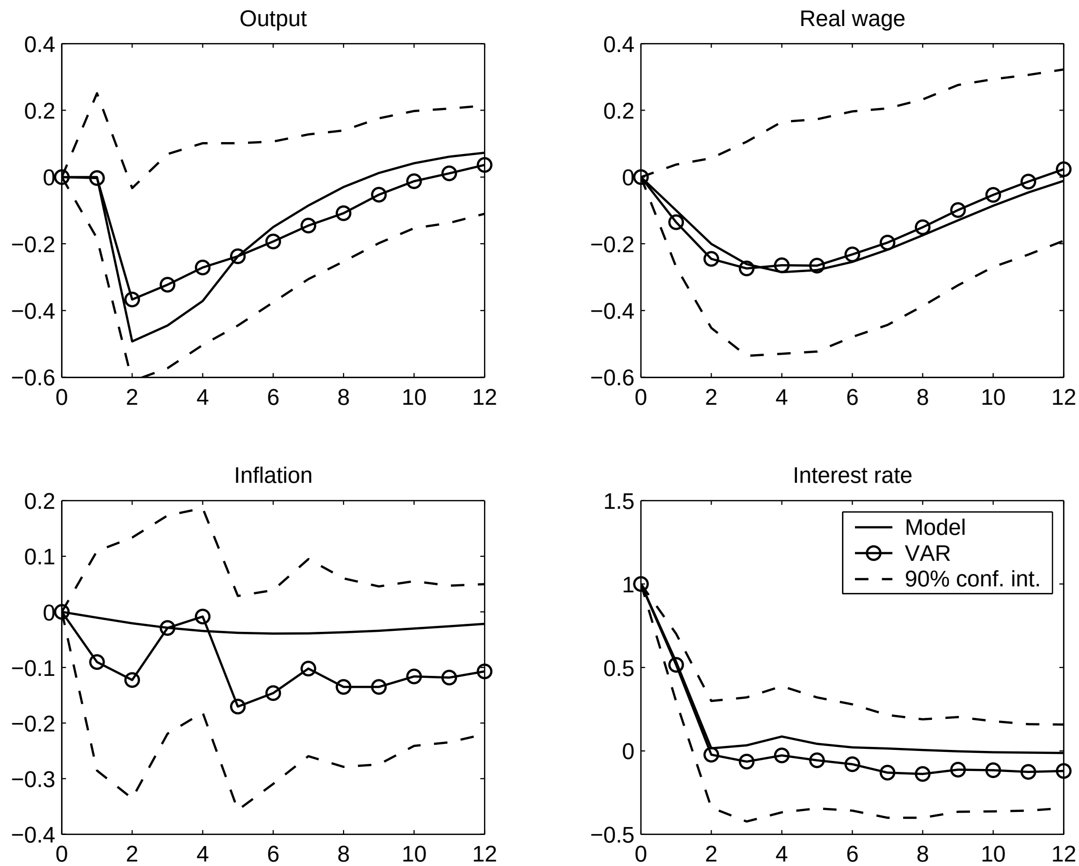

Figure A.1: Impulse responses in the absence of habit persistence ($\eta = 0$).

A The Quantitative Model of the U.S. Economy: Alternative Specifications

A.1 Impulse Responses for the Restricted Models

Section 2.3 of the paper reports the estimated parameters for several restricted cases of the general model. Figures A.1 – A.3 here show the predicted impulse response functions to a monetary policy shock by the model for these restricted models.

In the absence of habit persistence (Figure A.1), output falls sharply two periods following an unexpected increase in the interest rate, and then returns back to the initial situation. In

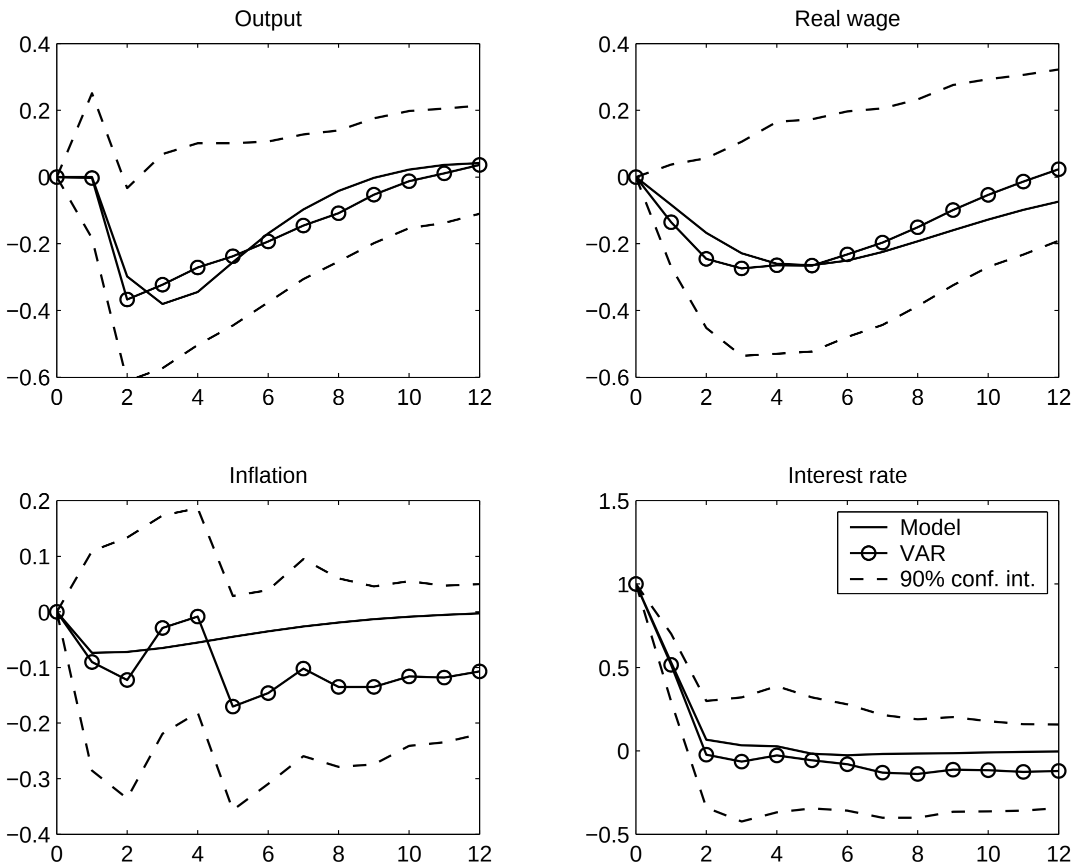

Figure A.2: Impulse responses in the absence of indexation ($\gamma_p = \gamma_w = 0$).

contrast, in the presence of habit persistence as can be seen from Figure 6 in the text, output falls by less two periods following the shock, but then continues to decrease before returning to the initial level. Figure A.2 shows that in the absence of indexation to lagged inflation ($\gamma_p = \gamma_w = 0$), inflation falls one period after the unexpected increase in the interest rate but then returns gradually to the initial level. Instead, Figure 3 in the text indicates that in the presence of indexation, inflation declines gradually but more persistently before returning to the initial level. Finally, in the case of flexible wages, Figure A.3 below shows that the real wage decreases more than in the case of wage stickiness in the first few quarters following a monetary policy shock. This is associated also with a sharper reduction in output than is the case when wages are sticky.

