum_{t=s+1}^{\infty} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{Y_t}{Y_s} \right) (1 - \kappa) (1 - e^{-(1+\chi)u_t}) \varpi^{t-s} \right\}.$$

It will be useful to define

$$\theta^e \equiv \frac{E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{\pi_t^I}{\pi_s^I} \right)}{E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{(Y_t/N_t)^{(1-\psi)(1-\gamma)}}{(Y_s/N_s)^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s} \right)^{-\frac{1}{\gamma}}} \quad (59)$$

and

$$\theta^b \equiv E_s \sum_{t=s+1}^{\infty} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{Y_t}{Y_s} \right) (1-\kappa) (1 - e^{-(1+\chi)u_t}) \varpi^{t-s}. \quad (60)$$

The maintained conjecture that ξ_{t+1}/ξ_t is a deterministic function of ε_{t+1} and u_{t+1} and equation (28) imply that θ^e and θ^b are both constants. Using (29) inside (14),

$$c_{s,s}^e = \frac{\theta^e \alpha (1-\alpha) Y_s}{\phi \lambda} \left\{ \kappa (1 - e^{-(1+\chi)u_s}) + \left(\frac{1-\varpi}{\varpi} \right) \theta^b \right\}. \quad (61)$$

Combining (56) and (61) and noting that s in equations (61) and (56) is arbitrary, we obtain

$$\begin{aligned} \sum_{i \in \{w, e\}} \phi^i \frac{c_{t+1, t+1}^i}{Y_{t+1}} &= \theta^e \frac{1}{\lambda} \left\{ \kappa (1 - e^{-(1+\chi)u_{t+1}}) + \left(\frac{1-\varpi}{\varpi} \right) \theta^b \right\} \alpha (1-\alpha) \\ &\quad + \bar{h} \theta^w (1 - (1-\lambda)(1+\delta) e^{-\rho u_{t+1}}) (\alpha^2 + 1 - \alpha), \end{aligned} \quad (62)$$

which is a deterministic function of u_{t+1} . Using (62) and $\bar{h} = 1/\lambda$ inside (32) verifies the conjecture that there exists an equilibrium with $\frac{\xi_{t+1}}{\xi_t} = \beta \left(\frac{Y_{t+1}}{Y_t} \right)^{-1+\psi(1-\gamma)} \times f(u_{t+1})$ where $f(u_{t+1})$ is given by $f(u_{t+1}) = \left[\frac{1}{1-\lambda} v(u_{t+1}; \theta^e, \theta^w, \theta^b) \right]^{-\gamma}$. This proves equation (54).

To obtain equations (50), (51) and (52) we start by using (54) to compute the term inside square brackets in equation (58). Since $\left(\frac{\xi_{i+1}}{\xi_i} \right) \left(\frac{\pi_{i+1}^I}{\pi_i^I} \right)$ is an i.i.d random variable for any i , it follows that

$$\begin{aligned} E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{\pi_t^I}{\pi_s^I} \right) &= \sum_{t=s}^{\infty} E_s \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{\pi_t^I}{\pi_s^I} \right) = \sum_{t=s}^{\infty} E_s \prod_{i=s}^{t-1} \left(\frac{\xi_{i+1}}{\xi_i} \right) \left(\frac{\pi_{i+1}^I}{\pi_i^I} \right) \\ &= \sum_{t=s}^{\infty} \prod_{i=s}^{t-1} E_s \left(\frac{\xi_{i+1}}{\xi_i} \right) \left(\frac{\pi_{i+1}^I}{\pi_i^I} \right) = \sum_{t=s}^{\infty} \left[E_s \left(\frac{\xi_{s+1}}{\xi_s} \right) \left(\frac{\pi_{s+1}^I}{\pi_s^I} \right) \right]^{t-s} \\ &= \frac{1}{1 - E_s \left(\frac{\xi_{s+1}}{\xi_s} \right) \left(\frac{\pi_{s+1}^I}{\pi_s^I} \right)} \\ &= \frac{1}{1 - \zeta E_s \left[e^{((1-\alpha)\psi(1-\gamma)-(1+\chi)u_{s+1})} v(u_{s+1}; \theta^e, \theta^w, \theta^b)^{-\gamma} \right]}, \end{aligned} \quad (63)$$

where the last equality follows from (54):

$$\begin{aligned} E_s \left[\frac{\xi_{s+1} \pi_s}{\xi_{s+1} \pi_s} \right] &= \beta E_s \left[(Z_{s+1} e^{(1-\alpha)u_{s+1}})^{\psi(1-\gamma)-1} (Z_{s+1} e^{-(\alpha+\chi)u_{s+1}}) \right] \\ &= \beta E_s \left[Z_{s+1}^{\psi(1-\gamma)} e^{((1-\alpha)\psi(1-\gamma)-(1+\chi))u_{s+1}} \right]. \end{aligned}$$

