The Global Liquidity Trap

Olivier Jeanne

Johns Hopkins University, NBER and CEPR

October 2009

Abstract

This paper presents a two-country model of the world economy with money and nominal stickiness in which countries may be affected by demand shocks. We show that a negative demand shock in one country may push the world economy in a global liquidity trap with unemployment and zero nominal interest rates in both countries. Global monetary stimulus (a temporary increase in both countries' inflation targets) may restore the first-best level of employment and welfare. Fiscal stimulus may restore full employment but distorts the allocation of consumption between private and public goods. We also study the international spillovers associated with each policy, and the risk that they lead to trade protectionism.

1 Introduction

Figure 1 shows the monetary policy interest rates and the inflation rates in the United States, the Euro area and Japan since 2005. Everywhere the rates of inflation have decreased to negative levels. The policy interest rate, which has been very low for a long time in Japan, is close to zero in the United States. Several monetary authorities, including the United States and the United Kingdom, have sharply increased their supply of base money, as Japan did in with its policy of "quantitative easing". It is difficult to look at Figure 1 without wondering whether monetary policy in a large part of the world is converging toward a Japanese-style situation—what one might call a "global liquidity trap".[1]

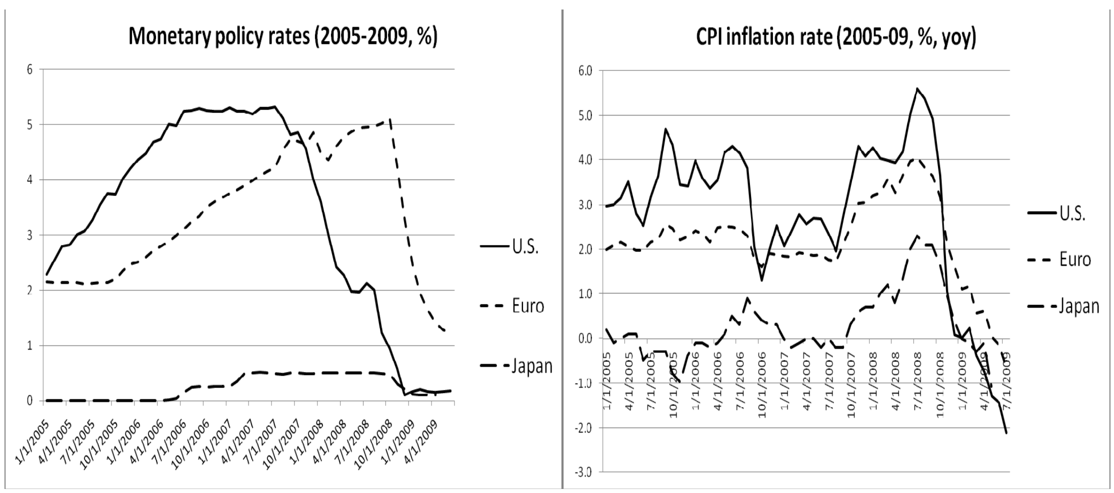

Figure 1: Monetary Policy Rate and CPI Inflation Rate in the U.S., the Euro area and Japan. Monthly data. CPI inflation rate is non-core, year-on-year (source IFS). Interest rates from national sources, through Datastream.

The purpose of this paper is to explore the mechanisms involved in a global liquidity trap. An important wave of literature (to be discussed later in this introduction) has

[1] A number of developing or emerging market countries do not show the features presented in Figure 1. Although the Chinese CPI inflation rate has recently become negative, India or Russia, have inflation rates in excess of 10 percent. In addition, the expected inflation rate remains positive in the U.S., the euro area and Japan. The statement is not that the global economy is now in a liquidity trap, but that this scenario is worth considering, looking forward.

been inspired by the liquidity trap in one country and one period—Japan in the 1990s. The present paper builds on, and extends this literature to the case where several countries simultaneously fall in a liquidity trap. The interesting new dimension is the international spillovers which—I will argue—are essential to a correct understanding of a global liquidity trap. A global liquidity trap cannot be properly understood as a juxtaposition of countries that happen to be in a liquidity trap at the same time.

I look at this question using a dynamic general equilibrium model with two countries and two goods. International spillovers are involved, first, in the entry into a liquidity trap. In a closed economy, a liquidity trap is caused by a negative demand shock that lowers the "natural" rate of interest consistent with full employment. I show that in the open economy, the natural rate of interest is reduced not only in the country that is hit by the shock but also in the rest of the world. Thus, the conditions leading to a liquidity trap in one country tend to spill over to the rest of the world.

Second, I study the optimal macroeconomic policies to exit the global liquidity trap. As the earlier literature on Japan has shown, the channel of monetary policy, in a liquidity trap, relies entirely on expectations—a monetary stimulus is, essentially, an "expectational stimulus" that works by raising the expected inflation rate rather than the quantity of money per se. It is always possible to exit the liquidity trap if the monetary authorities can raise the expected inflation rate to a sufficiently high level.

One possible problem with such a policy, in a multi-country world, is that it has beggar-thy-neighbor effects. In my model, increasing the expected inflation rate raises domestic unemployment and welfare at home but has the opposite effects abroad because of a depreciation of the home currency. However, I show that these beggar-thy-neighbor effects should not prevent the two countries from doing monetary stimulus, if both countries can do it. Uncoordinated monetary stimulus leads to full employment and the first-best level of welfare, as the beggar-thy-neighbor effects cancel out when both countries increase their inflation targets.

As for fiscal policy, a decrease in taxes that leaves public expenditures unchanged has no effect on employment and welfare, but an increase in public expenditures stimulates demand if private consumption and public consumption are not perfectly substitutable. A fiscal stimulus, thus, can be used to reach full employment in a global liquidity trap. It does not lead to the first-best level of welfare because the allocation of spending between private and public consumption is distorted—fiscal stimulus leads to overconsumption of public goods.

This paper is related to the literature dealing with the liquidity trap in Japan. It belongs to the neo-Wicksellian approach which, starting with Krugman (1998), explains the liquidity trap by a fall in the natural rate of interest.[2] One important point (originally made by Krugman, 1998, and later formalized by Eggertsson and Woodford, 2003, and Eggertsson 2006) is that exiting a liquidity trap amounts to a credibility problem. The problem is to make it credible that the higher inflation rate will be implemented, even though it may be known that the central banks has anti-inflationary preferences. We do not address this issue here and simply assume that commitment to the inflation target is possible. We focus instead on the international spillovers involved in the exit from the liquidity trap.

Most of the literature looks at the liquidity trap in a closed-economy context. Krugman (1998) argued that the intuition from a closed-economy model survives openness if there is a large nontradable sector. Formal open-economy models of the liquidity trap were used by Svensson (2001) and Jeanne and Svensson (2007) to study the role of the exchange rate in optimal exits of a liquidity trap. But these are small open-economy models that cannot be used to study the international spillovers involved in a global

[2]The relationship to Wicksell's theory of the natural rate of interest is discussed by Woodford (2003). Benhabib, Schmitt-Grohé and Uribe (2002) present another approach in which the liquidity trap results from a self-fulfilling switch to deflationary expectations.

liquidity trap. Coenen and Wieland (2002) study the Japanese liquidity trap in the context of a three-country model of the global economy. They point to the beggar-thy-neighbor effects that a monetary stimulus in Japan would imply for the U.S. economy, but do not look at the case where there is a liquidity trap in more than one country.

The paper is structured as follows. Section 2 presents the main assumptions of the model. Section 3 looks at the response of the natural rates of interest (at home and abroad) to asymmetric shocks. In section 4 we introduce nominal stickiness into the model and look at the global liquidity trap. Section 5 focuses on the policies to exit the liquidity trap and section 6 concludes.

2 Assumptions

The models features two countries, home (H) and foreign (F), and two goods. Each country is populated by an infinitely-lived representative consumer. I present the assumptions for the home country—the assumptions for the foreign country are symmetric.

The utility of the Home country's representative consumer in period 1 is given by,

$$U_1 = \Delta \cdot \left[u(C_1) - f(L_1) + v\left(\frac{M_1}{P_1^c}\right)\right] + \sum_{t=1}^{+\infty} \beta^t \left[u(C_t) - f(L_t) + v\left(\frac{M_t}{P_t^c}\right)\right] \tag{1}$$

where C is consumption, $f(L)$ is the disutility of labor, M/P^c is the level of real money holdings, and Δ is an exogenous demand-shifting factor that raises or lowers the utility of period-1 consumption relative to future consumption. The utility of consumption has a constant elasticity of intertemporal substitution γ,

$$u(C) = \frac{C^{1-1/\gamma}}{1-1/\gamma}, \tag{2}$$

with $u(C) = \log(C)$ if $\gamma = 1$.

Home consumption is a CES index of consumption of home good (C_H) and consumption of imported foreign good (C_F),

$$C = \left[(1-\eta)^{1/\sigma} C_H^{(\sigma-1)/\sigma} + \eta^{1/\sigma} C_F^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}. \tag{3}$$

The law of one price applies, so that the price index for home consumption is given by,

$$P^c = \left[(1-\eta) P^{1-\sigma} + \eta (SP^*)^{1-\sigma}\right]^{1/(1-\sigma)}, \tag{4}$$

where P is the price of the home good in terms of home currency, P^* is the price of the foreign good in terms of foreign currency, and S is the exchange rate (the price of the foreign currency in terms of domestic currency).

The marginal disutility of labor $f'(L)$ is positive and increasing. I specify $f(L)$ in such a way that there is a fixed level of labor, denoted by $\overline{L}$, corresponding to "full employment". We assume that the marginal disutility of labor discontinuously jumps up in $\overline{L}$, and that it is sufficiently low for $L < \overline{L}$, and sufficiently high for $L > \overline{L}$, that $\overline{L}$ is the optimal quantity of labor in equilibrium. Thus we can say that there is full employment if $L = \overline{L}$, and underemployment (overemployment) if $L < \overline{L}$ ($L > \overline{L}$).