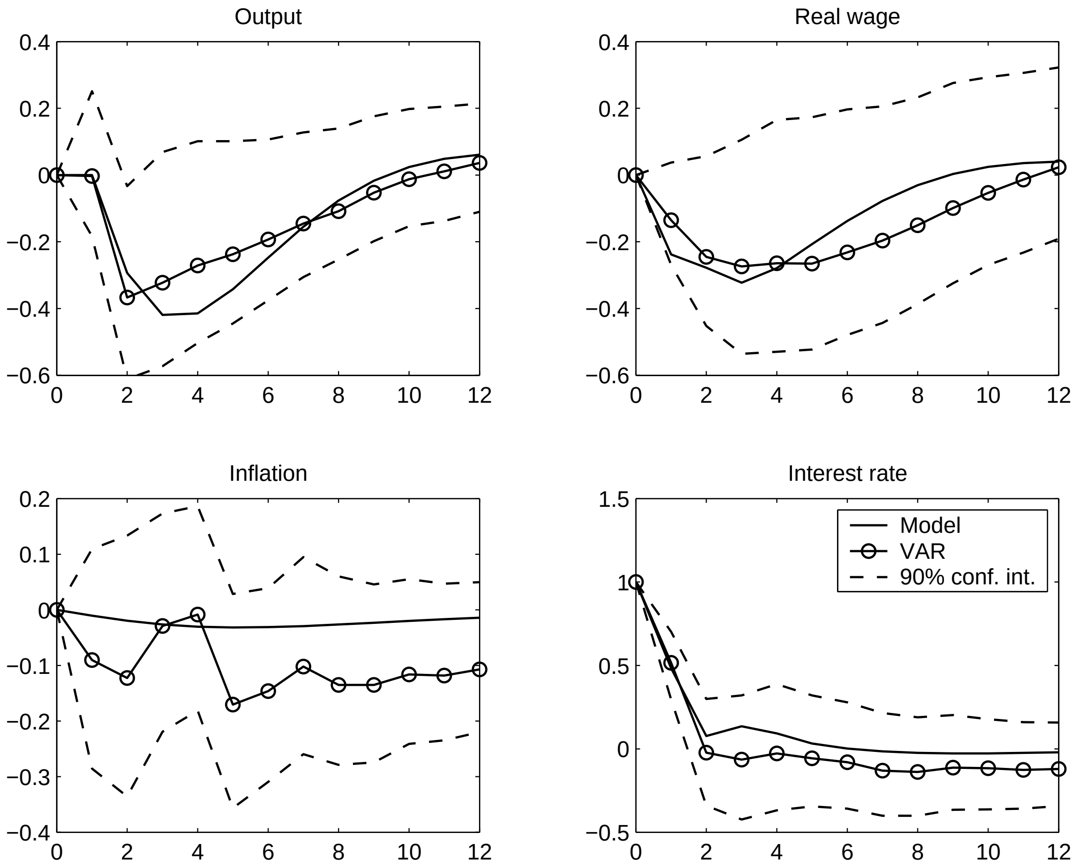

Figure A.3: Impulse responses with flexible wages $\left(\xi_w^{-1} = 0\right)$.

A.2 Estimated Parameters for Alternative Horizons

The parameter estimates reported in Table 3 of the text are based on impulse response functions with a horizon of 12 quarters following the shock. In Table A.1 [printed at the end of this appendix], we report as a robustness check the parameter estimates for the baseline model based alternative horizons. The following Figures A.4 – A.8 show the corresponding predicted impulse responses to a monetary policy shock in the baseline model, for the various horizons.

	Horizon				
	6	8	12	16	20
Estimated parameters					
$\psi \equiv \frac{\varphi^{-1}}{1+\beta\eta^2}$	0.8756	0.7574	0.6715	0.6475	0.6698
	(0.1044)	(0.1623)	(0.3330)	(0.2089)	(0.0458)
$\tilde{\eta} \equiv \frac{\eta}{1+\beta\eta^2}$	0.5025	0.5025	0.5025	0.5025	0.5025
	(0.0278)*	(0.0441)*	(0.0692)*	(0.0304)*	(0.0146)*
ξ_p	0.0065	0.0036	0.0020	0.0017	0.0013
	(0.0012)	(0.0006)	(0.0009)	(0.0006)	(0.0002)
ξ_w	0.0073	0.0056	0.0042	0.0081	0.0203
	(0.0961)	(0.4126)	(0.1343)	(0.0227)	(0.0192)
ω_w	19.559	19.545	19.551	9.4925	4.2794
	(244.8)	(1360.1)	(595.1)	(23.70)	(2.9934)
γ_p	0.9374	1	1	1	1
	(0.0707)	(0.4438)*	(0.3800)*	(0.1130)*	(0.0463)*
γ_w	1	1	1	1	1
	(1.9813)*	(18.578)*	(10.908)*	(1.7840)*	(1.4887)*
Implied parameters					
φ	0.5739	0.6635	0.7483	0.7760	0.7502
η	1	1	1	1	1
$\kappa_p \equiv \xi_p\omega_p$	0.0022	0.0012	0.0007	0.0006	0.0004
$\omega \equiv \omega_p + \omega_w$	19.893	19.878	19.884	9.8258	4.6127
$\nu \equiv \omega_w/\phi$	14.700	14.659	14.663	7.1193	3.2096
$\mu_p \equiv \frac{\theta_p}{\theta_p-1}$	1.0127	1.0069	1.0039	1.0032	1.0025
$\mu_w \equiv \frac{\theta_w}{\theta_w-1}$	2.6976	1.9062	1.5361	1.5066	1.7018
Weights in loss function					
λ_p	0.9870	0.9932	0.9960	0.9985	0.9997
λ_w	0.0130	0.0068	0.0040	0.0015	0.0003
$(\lambda_x)\,16$	0.0269	0.0082	0.0026	0.0010	0.0003
δ	0.0273	0.0313	0.0351	0.0686	0.1248
Obj. function val.	3.419	5.979	13.110	20.310	27.035

