Banks, Sovereign Debt and the International Transmission of Business Cycles

Luca Guerrieri
Federal Reserve Board

Matteo Iacoviello
Federal Reserve Board

Raoul Minetti*
Michigan State University

Abstract

This paper studies the international transmission of bank credit crunches triggered by sovereign debt defaults. We posit a two-country economy where capital constrained banks grant loans to firms and invest in bonds issued by the domestic and the foreign government. The model economy is calibrated to data from the euro area, the U.K. and Switzerland, with the two countries representing the core and the periphery countries, respectively. Large contractionary shocks in the periphery trigger sovereign default. Sizing the default to match recent credit events in Greece, we find sizable spillover effects to the core bloc through a drop in the volume of credit extended by the banking sector.

1 Introduction

In the last fifteen years, following the introduction of the euro and the resulting elimination of exchange rate risk among euro area members, European banks have increasingly, "happily owned regional, rather than merely national, government bond portfolios" (The Economist, 2012). In particular, banks of the core European countries (e.g., France and Germany) have turned into major holders of the sovereign debt of periphery countries, such as Portugal, Italy, Greece, Spain (the so-called PIGS). For instance, combining data from the Bank for International Settlements with data from the Bank of France reveals

*Corresponding author. Department of Economics, Michigan State University. E-mail: minetti@msu.edu. Phone: +1-517-355-7349. Address: MSU Department of Economics, Marshall-Adams Hall, 486 W Circle Dr. Rm 110, East Lansing, MI 48824. The views expressed in this paper are those of the authors only, and do not necessarily represent the views of the Federal Reserve Board or the Federal Reserve System.

that in the last quarter of 2009, just before the outbreak of the eurozone sovereign debt crisis, the ratio of French banks' holdings of PIGS sovereign debt over their holdings of French government debt equaled 56%, up from 19% in the first quarter of 2005. Figure 1 plots various ratios for French banks' holdings of PIGS sovereign debt from the first quarter of 2005 (the first for which data are available) to the third quarter of 2011 (analogous data for German banks are available only starting in 2010Q4). The figure shows a dramatic increase in the holdings of PIGS sovereign debt by French banks, whether these are normalized by banks' total assets, their holdings of French government debt or by the total sovereign debt of PIGS countries.[1]

During the current sovereign debt crisis of the eurozone, the large holdings of government bonds accumulated in recent years have significantly exposed European banks to the default risk of periphery countries. This has been exacerbated by the fact that, as shown by the restructuring of Greek sovereign debt in March 2012, banks and other private investors are treated as junior creditors relative to official investors (e.g., the central banks of the Eurosystem) during sovereign debt restructurings.[2] European banks have thus suffered from a sizable erosion of their capitalization and a severe difficulty to tap wholesale funding and interbank markets. Allegedly, this has in turn resulted into reduced ability and propensity to extend credit to firms (credit crunch).

The propagation of shocks to the debt service capacity of sovereigns to the banking sector and, ultimately, to the corporate sector poses challenges that have been largely unexplored thus far. A first critical question regards the magnitude of the possible effects. How large can a bank credit crunch induced by a sovereign debt crisis be and how important the international spillovers? Second, how does the degree of banks' exposure to foreign sovereign debt affect the transmission of shocks across countries?[3] If banks tend to deleverage especially by selling government bonds this will aggravate the financing problems of the government, but it will shield firms from the credit crunch. If instead banks tend to deleverage especially by contracting credit to the corporate sector, this will alleviate the pressures on the government, but it will exacerbate firms' financing problems.[4] In an international

[1] The holdings of Belgian, Italian and Spanish government debt exceeded the tier 1 capital of banks in the three countries and 50% of the tier capital of banks in France and Germany (see Shambaugh, 2012).

[2] The restructuring of Greek debt in March 2012 involved bonds for a total value of about 199 billion Euros. The exchange of new bonds for old ones reduced the face value of bonds by almost 55%.

[3] In the analysis, we abstract from the interbank market and posit that banks fund themselves through retail deposits.

[4] During the eurozone crisis, banks of core European countries have only slightly decreased or increased their holdings of domestic government debt. By contrast, they have reduced both their holdings of foreign government bonds and the credit extended to firms.

perspective, these issues are compounded by the fact that banks hold both domestic and foreign government bonds. This elicits the question not only whether banks choose to reduce government bond holdings or firm loans, but also whether they especially scale down their holdings of foreign government bonds. In the light of all these issues, it becomes critical to analyze the international transmission of business cycles when banks hold both loans to the private sector and domestic and foreign government bonds in their portfolios.

For our analysis, we posit a two-country dynamic stochastic general equilibrium model with flexible prices and wages. In each country there are households, entrepreneurs, final good firms, and a government. In addition, in each country a banking sector intermediates funds between households and entrepreneurs. Households supply labor to domestic entrepreneurs and deposit savings in local banks. Banks, in turn, grant loans to entrepreneurs and invest in bonds issued by the domestic and the foreign government. Agents in both countries derive utility from final consumption goods. In each country, entrepreneurs produce final goods using capital and labor (non-tradeable internationally). Banks are subject to an exogenous capital requirement that constrains their loanable funds. The government maximizes the utility from its expenditure, finances itself through taxes and bonds and can choose to default on its outstanding debt. In each period, its debt capacity is related to its repayment history, reflecting the possibility that default induces punishment in the form of partial exclusion from financial markets.

The model is calibrated to data for the PIGS countries on one side and for the rest of the euro area, the U.K., and Switzerland on the other. We perturb the economy with shocks to the total factor productivity of the PIGS bloc, and compare the effects of two contractionary shocks that only differ in size. One shock is too small to cause sovereign default. Another shock is large enough to induce default. Our main objective is to investigate whether country-specific shocks that trigger sovereign default in the PIGS bloc can generate international contagion, inducing a contraction of credit, firms' investment and output in the core bloc. We find that a shock that induces a default in the PIGS bloc can have a sizable impact on output in the core bloc: a 200 billion euros partial default on the sovereign debt of the PIGS, roughly twice the size of the Greek debt restructuring in March 2012, lowers non-PIGS GDP by about a 1/4 percent after two years. Intuitively, default erodes the capitalization of both domestic and foreign banks. In the presence of binding capital requirements, this forces banks to contract their loans, thus causing a drop of investment and output. Overall, the simulations reveal that inside the PIGS bloc the sovereign default may slightly mitigate the effects of the negative technological shock

on output. Together with the finding that the output of the core bloc is negatively hit, this suggests that the PIGS sovereign default could have the effect of a "beggar-thy-neighbor" policy.

The remainder of the paper is organized as follows. Section 2 relates the paper to the prior literature. Section 3 describes the model. Section 4 presents the calibration of the parameters. In Section 5, we present the simulation results. Section 6 concludes. The Appendix provides details on the solution of the model while the formulae for the steady state equilibrium are relegated to a Supplement.

2 Related Literature

There are very few studies on the role of banks and supply-side credit market imperfections in global economies. In addition, these studies do not focus on sovereign debt problems. Devereux and Yetman (2010) study a two-country economy in which investors hold assets in the domestic and the foreign country but are exposed to leverage constraints. They find that if international financial markets are highly integrated, productivity shocks will be propagated through investors' financial portfolios. In turn, this will generate a strong output comovement between the two countries. Mendoza and Quadrini (2010) consider a two-country model in which the two countries feature a different degree of financial development, as captured by households' ability to insure against income shocks. Then, they investigate the propagation of shocks to bank capital from one country to the other. Kollmann, Enders and Muller (2011) and Kalemli-Ozcan, Papaioannou and Perri (2011) consider two-country environments with a global banking sector. When a shock erodes the capitalization of the global banks, it reduces credit supply and depresses economic activity in both countries.[5] In particular, banks' losses raise bank intermediation costs in both countries, triggering synchronized business fluctuations. Kamber and Thonissen (2012) analyze the international transmission of shocks in a global economy in which banking sectors are mostly independent: banks in the large economy do not lend to firms in the small economy. Ueda (2012) constructs a two-country model in which financial intermediaries stipulate chained credit contracts domestically and abroad (that is, they engage in cross-border lending by undertaking cross-border borrowing from investors). His analysis reveals that negative shocks to

[5] Gilchrist (2004) models a financial accelerator in a two-country model where the two countries feature different leverage. Shocks are transmitted internationally by affecting foreign borrowers' net worth.

one country propagate to the other, generating comovement of business cycles.[6]

This paper also relates more broadly to the literature on financial imperfections in open economies. A growing body of research finds that credit market imperfections help explain some of the features of the international transmission of business cycles that cannot be explained by RBC models. Backus, Kehoe and Kydland (1992), Baxter and Crucini (1995) and Heathcote and Perri (2002) find that restrictions in the trade of financial assets can account for the positive output correlation across countries by reducing international capital mobility. More recently, such papers as Kehoe and Perri (2002), Iacoviello and Minetti (2006) and Gilchrist, Hairault and Kempf (2002) have analyzed models in which countries face borrowing constraints when tapping international financial markets. The presence of borrowing constraints amplifies the international transmission of shocks. In Dedola and Lombardo (2012) and Perri and Quadrini (2010) firms face borrowing constraints due to limited credit contract enforceability. In their environments, tighter borrowing constraints in one country can induce a contraction in economic activity in the other country.[7]

Finally, the analysis is related to the growing literature on the role of banks and bank capital in general equilibrium closed-economy models. Papers in this literature include Gertler and Karadi (2010) and Gertler and Kiyotaki (2010).[8] Banks do not hold government debt in these papers. We will further discuss the relation with these studies.