The home consumption good is produced using a continuum of differentiated inputs with the CES production function

$$Y = \left(\int_0^1 (Y_j)^{(\theta-1)/\theta} dj\right)^{\theta/(\theta-1)}, \quad \theta > 1. \tag{5}$$

There is perfect competition between the producers of home consumption good. Each producer of input uses labor with a linear technology that transforms one unit of labor into one unit of good,

$$Y_j = L_j. \tag{6}$$

The budget constraint of the home consumer is

$$P_t^c C_t + P_t^c \frac{B_{t+1}}{R_t} + S_t P_t^{*c} \frac{B_{t+1}^*}{R_t^*} + M_t = W_t L_t + \Pi_t + P_t^c B_t + S_t P_t^{*c} B_t^* + M_{t-1} + N_t, \tag{7}$$

where B_t and B_t^* denote the home holdings of bonds respectively denominated in home consumption good and foreign consumption good, W_t is the nominal wage, Π_t is the profit of home firms, and $N_t = M_t - M_{t-1}$ is a lump-sum money transfer from the home central bank.

The assumptions for the foreign country are symmetric. Foreign variables are generally denoted with an asterisk. The foreign consumption index is given by,

$$C^* = \left[(1-\eta)^{1/\sigma} C_F^{*(\sigma-1)/\sigma} + \eta^{1/\sigma} C_H^{*(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)},$$

where C_F^* is foreign consumption of foreign good and C_H^* is foreign consumption of home good. Parameter η is smaller than 1/2: the two countries have the same bias for consuming the good that is produced domestically. The foreign consumption price index is given by,

$$P^{*c} = \left[(1-\eta)P^{*1-\sigma} + \eta(P/S)^{1-\sigma}\right]^{1/(1-\sigma)}, \tag{8}$$

and the budget constraint of the foreign consumer is

$$P_t^{*c} C_t^* - \frac{P_t^c}{S_t}\frac{B_{t+1}}{R_t} - P_t^{*c}\frac{B_{t+1}^*}{R_t^*} + M_t^* = W_t^* L_t^* + \Pi_t^* - \frac{P_t^c}{S_t} B_t - P_t^{*c} B_t^* + M_{t-1}^* + N_t^*.$$

The foreign bond holdings are the home levels with a negative sign (since one country's asset is the other country's liability). I assume that the foreign country has the same labor endowment as the home country, and normalize it to 1 ($\overline{L}^* = \overline{L}$).

The economy starts from a symmetric steady state with no asset and no liabilities ($B_1 = B_1^*$). In period 1 the economy is unexpectedly disturbed by demand shifts in the home and foreign country, Δ and Δ^*.[3] We look at the dynamic response of the world economy to those demand shocks, first in the case of flexible prices (section 3) and then in the case with nominal stickiness (section 4).

3 Flexible Price Equilibria

Although a liquidity trap can arise only if there is some nominal stickiness, the flexible price equilibrium is interesting to look at because it shows us how the "natural" real rates of interest respond to the demand shocks. An economy is in liquidity trap when the real rate of interest cannot be lowered to the natural level because of the zero bound on the nominal interest rate.

I first look at the relationship between the terms of trade and the other important variables in the economy (section 3.1). The following section reports the results of numerical simulations in which the home country is hit by a negative demand shock.

3.1 The Marshall-Lerner condition

The foreign terms of trade (the price of home imports in terms of home exports) are given by,

$$Q_t = \frac{S_t P_t^*}{P_t}.$$

[3]The assumption that the demand shifts are unexpected (i.e., that they are shocks) is not essential for the analysis, but is natural given our focus on the post-shock equilibrium. If the demand shifts were expected, the economy would not be in a steady state before period 1. What we want to focus on, however, is how the economy responds to the realization of demand shocks, rather than how it behaves in anticipation of those shocks.

The home real exchange rate

$$\frac{S_t P_t^{*c}}{P_t^c} = \left[\frac{(1-\eta)Q_t^{1-\sigma} + \eta}{1-\eta+\eta Q_t^{1-\sigma}}\right]^{1/(1-\sigma)}, \tag{9}$$

is increasing with Q because there is domestic bias in consumption ($\eta < 1/2$). An improvement in the foreign terms of trade corresponds to a real depreciation at home.

There are simple equilibrium relationship between Q_t and the other time-t variables. Let us drop the time subscripts to alleviate the notations. Given that one unit of labor is transformed into one unit of production input, the output of each good is equal to the labor endowment of the producing country in a full-employment equilibrium. The two countries having the same labor endowment $\overline{L}$, the equality between supply and demand can then be written

$$\overline{L} = \widetilde{C}_H(C, Q) + \widetilde{C}_H^*(C^*, Q), \tag{10}$$

$$\overline{L} = \widetilde{C}_F(C, Q) + \widetilde{C}_F^*(C^*, Q), \tag{11}$$

where the right-hand sides of equations (10) and (11) sum up the demands for the home good and for the foreign good respectively. The demand for a given good in a given country depends on this country's total consumption and on the relative price between the two goods, which itself depends on Q. For example, the home demand for the home good is given by

$$\widetilde{C}_H(C, Q) = (1-\eta)\left(\frac{P}{P^c}\right)^{-\sigma} C = (1-\eta)\left[(1-\eta) + \eta Q^{1-\sigma}\right]^{\sigma/(1-\sigma)} C.$$

One can derive similar expressions for the other components of demand on the right-hand side of equations (10) and (11). Those expressions can then be inverted to give home and foreign total consumption demands in terms of the real exchange rate,

$$C = \widetilde{C}(Q), \tag{12}$$

$$C^* = \widetilde{C}(1/Q), \tag{13}$$

(see the appendix for closed-form expressions). Home consumption decreases with the real exchange rate because of the substitution effect, $\widetilde{C}'(Q) < 0$.

Home net exports can be written in terms of the home consumption good,

$$X = \frac{P}{P^c}(EX - IM), \tag{14}$$

where home exports and imports in terms of the home good are respectively given by

$$EX = \eta \left(\frac{P/S}{P^{*c}} \right)^{-\sigma} C^* = \eta \left[(1-\eta) Q^{1-\sigma} + \eta \right]^{\sigma/(1-\sigma)} C^*,$$

$$IM = \eta Q \left(\frac{SP^*}{P^c} \right)^{-\sigma} C = \eta Q \left[(1-\eta) Q^{-(1-\sigma)} + \eta \right]^{\sigma/(1-\sigma)} C,$$

Starting from a symmetric equilibrium with $Q = C = C^* = 1$, the elasticity of the home trade balance with respect to Q is given by

$$\frac{\partial X}{\partial Q} = \overbrace{\eta \left[2(1-\eta)\sigma - 1 \right]}^{\chi}. \tag{15}$$

Thus, the home trade balance increases with a real depreciation if and only if $\chi > 0$, that is

$$2(1-\eta)\sigma > 1. \tag{16}$$

This is the Marshall-Lerner condition.[4] Using (12) and (13) we can also substitute

[4] The textbook version of the Marshall-Lerner condition is that the sum of price elasticity of

out C and C^* and express net exports as a function $\widetilde{X}(Q)$. We derive a closed-form expression for $\widetilde{X}(\cdot)$ in the appendix, and show that it is increasing in Q if the Marshall-Lerner condition is satisfied.

3.2 Impact of a demand shock at home

The demand shocks occur in period 1. From period 2 onwards the economy is in a new steady state in which each country has a constant current account balance determined by the international assets and liabilities accumulated in period 1. The net supply of bonds denominated in foreign good is set to zero ($B_2^* = 0$), so that the international assets and liabilities are characterized by one variable, B_2.[5]

The equilibrium conditions are derived from the home and foreign consumer's optimization problems in the appendix. Given that the economy is in a steady state from period 2 onwards we focus on the equilibrium in period 1 and period 2. The equilibrium can be characterized by five conditions

$$\Delta \cdot u'\left(\widetilde{C}(Q_1)\right) = \beta R_1 u'\left(\widetilde{C}(Q_2)\right), \tag{17}$$

$$\Delta^* \cdot u'\left(\widetilde{C}(1/Q_1)\right) = \beta R_1^* u'\left(\widetilde{C}(1/Q_2)\right), \tag{18}$$

$$R_1 = R_1^* \frac{RER(Q_2)}{RER(Q_1)}, \tag{19}$$

$$B_2 = R_1 \widetilde{X}(Q_1), \tag{20}$$

$$\widetilde{X}(Q_2) + (1-\beta)B_2 = 0, \tag{21}$$

exports and imports (in absolute value) must be greater than 1. Here, the price elasticity of exports (expressed in terms of home good) is the same as for imports and is equal to $(1-\eta)\sigma$.

[5] The consumers were given the choice between home and foreign bonds to derive the interest parity equation. Once interest parity is satisfied the two kinds of bonds are perfectly substitutable in equilibrium since there is no uncertainty from period 1 onwards.

which jointly determine five unknown variables, Q_1, Q_2, R_1, R_1^* and B_2. Equations (17) and (18) are the Euler conditions for the home and foreign countries respectively. Equation (19) is the real interest parity condition, where $RER(\cdot)$ is the function mapping the foreign terms of trade into the home real exchange rate, given by equation (9). Equation (20) equates the home country's foreign assets in period 2 to its period-1 trade balance times the interest factor between period 1 and period 2 (remember that $B_1 = 0$). The last equation states that from period 2 onwards the home country runs a trade deficit equal to its net income from abroad. The appendix explains how this system of equations can be solved numerically as a fixed point for B_2.