Table A.1: Estimated structural parameters for the baseline model with different horizons

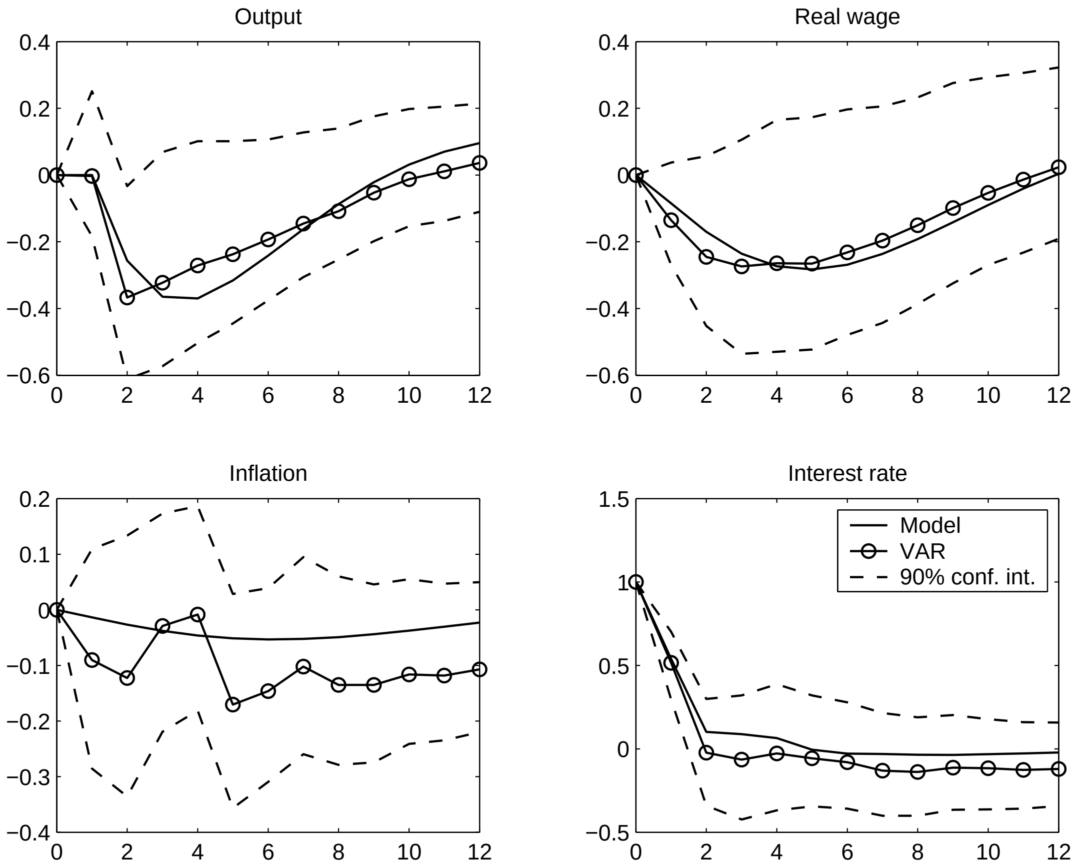

Figure A.4: Impulse responses to a monetary policy shock with a horizon of 6 quarters.

B Welfare Criterion for the Quantitative Model

We assume that the policymaker maximizes the expectation of the unweighted average of household utility functions

$$\mathcal{W}_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U_t \right\} \tag{B.1}$$

where

$$U_t \equiv u\left(C_t - \eta C_{t-1}; \xi_t\right) - \int_0^1 v\left(H_t^h; \xi_t\right) dh. \tag{B.2}$$

Recall that consumption is identical for all households, while labor supplied may vary across households. In the text, we determine the equilibrium evolution of inflation, output, interest rates using log-linear approximations to the exact equilibrium conditions. Thus we have

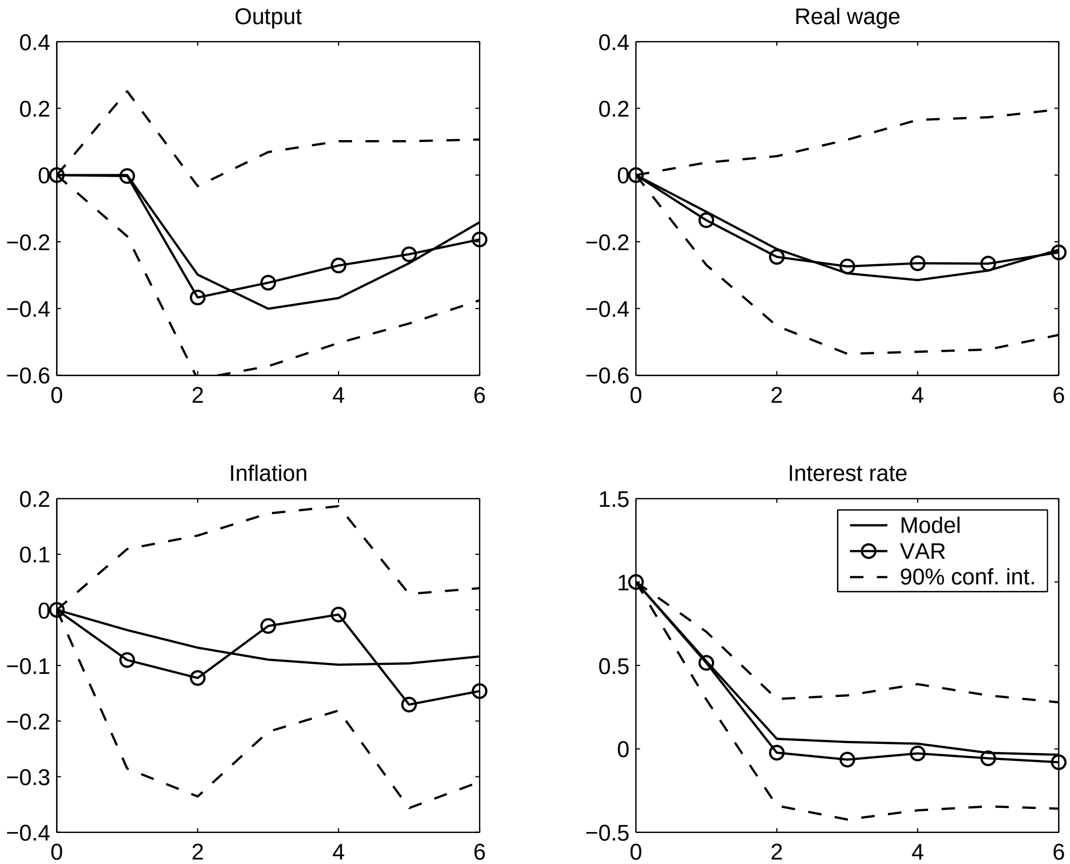

Figure A.5: Impulse responses to a monetary policy shock with a horizon of 12 quarters.