3 The Model

This section describes the set-up of the model, solves for agents' decisions, and characterizes the equilibrium.

3.1 Environment

The world economy consists of two countries, Home and Foreign. In each country there are infinitely-lived households, entrepreneurs (capital good producers), final good producers, bankers. All agents of a given type are homogeneous. In addition, in each country there is a government that purchases final

[6] Other papers in this literature include Olivero (2010) and Gerali, Neri, Sessa and Signoretti (2010). Olivero (2010) investigates an imperfectly competitive global banking system and examines how changes in banks' markup can propagate shocks internationally.

[7] In Dedola and Lombardo (2012) investors hold both domestic and foreign capital stock but can only borrow from the domestic capital market in a model with financial frictions as in Bernanke, Gertler and Gilchrist (1999).

[8] See also, e.g., Den Haan, Ramey and Watson (2004), Minetti (2007), and Iacoviello (2011).

goods financing its expenditures with debt and lump-sum taxes. In the world economy there is one final good (tradeable internationally at no cost). The final good is produced using labor (non-tradeable internationally) and capital. All markets are competitive. The home and the foreign country have symmetric preferences and technology. In what follows, we concentrate on the description of the home country, denoting the variables of the foreign country by an asterisk (*).[9]

Agents' activities are as follows. In each period, households supply labor to entrepreneurs. Households can save in three ways: they can invest in physical capital, which they rent to final good producers; they can hold deposits in domestic banks; they can purchase noncontingent bonds issued by the domestic government.[10] Entrepreneurs receive loans from banks and invest into physical capital, which they rent to final good producers. Final good producers produce the final good using labor and capital. Bankers receive deposits, make loans to domestic entrepreneurs and purchase bonds issued by the domestic and the foreign government. In each period, final good producers interact with households in the labor market and in the market for capital and with entrepreneurs in the market for capital; bankers interact with entrepreneurs in the credit market.

3.2 Households

There is a continuum of identical infinitely-lived households, who derive utility from consumption and from leisure. The representative household maximizes its expected utility

$$E_s \left\{ \sum_{t=s}^{\infty} \beta_H^{t-s} \left(\frac{C_{H,t}^{1-\sigma_H}}{1-\sigma_H} - \frac{N_{H,t}^{1+\eta}}{1+\eta} \right) \right\}, \tag{1}$$

where E_s denotes the expectation operator conditional on the information available in period s, β_H is the household's subjective discount factor, $C_{H,t}$ is its consumption, $N_{H,t}$ is its labor supply, σ_H is the household's intertemporal elasticity of substitution of consumption, η is the parameter that governs the labor supply elasticity. As explained below, we allow the subjective discount factor to differ among households, bankers and entrepreneurs.

Equation (2) shows the budget constraint of the representative household:

[9] Our model features perfect substitutability between goods produced at home and abroad.

[10] For simplicity, we do not allow households to hold foreign government debt and foreign deposits. The amount of foreign government bonds held by households directly (i.e., without the intermediation of the banking sector) is very small.

$$
\begin{aligned}
& C_{H,t} + D_t + B_{H,t} + K_{H,t} + T_t \qquad (2)\\
= \;& R_{D,t} D_{t-1} + R_{B,t-1} B_{H,t-1} - \frac{B_{H,t-1}}{B_{t-1}} \zeta_t + \left(R_{KH,t} + 1 - \delta\right) K_{H,t-1} + W_t N_{H,t} \; + AC_{H,t},
\end{aligned}
$$

where D_t are the household's holdings of deposits in domestic banks, $B_{H,t}$ are the holdings of domestic government bonds, $K_{H,t}$ is the amount of capital owned by the household. $R_{D,t}$ and $R_{B,t-1}$ are respectively the gross interest rates on deposits and government bonds between period $t-1$ and period t. Notice that the return on deposits is assumed to be predetermined. $R_{KH,t}$ is the rental price of capital paid to households by final good producers, δ is the capital depreciation rate, W_t is the wage rate, and T_t is a lump-sum tax imposed on households by the government. In any period, after agents purchase government debt, the government may decide to default. The term ζ_t captures the loss borne by households from government default, where $\frac{B_{H,t-1}}{B_{t-1}}$ is the share of outstanding government bonds B_{t-1} held by households. ζ_t takes non-negative values and is equal to zero unless partial sovereign default occurs. Finally, the term $AC_{H,t}$ denotes quadratic portfolio adjustment costs which are paid by the household for changing the holdings of D_t and $K_{H,t}$ between one period and the next, and for changing the holdings of $B_{H,t}$ relative to a target steady state level which we calibrate from the data. These costs make the households' supply of deposits, bonds and capital less sensitive to interest rate differentials, and, in the case of $B_{H,t}$, pin down the steady state amount of government debt held by households.[11]

Using the first-order conditions for the household's problem with respect to consumption and government debt holdings, one obtains the following Euler condition:

$$
\left(C_{H,t}\right)^{-\sigma_H} = \beta_H E_t \left(R_{BH,t} - \frac{\zeta_{t+1}}{B_t} \right) \left(C_{H,t+1}\right)^{-\sigma_H} . \qquad (3)
$$

From this condition, it is easy to see that the expectation of default affects the effective rate of return from bond holding. Accordingly, equilibrium interest rates will adjust to the extent that government default is anticipated.

The remaining optimality conditions for the households with respect to bank deposits, capital holdings and labor supply are standard and are in the Appendix. The households' optimality conditions

[11] We solve our model using first-order approximations around the deterministic steady state. In principle, given the total supply of government debt, the relative holdings of households and banks could be solved for endogenously using the methods described in Devereux and Sutherland (2011); this would in turn require specifying a rich stochastic structure for the shocks hitting our model economy, an approach that we do not pursue here.

will imply that in an interior steady state the households will demand both deposits and government bonds when their return equals the households' discount rate, $1/\beta_H - 1$.

3.3 Bankers

Bankers collect deposits from domestic households, make loans to domestic entrepreneurs, and hold domestic and foreign government bonds. Since the banking sector is perfectly competitive, banks take interest rates prevailing in the domestic and foreign bond and credit markets as given. The representative banker maximizes its expected discounted utility

$$E_s \left[\sum_{t=s}^{\infty} \beta_B^{t-s} \frac{(C_{B,t})^{1-\sigma_B}}{1-\sigma_B} \right], \tag{4}$$

where $C_{B,t}$ denotes the banker's consumption, σ_B is the banker's intertemporal elasticity of substitution of consumption, and β_B is its discount factor. The flow budget constraint of a banker is

$$\begin{aligned} & C_{B,t} + R_{D,t} D_{t-1} + L_t + B_{B,t} + B_{F,t} \\ = \; & D_t + R_{L,t} L_{t-1} + R_{B,t-1} B_{B,t-1} + R^*_{B,t-1} B_{F,t-1} \\ & - \frac{B_{B,t-1}}{B_{t-1}} \zeta_t - \frac{B_{F,t-1}}{B^*_{t-1}} \zeta^*_t + AC_{B,t}, \end{aligned} \tag{5}$$

where L_t denotes loans to entrepreneurs, $B_{B,t}$ and $B_{F,t}$ are the banker's holdings of domestic and foreign government bonds respectively. The terms ζ_t and ζ^*_t measure the losses that are borne by the bankers in case of domestic and foreign sovereign debt defaults. Like for the household problem, $AC_{B,t}$ denotes quadratic adjustment costs for changing L_t, $B_{B,t}$, $B_{F,t}$, and D_t.

Bankers are subject to a capital constraint that specifies that the value of the loans they extend cannot exceed a certain fraction of the value of their net worth. The capital constraint reads as:

$$D_t \leq \gamma_L L_t + B_{B,t} + B_{F,t}, \tag{6}$$

where $\gamma_L < 1$. The capital constraint is aimed at capturing both regulatory-driven and market-driven capital requirements. In the case of loans, the constraint puts a limit on the leverage of the bank since it requires that, for each unit of loans extended, the bank sets aside $1 - \gamma_L$ units of goods as bank capital. As a consequence, the equilibrium return on loans will be higher than the return on government bonds (and higher than the cost of deposits) in order to compensate the bank for the relative illiquidity/non-pledgeability of loans relative to government bonds (see the Appendix for more

details).[12] We also assume that government bonds are fully collateralizable by the bank (that is, the bank can collect one dollar of deposits and buy one dollar of government bonds without having to set aside any net worth), in line with standard features of capital regulation.[13]

We assume that the bankers' discount factor β_B is lower than the households'. This relative impatience assumption implies that the bankers' capital constraint binds with equality in a neighborhood of the steady state.

3.4 Entrepreneurs

In the home country, there is a continuum of identical infinitely-lived entrepreneurs who maximize the following expected lifetime utility

$$E_s \left[\sum_{t=s}^{\infty} \beta_E^{t-s} \frac{(C_{E,t})^{1-\sigma_E}}{1-\sigma_E} \right], \tag{7}$$

where β_E is the entrepreneur's subjective discount factor, σ_E is the entrepreneur's intertemporal elasticity of substitution of consumption, and $C_{E,t}$ is the entrepreneur's consumption. Entrepreneurs are less patient than households and bankers.[14] Entrepreneurs transform loans into capital and into consumption using a one-for-one technology. The capital good they produce is rented out to final good producers. In each period, the representative entrepreneur borrows from bankers, produces and rents

[12] The assumption that $\gamma_L < 1$ implies that loans and deposits are not perfect substitutes for the bank, and that loans provide, loosely speaking, lower utility to the bank, the lower γ_L is. For the banker to be indifferent between financing its operations through loans and deposits, the interest rate on loans adjusts in equilibrium to compensate the banker for the relative illiquidity of loans. This happens because all bankers are identical (and subject to the same constraint) and because markets are incomplete – only bankers have the technology to make loans.