Table 1.Benchmark calibration

β	γ	η	σ
$e^{-0.03}$	0.5	0.3	1.5

The impact of a demand shock at home is illustrated with a calibrated version of the model. The parameter values are given in Table 1. The calibration of γ corresponds to a relative risk aversion of 2, in the interval of values $[1, 10]$ usually considered in the literature. The share of the foreign good in home consumption is 30 percent. The value of the elasticity of substitution between the home good and the foreign good, σ, is also in the range of values used in the literature. Like in Obstfeld and Rogoff (2005), the baseline choice for the value of σ is a compromise between two sources of evidence.[6] Studies based on based on disaggregated data tend to find higher values (up to 4 for 3-digit SIC good categories, see Broda and Weinstein, 2006). By contrast, the estimates based on macroeconomic evidence are generally close to 1. For example, the sum of the trade elasticities found in the literature that tests the Marshall-Lerner

[6]Obstfeld and Rogoff (2005) use a model with traded and nontraded goods. They take a benchmark value of 2 for the elasticity of substitution between tradable goods, and of 1 for the elasticity of substitution between tradable and nontradable goods.

condition typically ranges from 1 to 2 (depending on whether one looks at the short-run or the long-run elasticities, see Hooper et al, 2000). Given that the price elasticity of imports or exports is $(1-\eta)\sigma$, this implies that σ would lie between 0.71 and 1.43.

Without restriction of generality (since the model is symmetric), I set the foreign demand shock to zero and look at the response of the global economy to a demand shock at home.[7] Figure 2 shows how the period-1 level of consumption (higher panel) and the real interest rate (middle panel) vary with home demand in both countries. Consumption varies in opposite directions at home and abroad. A decrease in home demand depresses consumption at home but increases foreign consumption. By contrast, the real interest rates vary in the same direction in the two countries: a decrease in home demand lowers the real interest rate both at home and abroad. The real interest rate responds more at home than in the foreign country. If the home demand shock exceeds 6 percent the natural rate of interest becomes negative in both countries.

One remarkable feature of this simulation is how close the two interest rates stay to each other. The interest rate differential between the two countries never exceeds 0.6 percent even though the interest rate levels vary between -2 and +6 percent in the simulation. As a result, the terms of trade and the real exchange rate do not move a lot (see lower panel of Figure 2).[8]

The impact of the negative demand shock on the real interest rate is almost as large in the foreign country as in the home country. This result is important because it suggests that the conditions that tend to create a liquidity trap in one country (by making the natural rate of interest negative) tend to spill over to the rest of the world. The intuition behind this result will be clarified in the following section.

[7] No attempt is made here to reproduce the main features of the current global crisis. The U.S. entered the crisis with very large current account deficits. Here, the shock disturbs a steady state with a zero trade balance.

[8] The real exchange rate depreciation is equal to the real interest rate differential. The figure shows the terms of trade Q, which are more responsive to the interest rate differential than the real exchange rate.

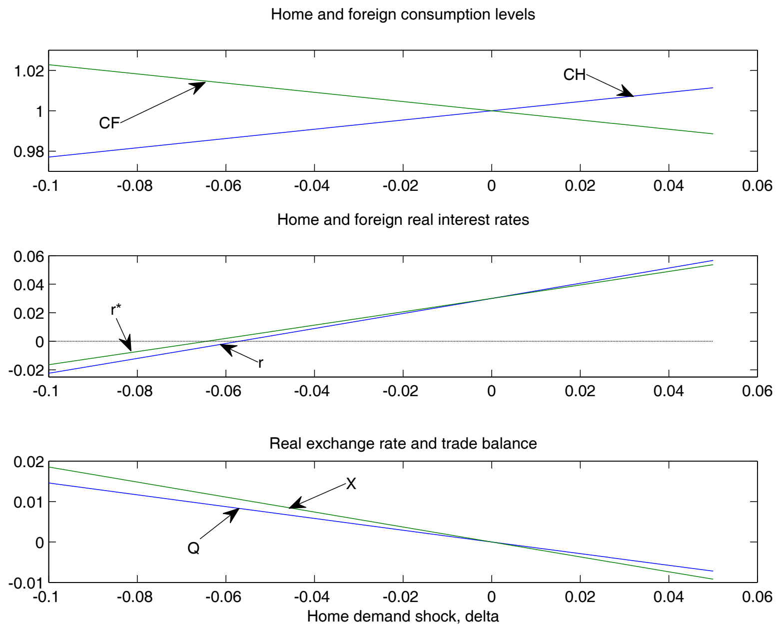

Figure 2: Response of period-1 variables to a demand shock at home.

4 The Global Liquidity Trap

I now introduce nominal stickiness into the model by assuming that the producers of intermediate goods set their prices one period ahead. This assumption may lead to a deviation from the full employment equilibrium in a period at which there is an unexpected deviation from the perfect foresight equilibrium (period 1).

The first subsection provides a simple characterization of monetary policy in terms of two variables: the inflation target and the (real or nominal) interest rate. The following subsections analyse the relationship between real interest rates and employment, and between employment and welfare. Finally, we derive the Nash equilibrium of the monetary policy game assuming that each country maximizes domestic welfare.

The logarithm of variables are denoted with lower case letters, e.g., $r_t = \log R_t$. The time subscript for period-1 variables is dropped when it is not necessary. For example, the period-1 real interest rates will be denoted by r and r^*.

4.1 Monetary policy

The demand for money is given by

$$v'\left(\frac{M_t}{P_t^c}\right) = u'(C_t)\left(1 - e^{-i_t}\right), \tag{22}$$

where i_t, the logarithmic nominal interest rate, is the sum of the real interest rate and the inflation rate

$$i_t = \log\left(R_t \frac{P_{t+1}^c}{P_t^c}\right) = r_t + \pi_t. \tag{23}$$

We assume that the utility of money is satiable, i.e., there is a finite level of real money holdings, m^*, such that the marginal utility of money $v'(m)$ is equal to zero for $m \geq m^*$. This implies that if the nominal interest rate is equal to 0, the demand for real money balances is indeterminate above m^*.

In period 1, monetary policy consists in the announcement of a path for current and future money supply: $(M_t)_{t\geq 1}$ in the home country and $(M_t^*)_{t\geq 1}$ in the foreign country. We assume that monetary policy is implemented in the context of a *strict* inflation targeting framework. The home and foreign central banks are endowed with positive inflation targets, respectively denoted by $\widehat{\pi}$ and $\widehat{\pi}^*$. Achieving those targets lexicographically dominates other objectives that the central banks might care about, such as employment or welfare. In other terms, each central bank picks a money supply path in the subset that leads to the domestic inflation rate always being equal to the target.

Under these assumptions, one can show that the monetary policy of a country in period 1 can be characterized in terms of two variables: its inflation target and the nominal interest rate in period 1. This is stated more formally in the following proposition (where $\overline{r} = 1/\beta - 1$ denotes the steady state level of the real interest rate).

Proposition 1 *Any inflation target $\widehat{\pi} \geq -\bar{r}$ and period-1 nominal interest rate $i \geq 0$ can be implemented in the home country with a unique money supply path $(M_t)_{t\geq 1}$ (taking $\widehat{\pi}^*$ and i^* as given). Symmetrically, any inflation target $\widehat{\pi}^* \geq -\bar{r}$ and period-1 nominal interest rate $i^* \geq 0$ can be implemented in the foreign country with a unique money supply path $(M_t^*)_{t\geq 1}$ (taking $\widehat{\pi}$ and i as given). Conditional on $(\widehat{\pi}, i, \widehat{\pi}^*, i^*)$ the equilibrium is uniquely determined.*

Proof. See the appendix.■

The proposition has two parts. The first part simply states the zero-bound constraint: any period-1 nominal interest rate is implementable, provided it is non-negative. The second part of the proposition provides a simple characterization of monetary policy in terms of two variables per country: the inflation target and the first-period nominal interest rate. If the inflation targets are taken as given because they are pre-set in the central banks' mandates, monetary policy can be summarized, in each country, by the period-1 nominal interest rate.

Figure 3 illustrates the relationship between period-1 money supply and interest rates in the home country (keeping the foreign nominal interest rate and inflation constant). The nominal interest rate i is decreasing with money supply M_1 and—as the utility of money is satiable—there is a finite level of money supply above which the nominal interest rate is equal to zero. Note that as M_1 increases, the path of future money supply $(M_2, M_3, ...)$ is adjusted so as to keep the inflation rate always equal to the target. The period-1 real interest rate, thus, decreases one-for-one with the nominal interest rate and reaches its minimum, equal to the opposite of the inflation target, when the demand for money is satiated.

4.2 Real interest rates and employment: the (r, r^*) diagram

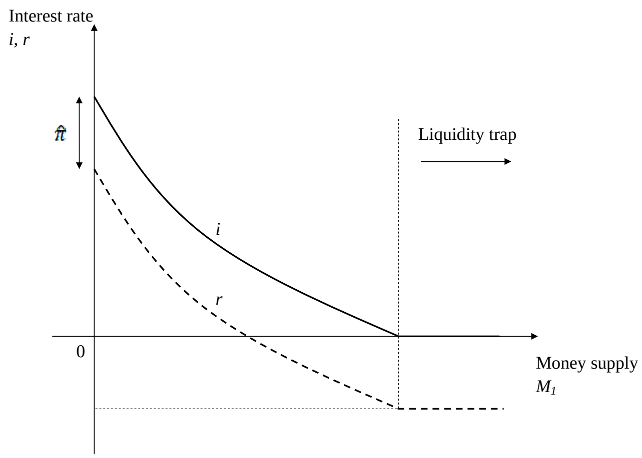

Figure 3: Monetary Supply and Interest Rates.