characterized equilibrium fluctuations in those variables up to a residual of order $O\left(\|\xi\|^2\right)$, where $\|\xi\|$ is a bound on the amplitude of exogenous disturbances. As shown in Woodford (2003, ch. 6), we may compute a second-order approximation of (B.1) – (B.2) using a log-linear approximation to the equilibrium conditions, provided that we expand around a steady-state that is close to being optimal in the sense of achieving the maximum expected utility. We thus assume that the steady-state level of output with zero inflation, $\bar{Y}$, is near the efficient level of output.

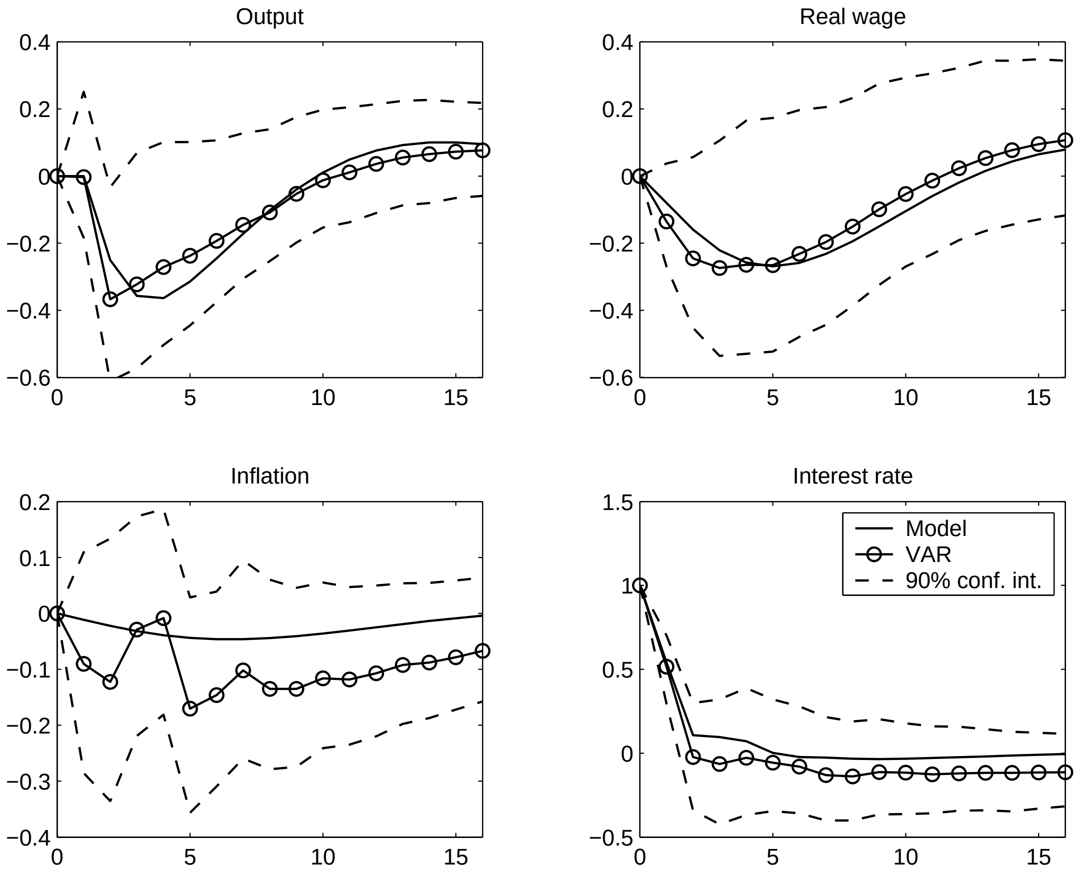

Figure A.6: Impulse responses to a monetary policy shock with a horizon of 16 quarters.

B.1 Natural Rate of Output

Before performing the approximation of (B.1) – (B.2), we determine the natural rate of output, i.e., the equilibrium level of output under flexible prices, flexible wages, constant levels of distorting taxes and of desired markups in the labor and products markets, and with wages, prices and spending decisions predetermined only by one period. As mentioned in footnote 36 of the text, we may alternatively have defined the natural rate of output as the equilibrium level of output under flexible prices, flexible wages, when none of the pricing or spending decisions are predetermined. This assumption would not affect any of our conclusions about optimal monetary policy, as it is only the forecastable component of the output gap that is forecastable one period in advance that matters both for the structural