The nature of the bank problem described here extends the closed-economy formulation of Iacoviello (2011) to an open economy setting and to the case where banks can hold more than one asset.

[13] For the households' problem to have an interior solution, one requires that the steady state return on government bonds equals the steady state return on deposits. Since bankers receive deposits from households and also purchase government bonds, imposing that government bonds are not fully collateralizable for the bank would imply that banks would not want to purchase government bonds, since deposit would always be cheaper as a source of financing.

[14] We assume that entrepreneurs are impatient relative to bankers and that bankers are impatient relative to households. Implicitly, we also assume that the markets are segmented, so that banks are essential in order to intermediate funds between households who want to save and entrepreneurs who want to borrow. These assumptions ensure that in the steady state equilibrium there is borrowing and lending between the household and the bankers (through deposits) on the one hand, and borrowing and lending (through loans) between the bankers and the entrepreneurs.

capital, repays debt, and consumes. The flow budget constraint is:

$$C_{E,t}+K_{Et}+R_{L,t}L_{t-1} = R_{K,t}K_{Et-1}+(1-\delta)K_{Et-1}+L_t+AC_{E,t}, \tag{8}$$

where K_{Et} is the capital stock owned by the entrepreneur, R_t^L is the gross interest rate on loans between period $t-1$ and period t, $R_{K,t}$ is the rental rate of capital paid to entrepreneurs by final good producers, and $AC_{E,t}$ are quadratic adjustment costs for changing K_E and L_t between one period and the next. Entrepreneurs are subject to a working capital constraint that limits their leverage to a fraction m of their capital holdings:

$$L_t = \rho_E L_{t-1} + (1-\rho_E)\, mK_{Et}, \tag{9}$$

where the parameter ρ_E dictates how elastic is the loan limit to the current capital choice of the entrepreneur.

3.5 Final good producers

The representative final good producer operates a Cobb-Douglas production function given by

$$Y_t = F(K_{Ht-1}, K_{Et-1}, N_t, A_t) = A_t K_{Ht-1}^{\alpha(1-\mu)} K_{Et-1}^{\alpha\mu} N_t^{1-\alpha}, \tag{10}$$

where Y_t is the output of domestic final good, K_{Ht} is the capital stock rented from households, K_{Et} is the capital stock rented from entrepreneurs, N_t is the labor input and A_t denotes a stochastic process for productivity. $1-\alpha$ is the labor income share, $\alpha(1-\mu)$ the share of household capital, $\alpha\mu$ the share of entrepreneur capital. Producers simply maximize period by period revenues, make zero profits because of perfect competition and constant returns to scale, and equate the marginal product of each factor to its rental cost.

3.6 Government

The government carries out public expenditure and finances its expenditure through taxes on households and debt. The government budget constraint is

$$G_t + R_{B,t-1}B_{t-1} = B_t + T_t + \zeta_t, \tag{11}$$

where G_t is government consumption and B_t is government debt. Taxes T_t are governed by the following reaction function:

$$T_t = (1-\rho_T)\,\bar{t}Y_t + \rho_T T_{t-1}. \tag{12}$$

Government debt is held by households, domestic and foreign banks (that is, $B_t = B_{H,t}+B_{B,t}+B^*_{Ft}$).We assume that the government is myopic and seeks to maximize current expenditure (see the Appendix for details). The term ζ_t in equation (11) captures the partial default on previously contracted debt obligations. We restrict ζ_t to take only two values – either zero in the non-default state, or some positive value ζ if partial default occurs.

Given its myopic objective, the government could in principle try and borrow as much as possible today in order to maximize its expenditures G_t. We impose a limit to the government ability to raise funds in financial markets, with the tightness of the constraint being related to the government debt repayment history. The debt-ceiling constraint takes the form

$$B_t \leq (1-\rho_B)\,\overline{b}Y_t + \rho_B B_{t-1} - b_t. \tag{13}$$

When the parameter ρ_B is zero, the debt ceiling only depends on current output Y_t. Values of ρ_B greater than zero allow for dependence of the current debt ceiling on the limit in the previous period. In the event of default, the term b_t denotes a punishment triggered through an immediate reduction in the debt ceiling, where

$$b_t = \begin{cases} (1-\rho_B)\,\underline{b}Y_t \text{ at time } t \\ \rho_b b_{t-1} \text{ in periods after } t \end{cases}. \tag{14}$$

The parameter ρ_b governs the persistence of the punishment. With $\rho_b < 0$, eventually b_t returns to zero. The parameters $\underline{b}$ and ζ are chosen so that, in the non-stochastic steady state, default is punished with a reduction of the debt ceiling large enough to imply a fall in government spending. Only when shocks, through their effects on Y_t, lead to a large enough reduction in the debt ceiling does the government find it convenient to default.[15]

This specification of the government's problem allows to endogenize the government's decision to default. This endogenization relates the occurrence and persistence of default episodes to the evolution of key macroeconomic variables.

[15] Suppose for instance that $T_t = \overline{t}Y_t$ and $\rho_B = 0$. The government maximum expenditure without default is

$$G_t^{ND} = \overline{b}Y_t - R_{t-1}B_{t-1} + \overline{t}Y_t,$$

while expenditure with default is

$$G_t^{D} = \underline{b}Y_t + \zeta - R_{t-1}B_{t-1} + \overline{t}Y_t.$$

If $Y_t < \frac{\zeta}{\overline{b}-\underline{b}}$, then $G^D > G^{ND}$, and the government finds it convenient to default.

3.7 Equilibrium characterization

Given initial capital stock of households and entrepreneurs $\{K_H, K_E\}$, bond holdings of domestic and foreign banks and households $\{B_B, B_F, B_H\}$, and a sequence of productivity realizations $\{A_t\}$, an equilibrium of our economy is a sequence of allocations $\{Y, C_B, C_E, C_H, K, L, D, B, B_B, B_H, T, G,\}$ and prices $\{R_D, R_K, R_L, W, \lambda_B, \lambda_E, R_B, \mu\}$ such that (i) households, entrepreneurs, final good producers, bankers, the government solve their optimization problems and (ii) the market for final goods, the domestic and the foreign labor market, the domestic and the foreign credit market, the domestic and the foreign deposit market, and the domestic and the foreign government bond market clear.[16]

4 Calibration and Solution

We choose the relative size of technology in the two country blocs so that the relative size of output is 1/3 in the non-stochastic steady state. This choice reflects the size of nominal GDP at the end of 2010 for Portugal, Italy, Greece, and Spain relative to the rest of the euro area plus the United Kingdom and Switzerland. The parameters σ_H, σ_B, and σ_E are set to imply an intertemporal elasticity of substitution for consumption equal to 1. The Households' discount factor, β_H, is set to 0.9925, implying an annual interest rate on deposits and bonds of 3%, given that the unit of time is one quarter. The discount factors of bankers and entrepreneurs are instead set equal to 0.965 and 0.96, respectively. The households' labor supply elasticity is set at 2 (that is, $\eta = 1/2$).

On the production side, the depreciation rate δ is 0.03 and the capital share in production α is 0.33. The value of the capital adjustment cost parameter is set equal to 0.5,[17] and all the capital is assumed to be owned by the entrepreneurs, so that $\mu = 1$. We let the parameter governing entrepreneurs' working capital constraint, m, entail a loan-to-value ratio of 0.90, and set $\rho_E = 0.75$.

Moving to the government, the parameter $\bar{b}$ is set to 3.4. This choice implies a debt-to-GDP ratio equal to 0.85 in the PIGS bloc and 0.6 in the rest-of-Europe bloc (euro area excluding PIGS plus Switzerland and the United Kingdom), in line with data from the IMF Economic Outlook database for the end of 2010. The value for $\underline{b}$ is instead set at 3. For the choice of the tax rate reaction function,

[16] The formulae for the steady state are in a Supplement available from the authors.

[17] The capital adjustment cost function takes the form $\phi_{KE}\left(\frac{K_t}{K_{t-1}} - 1\right)^2$. We set $\phi_{KE} = 0.5$. Entrepreneurs also pay a convex cost for adjusting loans of the form $\phi_{LE}\left(\frac{L_t}{L_{t-1}} - 1\right)^2$. We set $\phi_{LE} = 0.05$.

we set $\rho_T = 0.5$ and $\bar{t} = 0.24$. In the government debt ceiling equation,

$$B_t \leq (1 - \rho_B)\,\bar{b} Y_t + \rho_B B_{t-1} - b_t, \tag{15}$$

we set $\rho_B = 0.8$ in the no-default case.