I now present a linearized version of the model that is useful to interpret the results of the numerical simulation in the previous section and will help us understand the global liquidity trap. As shown in the appendix, we can find closed-form expressions for all the period-1 variables in terms of the real interest rates r and r^*. To any pair of real interest rates (r, r^*) one can associate demands for home labor and foreign labor, respectively denoted by $L(r, r^*)$ and $L^*(r, r^*)$, which may be smaller or larger than $\overline{L}$ since the real interest rates are not necessarily at the natural level.

Solving for the equilibrium is considerably simplified if we assume that the impact of period-1 capital flows on foreign assets and liabilities is neutralized by a "compensating transfer" that sets B_2 to zero in period 2. The transfer is lump-sum and taken as exogenous by the consumers of both countries. As a result, one no longer has to solve the fixed-point problem for B_2 and the equilibrium can be parameterized by the period-1 real interest rates. This does not significantly affect the quantitative properties of the model (Q_2 was varying by less than 0.05 percent in the numerical simulation of the previous section).

As shown in the appendix, linearizing the model around the steady state gives the following expression for the demand for home labor

$$\frac{L}{\overline{L}} = 1 + \widetilde{L}_r \cdot (r - \overline{r}) + \widetilde{L}_{r^*} \cdot (r^* - \overline{r}) + \widetilde{L}_\delta \cdot \delta + \widetilde{L}_{\delta^*} \cdot \delta^*, \tag{24}$$

where the coefficients are the functions of the underlying parameters

$$\begin{aligned}
\widetilde{L}_r &= -(1-\eta)\left(\frac{2\eta\sigma}{1-2\eta} + \gamma\right), \\
\widetilde{L}_{r^*} &= \eta\left(\frac{2(1-\eta)\sigma}{1-2\eta} - \gamma\right), \\
\widetilde{L}_\delta &= (1-\eta)\gamma, \\
\widetilde{L}_{\delta^*} &= \eta\gamma.
\end{aligned}$$

We observe that $\widetilde{L}_r < 0$. Increasing the real interest rate in one country unambiguously decreases the demand for this country's good—and labor—by reducing the country's level of consumption and by appreciating its real exchange rate. By contrast, the sign of $\widetilde{L}_{r^*}$ is ambiguous in general. Changing the real interest rate in one country affects the demand for the *other* country's labor through two channels that work in opposite directions. Raising the foreign real interest rate reduces the demand for the home good by lowering foreign imports, but increases the demand for the home good by appreciating the foreign currency. The substitution effect dominates if

$$2\frac{1-\eta}{1-2\eta}\sigma > \gamma. \tag{25}$$

If this condition is satisfied, decreasing the real interest rate in one country has a beggar-thy-neighbor effect: it increases the demand for domestic labor but reduces the demand for the other country's labor. Condition (25) is satisfied as soon as $\sigma \geq 1/2$ and $\gamma \leq 1$, that is, for standard calibrations of the model.

The expression for the demand for foreign labor is symmetric,

$$\frac{L^*}{\overline{L}} = 1 + \widetilde{L}_r \cdot (r^* - \overline{r}) + \widetilde{L}_{r^*} \cdot (r - \overline{r}) + \widetilde{L}_\delta \cdot \delta^* + \widetilde{L}_{\delta^*} \cdot \delta. \tag{26}$$

Equations (24) and (26) allow us to analyze the international spillovers generated by a negative shock at home with a simple diagram (Figure). The lines labelled H and F are the $L(r, r^*) = \overline{L}$ and the $L^*(r, r^*) = \overline{L}$ loci, i.e., the combinations of real interest rates for which there is full employment respectively at home and abroad. On the H line, there is full employment at home, and on the F line there is full employment in the foreign country. I assume that (25) is satisfied, so that both lines have a positive slope. The slope of the H line is larger than 1, since if the home real interest rate increases by one percent, the foreign real interest rate must increase by more than one percent to produce a real depreciation that maintains full employment at home. Symmetrically, the slope of the F line is smaller than 1. The two loci intersect for the natural rates of interest in point A, the only point where there is full employment in both countries. The interior of the cone delimited by H and F is the region for which there is less than full employment.[9]

The impact of a negative demand shock at home is illustrated by Figure 4. The shock shifts the H line to the left: other things equal, maintaining the demand for the home good at its pre-shock level requires a lower real interest rate at home. It also lowers the F line downward since the shock lowers the demand for foreign exports. Full employment is achieved when the global economy has converged to an equilibrium where the real interest rate is lower not only at home—where the negative demand shock originated—but also in the rest of the world (point A'). The coordinates of point A' are the natural real interest rates at home and abroad, r^n and r^{*n}.

[9]The region outside of this cone corresponds to a situation of overemployment in at least one country.

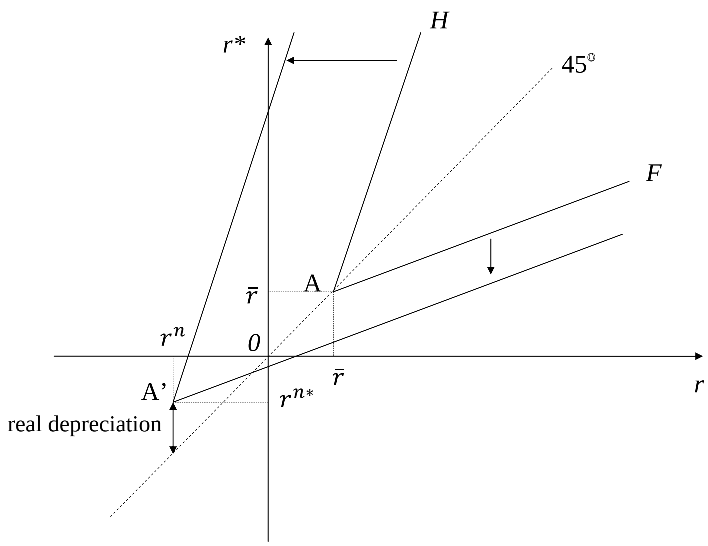

Figure 4: Impact of negative demand shock at home on real interest rates.

Demand shocks are transmitted internationally because the countries trade with each other, a channel that is magnified by the beggar-thy-neighbor effect. The natural interest rates are obtained by setting $L = L^* = \overline{L}$ in equations (24) and (26). Taking the sum and the difference of the two equations gives us some expressions for how the world average real interest rate and the interest rate differential between the home country and the foreign country are affected by demand shocks,

$$\frac{r^n + r^{n*}}{2} = \overline{r} + \frac{\delta + \delta^*}{2}. \tag{27}$$

$$r^n - r^{n*} = \frac{1 - 2\eta}{1 + 2\widetilde{L}_{r^*}/\gamma}(\delta - \delta^*). \tag{28}$$

Under autarky ($\eta = \widetilde{L}_{r^*} = 0$) we have $r^n = \overline{r} + \delta$ and $r^{n*} = \overline{r} + \delta^*$ and there is no international transmission of demand shocks. We observe from equation (27) that the impact of the demand shocks on the world average real interest rate is the same as under autarky. By contrast, the impact of demand shocks on the interest rate differential is smaller than under autarky—and for plausible calibrations of the parameters, much smaller. For the benchmark calibration given in Table 1 we have $r^n - r^{n*} = 0.06 \cdot (\delta - \delta^*)$, that is, only 6 percent of the effect that we would observe under autarky.

There are two channels of international transmission of demand shocks. First, a share η of the decrease in home demand falls on the foreign country's exports. The shock is thus allocated to the home and foreign real interest rates with weights $1 - \eta$ and η respectively, which explains the factor $1 - 2\eta$ in the numerator of the fraction in the right-hand-side of (28). With $\eta = 0.3$, this explains why the differential $r - r^*$ is only 40 percent of the level that would be observed in autarky. In addition, the beggar-thy-neighbor effect shifts labor demand from the foreign country to the home country, implying that the real interest rate differential must be lower for both countries to be in full employment (this captured by the term in $\widetilde{L}_{r^*}$ in (28)). For the benchmark calibration of Table 1 we have $\widetilde{L}_{r^*} = 1.42$, which is sufficient to reduce the equilibrium interest rate differential from 40 percent to 6 percent of the autarky level. The larger the beggar-thy-neighbor effect is, the smaller is the response of the real interest rate differential to asymmetric demand shocks in equilibrium.

4.3 Employment and welfare

The only thing that is missing, in order to have a well-defined game between the two central banks, is the specification of their objectives. I assume that each central bank maximizes domestic welfare conditional on meeting its inflation target.[10] The instrument of monetary policy being the nominal interest rate, the home central bank's

[10] The inflation targets cannot be changed by the central banks. Section 5 will analyze the game in which countries can reset their inflation targets.

problem can be written

$$\begin{cases} \max_i U = \Delta \cdot (u(C_1) - f(L_1)) + \sum_{t=1}^{+\infty} \beta^t \left(u(C_t) - f(L_t)\right), \\ \text{subject to } i^*, \forall t \geq 1, \ \pi_t = \widehat{\pi} \text{ and } \pi_t = \widehat{\pi}^*. \end{cases}$$

The central bank's objective U is the period-1 welfare of the home consumer excluding the utility of real money balances. The foreign central bank has a symmetric objective.

Conditional on the inflation rate always being equal to the target, setting the period-1 nominal interest rate, i, is equivalent to setting the real interest rate, r. Furthermore, welfare, like employment, can be written as a reduced-form function of the period-1 real interest rates, r and r^*. Thus, we can rewrite the home central bank's problem in the simple following form

$$(P) \begin{cases} \max_r U(r, r^*) \\ r \geq -\widehat{\pi}. \end{cases}$$

It would be natural to assume that welfare is maximized when there is neither overemployment nor underemployment. This is not necessarily true, however, because of the monopolistic distortion, which may make it optimal to increase labor above the flexible-price equilibrium level. Because of this effect, the central bank might always be tempted to raise employment above the full employment level—in which case an equilibrium with full employment is not time-consistent under sticky prices.