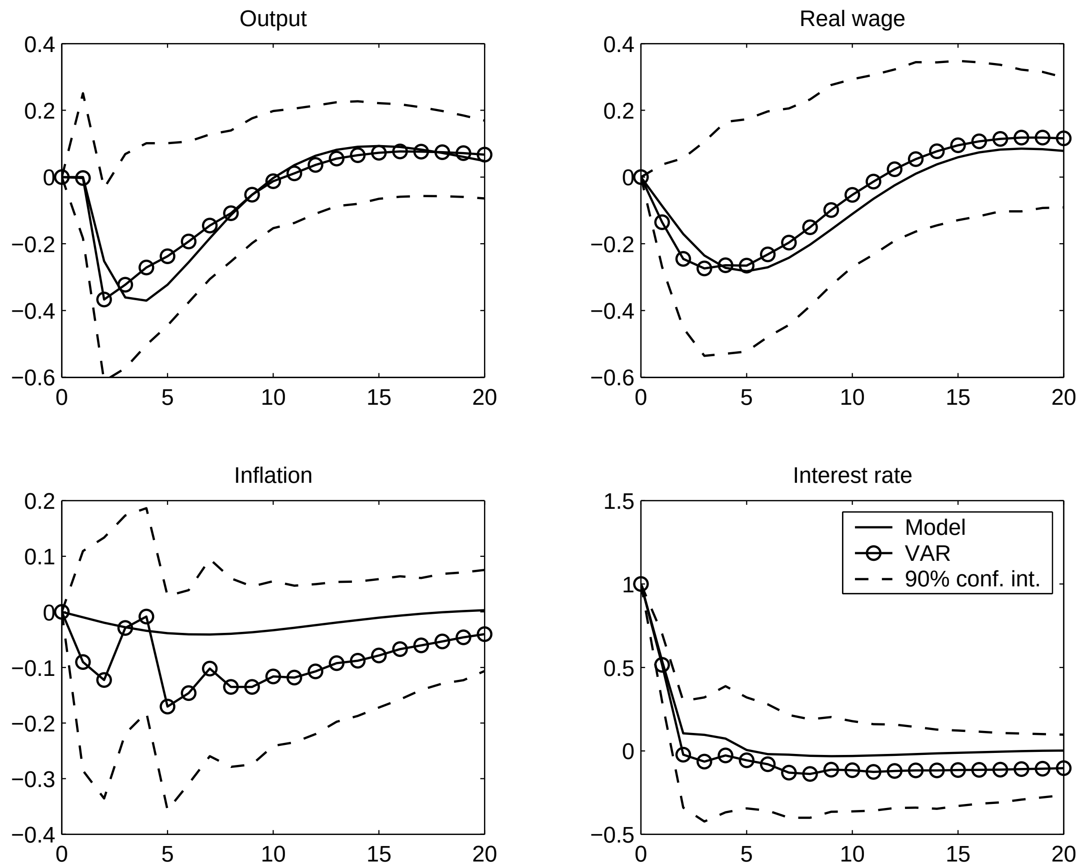

Figure A.7: Impulse responses to a monetary policy shock with a horizon of 20 quarters.

equations of the model and for evaluating welfare under alternative policies. For simplicity, we thus assume that the natural rate of output is predetermined one period in advance, so that the output gap x_t is also predetermined.

To determine the natural rate of output, we first note that the first-order condition for the optimal supply of labor by household h is given by

$$\frac{v_h\left(H_t^h;\xi_t\right)}{\lambda_t}=\frac{w_t\left(h\right)}{P_t} \tag{B.3}$$

at all dates t.

Next, the firm's profits are given by

$$\Pi_t\left(z\right) \equiv \left(1+\tau_p\right)p_t\left(z\right)y_t\left(z\right)-W_tH_t\left(z\right)$$

$$= \quad (1+\tau_p)\, p_t(z)^{1-\theta_p} P_t^{\theta_p} Y_t - W_t f^{-1}\left(p_t(z)^{-\theta_p} P_t^{\theta_p} Y_t / A_t\right).$$

where $0 \leq \tau_p < 1$ is a subsidy for output that offsets the effect on imperfect competition in goods markets on the steady-state level of output. To derive the last equation, we use the usual Dixit-Stiglitz demand for good z, $y_t(z) = Y_t (p_t(z)/P_t)^{-\theta_p}$ and we invert the production function $y_t(z) = A_t f(H_t(z))$.

In the case that prices are flexible but predetermined by one period, the optimal pricing decision for the firm z, i.e., the price that would maximize profits at each period is given by

$$p_t(z) = E_{t-1}\left[\frac{\mu_p}{1+\tau_p} \frac{W_t}{A_t f'\left(f^{-1}\left(y_t(z)/A_t\right)\right)}\right],$$

where the desired markup $\mu_p \equiv \frac{\theta_p}{\theta_p - 1}$. Using again the demand for good z, we note that the relative supply of good z must in turn satisfy

$$\left(\frac{y_t(z)}{Y_t}\right)^{-1/\theta_p} = E_{t-1}\left[\frac{\mu_p}{1+\tau_p} \frac{W_t}{P_t} \frac{1}{A_t f'\left(f^{-1}\left(y_t(z)/A_t\right)\right)}\right].$$

Because all wages are the same in the case of flexible wages, we have $w_t(h) = W_t$ and $H_t^h = H_t$ for all h. Thus (B.3) implies that when wages and prices are flexible, all sellers supply a quantity Y_t^n, determined at date $t-1$, satisfying

$$1 = E_{t-1}\left[\frac{\mu_p}{1+\tau_p} \frac{v_h\left(f^{-1}\left(Y_t^n/A_t\right);\xi_t\right)}{\lambda_t^n} \frac{1}{A_t f'\left(f^{-1}\left(Y_t^n/A_t\right)\right)}\right], \tag{B.4}$$

where $\lambda_t^n = E_{t-1}\lambda_t^n$ denotes the marginal utility of income at date t in the case of flexible prices, flexible wages, and in the case that prices and spending decisions predetermined by one period. Note that in steady-state, (B.4) reduces to

$$\frac{v_h}{\bar{\lambda} f'} = \frac{1+\tau_p}{\mu_p} \equiv 1 - \Phi$$

where Φ is a measure of inefficiency in the economy, at steady-state. As in Woodford (2003), we assume that Φ is of order $O(||\xi||)$. Furthermore, using (2.4), we observe that in the steady state, $u_c(1-\beta\eta) = \bar{\lambda}$, so that

$$v_h = (1-\Phi)(1-\beta\eta)\, u_c f'. \tag{B.5}$$

Log-linearizing (B.4) about this steady-state and solving for $\hat{Y}_t^n$ yields

$$\omega\hat{Y}_t^n = E_{t-1}\left[(1+\omega)\,a_t - \frac{v_{h\xi}}{v_h}\xi_t + \hat{\lambda}_t^n\right] \tag{B.6}$$

where $\hat{\lambda}_t^n \equiv \log\left(\lambda_t^n/\bar{\lambda}\right)$, and $a_t \equiv \log A_t$.

In the case of flexible prices and wages, and in the case that consumption decisions for period t are that prices and spending decisions predetermined by one period, the variable μ_t, defined as $\mu_t \equiv \hat{\lambda}_t - \varphi\left(\tilde{g}_t - \tilde{Y}_t\right)$, satisfies $E_{t-1}\mu_t = 0$ at all dates. It thus follows that

$$\hat{\lambda}_t^n = -\varphi E_{t-1}\left(\tilde{Y}_t^n - \tilde{g}_t\right). \tag{B.7}$$

Using this to substitute for $\hat{\lambda}_t^n$ in (B.6), we obtain

$$E_{t-1}\left\{\left[\omega + \varphi\,(1-\eta L)\left(1-\beta\eta L^{-1}\right)\right]\hat{Y}_t^n\right\} = E_{t-1}\left[(1+\omega)\,a_t - \frac{v_{h\xi}}{v_h}\xi_t + \varphi\tilde{g}_t\right] \tag{B.8}$$

which implicitly determines the natural rate of output $\hat{Y}_t^n = E_{t-1}\hat{Y}_t^n$.

B.2 Approximation of Welfare Criterion

We now turn to the second-order Taylor expansion of each term on the right-hand side of (B.2).