We pin down the steady state relative holdings of sovereign debt by imposing a quadratic adjustment cost relative to a target steady state level. The adjustment cost function for sovereign bonds held by households takes the form $\phi_{BH}\left(B_{H,t}/B_H - 1\right)^2$, where B_H denotes the households' steady state holdings of government debt. We set $\phi_{BH} = 0.05$.[18] In the bank problem, the adjustment cost takes the same form: the two parameters measuring the convexity of the adjustment cost function, ϕ_{BB} and ϕ_{BF}, denote how costly it is for the banks to adjust domestic and foreign bonds: we set $\phi_{BB} = 0.05$ and $\phi_{BF} = 0.5$, denoting larger adjustment costs for foreign debt.[19] Next, to apportion the holdings of government debt to households, domestic banks, and foreign banks, we use several data sources. For the PIGS bloc, we obtained data on the amount of government debt held by foreign counter-parties from the World Bank Quarterly External Debt Statistics. At the end of 2010, 50% of the PIGS sovereign debt was held outside the PIGS. In the absence of a country-by-country breakdown on holdings of government debt, we apportioned the holdings of foreign debt for the PIGS bloc to European and non-European countries using the Consolidated Banking Statistics maintained by the Bank for International Settlements. At the end of 2010, on an ultimate-risk basis, 84% of the PIGS sovereign debt held by banks was held by European banks.[20] Accordingly, we imposed that in the non-stochastic steady state 42% (that is, 84% times 50%) of the public debt of the PIGS is held by the European bloc. Using data from national flow of funds statistics for the PIGS we estimated that 22% of the outstanding government debt was held by PIGS households. We chose a steady-state allocation for the holding of debt by households in the PIGS bloc of 30% of the outstanding debt, equal to the estimate of 22% from the flow of funds plus the 8% apportioned to non-European financial institutions using BIS data. The remainder of the PIGS government debt, 28% of the total, is held by PIGS banks.

The calibration of the households' holding of government debt for the non-PIGS European bloc

[18] Households pay a cost for adjusting deposits of the form $\phi_{DH}\left(\frac{D_t}{D_{t-1}} - 1\right)^2$. We set $\phi_{DH} = 0.05$.

[19] Banks also pay a cost for adjusting deposits and loans. The cost functions are $\phi_{DB}\left(\frac{D_t}{D_{t-1}} - 1\right)^2$ for deposits, and $\phi_{LB}\left(\frac{L_t}{L_{t-1}} - 1\right)^2$ for loans. We set $\phi_{DB} = \phi_{LB} = 0.05$.

[20] For the European bloc, we selected the banks of all the countries in the euro area (excluding the PIGS) that respond to the BIS. The list of these countries included: Belgium, France, Germany, Switzerland, and the United Kingdom. The non-european bloc included all other countries in the BIS database (not including the PIGS).

is based on national flow of funds statistics for France, Germany, Switzerland, and the United Kingdom. 11% of their collective sovereign debt is held by households in those countries.[21] To ensure balanced trade in steady state, we forced the foreign debt of the non-PIGS countries held by PIGS banks to match the level of its counterpart. The remainder of the outstanding debt of the non-PIGS European bloc, 70% of the total, was assigned to domestic banks.

For the parameters governing bank leverage, we set $\gamma_L = 0.9$. Given the relative shares of loans and bonds in the bank balance sheet, this parameter implies a steady state bank capital to asset ratio of 0.08, in line with the data.

Moving to the solution method, we use a piece-wise linear solution approach as is common in the expanding literature on the zero lower bound on nominal interest rates.[22] We treat the possibility of default as an alternative regime. The same numerical method can handle occasionally binding constraints or regime switches that depend on the evolution of endogenous variables. In case of no default, the linearized system of necessary conditions for an equilibrium can be expressed as

$$\mathcal{A}_1 E_t X_{t+1} + \mathcal{A}_0 X_t + \mathcal{A}_{-1} X_{t-1} = 0, \tag{16}$$

where $\mathcal{A}_1$, $\mathcal{A}_0$, and $\mathcal{A}_{-1}$ are square matrices of coefficients, conformable with the vector X. In turn, X is a vector of all the variables in the model expressed in deviation from the steady state for the regime without default. Similarly, in case of default, the linearized system can be expressed as

$$\mathcal{A}_1^* E_t X_{t+1} + \mathcal{A}_0^* X_t + \mathcal{A}_{-1}^* X_{t-1} + \mathcal{C}^* = 0, \tag{17}$$

where $\mathcal{C}^*$ is a vector of constants. In the absence of default, we use standard linear solution methods to express the decision rule for the model as

$$X_t = \mathcal{P} X_{t-1}. \tag{18}$$

With default, we shoot back towards the initial conditions, from the first period when no default is posited. For example, to check for no default in period $t+1$ and default in period t, the decision rule between period $t-1$ and t can be expressed as:

$$\begin{aligned} \mathcal{A}_1^* \mathcal{P} X_t + \mathcal{A}_0^* X_t + \mathcal{A}_{-1}^* X_{t-1} + \mathcal{C}^* = 0, \\ X_t = -\left(\mathcal{A}_1^* \mathcal{P} + \mathcal{A}_0^*\right)^{-1} \left(\mathcal{A}_{-1}^* X_{t-1} + \mathcal{C}^*\right). \end{aligned} \tag{19}$$

[21] For both the PIGS bloc and the European bloc (ex PIGS), when national flow of funds data did not split out explicitly the holdings of government debt by households, we used broader holdings of securities excluding shares.

[22] For instance, see Eggertson and Woodford (2003) and Bodenstein, Guerrieri and Gust (2010).

We proceed in a similar fashion to construct the time-varying decision rules for the case when default is posited to last multiple periods or happen starting in periods beyond t. Under this solution, government bond yields reflect the possibility of default only when default is perfectly anticipated. Since anticipated future defaults imply a default premium that completely compensates bond holders for default, we conjecture and verify that default is only chosen for one period in response to contractionary unanticipated shocks. Intuitively, if anticipated, default is priced into bond yields and the government does not find it convenient to default.

5 Results

We are interested in investigating the impact of unexpected technological shocks that induce partial default by the government of the Home country (the PIGS bloc). Our objective is to address the following questions: how do shocks to a government's debt service capacity propagate internationally? To what extent can they trigger a global credit crunch? And what are the factors that shape banks' responses to the government default shock, especially the relative contractions of loans and government bond holdings? To answer these questions, in this section we report the impulse responses of our model economy to country-specific shocks to productivity. To establish a useful benchmark, we first consider a scenario in which the shock is sufficiently small so that the government does not default on its debt obligations. Then, we turn to the scenario in which the shock is large enough that the government defaults.

5.1 No Default

To isolate the transmission of country-specific productivity shocks, we first consider the effects of a 1% decrease in the process A_t in the PIGS bloc, with autoregressive persistence equal to 0.95. The solid lines in Figure 2 show the responses for the benchmark calibration; the dashed lines show the responses for a calibration that differs in the time-t income elasticity of the debt-ceiling for the government. In the latter case, ρ_B is set at 0.99: the debt ceiling has greater lagged dependence and becomes relatively insensitive to changes in output.

In both cases, as households anticipate a future rebound in output, consumption drops less than output up front, reflecting households' incentive to smooth consumption in response to the shock. As a result, the supply of households' savings contracts. The equilibrium bond rate depends on the response of the government supply of debt. Under our benchmark parameterization, the debt ceiling

is sufficiently elastic with respect to output that the drop in supply exceeds the drop in savings at constant interest rates. Therefore, to reestablish the equilibrium in the bonds market, the yield on public debt falls. Lower interest rates on bonds lead banks to skew their portfolio towards loans to entrepreneurs, cushioning the drop in investment in the PIGS bloc and leading to an expansion in investment abroad. In this respect, the implications for the spillover effects of country-specific shocks in our baseline model with no default are not very different from those in a canonical international real business cycle model with complete markets.

By contrast, when the demand for bonds (or, equivalently, the supply of savings) from households drops relatively more than the supply of bonds from the government – as shown in the case of the alternative calibration with inelastic debt ceiling – the yield on government bonds can rise. Investment drops more severely at home and drops abroad, too, as banks move their portfolio into government bonds and away from loans to entrepreneurs.

5.2 Default

We now turn to investigate a scenario where government default occurs. Figure 3 shows the effects of a negative 5 percent technology shock. The solid line shows the piece-wise linear solution that reflects the partial default on government debt in the PIGS bloc. The dashed line shows the response in a model that excludes the possibility of default, but is otherwise identical to the benchmark. The amount of default is governed by the parameter ζ in the budget constraint of the government.

We choose ζ to mimic a scenario somewhat worse than the March 2012 Greek debt restructuring. In the March 2012 restructuring, the face value of Greek sovereign debt held by private investors was cut by about 100 billion euros. Given our calibration, such a scenario would correspond to a (partial) default equal to roughly 4 percent of the PIGS annual GDP, and, taking into account country sizes, 1.5 percent of non-PIGS GDP. In our experiment, we size ζ so that the non-repayment is equal to roughly twice as much as the March amount. As a consequence, default amounts to about 3 percent of non-PIGS GDP, and, given the amount of PIGS debt held by foreign banks, the losses for non-PIGS banks are about 1.25 percent of non-PIGS GDP. Ultimately, the main impact of the default on non-PIGS economies is to erode bank capital by this amount, which is about 80 billion euros.

In this experiment, the government does not pay back in full the holders of its bonds. Such a negative shock to the government repayment rate can effectively be thought of as a wealth transfer from the banking sector (domestic and foreign banks) to households. In fact, in an economy in which

households are the only holders of government bonds a repudiation of sovereign debt would effectively be a wash out. Households know that government debt today entails taxes in the future, that is, by Ricardian equivalence they do not treat government bonds as net worth. When instead both banks and households hold sovereign debt, a repudiation of part of this debt implies lower taxes for households in the future. Therefore, the repudiation effectively results into a transfer from banks to households.