The welfare consequences of a game in which each country is chronically tempted to raise employment above the flexible-price level have been studied in the earlier literature, and are not the focus of this paper.[11] Thus, I rule out this possibility by

[11] See Corsetti and Pesenti (2001) and Obstfeld and Rogoff (2002). Corsetti and Pesenti (2001) emphasize the fact that the welfare gains from an opportunistic monetary expansion are mitigated by the exchange rate depreciation. Obstfeld and Rogoff (2002) find that the welfare gains from coordination are small.

assuming that the marginal disutility of labor is high enough for $L > \overline{L}$ to dissuade central banks from raising employment above the full-employment level. A closed-form condition on $f'(\overline{L}^+)$ is derived in the appendix by linearizing the model.

Proposition 2 *If the marginal disutility of labor is large enough for $L > \bar{L}$, maximizing domestic welfare is equivalent to achieving full employment.*

Proof. See the appendix.■

Hereafter I will assume that the condition on the disutility of labor underlying this proposition are satisfied. Thus the home central bank's problem may be rewritten in terms of employment as

$$(P') \begin{cases} \min_r |L(r, r^*) - \overline{L}| \\ r \geq -\widehat{\pi}, \end{cases}$$

with a symmetric problem for the foreign central bank.

4.4 Nash equilibrium

Given that maximizing domestic welfare is equivalent to achieving full employment, the Nash equilibrium can be analyzed with the diagram that we used to look at employment. Let us assume that the global economy has been hit by negative demand shocks that have led the natural (full employment) levels of the real interest rates, r^n and r^{*n}, into negative territory (see Figure 5). Inside the unemployment cone, each central bank attempts to reduce the domestic real interest rate to boost domestic employment and welfare. This may lead to full employment in both countries if the zero-bound constraints do not bind. In the case illustrated by Figure 5, however, the Nash equilibrium is constrained by the zero-bound on the nominal interest rate. Both countries fall in a liquidity trap with less than full employment—a "global liquidity trap". This is a Nash equilibrium because although each country would like to increase its employment at the expense of its neighbor, neither can.

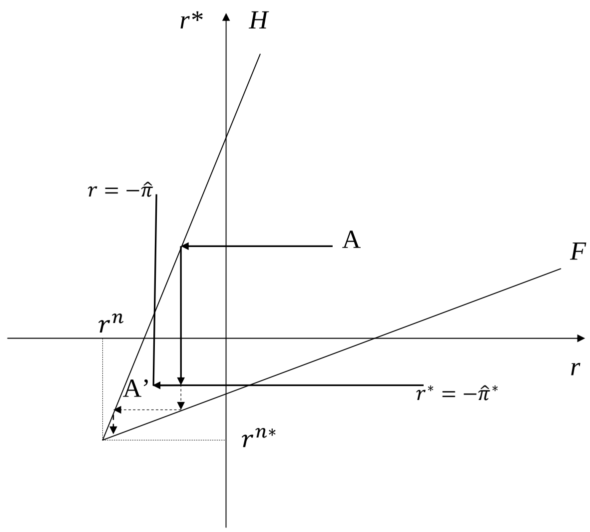

Figure 5: The global liquidity trap.

More generally, negative demand shocks will result in a liquidity trap if full employment at the global level is inconsistent with at least one zero-bound constraint, because $r^n + \widehat{\pi} < 0$ or $r^{*n} + \widehat{\pi}^* < 0$. Interestingly, the liquidity trap does not necessarily occur in the country that is hit by the larger demand shock. The liquidity trap could be "exported" to the other country if that country has a lower inflation target. Indeed, the liquidity trap could be exported to the foreign country even if it has no demand shock at all. This is the case if $r^n + \widehat{\pi} > 0$ and $r^{*n} + \widehat{\pi}^* < 0$. More generally, the condition for a country to fall in a liquidity trap is stated in the following proposition.

Proposition 3 *There is one unique Nash equilibrium. Following negative demand shocks that lower the natural rates of interest r^n and r^{*n}, the home country falls in a* *liquidity trap with underemployment and a zero nominal interest rate if and only if*

$$r^n + \widehat{\pi} < \min\left(0, \frac{r^{*n} + \widehat{\pi}^*}{1 + \gamma/\widetilde{L}_{r^*}}\right), \tag{29}$$

and the foreign country falls in a liquidity trap if and only if

$$r^{*n} + \widehat{\pi}^* < \min\left(0, \frac{r^n + \widehat{\pi}}{1 + \gamma/\widetilde{L}_{r^*}}\right). \tag{30}$$

Proof. The Home country falls in a liquidity trap if and only if $r^n + \widehat{\pi} < 0$ and the point $(-\widehat{\pi}, -\widehat{\pi}^*)$ is below the H line (see Figure 5), that is if

$$\widetilde{L_r}(-\widehat{\pi} - r^n) + \widetilde{L}_{r^*}(-\widehat{\pi}^* - r^{n*}) < 0,$$

which, after simple manipulations, gives (29). Condition (30) is derived in a similar way, by noting that the Foreign country falls in a liquidity trap if and only if $r^{*n} + \widehat{\pi}^* < 0$ and the point $(-\widehat{\pi}, -\widehat{\pi}^*)$ is above the F line.■

5 Exit Policies

This section studies the policies to exit a global liquidity trap, looking first at monetary policy and then at fiscal policy. The third subsection studies the case where a country, being unable or unwilling to use fiscal or monetary policy, resorts instead to tariffs on imports.

5.1 Monetary stimulus

The only way that monetary policy can stimulate the economy, in a liquidity trap, is by raising the expected rate of inflation. In our model, where (by assumption) the inflation rate is always equal to the target, this means raising the inflation target. The target must be raised temporarily, not forever, since the objective is to raise the expected inflation rate in the short run (between period 1 and period 2).[12]

Raising the expected inflation rate may be more or less difficult in practice, depending on the institutional framework of monetary policy. Although the medium- or long-run inflation objective of the monetary authorities is meant to provide a stable anchor for expectations—and as such should not be changed in response to a crisis—there may be room of manoeuver in setting the short-term inflation objective. In inflation targeting regimes, the central bank is supposed to aim the inflation target at a certain horizon through a path that is determined with some consideration paid to the level of economic activity. As for regimes with no formal inflation targets, the monetary authorities can influence expectations by communicating about the rate of inflation that they will be aiming at.[13] In spite of the academic literature's emphasis on the difficulty of "committing to being irresponsible", it is likely that such policy actions, if they were tried, would have some effect on inflation expectations.

Going back to the model, the short-run inflation targets (between period 1 and 2), denoted by $\widehat{\pi}_1$ and $\widehat{\pi}_1^*$, could be different from the long-run targets $\widehat{\pi}$ and $\widehat{\pi}^*$. We

[12]In the real world, unlike in the model, the central bank does not perfectly control inflation in the short run and it may take some time for the inflation rate to reach the target. Consequently the economy may have to stay for some time in the liquidity trap. In this case, what the central bank raises is the inflation target at the exit of the liquidity trap. In practice, the monetary authorities announce, as part of their "exit strategy", that the policy interest rate will not be raised until the inflation rate reaches the higher target—while reassuring the public that the inflation target remains equal to a lower level in the long run.

[13]Even an independent central banker might derive some reputational benefits from delivering on his promises. Here again, the policy announcement would have to be justified in the context of the monetary authorities' mandate.

capture the fact that inflation targets can be adjusted by making the stark but simple assumption that each country chooses its inflation target so as to maximize domestic welfare. The Nash equilibrium thus involves two policy variables for each country: the short-run inflation target, and the nominal interest rate.[14] It is easy to see that under those assumptions, the Nash equilibrium leads to full employment. There cannot be unemployment in the Nash equilibrium, since the country with unemployment would be better off raising its short-run inflation target to relax the zero-bound constraint and increase its level of employment and welfare. The result is highlighted in the following proposition.

Proposition 4 *If each country can set its short-run inflation target in addition to the nominal interest rate, the Nash equilibrium leads the world economy to full employment and the first-best level of welfare.*

Proof. See the discussion above.

Note that the first-best level of welfare is achieved in the *uncooperative Nash equilibrium*, so that there is no reason for coordinating monetary policies.[15] This is so even though unilateral relaxation of the inflation target is a beggar-thy-neighbor policy that reduces employment in the other country. The beggar-thy-neighbor effect does not lead to a prisoner dilemma situation in a global liquidity trap, because a monetary stimulus in one country increases domestic employment more than it reduces foreign employment, as can be seen by adding up equations (24) and (26),

$$\frac{L + L^*}{\overline{L}} = 1 + \gamma \cdot \left[(\delta + \delta^*) - (r - \overline{r}) - (r^* - \overline{r}) \right].$$

[14] It does not matter whether the inflation target is set first or at the same time as the interest rate.

[15] The results would be different if there were gains from marginally increasing employment above the full employment level. However, Obstfeld and Rogoff (2002) show that even in this case the gains from coordination are small.

A global monetary stimulus that lowers r and r^* by the same amount raises global employment in the same way as a monetary stimulus does in a closed economy.

5.2 Fiscal stimulus

I now assume that in each country the domestic government can finance a public expenditure by raising taxes or issuing debt. The budget constraint of the home government is

$$G_t + D_t = T_t + \frac{D_{t+1}}{R_t},$$

where G is the level of government expenditure, D is the level of government debt and T is a lump-sum tax on the domestic consumers. All variables are expressed in terms of home consumption good.