First term. The first term can be approximated as follows

$$\begin{aligned} u\left(C_t - \eta C_{t-1};\xi_t\right) &= \bar{Y}u_c\left\{\left(\hat{Y}_t - \eta\hat{Y}_{t-1}\right) + \frac{1}{2}\left[\left(\hat{Y}_t^2 - \eta\hat{Y}_{t-1}^2\right) - \sigma^{-1}\left(\hat{Y}_t - \eta\hat{Y}_{t-1}\right)^2\right]\right. \\ &\left. + \sigma^{-1}\left(\hat{Y}_t - \eta\hat{Y}_{t-1}\right)\left(g_t - \eta\hat{G}_{t-1}\right)\right\} + \text{t.i.p.} + \text{unf} + O\left(\|\xi\|^3\right) \end{aligned} \tag{B.9}$$

where $\sigma \equiv -\frac{u_c}{u_{cc}\bar{Y}}$, "t.i.p." denotes terms independent of the actual policy such as constant terms and terms involving only exogenous variables, and "unf" represents an unforecastable term, i.e., a term z_t such that $E_{t-2}z_t = 0$. To obtain (B.9), we have used the second-order Taylor expansion

$$z_t/\bar{z} = 1 + \hat{z}_t + \frac{1}{2}\hat{z}_t^2 + O\left(\|\xi\|^3\right) \tag{B.10}$$

where $\hat{z}_t \equiv \log\left(z_t/\bar{z}\right)$, for any variable z_t around its steady-state $\bar{z}$.

Second term. A second-order approximation of $v\left(H_t^h;\xi_t\right)$, integrated over the continuum of different types of labor, yields

$$\int_0^1 v\left(H_t^h;\xi_t\right)dh = \bar{H}v_h\left[\hat{H}_t + \frac{1}{2}(1+\nu)\hat{H}_t^2 - \nu\bar{h}_t\hat{H}_t + \frac{1}{2}\theta_w(1+\nu\theta_w)\,\mathrm{var}_h\log w_t(h)\right] + \text{t.i.p.} + O\left(\|\xi\|^3\right) \tag{B.11}$$

as in Woodford (2003, chap. 6). To obtain this equation, we used (B.10), a second-order approximation of (2.10)

$$\hat{H}_t = E_h\hat{H}_t^h + \frac{1}{2}\left(1-\theta_w^{-1}\right)\mathrm{var}_h\hat{H}_t^h + O\left(\|\xi\|^3\right),$$

and the fact that a log-linear approximation of the demand for labor of type h by firm z, $H_t^h(z) = H_t(z)\left(w_t(h)/W_t\right)^{-\theta_w}$, implies

$$\mathrm{var}_h\hat{H}_t^h = \theta_w^2\mathrm{var}_h\log w_t(h) + O\left(\|\xi\|^3\right).$$

We note $E_h z_t(h)$ for the mean value of $z_t(h)$ across all h's and $\mathrm{var}_h z_t(h)$ for the corresponding variance. We furthermore define

$$\bar{h}_t \equiv -\nu^{-1}\frac{v_{h\xi}}{v_h}\xi_t$$

and

$$\nu \equiv \frac{\bar{H}v_{hh}}{v_h} > 0.$$

Following again Woodford (2003, chap. 6) we find, using an approximation of the production function, that the aggregate demand for the composite labor input satisfies

$$\hat{H}_t = \phi\left(\hat{Y}_t - a_t\right) + \frac{1}{2}(1+\omega_p-\phi)\phi\left(\hat{Y}_t - a_t\right)^2 + \frac{1}{2}(1+\omega_p\theta_p)\theta_p\phi\,\mathrm{var}_z\log p_t(z) + O\left(\|\xi\|^3\right),$$

where $\phi \equiv f/\left(\bar{H}f'\right) > 0$ and $\omega_p \equiv -f''\bar{Y}/(f')^2 > 0$. Combining this with (B.11), we obtain

$$\int_0^1 v\left(H_t^h;\xi_t\right)dh = \bar{H}v_h\phi\left[\hat{Y}_t + \frac{1}{2}(1+\omega)\left(\hat{Y}_t - a_t\right)^2 - \nu\bar{h}_t\hat{Y}_t + \frac{1}{2}\theta_p(1+\omega_p\theta_p)\,\mathrm{var}_z\log p_t(z) + \frac{1}{2}\theta_w\phi^{-1}(1+\nu\theta_w)\,\mathrm{var}_z\log w_t(z)\right] + \text{t.i.p.} + O\left(\|\xi\|^3\right), \tag{B.12}$$

where $\omega = \omega_p + \omega_w$, and $\omega_w = \phi\nu$.

Assuming as in the text that prices and wages are reoptimized in each period with probability $(1-\alpha_p)$ and $(1-\alpha_w)$ respectively allows us to express $\text{var}_z \log p_t(z)$ and $\text{var}_z \log w_t(z)$ in terms of the variability of aggregate inflation and aggregate wage inflation as follows. We let

$$\bar{P}_t \equiv E_z \log p_t(z), \qquad \text{and} \qquad V_t^p \equiv \text{var}_z \log p_t(z),$$

and note that

$$\begin{aligned} \bar{P}_t - \bar{P}_{t-1} &= E_z \left[\log p_t(z) - \bar{P}_{t-1}\right] \\ &= \alpha_p E_z \left[\log p_{t-1}(z) + \gamma_p \pi_{t-1} - \bar{P}_{t-1}\right] + (1-\alpha_p)\left(\log p_t^* - \bar{P}_{t-1}\right) \\ &= (1-\alpha_p)\left(\log p_t^* - \bar{P}_{t-1}\right) + \alpha_p \gamma_p \pi_{t-1}. \end{aligned}$$

Similarly,

$$\begin{aligned} V_t^p &= \text{var}_z \left[\log p_t(z) - \bar{P}_{t-1}\right] \\ &= E_z \left\{\left[\log p_t(z) - \bar{P}_{t-1}\right]^2\right\} - \left[E_z \log p_t(z) - \bar{P}_{t-1}\right]^2 \\ &= \alpha_p E_z \left\{\left[\log p_{t-1}(z) + \gamma_p \pi_{t-1} - \bar{P}_{t-1}\right]^2\right\} + (1-\alpha_p)\left(\log p_t^* - \bar{P}_{t-1}\right)^2 - \left(\bar{P}_t - \bar{P}_{t-1}\right)^2 \\ &= \alpha_p V_{t-1}^p + \frac{\alpha_p}{1-\alpha_p}\left(\bar{P}_t - \bar{P}_{t-1} - \gamma_p \pi_{t-1}\right)^2. \end{aligned}$$