As shown in Figure 3, sovereign default initially cushions the contraction in government spending – in the first period, the black line in panel 3 is above the dashed line. Mechanically, default occurs because the contraction in output lowers the debt ceiling even in the absence of default and because the punishment for default is in the form of a fixed proportional reduction in the debt ceiling: as the economy contracts, the punishment shrinks in absolute terms. As the punishment is persistent, and the debt ceiling is lag dependent, the government debt initially continues to shrink. Then, as output gradually recovers, the government ability to borrow also bounces back. In spite of a reduced ability to borrow, lower interest payments eventually allow government spending to expand above the steady state level.

The evolution of investment in the PIGS is closely linked to loan rates. In turn, loan rates are influenced by two opposing effects, a portfolio substitution effect and a hit to the equity position of banks. Consider the latter. Both in the home and in the foreign economy, banks directly suffer losses when the default rate of the government rises above what expected when they invested in bonds and granted loans. This loss induces a decline in banks' capitalization and in turn, in the presence of binding capital requirements, this generates a contraction in credit supply, raising loan rates. Remember that entrepreneurs use bank loans to finance the production of capital goods. The increase in loan rates thus triggers a contraction in the supply of capital goods, a rise in the rental price of capital and, hence, a drop in investment and in the capital stock. Consider next the portfolio substitution effect. We need to consider the equilibrium rates in the sovereign bonds market. As seen above, even without default, a negative productivity shock lowers the returns on government bonds, because the supply of bonds falls more than the demand for bonds and because no further default is anticipated for the future. By curtailing the government's ability to issue debt, the punishment from default deepens the initial drop in bond yields. Due to lower yields, banks find it optimal to substitute away from government bonds into loans to entrepreneurs. In the PIGS bloc, the portfolio effects dominate initially, loan rates fall and entrepreneurs' investment expands.[23]

[23] There are various forces affecting the labor supply. As noted, households feel wealthier after the government default

Similar considerations apply to the foreign loan rates, but the key difference is the strength of the portfolio substitution effect. As PIGS sovereign debt accounts for a relatively smaller fraction of the portfolio of foreign banks, the direct hit from default on their equity positions dominates. Foreign banks experience a shrinkage of their capitalization and cut their loans to firms in the foreign country. Loan rates rise and entrepreneurs' investment contracts. Hence, the overall spillover effects of sovereign default on foreign output are negative.

How large and persistent are the effects of a government default shock and their international spillovers? In the non-PIGS bloc, the shock elicits a large and persistent drop in firms' investment, as induced by a significant increase in interest rate on loans determined by the contraction in credit supply. The contraction in investment and capital is also persistent: investment is not back to its steady state until after 10 quarters. Finally, output drops on impact and remains below its steady state level for several quarters. The additional decline in output induced by default is roughly 0.25 percent, as given by the difference between the dashed and solid lines in panel 7.

5.3 Sensitivity Analysis

We conduct a number of sensitivity experiments.

A first sensitivity experiment consists of modifying the financial openness of the two countries (that is, the degree of international integration of bond markets). Figure 4 considers an alternative calibration that implies financial autarky, that is banks do not hold foreign government bonds. We achieve autarky by reassigning the PIGS debt held in the foreign bloc to PIGS households and pushing up the adjustment costs on foreign debt. Imposing financial autarky effectively shuts down the mechanism of transmission of shocks through the financial sector. As the figure shows, compared to the baseline calibration, the spillover of the shock from the PIGS block to the Europe ex-PIGS block fades away. The output of the non-PIGS block rises very slightly after the shock, and then quickly reverts to the steady state level. The effects of the shock in the PIGS block are roughly comparable with those in the baseline case.

Figure 5 considers an alternative calibration in which we decrease the short-run substitutability between domestic and foreign bonds by increasing the adjustment cost on domestic and foreign bonds

because they expect lower taxes in the future. This tends to reduce their labor supply. However, an increase in the interest rate earned on deposits and bonds induces households to substitute current leisure with future leisure. Finally, the contraction in output induces to increase labor supply, too. The simulations reveal that labor supply increase modestly on impact and then progressively goes back to its steady state level.

to 10. The differences from the baseline calibration can be grasped by considering the response of the loan rate (not shown) in the non-PIGS bloc. The loan rate increases more than in benchmark case, and in turn this induces a larger contraction in investment and output than in the baseline scenario. Intuitively, in this case banks' substitution from government bonds to loans is smaller than in the baseline case because the government bond rate drops less.

Figure 6 increases the persistence of the reduction in the debt ceiling following default from $\rho_b = 0.5$, in the benchmark calibration, to $\rho_b = 0.99$. With the more persistent punishment, the contraction in the debt ceiling is more persistent and deeper. The deeper contraction in the supply of bonds accentuates the drop in bond yields and gives strength to the portfolio substitution effect discussed in the benchmark default experiment described above. The stronger substitution effect lowers the loan rates relative to the benchmark case. Consequently investment expands by more in the PIGS bloc and contracts by less in the European ex-PIGS bloc.

6 Conclusion

This paper has investigated the international transmission of government default shocks to the corporate sector through the banking system. We have considered a two-country global economy where banks grant loans and invest in government bonds both domestically and abroad. We have calibrated the model to data from the eurozone, treating one country as representing the core European countries, the other as representing the PIGS countries.

The results reveal that a government default shock in the PIGS bloc may have a large impact on the amount of credit extended both by domestic and by foreign banks and trigger a sizable contraction in output in the core bloc. In our benchmark experiment, a sovereign default of roughly 200 billion euros (that is, 7 percent of the outstanding PIGS sovereign debt) reduces GDP in the non-PIGS bloc by about 1/4 percent after two years. Larger restructuring could cause, obviously, even larger declines in economic activity abroad: for instance, given a rough linearity of the effects, a default of 25% on the sovereign debt of the PIGS countries could induce almost a 1 percent decline in the GDP of the rest of the euro area. We believe these estimates are conservative: for instance, our model lacks nominal rigidities, exchange rate effects and confidence channels that could make the spillovers even larger.

The emphasis of the analysis has been on the solvency of governments and its impact on the credit market in the presence of tight bank capital requirements. However, the increase in the sovereign default risk of periphery countries has also turned into a major liquidity crisis in the European interbank

market. As a result of the increase in the riskiness of their portfolios, the banks of core European countries have experienced a liquidity "freeze" in the interbank market. A pressing step for future research is to embed an interbank market into our environment and allow for the presence of a lender of last resort. Such an analysis would help shed light on the way the interventions of central banks can shape the transmission of a credit crunch induced by a sovereign debt crisis, for example by affecting banks' asset portfolio choices. During the current crisis, for instance, after engaging initially in purchases of sovereign bonds in the secondary market, the European Central Bank has shifted to a policy of liquidity provision to banks. It has granted large amounts of loans to banks, accepting government bonds, including those of high risk sovereigns, as collateral. As a result of these Long Term Refinancing Operations, banks have allegedly tilted their portfolios towards liquid government bonds usable as collateral, despite the persisting high risk of these bonds. This has stopped the run on government debt, but has so far allegedly had ambiguous effects on the credit crunch on firms. We leave the analysis of this and other issues for future research.

References

[1] Backus, D. K., P. J. Kehoe, and F.E. Kydland (1992) "International real business cycles," Journal of Political Economy 100, 745–775.

[2] Baxter, M., and M. J. Crucini (1995) "Business cycles and the asset structure of foreign trade," International Economic Review 36, 821–854.

[3] Bernanke, B. S., M. Gertler, and S. Gilchrist (1999) "The financial accelerator in a quantitative business cycle framework," in: J. B. Taylor & M. Woodford, eds., Handbook of Macroeconomics 1, 1, pp. 1341–1393.

[4] Bodenstein, M., L. Guerrieri, and C. Gust (2010) "Oil shocks and the zero bound on nominal interest rates," Board of Governors of the Federal Reserve System, International Finance Discussion Papers, Number 1009.

[5] Dedola, L. and G. Lombardo (2012) "Financial frictions, financial integration and the international propagation of shocks," Economic Policy 70, 319–359.

[6] Den Haan, W. J., G. Ramey, and J. Watson (2003) "Liquidity flows and fragility of business enterprises," Journal of Monetary Economics 50, 1215–1241.

[7] Devereux, M. B. and A. Sutherland (2011) "Country portfolios In open economy macro-models," Journal of the European Economic Association 9, 337–369.

[8] Devereux, M. B. and J. Yetman (2010) "Leverage constraints and the international transmission of shocks," Journal of Money, Credit and Banking 42 (s1), 71–105.

[9] Eaton, J., and M. Gersovitz (1981) "Debt with potential repudiation: Theoretical and empirical analysis," Review of Economic Studies 48, 289–309.

[10] Eggertsson, G., and M. Woodford (2003) "The Zero Bound on Interest Rates and Optimal Monetary Policy," Brookings Papers on Economic Activity 34, 139–235.

[11] Gerali, A., S. Neri, L. Sessa, and F. M. Signoretti (2010) "Credit and banking in a dsge model of the euro area," Journal of Money, Credit and Banking 42 (s1), 107–141.

[12] Gertler, M. and P. Karadi (2011) "A model of unconventional monetary policy," Journal of Monetary Economics 58, 17–34.

[13] Gertler, M. and N. Kiyotaki (2010) "Financial intermediation and credit policy in business cycle analysis," in: B. M. Friedman and M. Woodford, eds., Handbook of Monetary Economics 1, 3, pp. 547–599.

[14] Gilchrist, S. (2004) "Financial markets and financial leverage in a two-country world economy," Central Banking, Analysis, and Economic Policies Book Series, in: L. A. Ahumada, J. R. Fuentes, N. Loayza and K. Schmidt-Hebbel, eds., Banking market structure and monetary policy 1, 7, pp. 27–58, Central Bank of Chile.