The impact of an increase in public spending G crucially depends on the substitutability between public consumption and private consumption—as noted by Eggertsson (2009). If the two forms of consumption are perfectly substitutable, i.e., if the consumer's flow utility of consumption is given by $u(C+G)$, it is easy to see that a fiscal stimulus has no impact on the equilibrium because increases in public spending crowd out private consumption one-for-one, leaving total spending $C + G$ unchanged. But in the general case, public spending does not crowd out private consumption one-for one—and even crowds it *in* if public spending and private consumption are complements. The level of private consumption being pinned down by the Euler equation and the level of intertemporal income, the effect of the fiscal stimulus does not depend on the extent to which the additional public spending is financed by taxes or by debt. In particular, a tax cut (keeping public spending the same) has no stimulative impact. A fiscal stimulus, thus, will hereafter mean an increase in G.

Although a fiscal stimulus can raise the level of employment, its welfare properties are not as appealing as that of a monetary stimulus, because it distorts the allocation of spending between the private good and the public good. To see this without introducing unnecessary complications into the model, let us assume that the consumer's flow utility of consumption is given by $u(C)+g^{1/\gamma}u(G)$, where g is an exogenous parameter. Given that the rate of transformation between private consumption and public consumption is one, the optimal level of public consumption satisfies the first order condition $u'(C) = g^{1/\gamma}u'(G)$ or

$$G = gC. \tag{31}$$

Parameter g is the optimal ratio of public consumption to private consumption.

Let us assume that starting from a steady state with constant levels of private and public consumption, a negative demand shock puts the home economy in a liquidity trap in period 1. Private consumption is consequently lower than expected ($C_1 < C_1^e$). If public consumption is maintained at the level that was expected before the shock, G_1^e, there is overconsumption of public goods relative to private goods ($G_1^e > gC_1$).

The home government can stimulate domestic production by increasing public spending G_1 above G_1^e. The real interest rate being equal to the opposite of the inflation target in a liquidity trap, private consumption satisfies the Euler equation

$$\Delta \cdot u'(C_1) = \beta e^{-\widehat{\pi}} \cdot u'(C_2). \tag{32}$$

Thus the fiscal stimulus affects period-1 consumption only to the extent that it affects C_2. The fiscal stimulus may lower C_2 and so C_1 by inducing a trade deficit in period 1. But this crowding out effect is quantitatively small so that the sum of private consumption and public consumption, $C_1 + G_1$, on balance increases with G_1. Note however that the fiscal stimulus tends to worsens the distortion coming from the overconsumption of public goods, since private consumption C_1 decreases while public consumption increases, starting from a situation in which the ratio of public consumption to private consumption was already too high.

To illustrate, let us compare the impact of a fiscal stimulus and a monetary stimulus on employment and welfare if the economy falls in a global liquidity trap because of a symmetric demand shock ($\widehat{\pi} = \widehat{\pi}^*$ and $\delta = \delta^* < -(\overline{r}+\widehat{\pi})$). By symmetry, foreign assets and liabilities are equal to zero in period 2 ($B_2 = 0$), implying that $C_2 = C_2^* = 1/(1+g)$, the optimal level of private consumption. The Euler equation (32) can be written

$$C_1 = \frac{e^{\overline{r}+\widehat{\pi}+\delta}}{1+g}.$$

Full employment is reached in period 1 if $C_1 + G_1 = 1$, so that the ratio of public spending to private spending is given by

$$\frac{G_1}{C_1} = (1+g)e^{-(\overline{r}+\widehat{\pi}+\delta)} - 1,$$

which is larger than the optimal level, g, if the economy is in a liquidity trap ($\overline{r}+\widehat{\pi}+\delta < 0$). Thus there is overconsumption of public goods.

5.3 Tariffs

One concern that is often expressed with regards to beggar-thy-neighbor depreciations is that they may lead to protectionism. In order to look into this question, assume that countries can impose a tariff on imports, denoted by τ and τ^*. The price of the foreign good at home is $e^{\tau}SP^*$ and the price of the home good in the foreign country is $e^{\tau^*}P/S$.

First, let us consider a global liquidity trap with zero nominal interest rates and underemployment in both countries. We assume that the countries do not rely on macroeconomic policies and resort instead to tariffs in order to boost domestic employment. We look for the Nash equilibrium in which each country sets its tariff rate taking the other country's tariff policy as given.

The Nash equilibrium now depends on whether countries try to reach full employment or maximize welfare. Each country can increase its level of employment by raising its tariff on imports and in fact, both countries can simultaneously reach full employment by using tariffs. Raising employment with tariffs, however, is welfare-decreasing. This can easily be seen by considering (as in the previous section) a symmetric global liquidity trap. Given $B_2 = 0$ and $C_2 = C_2^* = 1$, the period-1 levels of consumption are pinned down by the Euler equations

$$\begin{aligned} u'(C_1) &= \beta e^{-\widehat{\pi}}, \\ u'(C_1^*) &= \beta e^{-\widehat{\pi}^*}, \end{aligned}$$

and so are not affected by the tariffs. However, tariffs can increase labor in both countries all the way up to full employment (see the appendix). The only effect of tariffs, thus, is to increase the quantity of labor that must be used to achieve the same level of consumption. On a net basis, welfare decreases because of the disutility of labor. Tariffs increase the demand for labor by wasting it.

6 Conclusions

I have presented a model showing how a two-country world economy responds to demand shocks, and how—in the presence of nominal stickiness—macroeconomic policies can help to restore full employment and the first-best level of welfare. If the global economy falls into a liquidity trap, full employment and the first-best level of welfare can be achieved increasing the inflation targets of the two countries. Full employment can also, under some conditions, be achieved by increasing public expenditures, but fiscal stimulus distorts the allocation of spending between private and public goods, and thus leaves welfare below the first-best level.

There are several caveats to the conclusion that monetary stimulus is preferable to fiscal stimulus to deal with a global liquidity trap. First, increasing the inflation targets, even temporarily, may compromise the long-term credibility of the monetary framework. Second, monetary stimulus has a beggar-thy-neighbor effect if it is not implemented in all the countries where it is warranted, possibly leading to protectionist policies that may be difficult to reverse. Third, a monetary stimulus works only if the announcement of a higher inflation target is credible, which may not be the case if the central banker is known to have strong anti-inflationary preferences. Fiscal stimulus does not have these problems and might be preferable to monetary stimulus once they are taken into account.

One potential problem with fiscal stimulus, however, is the duration of the liquidity trap. Fiscal stimulus seems appropriate to stimulate the economy during a short-lived liquidity trap (i.e., if the natural real interest rate is negative for a short time). But a fiscal stimulus may be difficult to withdraw and result in unsustainable levels of debt if the liquidity trap lasts for a long time. In other terms, the fiscal cure may be appropriate to treat a liquidity trap from which the economy will recover quickly anyway, but persistent liquidity traps may require a monetary cure.

This remark leads us to a direction in which the model could be usefully extended. Nominal stickiness, in our model, was one-period ahead, like in Krugman (1998) or Jeanne and Svensson (2007). This simplified the analysis—allowing us to analyze the equilibrium with simple diagrams—but prevented the model from shedding light on the dynamics of the exit from the liquidity trap. By construction, the liquidity trap could not last more than one period. It would be interesting to study a variant of the model with price or wage staggering of nominal stickiness (like Eggertsson and Woodford, 2003, or Auerbach and Obstfeld, 2005).

References

Auerbach, Alan J. and Maurice Obstfeld, 2005, "The case for Open-Market Purchases in a Liquidity Trap," *American Economic Review* 95(1), 110-137.

Benhabib, Jess, Schmitt-Grohé, Stephanie and Martín Uribe, 2002, "Avoiding Liquidity Traps," *Journal of Political Economy* 110(3), 535-563.

Broda, Christian and David E. Weinstein, 2006, "Globalization and the Gains from Variety," *Quarterly Journal of Economics* 121(2), 541-585.

Christiano, Lawrence, Eichenbaum, Martin and Sergio Rebelo, 2009, "When is the Government Spending Multiplier Large?", manuscript, Northwestern University.

Coenen, Günter and Volker Wieland, 2002, "The Zero-Interest-Rate Bound and the Role of the Exchange Rate for Monetary Policy in Japan," *Journal of Monetary Economics* 50(5), 1071-1101.

Corsetti, Giancarlo and Paolo Pesenti, 2001, "Welfare and Macroeconomic Interdependence," *Quarterly Journal of Economics* 116(2), 421-45.

Eggertsson, Gauti B., 2006, "The Deflation Bias and Committing to Being Irresponsible," *Journal of Money, Credit and Banking* 38(2), 283-321.

Eggertsson, Gauti B., 2009, "What Fiscal Policy is Effective at Zero Interest Rates?", manuscript., Federal Reserve Bank of New York.

Eggertsson, Gauti B. and Michael Woodford, 2003, "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity* 2003:1, 139-211.

Hooper, Peter, Johnson, Karen and Jaime Marquez, 2000, *Trade Elasticities for the G-7 Countries*, Princeton Studies in International Economics (International Economics Section, Department of Economics, princeton University).

Jeanne, Olivier and Lars Svensson, 2007, "Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank", *American Economic Review* 97, 474-490.

Krugman, Paul, 1998, "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity* 1998:2, 137-87.

Obstfeld, Maurice and Kenneth Rogoff, 2002, "Global Implications of Self-Oriented National Monetary Rules," *Quarterly Journal of Economics* 117(2), 503-535.

Obstfeld, Maurice and Kenneth Rogoff, 2005 , "Global Current Account Imbalances and Exchange Rate Adjustments," *Brookings Papers on Economic Activity* 1:2005, 67-146.