Using the log-linear approximation

$$\bar{P}_t = \log P_t + O\left(\|\xi\|^2\right),$$

we obtain

$$\begin{aligned} V_t^p &= \alpha_p V_{t-1}^p + \frac{\alpha_p}{1-\alpha_p}\left(\pi_t - \gamma_p \pi_{t-1}\right)^2 + O\left(\|\xi\|^3\right) \\ &= \frac{\alpha_p}{1-\alpha_p}\sum_{s=0}^{t} \alpha_p^{t-s}\left(\pi_s - \gamma_p \pi_{s-1}\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right), \end{aligned}$$

where we note that the price dispersion before the first period (period 0) is independent of the policy that is chosen to apply in periods $t \geq 0$. Taking the present discounted sum on both sides of the last equation, we obtain

$$\sum_{t=0}^{\infty} \beta^t V_t^p = \frac{\alpha_p}{(1-\alpha_p)(1-\alpha_p\beta)} \sum_{t=0}^{\infty} \beta^t \left(\pi_t - \gamma_p \pi_{t-1}\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right).$$

Following the same steps with nominal wages, we obtain

$$\sum_{t=0}^{\infty}\beta^t V_t^w = \frac{\alpha_w}{(1-\alpha_w)(1-\alpha_w\beta)}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^w-\gamma_w\pi_{t-1}\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right).$$

where

$$V_t^w \equiv \text{var}_z \log w_t(z).$$

Using this, and taking the present discounted sum on both sides of (B.12), we obtain

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\left[\int_0^1 v\left(H_t^h;\xi_t\right)dh\right] &= \bar{H}v_h\phi\sum_{t=0}^{\infty}\beta^t\left[\hat{Y}_t+\frac{1}{2}(1+\omega)\left(\hat{Y}_t-a_t\right)^2-\nu\bar{h}_t\hat{Y}_t\right.\\ &\left.+\frac{1}{2}\theta_p\xi_p^{-1}\left(\pi_t-\gamma_p\pi_{t-1}\right)^2+\frac{1}{2}\theta_w\phi^{-1}\xi_w^{-1}\left(\pi_t^w-\gamma_w\pi_{t-1}\right)^2\right]\\ &+\text{t.i.p.}+O\left(\|\xi\|^3\right),\end{aligned}$$

where ξ_w and ξ_p are defined in (2.12) and (2.15) respectively. Next, using (B.5), and recalling that Φ is of order $O(||\xi||)$, we obtain

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\left[\int_0^1 v\left(H_t^h;\xi_t\right)dh\right] &= \bar{Y}u_c(1-\beta\eta)\sum_{t=0}^{\infty}\beta^t\left[(1-\Phi)\hat{Y}_t+\frac{1}{2}(1+\omega)\left(\hat{Y}_t-a_t\right)^2-\nu\bar{h}_t\hat{Y}_t\right.\\ &\left.+\frac{1}{2}\theta_p\xi_p^{-1}\left(\pi_t-\gamma_p\pi_{t-1}\right)^2+\frac{1}{2}\theta_w\phi^{-1}\xi_w^{-1}\left(\pi_t^w-\gamma_w\pi_{t-1}\right)^2\right]\\ &+\text{t.i.p.}+O\left(\|\xi\|^3\right),\end{aligned}$$

Combining the two terms. Taking the present discounted sum on both sides of (B.9), and subtracting the previous equation, we obtain

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t U_t &= \bar{Y}u_c\sum_{t=0}^{\infty}\beta^t\left\{\left(\hat{Y}_t-\eta\hat{Y}_{t-1}\right)+\frac{1}{2}\left[\left(\hat{Y}_t^2-\eta\hat{Y}_{t-1}^2\right)-\sigma^{-1}\left(\hat{Y}_t-\eta\hat{Y}_{t-1}\right)^2\right]\right.\\ &+\sigma^{-1}\left(\hat{Y}_t-\eta\hat{Y}_{t-1}\right)\left(g_t-\eta\hat{G}_{t-1}\right)\\ &\left.-(1-\beta\eta)\left[(1-\Phi)\hat{Y}_t+\frac{1}{2}(1+\omega)\left(\hat{Y}_t-a_t\right)^2-\nu\bar{h}_t\hat{Y}_t\right]-\frac{(1-\beta\eta)}{2}\tilde{L}_t\right\}\\ &+\text{t.i.p.}+O\left(\|\xi\|^3\right),\end{aligned}$$

where

$$\tilde{L}_t \equiv \theta_p\xi_p^{-1}\left(\pi_t-\gamma_p\pi_{t-1}\right)^2+\theta_w\phi^{-1}\xi_w^{-1}\left(\pi_t^w-\gamma_w\pi_{t-1}\right)^2.$$

Using (B.8), and given that

$$\sum_{t=0}^{\infty} \beta^t z_{t-1} = z_{-1} + \beta \sum_{t=0}^{\infty} \beta^t z_t = \beta \sum_{t=0}^{\infty} \beta^t z_t + t.i.p.$$

for any variable z_t, we can rewrite our welfare function as

$$\begin{aligned} \mathcal{W}_0 &= E_0 \sum_{t=0}^{\infty} \beta^t U_t = -\bar{Y} u_c (1-\beta\eta) E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ -\Phi \hat{Y}_t + \frac{1}{2}\left[\omega + \varphi\left(1+\beta\eta^2\right)\right] \hat{Y}_t^2 - \eta\varphi \hat{Y}_t \hat{Y}_{t-1} \\ & - \left[\omega + \varphi\left(1+\beta\eta^2\right)\right] \hat{Y}_t^n \hat{Y}_t + \varphi\beta\eta \hat{Y}_{t+1}^n \hat{Y}_t + \varphi\eta \hat{Y}_{t-1}^n \hat{Y}_t + \frac{1}{2}\tilde{L}_t \Big\} + \text{t.i.p.} + O\left(\|\xi\|^3\right) \end{aligned} \quad \text{(B.13)}$$

We now conjecture that there exist constants δ, δ_0 and $\hat{x}^*$ such that the previous expression can be expressed in terms of

$$\begin{aligned} \frac{1}{2}\left[\left(\hat{Y}_t - \hat{Y}_t^n\right) - \delta\left(\hat{Y}_{t-1} - \hat{Y}_{t-1}^n\right) - \hat{x}^*\right]^2 &= -\hat{x}^* \hat{Y}_t + \hat{x}^* \delta \hat{Y}_{t-1} + \frac{1}{2}\left(\hat{Y}_t^2 + \delta^2 \hat{Y}_{t-1}^2\right) - \delta \hat{Y}_t \hat{Y}_{t-1} \\ & -\hat{Y}_t^n \hat{Y}_t - \delta^2 \hat{Y}_{t-1}^n \hat{Y}_{t-1} + \delta \hat{Y}_t^n \hat{Y}_{t-1} + \delta \hat{Y}_t \hat{Y}_{t-1}^n + \text{t.i.p.} \end{aligned}$$