Following a similar reasoning,

$$\begin{aligned} E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{w_t}{w_s} \right) (1+\delta)^{t-s} \left(\frac{A_t}{A_s} \right)^{-\rho} \\ = \frac{1}{1 - (1-\lambda)(1+\delta) \zeta E_s [e^{((1-\alpha)\psi(1-\gamma)-\rho)u_{s+1}} \nu(u_{s+1}; \theta^e, \theta^w, \theta^b)^{-\gamma}]} \end{aligned} \quad (64)$$

and

$$\begin{aligned} E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{Y_t^{(1-\psi)(1-\gamma)}}{Y_s^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s} \right)^{-\frac{1}{\gamma}} \\ = \frac{1}{1 - \zeta E_s [e^{\psi(1-\alpha)(1-\gamma)u_{s+1}} \nu(u_{s+1}; \theta^e, \theta^w, \theta^b)^{1-\gamma}]} \end{aligned} \quad (65)$$

Finally,

$$\begin{aligned} E_s \sum_{t=s+1}^{\infty} \left(\frac{\xi_t}{\xi_s} \right) \left(\frac{Y_t}{Y_s} \right) (1 - e^{-(1+\chi)u_t}) (1 - \kappa) \varpi^{t-s} \\ = E_s \sum_{t=s+1}^{\infty} \left[\prod_{i=s+1}^t \left(\frac{\xi_i}{\xi_{i-1}} \right) \left(\frac{Y_i}{Y_{i-1}} \right) \varpi \right] (1 - e^{-(1+\chi)u_t}) (1 - \kappa) \\ = \sum_{t=s+1}^{\infty} E_s \left\{ \left[\prod_{i=s+1}^{t-1} \left(\frac{\xi_i}{\xi_{i-1}} \right) \left(\frac{Y_i}{Y_{i-1}} \right) \varpi \right] \left(\frac{\xi_t}{\xi_{t-1}} \right) \left(\frac{Y_t}{Y_{t-1}} \right) \varpi (1 - e^{-(1+\chi)u_t}) (1 - \kappa) \right\} \quad (66) \\ = \left\{ \sum_{t=s+1}^{\infty} \left[\prod_{i=s+1}^{t-1} E_s \left(\frac{\xi_i}{\xi_{i-1}} \right) \left(\frac{Y_i}{Y_{i-1}} \right) \varpi \right] \right\} \times E_t \left[\left(\frac{\xi_{t+1}}{\xi_t} \right) \left(\frac{Y_{t+1}}{Y_t} \right) \varpi (1 - e^{-(1+\chi)u_{t+1}}) (1 - \kappa) \right] \\ = \frac{\varpi \zeta E_s [e^{(1-\alpha)\psi(1-\gamma)u_{s+1}} \nu(u_{s+1}; \theta^e, \theta^w, \theta^b)^{-\gamma} (1 - e^{-(1+\chi)u_{s+1}}) (1 - \kappa)]}{1 - \varpi \zeta E_s \{e^{(1-\alpha)\psi(1-\gamma)u_{s+1}} \nu(u_{s+1}; \theta^e, \theta^w, \theta^b)^{-\gamma}\}} \end{aligned}$$

Combining (66) with (60) leads to (52). Similarly, combining (59) with (65) and (63) leads to (50), while combining (57), (64), and (65) implies (51). ■

Proof of Lemma 1. To establish that the equity premium is non-zero in the limit, it suffices to show that

$$\lim_{\alpha \rightarrow 1} \text{cov} \{R_t, (\xi_{t+1}/\xi_t)\} \neq 0. \quad (67)$$

Since $\kappa = 1$, all stocks have rate of return

$$R_t = \frac{(\pi_{t+1}^I/\pi_t^I) + (\Pi_{t+1}/\pi_{t+1})}{\Pi_t/\pi_t}.$$

Equation (58) implies that (Π_t/π_t) is a constant. Therefore, in order to establish (67) it suffices to show that $\lim_{\alpha \rightarrow 1} \text{cov} \{(\pi_{t+1}^I/\pi_t^I), (\xi_{t+1}/\xi_t)\} \neq 0$. To see that this is the case note that $\lim_{\alpha \rightarrow 1} (\pi_{t+1}^I/\pi_t^I) = e^{\mu - (1+\chi)u_{t+1}}$. Hence, in order to establish (67), we need to show that ξ_{t+1}/ξ_t is a non-degenerate function of u_{t+1} as $\alpha \rightarrow 1$. Given that