[15] Gilchrist, S., J. O. Hairault, and H. Kempf (2002) "Monetary policy and the financial accelerator in a monetary union," European Central Bank w.p. 175.

[16] Heathcote, J., and F. Perri (2002) "Financial autarky and international business cycles," Journal of Monetary Economics 49, 601–627.

[17] Iacoviello, M. (2011) "Financial business cycles", Working Paper, Federal Reserve Board.

[18] Iacoviello, M., and R. Minetti (2006) "International business cycles with domestic and foreign lenders," Journal of Monetary Economics 53, 2267–2282.

[19] Kamber, G. and C. Thonissen (2012) "Financial intermediation and the international business cycle: The case of small countries with large banks," Working Paper, Reserve Bank of New Zealand.

[20] Kehoe, P. J., and F. Perri (2002) "International business cycles with endogenous incomplete markets," Econometrica 70, 907–928.

[21] Kalemli-Ozcan, S., E. Papaioannou, and F. Perri (2011) "Global banks and crisis transmission," Working paper, University of Houston.

[22] Kiyotaki, N., and J. Moore (1997) "Credit cycles," Journal of Political Economy 105, 211–248.

[23] Kollmann, R., Z. Enders, and G. Muller (2011) "Global banking and international business cycles," European Economic Review 55, 407–426.

[24] Mendoza, E. and V. Quadrini (2010) "Financial globalization, financial crisis and contagion," Journal of Monetary Economics 57, 24–39.

[25] Minetti, R. (2007) "Bank capital, firm liquidity, and project quality," Journal of Monetary Economics 54, 2584–2594.

[26] Olivero, M. P. (2010) "Market power in banking, Countercyclical margins and the international transmission of business cycles," Journal of International Economics 80, 292–301.

[27] Paasche, B. (2001) "Credit constraints and international financial crises," Journal of Monetary Economics 48, 623–650.

[28] Perri, F. and V. Quadrini (2011) "International recessions," NBER working paper 17201.

[29] Shambaugh, J. C (2012) "The euro's three crises," Brookings Papers on Economic Activity, Brookings Institution, Washington D.C.

[30] The Economist (2012) "Oat cuisine," February 11th.

[31] Ueda, K. (2012) "Banking globalization and international business cycles: Cross-border chained credit contracts and financial accelerators," Journal of International Economics 86, 1–16.

7 Appendix (Not for Publication)

This Appendix presents details on the solution of agents' problems, the market clearing conditions and the steady state equilibrium.

7.1 Agents' Decisions

7.1.1 Bankers

Bankers maximize their expected lifetime utility

$$\sum_{t=0}^{\infty} \beta_B^t \frac{(C_{B,t})^{1-\sigma_B}}{1-\sigma_B},$$

subject to

$$C_{B,t} + R_{D,t} D_{t-1} + L_t + B_{B,t} + B_{F,t} =$$
$$D_t + R_{L,t} L_{t-1} + R_{B,t} B_{B,t-1} + R^*_{B,t} B_{F,t-1} \tag{1}$$
$$D_t \leq \gamma_L L_t + B_{B,t} + B_{F,t}. \tag{2}$$

Let $\mu_{B,t}$ be the Lagrange multiplier on the budget constraint and $\lambda_{B,t}$ be the lagrange multiplier on the capital requirement. Bankers choose C, D, L, B_B, B_F to get

$$\mu_{B,t} = C_{B,t}^{-\sigma_B},$$

$$\mu_{B,t} - \beta_B \mu_{B,t+1} R_{D,t+1} - \lambda_{B,t} = 0,$$

$$-\mu_{B,t} + \beta_B \mu_{B,t+1} R_{L,t+1} + \lambda_{B,t} \gamma_L = 0,$$

$$-\mu_{B,t} + \beta_B \mu_{B,t+1} R_{B,t+1} + \lambda_{B,t} = 0,$$

$$-\mu_{B,t} + \beta_B \mu_{B,t+1} R^*_{B,t+1} + \lambda_{B,t} = 0.$$

Next, use a change in variables. Let $\tilde{\lambda}_{B,t} = \frac{\lambda_{B,t}}{\mu_{B,t}}$ or $\lambda_{B,t} = \mu_{B,t} \tilde{\lambda}_{B,t}$. Then, we can rewrite the conditions above

$$\mu_{B,t} = C_{B,t}^{-\sigma_B},$$

$$\left(1 - \tilde{\lambda}_{B,t}\right) \mu_{B,t} = \beta_B \mu_{B,t+1} R_{D,t+1},$$

$$\left(1 - \tilde{\lambda}_{B,t} \gamma_L\right) \mu_{B,t} = \beta_B \mu_{B,t+1} R_{L,t+1},$$

$$\left(1 - \tilde{\lambda}_{B,t}\right) \mu_{B,t} = \beta_B \mu_{B,t+1} R_{B,t+1},$$

$$\left(1-\tilde{\lambda}_{B,t}\right)\mu_{B,t}=\beta_B\mu_{B,t+1}R^*_{B,t+1}.$$

Let us express $R_{L,t+1}$ as a function of $R_{B,t+1}$. From above

$$\beta_B\mu_{B,t+1}R_{L,t+1}=\left(1-\tilde{\lambda}_{B,t}\gamma_L\right)\mu_{B,t}, \tag{3}$$

$$\beta_B\mu_{B,t+1}R_{B,t+1}=\left(1-\tilde{\lambda}_{B,t}\right)\mu_{B,t}. \tag{4}$$

Next, divide equation (3) by equation (4), obtaining

$$\frac{\beta_B\mu_{B,t+1}R_{L,t+1}}{\beta_B\mu_{B,t+1}R_{B,t+1}}=\frac{\left(1-\tilde{\lambda}_{B,t}\gamma_L\right)\mu_{B,t}}{\left(1-\tilde{\lambda}_{B,t}\right)\mu_{B,t}}.$$

Rearranging yields

$$\mu_{B,t+1}R_{L,t+1}=\mu_{B,t+1}R_{B,t+1}\frac{\left(1-\tilde{\lambda}_{B,t}\gamma_L\right)}{\left(1-\tilde{\lambda}_{B,t}\right)}.$$

Similarly, using

$$\left(1-\tilde{\lambda}_{B,t}\right)\mu_{B,t}=\beta_B\mu_{B,t+1}R^*_{B,t+1},$$
$$\left(1-\tilde{\lambda}_{B,t}\right)\mu_{B,t}=\beta_B\mu_{B,t+1}R_{B,t+1},$$

one obtains

$$\mu_{B,t+1}R^*_{B,t+1}=\mu_{B,t+1}R_{B,t+1}\frac{\left(1-\tilde{\lambda}_{B,t}\right)}{\left(1-\tilde{\lambda}_{B,t}\right)}.$$

To summarize, the conditions for an equilibrium from the bankers' problem are

$$\begin{aligned}
D_t &\leq \gamma_L L_t+\gamma_B B_{B,t}+\gamma_F B_{F,t},\\
\mu_{B,t+1}R_{L,t+1} &= \mu_{B,t+1}R_{B,t+1}\frac{\left(1-\tilde{\lambda}_{B,t}\gamma_L\right)}{\left(1-\tilde{\lambda}_{B,t}\right)},\\
\mu_{B,t+1}R^*_{B,t+1} &= \mu_{B,t+1}R_{B,t+1}\frac{\left(1-\tilde{\lambda}_{B,t}\right)}{\left(1-\tilde{\lambda}_{B,t}\right)}.
\end{aligned}$$

7.1.2 Entrepreneurs

Entrepreneurs transform loans into capital using a one-for-one technology and can convert loans back into consumption. They maximize their expected lifetime utility

$$\sum_{t=0}^{\infty}\beta_E^t\frac{(C_{E,t})^{1-\sigma_E}}{1-\sigma_E},$$

subject to

$$C_{E,t}+K_t+R_{L,t}L_{t-1} = R_{K,t}K_{t-1}+ (1-\delta) K_{t-1}+L_t, \tag{5}$$

$$L_t = \rho_E L_{t-1} + (1-\rho_E) mK_{Et}. \tag{6}$$

Introduce a change in variables. Let $\tilde{\lambda}_{E,t} = \frac{\lambda_{E,t}}{\mu_{E,t}}$ or $\lambda_{E,t} = \mu_{E,t}\tilde{\lambda}_{E,t}$. Then, the optimizing conditions can be written as, using $C_{E,t}^{-\sigma_E} = \mu_{E,t}$:

$$\left(1-\tilde{\lambda}_{E,t}\right) C_{E,t}^{-\sigma_E} = \beta_E \left(R_{L,t+1} - \rho_E \tilde{\lambda}_{E,t+1}\right) C_{E,t+1}^{-\sigma_E} \tag{7}$$

$$\left(1-\tilde{\lambda}_{E,t}\left(1-\rho_E\right) m\right) C_{E,t}^{-\sigma_E} = \beta_E \left(R_{K,t+1}+1-\delta\right) C_{E,t+1}^{-\sigma_E} \tag{8}$$

7.1.3 Government

The government is assumed be myopic and have utility given by $u_t = v(G_t)$ (any function increasing in G_t would do, given the simple nature of the problem). Under this setup, the government default if it can achieve greater spending today by defaulting relative to the no default case. Given the debt ceiling constraint of the government, default can take place if a reduction in GDP tightens the borrowing constraint so much that the government finds it convenient to repudiate its debt rather than to pay back its previous obligations.