Svensson, Lars E.O., 2001, "The Zero Bound in an Open Economy: A Foolproof Way of Escaping from a Liquidity Trap," *Monetary and Economic Studies* 19, Special Edition, 277-312.

Woodford, Michael, 2003, *Interest and Prices*, Princeton University Press.

APPENDIX

A1. The Marshall-Lerner condition

We derive closed-form expressions for the r.h.s. of (12) and (13). Demand is equal to supply for each good

$$\overline{L} = \widetilde{C}_H(C,Q) + \widetilde{C}^*_H(C^*,Q) = (1-\eta)\left(\frac{P}{P^c}\right)^{-\sigma} C + \eta\left(\frac{P/S}{P^{*c}}\right)^{-\sigma} C^*,$$

$$\overline{L} = \widetilde{C}^*_F(C^*,Q) + \widetilde{C}_F(C,Q) = (1-\eta)\left(\frac{P^*}{P^{*c}}\right)^{-\sigma} C^* + \eta\left(\frac{SP^*}{P^c}\right)^{-\sigma} C.$$

We invert this system to find expressions for C and C^*,

$$\begin{aligned} C &= \frac{1}{1-2\eta}\left(\frac{P^c}{P}\right)^{-\sigma}(1-\eta-\eta Q^\sigma)\,\overline{L}, \\ C^* &= \frac{1}{1-2\eta}\left(\frac{SP^{*c}}{P}\right)^{-\sigma}\left[(1-\eta)Q^\sigma-\eta\right]\overline{L}. \end{aligned}$$

Then using (4) this gives,

$$C = \frac{1}{1-2\eta}\left[1-\eta+\eta Q^{1-\sigma}\right]^{-\sigma/(1-\sigma)}\left[1-\eta-\eta Q^\sigma\right]\overline{L}.$$

This is a closed-form expression for function $\widetilde{C}(\cdot)$ in (12).

As for foreign consumption using (8) we have,

$$\begin{aligned} C^* &= \frac{1}{1-2\eta}\left(\frac{P^{*c}}{P^*}\right)^{-\sigma}\left[1-\eta-\eta Q^{-\sigma}\right]\overline{L}, \\ &= \frac{1}{1-2\eta}\left[1-\eta+\eta Q^{-(1-\sigma)}\right]^{-\sigma/(1-\sigma)}\left[1-\eta-\eta Q^{-\sigma}\right]\overline{L}, \\ &= \widetilde{C}(1/Q). \end{aligned}$$

The home trade balance is given by,

$$\begin{aligned} X &= \frac{P}{P^c}\left(C^*_H - QC_F\right) = \eta\frac{P}{P^c}\left[\left(\frac{P/S}{P^{*c}}\right)^{-\sigma} C^* - Q\left(\frac{SP^*}{P^c}\right)^{-\sigma} C\right], \\ &= \frac{\eta\overline{L}}{1-2\eta}\frac{(1-\eta)Q^\sigma+\eta Q-(1-\eta)Q^{1-\sigma}-\eta}{\left(1-\eta+\eta Q^{1-\sigma}\right)^{1/(1-\sigma)}}. \end{aligned}$$

This is the closed-form expression for $\widetilde{X}(Q)$. Differentiating with respect to Q at $Q = 1$ then gives

$$\frac{\partial X}{\partial Q} = \left(2\frac{1-\eta}{1-2\eta}\sigma - 1\right)\eta\overline{L}, \tag{33}$$

$$= \frac{\chi + 2\eta^2}{1-2\eta}\overline{L}. \tag{34}$$

This is positive if the Marshall-Lerner condition $\chi > 0$ is satisfied.

A2. First-order conditions

Consumers. The Lagrangian for the home consumer problem is

$$\pounds = \sum_{t=1}^{+\infty}\beta^t \left[\begin{array}{c} \Delta_t\left[u(C_t) - f(L_t) + v\left(\frac{M_t}{P_t^c}\right)\right] \\ +\lambda_t/P_t^c\left(\begin{array}{c} P_t^c B_t + S_t P_t^{*c} B_t^* + W_t L_t \\ +M_{t-1} - P_t^c C_t - P_t^c \frac{B_{t+1}}{R_t} - S_t P_t^{*c}\frac{B_{t+1}^*}{R_t^*} - M_t \end{array}\right)\end{array}\right],$$

where $\Delta_1 = \Delta$ and $\Delta_t = 1$ for $t \geq 2$.

FOC for C_t:

$$\lambda_t = \Delta_t u'(C_t). \tag{35}$$

FOC for L_t:

$$\Delta_t f'(L_t) = \lambda_t \frac{W_t}{P_t^c}. \tag{36}$$

FOC for B_{t+1}:

$$\lambda_t = \beta R_t \lambda_{t+1}. \tag{37}$$

FOC for B_{t+1}^*:

$$\lambda_t = \beta R_t^* \frac{S_{t+1}P_{t+1}^{*c}/P_{t+1}^c}{S_t P_t^{*c}/P_t^c}\lambda_{t+1}, \tag{38}$$

which with (37) implies real interest rate parity

$$R_t = R_t^* \frac{S_{t+1}P_{t+1}^{*c}/P_{t+1}^c}{S_t P_t^{*c}/P_t^c}. \tag{39}$$

FOC for M_t:

$$\frac{\Delta_t}{P_t^c}v'\left(\frac{M_t}{P_t^c}\right) = \frac{\lambda_t}{P_t^c} - \beta\left(\frac{\lambda_{t+1}}{P_{t+1}^c}\right), \tag{40}$$

which, using (35) and (37), implies equation (22).

Producers. The demand for intermediate input j is,

$$Y_j = \left(\frac{P_j}{P}\right)^{-\theta} Y, \tag{41}$$

where the price is given by,

$$P = \left(\int_0^1 (P_j)^{1-\theta} dj\right)^{1/(1-\theta)}. \tag{42}$$

Without flexible prices, the producer's problem at time $t-1$ is

$$\max_{P_{jt}} \left[\frac{\lambda_t}{P_t^c}\left(P_{jt} - W_t\right) Y_{jt}\right]$$
$$\text{s.t.} \qquad Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\theta} Y_t,$$

which (using $P_{jt} = P_t$) implies

$$P_t = \frac{\theta}{\theta - 1} W_t. \tag{43}$$

Combining (35), (36) and (43) gives a relationship between labor supply, consumption and the real exchange rate

$$f'(L_t) = \frac{\theta - 1}{\theta} \frac{u'(C_t)}{\left(1 - \eta + \eta Q_t^{1-\sigma}\right)^{1/(1-\sigma)}}.$$

With the step specification assumed for $f(\cdot)$, the equilibrium level of labor remains equal to $\overline{L}$ provided that

$$f'(\overline{L}^-) < \frac{\theta - 1}{\theta} \frac{u'(\widetilde{C}(Q_t))}{\left(1 - \eta + \eta Q_t^{1-\sigma}\right)^{1/(1-\sigma)}} < f'(\overline{L}^+).$$

This condition is satisfied locally if

$$f'(\overline{L}^-) < \frac{\theta - 1}{\theta} < f'(\overline{L}^+).$$

A3. Numerical resolution method

We explain how the system of equations (17)-(21) can be solved numerically as a fixed point for B_2. Eliminating R_1 and R_1^* between equations (17), (18) and (19) gives a relationship between Q_1 and Q_2

$$\psi(Q_1) = \left(\frac{\Delta}{\Delta^*}\right)^{\gamma} \cdot \psi(Q_2), \tag{44}$$

where function $\psi(\cdot)$ is defined by

$$\begin{aligned}\psi(Q) &= \frac{\widetilde{C}(Q)}{\widetilde{C}(1/Q)RER(Q)^{\gamma}}, \\ &= \left[\frac{(1-\eta)Q^{1-\sigma}+\eta}{1-\eta+\eta Q^{1-\sigma}}\right]^{(\sigma-\gamma)/(1-\sigma)} \frac{1-\eta-\eta Q^{\sigma}}{(1-\eta)Q^{\sigma}-\eta}\end{aligned}$$

Function $\psi(\cdot)$ is strictly decreasing with Q. (The real exchange rate tends to depreciate between 1 and 2 if there is a positive demand shock at home.)

Given the value of B_2^k from the iteration k, we compute Q_2 using equation (21) and then Q_1 from (44). Substituting out R_1 from (17) and (20) we can compute a new value for B_2

$$B_2^{k+1} = (1-\lambda)B_2^k + \lambda\frac{\Delta}{\beta}\left[\frac{\widetilde{C}(Q_2)}{\widetilde{C}(Q_1)}\right]^{1/\gamma}\widetilde{X}(Q_1).$$

Using a dampening factor $\lambda < 1$ may be necessary to ensure smooth convergence.

A.4. Proof of Proposition 1

From period $t = 2$ onwards, home consumption is constant and equal to $\widetilde{C}(Q_2)$, the home nominal interest rate i is constant and equal to $\overline{r} + \widehat{\pi}$, and the price of home consumption P_t^c grows at a constant rate $\widehat{\pi}$. It then follows from the money demand equation (22) that the supply of money must also grow at rate $\widehat{\pi}$,

$$\forall t \geq 2,\ M_t = M_2 e^{\widehat{\pi}(t-2)}.$$

The whole path $(M_t)_{t\geq 1}$ can then be derived from M_1 and M_2. The levels of M_1 and M_2 must be consistent with the exogenously set levels of the real interest rates, $R_1 = e^{i-\widehat{\pi}}$ and $R_1^* = e^{i^*-\widehat{\pi}^*}$. Equations (19), and (21) still apply with nominal stickiness in period 1, but not the other equations of system (18)-((21).