$$\lim_{\alpha \rightarrow 1} \left(\frac{\xi_{t+1}}{\xi_t} \right) = \beta (1 - \lambda)^\gamma e^{\mu(-1+\psi(1-\gamma))} [1 - \theta^w (1 - (1 - \lambda)(1 + \delta) e^{-\rho u_{t+1}})]^{-\gamma}, \quad (68)$$

the lemma holds as long as a solution $\theta^w > 0$ exists to Equation (51), an equation that simplifies to

$$\begin{aligned} \theta^w & \left(1 - \zeta (1 - \lambda) (1 + \delta) E \left[e^{-\rho u} (1 - \theta^w (1 - (1 - \lambda)(1 + \delta) e^{-\rho u_t}))^{-\gamma} \right] \right) \\ & = 1 - \zeta E \left[(1 - \theta^w (1 - (1 - \lambda)(1 + \delta) e^{-\rho u_t}))^{1-\gamma} \right]. \end{aligned} \quad (69)$$

By expanding the right-hand side of (69), the equation further simplifies to

$$1 = \zeta E \left[(1 - \theta^w (1 - (1 - \lambda)(1 + \delta) e^{-\rho u_t}))^{-\gamma} \right]. \quad (70)$$

As the right-hand side increases in θ^w , and the probability of the event $\{u_t \in (0, \epsilon)\}$ is strictly positive for all $\epsilon > 0$, conditions (34) and (35) are necessary and sufficient for the existence of a solution $\theta^w > 0$. (Note that $\theta^w \leq 1$.) ■

Proof of Lemma 2. The value of a growth firm is given by the value of all its assets in place and all its growth options.

$$\begin{aligned} P_{t,s} & = (1 - \eta)\kappa \left(\int_{A_{s-1}}^{A_s} \Pi_{j,t}^I dj \right) + \sum_{n=s+1}^t (1 - \varpi) (1 - \kappa) \varpi^{n-(s+1)} \left(\int_{A_{n-1}}^{A_n} \Pi_{j,t}^I dj \right) \\ & \quad + (1 - \varpi) \left[E_t \sum_{n=t+1}^{\infty} \left(\frac{\xi_n}{\xi_t} \right) \varpi^{n-(s+1)} \left(\int_{A_{n-1}}^{A_n} \Pi_{j,n}^I dj \right) (1 - \kappa) \right] \end{aligned}$$

Using the definition of Φ , and noting that $\int_{A_{s-1}}^{A_s} \pi_{j,t}^I dj = \left(\frac{A_s}{A_t} \right)^{1+\chi} (1 - e^{-(1+\chi)u_s})$ along with the definition of θ^b in equation (60) leads to (39). ■

Proof of Lemma 3. Equation (44) implies that

$$a_{s+1}^i - a_s^i = \log c_{s+1,s+1}^i - \log c_{s,s}^i + \frac{1}{\gamma} \log \left(C_{s+1}^{(1-\psi)(1-\gamma)} \beta^{-(s+1)} \xi_{s+1} \right) - \frac{1}{\gamma} \log \left(C_s^{(1-\psi)(1-\gamma)} \beta^{-s} \xi_s \right). \quad (71)$$

Using (54) inside (71) along with $C_s = Y_s$ and simplifying give

$$a_{s+1}^i - a_s^i = \log c_{s+1,s+1}^i - \log c_{s,s}^i - \log \left(\frac{C_{s+1}}{C_s} \right) - \log \left[\frac{1}{1-\lambda} v(u_{s+1}; \theta^e, \theta^w, \theta^b) \right].$$

Using the definitions of a_s and z_s along with (56), (61), noting that $C_s = Y_s$, and simplifying gives

$$a_{s+1} - a_s = -\log \left[\frac{1}{1-\lambda} v(u_{s+1}; \theta^e, \theta^w, \theta^b) \right] + z_{s+1} - z_s. \quad (72)$$

Iterating (72) forward leads to (47). ■

B Labor input as a composite service and human capital depreciation

This section provides a more elaborate model of the labor market, that reproduces the path of labor income over an agent's life, as postulated in equations (11) and (12). The main difference between the baseline model, and the model of this section, is that the labor income process results from general equilibrium wage effects, rather than assumptions on agents' endowments of labor efficiency units.

To draw this distinction, we assume that workers' efficiency units are only affected by aging and experience. Specifically, workers endowments of labor efficiency units evolve deterministically over their life according to $h_{t,s} = \bar{h} (1 + \delta)^{t-s}$. However, the innovation shocks u_t no longer have any effects on agents' endowment of labor efficiency units.