Figure 1
French Banks and PIGS Sovereign Debt

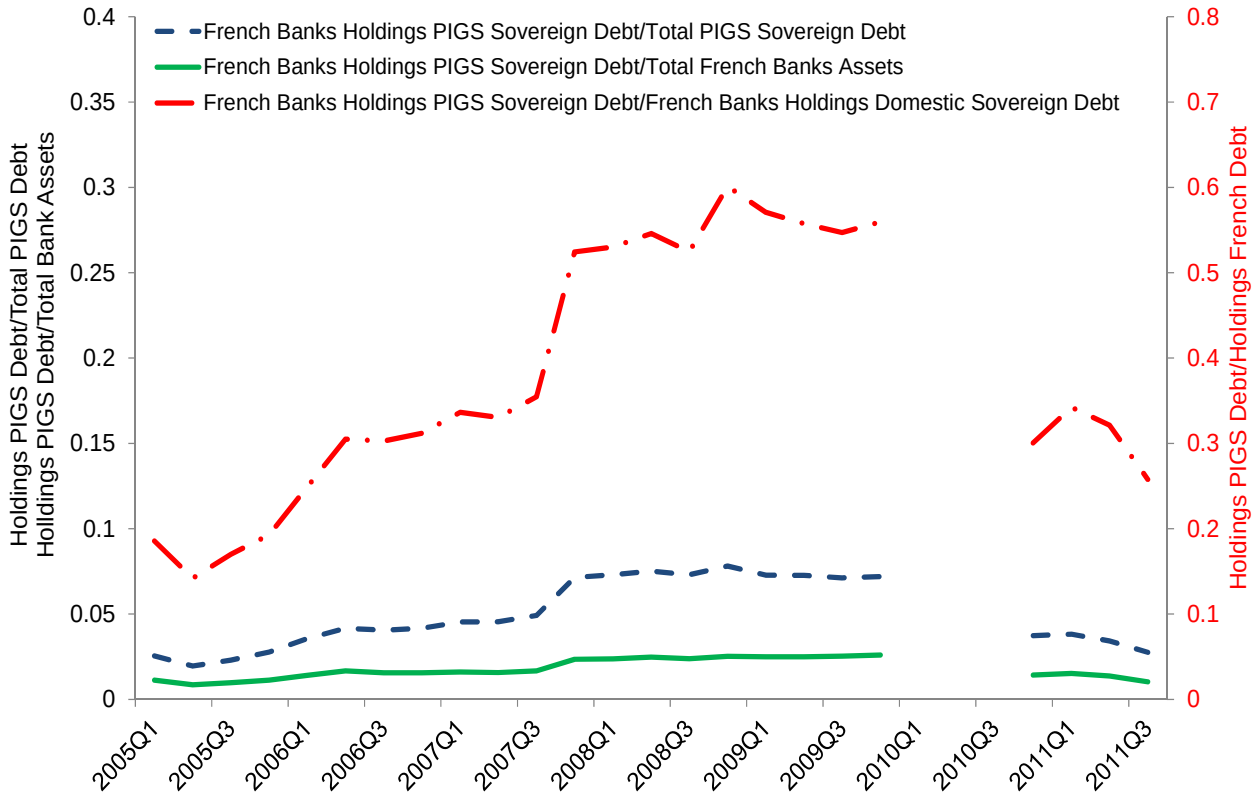

Figure 2
Decline in Productivity in the PIGS bloc -- No Default

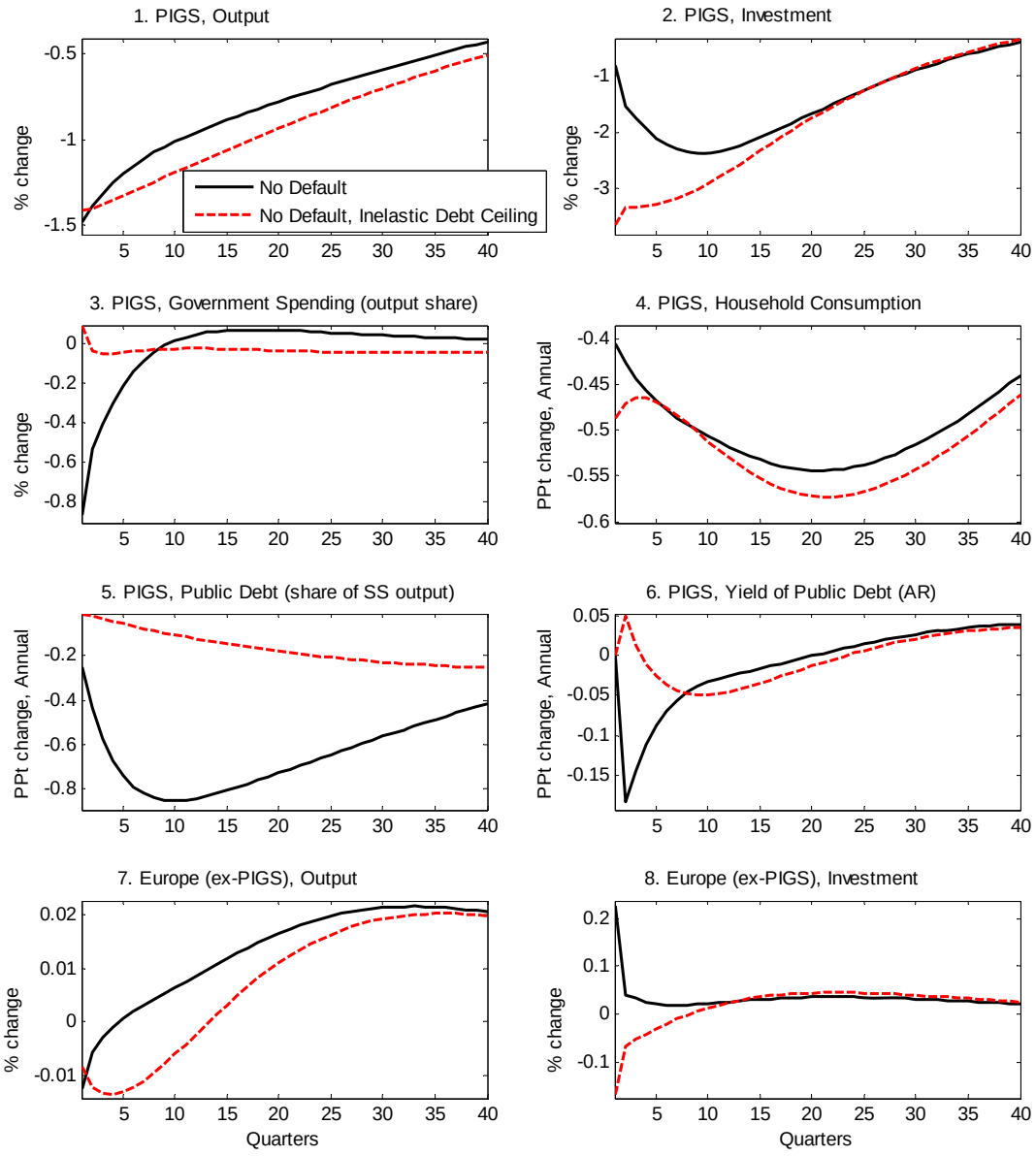

Figure 3
Decline in Productivity in the PIGS bloc -- Default

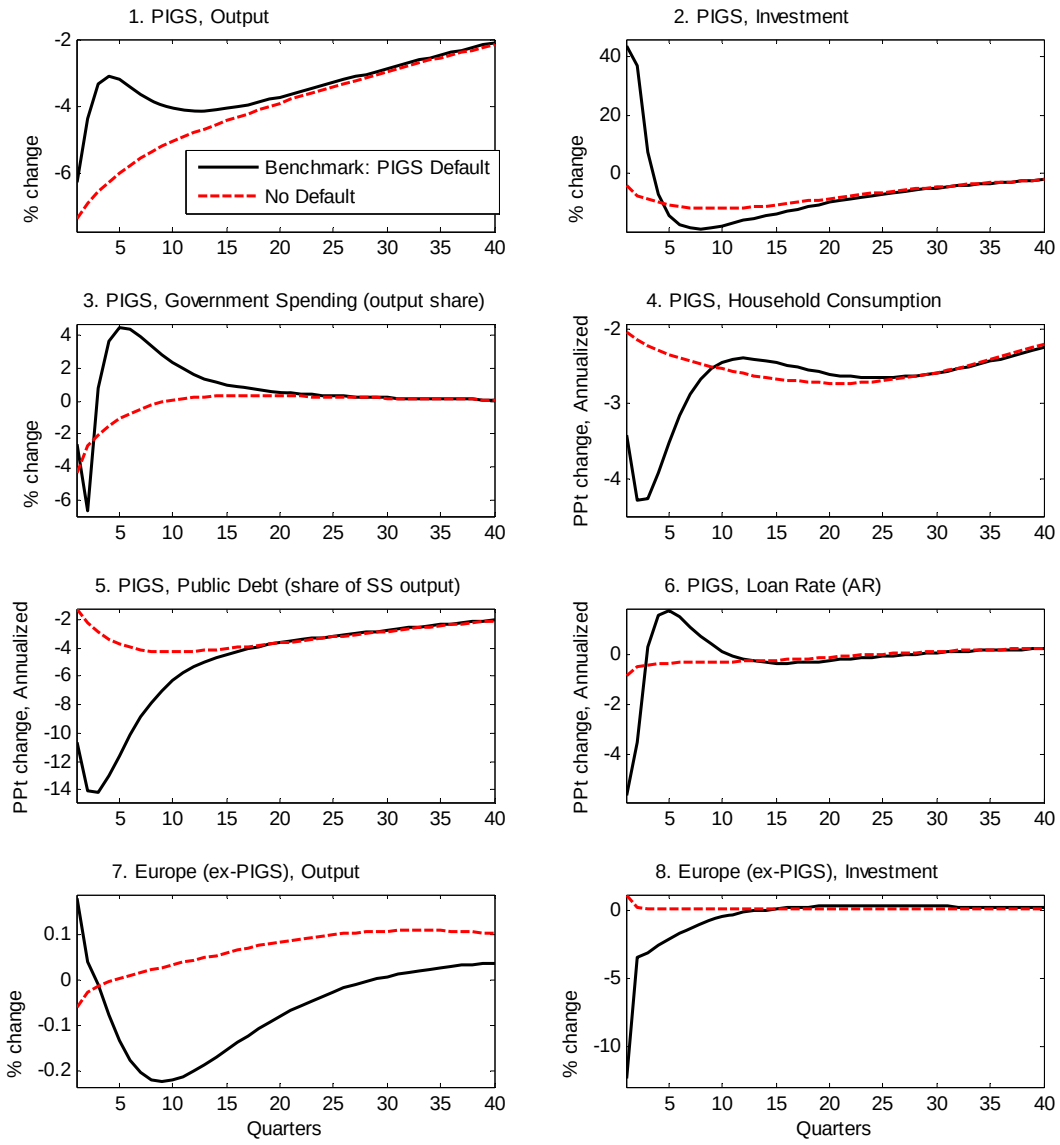

Figure 4