We show how period-1 and period-2 real variables can be determined given R_1 and R_1^*. We can solve for the equilibrium, again, as a fixed point for B_2. Given B_2, we derive Q_2 using (21) and then Q_1 using (19). Period-1 home and foreign consumptions then result from the Euler equations

$$\begin{aligned}\Delta \cdot u'(C_1) &= \beta R_1 u'(\widetilde{C}(Q_2)), \\ \Delta^* \cdot u'(C_1^*) &= \beta R_1^* u'(\widetilde{C}(1/Q_2)),\end{aligned}$$

which, note, are not the same as (17) and (18) because C_1 and C_1^* are not necessarily equal to the full employment levels. We then derive the new value of B_2 as $B_2 = R_1 X_1$ with

$$X_1 = \frac{\eta}{(1-\eta+\eta Q_1^{1-\sigma})^{1/(1-\sigma)}}\left\{\left[(1-\eta)Q_1^{1-\sigma}+\eta\right]^{\sigma/(1-\sigma)} C_1^* - Q_1\left[(1-\eta)Q_1^{-(1-\sigma)}+\eta\right]^{\sigma/(1-\sigma)} C_1\right\}.$$

The mapping thus defined (from B_2 to a new value of B_2) is decreasing. Thus there is one unique fixed point B_2.

Once we know the levels of C_1, Q_1 (and the level of P_1 being exogenous because of nominal stickiness), the levels of money supply in periods 1 and 2 are pinned down by the money demand equations

$$v'\left(\frac{M_1}{P_1\left[1-\eta+\eta Q_1^{1-\sigma}\right]^{1/(1-\sigma)}}\right)=u'(C_1)(1-e^{-i}),$$

$$v'\left(\frac{M_2}{P_1\left[1-\eta+\eta Q_1^{1-\sigma}\right]^{1/(1-\sigma)}e^{\widehat{\pi}}}\right)=u'(\widetilde{C}(Q_2))(1-e^{-(\overline{r}+\widehat{\pi})}).$$

Figure 4 shows the relationship between M_1 and i implied by the first equation. A decrease in i raises C_1 and Q_1, and so must be associated with an increase in M_1 (P_1 being given). When i goes to zero the supply of nominal money goes to the finite satiation level.

A5. Real interest rates and employment

We show that to any pair of real interest rates (r, r^*) one can associate demands for home good and foreign good—and so demands for home labor and foreign labor, respectively denoted by $L(r, r^*)$ and $L^*(r, r^*)$. We then study the variations of $L(r, r^*)$ and $L^*(r, r^*)$ in the linearized version of the model.

Given that $B_2 = 0$ (because of the compensating transfer) we have $Q_2 = C_2 = C_2^* = 1$. We can then derive C_1 and C_1^* from the Euler equations,

$$C_1 = (\beta R/\Delta)^{-\gamma}, \tag{45}$$

$$C_1^* = (\beta R^*/\Delta^*)^{-\gamma}, \tag{46}$$

whereas the real exchange rate Q_1 results from the interest parity condition (19) where $RER(Q)$ is given by (9),

$$\left[\frac{(1-\eta)Q_1^{1-\sigma}+\eta}{1-\eta+\eta Q_1^{1-\sigma}}\right]^{1/(1-\sigma)}=\frac{R^*}{R}, \tag{47}$$

or

$$Q_1=\left[\frac{(1-\eta)(R^*/R)^{1-\sigma}-\eta}{1-\eta-\eta(R^*/R)^{1-\sigma}}\right]^{1/(1-\sigma)}.$$

We can then compute the demand for the home good (which is equal to the level of employment in the home country),

$$\begin{aligned}
L &= \widetilde{C}_H(C_1, Q_1) + \widetilde{C}_H^*(C_1^*, Q_1), \\
&= (1-\eta)\left(\frac{P_1}{P_1^c}\right)^{-\sigma} C_1 + \eta\left(\frac{P_1/S_1}{P_1^{*c}}\right)^{-\sigma} C_1^*, \\
&= (1-\eta)\left[1-\eta+\eta Q_1^{1-\sigma}\right]^{\sigma/(1-\sigma)} C_1 + \eta\left[(1-\eta)Q_1^{1-\sigma}+\eta\right]^{\sigma/(1-\sigma)} C_1^*, \\
&= \left[\frac{1-2\eta}{1-\eta-\eta(R^*/R)^{1-\sigma}}\right]^{\sigma/(1-\sigma)} \left[(1-\eta)\left(\frac{\Delta}{\beta R}\right)^{\gamma} + \eta\left(\frac{R^*}{R}\right)^{\sigma}\left(\frac{\Delta^*}{\beta R^*}\right)^{\gamma}\right]. \quad (48)
\end{aligned}$$

Writing $\beta R/\Delta = \exp(r-\overline{r}-\delta)$, $\beta R^*/\Delta^* = \exp(r^*-\overline{r}-\delta^*)$, $R/R^* = \exp(r-r^*)$ and linearizing under the assumption that $r-\overline{r}$, $r^*-\overline{r}$, δ and δ^* are first order gives expression (24). The expression for the demand for foreign labor is symmetric (equation (26)).

A.5. Proof of Proposition 2

Period-1 welfare can be written

$$U = \Delta \cdot [u(C_1) - f(L_1)] + \frac{\beta}{1-\beta}\left[u(C_2) - f(\overline{L})\right],$$

from which it follows that

$$\frac{\partial U}{\partial r} \simeq \frac{\partial C_1}{\partial r} - f'(L_1)\frac{\partial L_1}{\partial r} + \frac{\beta}{1-\beta}\frac{\partial C_2}{\partial r}, \quad (49)$$

where we have used the fact that, to a first-order of approximation, $u'(C_1) \simeq u'(C_2) \simeq \Delta \simeq 1$.

We compute the partial derivatives for period-1 variables under the approximation that there is a compensating transfer in period 2. This implies

$$\frac{\partial C_1}{\partial r} = -\gamma,$$

and $\partial L_1/\partial r = \widetilde{L}_r$. We compute $\partial C_2/\partial r$ as

$$\frac{\partial C_2}{\partial r} = \widetilde{C}'(1)\frac{\partial Q_2}{\partial r}.$$

$\partial Q_2/\partial r$ results from the differentiation of the budget constraint $\widetilde{X}(Q_2) + (1-\beta)R_1X_1 = 0$, which around a steady state with $X_1 \simeq 0$ and $R_1 \simeq 1/\beta$ gives

$$\widetilde{X}'(1)\frac{\partial Q_2}{\partial r} + \frac{1-\beta}{\beta}\frac{\partial X_1}{\partial r} = 0,$$

Using the two previous equations to substitute out $\partial C_2/\partial r$ from (49) it follows that

$$\begin{aligned}\frac{\partial U}{\partial r} &= \frac{\partial C_1}{\partial r} - \frac{\widetilde{C}'(1)}{\widetilde{X}'(1)}\frac{\partial X_1}{\partial r} - f'(L_1)\widetilde{L}_r, \\ &= \frac{\partial C_1}{\partial r} + \frac{\chi+\eta}{\chi+2\eta^2}\frac{\partial X_1}{\partial r} - f'(L_1)\widetilde{L}_r.\end{aligned}$$

Under sticky prices we have

$$\begin{aligned}\frac{\partial X_1}{\partial r} &= \chi\frac{\partial Q_1}{\partial r} - \eta\frac{\partial C_1}{\partial r}, \\ &= -\frac{\chi}{1-2\eta} + \eta\gamma.\end{aligned}$$

Thus

$$\frac{\partial U}{\partial r} = -\frac{\chi(1-\eta)+\eta^2}{\chi+2\eta^2}\gamma - \frac{\chi(\chi+\eta)}{(\chi+2\eta^2)(1-2\eta)} - f'(L_1)\widetilde{L}_r.$$

This is negative for $L < \overline{L}$ and positive for $L > \overline{L}$ if and only if

$$f'(\overline{L}^-) < \frac{1}{\chi+2\eta^2}\left[\chi + \frac{\gamma\eta^2}{(\chi+\eta)/(1-2\eta)+\gamma(1-\eta)}\right] < f'(\overline{L}^+).$$

A.6. Tariffs

With tariffs the labor demands are given by

$$\frac{L}{\overline{L}} = (1-\eta)\left[1-\eta+\eta(e^{\tau}Q)^{1-\sigma}\right]^{\sigma/(1-\sigma)}C + \eta\left[(1-\eta)\left(e^{-\tau^*}Q\right)^{1-\sigma}+\eta\right]^{\sigma/(1-\sigma)}C^*,$$

$$\frac{L^*}{\overline{L}} = (1-\eta)\left[1-\eta+\eta(e^{\tau^*}/Q)^{1-\sigma}\right]^{\sigma/(1-\sigma)}C^* + \eta\left[(1-\eta)(e^{\tau}Q)^{-(1-\sigma)}+\eta\right]^{\sigma/(1-\sigma)}C.$$

Tariffs have an impact on the real exchange rate because they influence the consumption price indices. The real exchange rate can be written

$$RER(Q) = Q\left[\frac{1-\eta+\eta\left(e^{\tau^*}/Q\right)^{1-\sigma}}{1-\eta+\eta\left(e^{\tau}Q\right)^{1-\sigma}}\right]^{1/(1-\sigma)}.$$

This expression can be used to determine how the foreign terms of trade Q depend on the tariff rates τ and τ^*.

Let us consider a symmetric global liquidity trap in which countries attempt to boost employment by applying symmetric tariffs, $\tau = \tau^*$. Then real interest parity is satisfied for $Q = 1$. The consumption levels C and C^* being constant and equal, setting $\tau = \tau^*$ in the equation for $L/\overline{L}$ above shows that $L/\overline{L}$ goes to infinity as $\tau \to +\infty$. Hence full employment in both countries can be achieved for finite levels of tariff.