Assume moreover, that labor is not a homogenous service. Instead, the units of labor that enter the production function of final goods and intermediate goods are measured in terms of a composite service, which is a constant elasticity of substitution (CES) aggregator of the labor efficiency units provided by workers belonging to different cohorts. Specifically, one unit of (composite) labor is given by

$$L_t = \left(\sum_{s=-\infty}^t v_{t,s}^{\frac{1}{b}} (l_{t,s})^{\frac{b-1}{b}} \right)^{\frac{b}{b-1}}, \quad (73)$$

where $l_{t,s}$ denotes the labor input of cohort s at time t , $v_{t,s} > 0$ controls the relative importance of this input and $b > 0$ is the elasticity of substitution. The production function of final goods continues to be given by (3) and it still takes one unit of the composite labor service to produce one unit of the intermediate good. Equation (73) captures the idea that different cohorts have different skills and hence they are imperfect substitutes in the production process. Next, we let

$$v_{t,s}^{\frac{1}{b}} \equiv (1 - \phi)^{\left(\frac{b-1}{b}\right)} q_{t,s} h_{t,s}^{\frac{1}{b}}. \quad (74)$$

Before proceeding, we note that using (74) inside (73), recognizing that in equilibrium $l_{t,s} = h_{t,s}$, and noting that $\sum_{s=-\infty}^t q_{t,s} h_{t,s} = 1$ implies that the aggregate supply of (composite) labor efficiency units is constant and equal to $(1 - \phi)$.

Since labor inputs by agents belonging to different cohorts are imperfect substitutes, we need to solve for the equilibrium wage $w_{t,s}$ of each separate cohort. This process is greatly facilitated by first constructing a “wage index”, i.e. taking a set of cohort-specific wages as given, and then minimizing (over cohort labor inputs) the cost of obtaining a single unit of the composite labor input. As is well established in the literature, this wage index for CES production functions is given by

$$\bar{w}_t = \left(\sum_{s=-\infty}^t v_{t,s} (w_{t,s})^{1-b} \right)^{\frac{1}{1-b}}.$$

With this wage index at hand, the cohort specific input demands for a firm demanding a total of L_t units of the composite good are given by

$$w_{t,s} = \bar{w}_t v_{t,s}^{\frac{1}{b}} \left(\frac{l_{t,s}}{L_t} \right)^{-\frac{1}{b}}. \quad (75)$$

It is now straightforward to verify that an equilibrium in such an extended model can be determined by setting $\bar{w}_t = w_t$ (where w_t is given by [24]) and then obtaining the cohort-specific wages by setting $l_{t,s} = h_{t,s}$, and $L_t = (1 - \phi)$ in equation (75) and solving for $w_{t,s}$. To see this, note that by making these substitutions and using (74) inside (75) leads to the per-worker income process

$$\frac{w_{t,s} h_{t,s}}{(1 - \phi)} = \bar{w}_t q_{t,s}, \quad (76)$$

which coincides with the labor income process in the baseline model. Furthermore by setting $w_t = \bar{w}_t$, the market for total (composite) labor units clears by construction, whereas the cohort specific wages implied by (76) clear all cohort specific labor markets, since they satisfy equation (75) for all markets.

C Data Description

The CEX data are from the NBER website as compiled by Ed Harris and John Sabelhaus. See http://www.nber.org/ces_cbo/Cexfam.pdf for a detailed description of the data. In short, the data set compiles the results from the four consecutive quarterly interviews in the CEX into one observation for each household. We follow a large literature (see e.g. Vissing-Jørgensen (2002)) and drop from the sample households with incomplete income responses, households who haven't completed one of the quarterly interviews, and households that reside in student housing. To ensure that data selection does not unduly affect the results, we also ran all the regressions on the raw data including dummies for reporting status and the number of completed interview quarters. The results were not affected in any essential way.

A more delicate issue concerns the definition of consumption. As Fernandez-Villaverde and Krueger (2007), we used a comprehensive measure of consumption that potentially includes durables. Specifically, we used exactly the same definition as Harris and Sabelhaus. Our choice is motivated by our model; according to the model, cohort effects are determined by the intertemporal budget constraint of agents born at different times. Accordingly, as Fernandez-Villaverde and Krueger (2007), we use total consumption expenditure in the estimation of cohort effects. To test if this choice materially affects our results, we also ran the results using non-durable consumption (total consumption expenditure excluding medical and educational expenses, housing, furniture and automotive related expenses). Using non-durable consumption the volatility of cohort effects was larger; however, there was not a big difference between the variance of the permanent components of the cohort effects, no

matter which concept of consumption (total or only non-durable) we used.

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