Decline in Productivity in the PIGS bloc -- No Foreign Debt

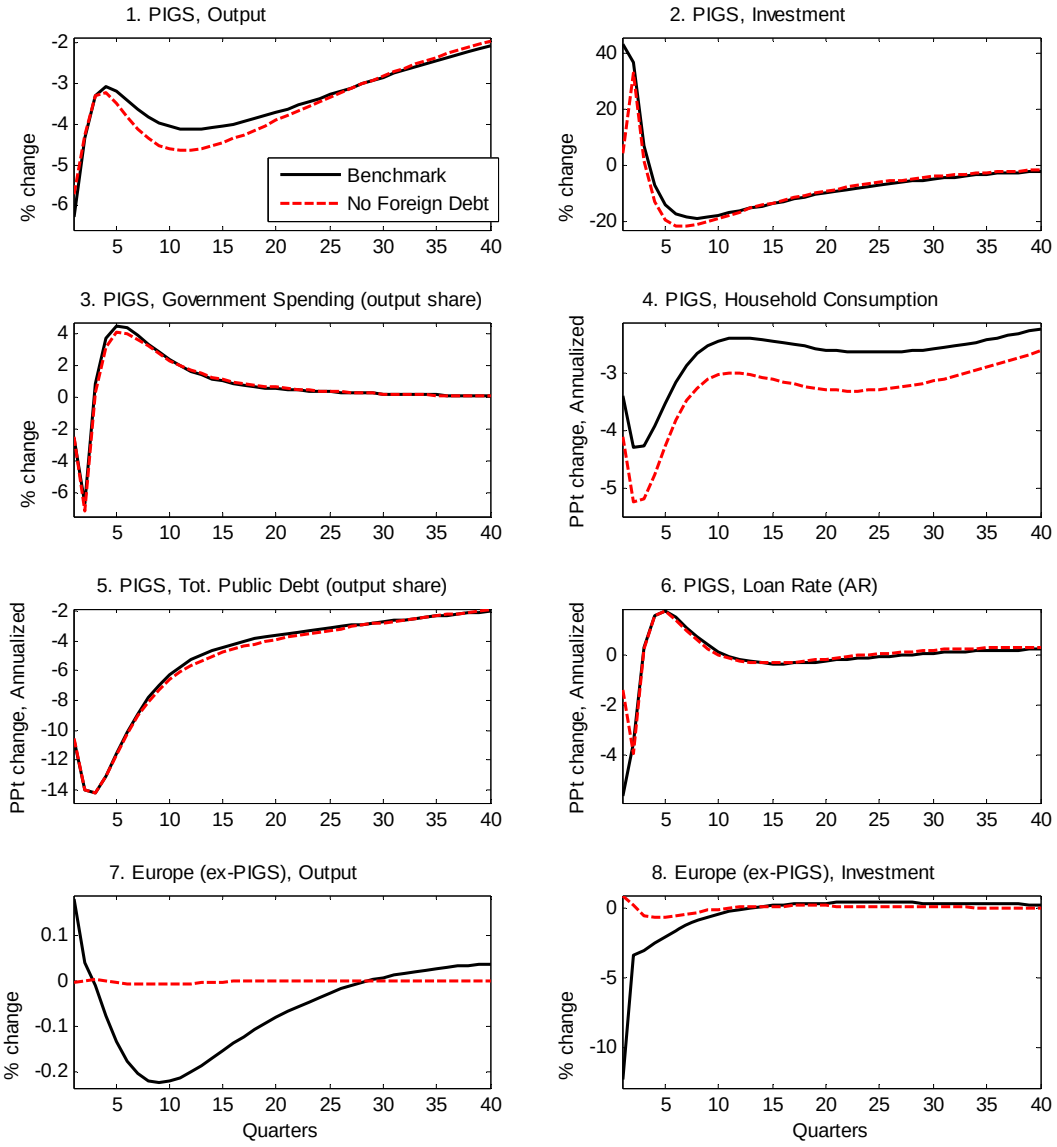

Figure 5
Decline in Productivity in the PIGS bloc
– Lower Substitutability between Domestic and Foreign Bonds

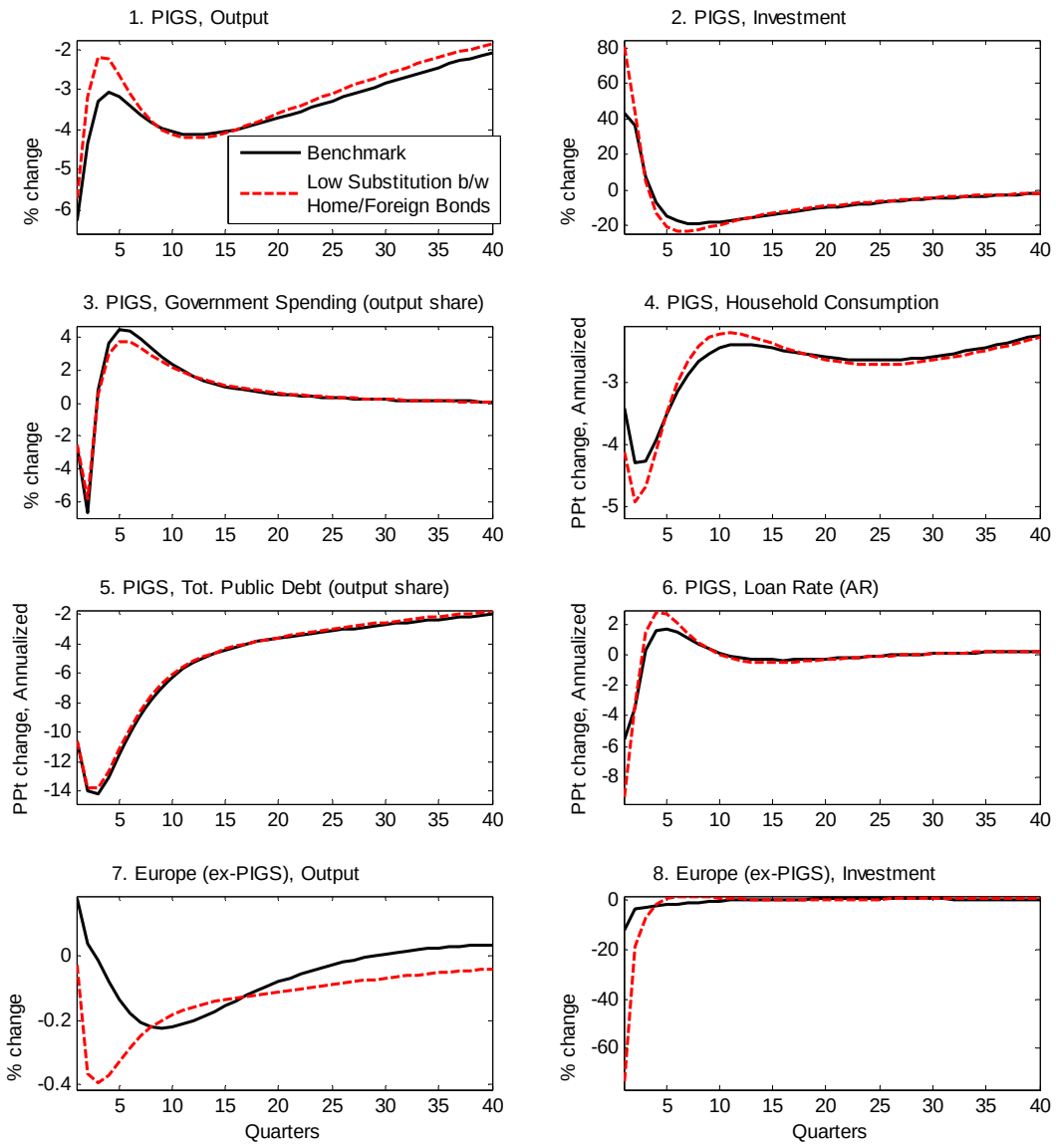

Figure 6
Decline in Productivity in the PIGS bloc – Persistent Punishment