This implies that

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t \frac{\delta_0}{2} \left(x_t - \delta x_{t-1} - \hat{x}^*\right)^2 &= \sum_{t=0}^{\infty} \beta^t \delta_0 \Big[-\hat{x}^* (1-\beta\delta) \hat{Y}_t + \hat{Y}_t^2 \left(1+\beta\delta^2\right)/2 - \delta \hat{Y}_t \hat{Y}_{t-1} \\ & - \left(1+\beta\delta^2\right) \hat{Y}_t^n \hat{Y}_t + \delta \hat{Y}_t \hat{Y}_{t-1}^n + \delta\beta \hat{Y}_{t+1}^n \hat{Y}_t \Big] + \text{t.i.p.} \end{aligned}$$

where $x_t \equiv \hat{Y}_t - \hat{Y}_t^n$. Matching the coefficients on the right-hand side of the last equation with the corresponding coefficients in (B.13) yields a set of three independent equations in the unknown δ_0, δ, and $\hat{x}^*$:

$$\Phi = \delta_0 \hat{x}^* (1-\beta\delta) \quad \text{(B.14)}$$

$$\omega + \varphi\left(1+\beta\eta^2\right) = \delta_0\left(1+\beta\delta^2\right) \quad \text{(B.15)}$$

$$\eta\varphi = \delta_0 \delta. \quad \text{(B.16)}$$

We know from (B.16) that $\delta_0 = \frac{\eta\varphi}{\delta}$, and from (B.15) that δ satisfies

$$\delta\left[\omega + \varphi\left(1+\beta\eta^2\right)\right] - \eta\varphi\left(1+\beta\delta^2\right) = 0 \quad \text{(B.17)}$$

or equivalently

$$P(\vartheta) \equiv \beta^{-1}\vartheta^2 - \chi\vartheta + \eta^2 = 0 \quad \text{(B.18)}$$

where

$$\vartheta \equiv \eta\delta^{-1}$$
$$\chi \equiv \frac{\omega + \varphi\left(1+\beta\eta^2\right)}{\beta\varphi} > 0.$$

Because $P(\vartheta)$ is a quadratic polynomial satisfying $P(0) = \eta^2 \geq 0$ and $P(1) = -\frac{\omega}{\beta\varphi} < 0$, the two roots of (B.18) satisfy

$$0 \leq \vartheta_1 < 1 < \vartheta_2$$

for all values of $\eta \in [0,1]$. Consider the larger root

$$\vartheta = \vartheta_2 = \frac{\beta}{2}\left(\chi + \sqrt{\chi^2 - 4\eta^2\beta^{-1}}\right) > 1.$$

Using the definition of ϑ, we have

$$\delta = \eta\vartheta^{-1},$$

so that δ satisfies $0 \leq \delta \leq \eta \leq 1$. Given a value for ϑ, we may then compute $\delta_0 = \vartheta\varphi$. Combining this with (B.14), we obtain

$$\hat{x}^* = \frac{\Phi}{\vartheta\varphi\left(1-\beta\delta\right)}.$$

We can thus rewrite the welfare criterion (B.13) as

$$\mathcal{W}_0 = -\Omega E_0 \sum_{t=0}^{\infty}\beta^t\left[\lambda_p\left(\pi_t - \gamma_p\pi_{t-1}\right)^2 + \lambda_w\left(\pi_t^w - \gamma_w\pi_{t-1}\right)^2 + \lambda_x\left(x_t - \delta x_{t-1} - \hat{x}^*\right)^2\right] + \text{t.i.p.} + O\left(\|\xi\|^3\right), \quad \text{(B.19)}$$

where

$$\Omega \equiv \frac{\bar{Y}u_c\left(1-\beta\eta\right)}{2}\left(\theta_p\xi_p^{-1} + \theta_w\phi^{-1}\xi_w^{-1}\right) > 0$$
$$\lambda_p \equiv \frac{\theta_p\xi_p^{-1}}{\theta_p\xi_p^{-1} + \theta_w\phi^{-1}\xi_w^{-1}} > 0, \qquad \lambda_w \equiv \frac{\theta_w\phi^{-1}\xi_w^{-1}}{\theta_p\xi_p^{-1} + \theta_w\phi^{-1}\xi_w^{-1}} > 0$$
$$\lambda_x \equiv \frac{\vartheta\varphi}{\theta_p\xi_p^{-1} + \theta_w\phi^{-1}\xi_w^{-1}} > 0,$$

and where the weights are normalized so that $\lambda_p + \lambda_w = 1$.

C Optimal Target Criterion for the Quantitative Model

This section characterizes the optimal target criterion in the estimated structural model of Section 2, along the lines proposed in Giannoni and Woodford (2002a, 2002b).

C.1 Analytical Derivation

The constraints relevant for optimal monetary policy are the aggregate supply equation (2.14) and the wage inflation equation (2.11). However, because there is no constraint on what the surprise component $E_{t-1}\mu_t$ may be (except that it must be unforecastable at date $t-2$), the only constraint implied by the wage inflation equation is

$$E_{t-1}\left(\pi^w_{t+1} - \gamma_w \pi_t\right) = \xi_w E_{t-1}\left(\omega_w x_{t+1} + \varphi \tilde{x}_{t+1}\right) + \xi_w E_{t-1}\left(w^n_{t+1} - w_{t+1}\right) + \beta E_{t-1}\left(\pi^w_{t+2} - \gamma_w \pi_{t+1}\right). \tag{C.1}$$

In addition, the identity

$$w_t = w_{t-1} + \pi^w_t - \pi_t \tag{C.2}$$

must be satisfied at all dates. The constraints (2.14), (C.1) – (C.2) generalize the constraints (1.29) – (1.31) of section 1.4.

Because of the delays assumed in the underlying model, the variables π_t, π^w_t, w_t, and x_t are all determined at date $t-1$. It will thus be convenient to define the variables $\bar{\pi}_t \equiv E_t \pi_{t+1} = \pi_{t+1}$ and $\bar{\pi}^w_t \equiv E_t \pi^w_{t+1} = \pi^w_{t+1}$, and $\bar{w}_t \equiv E_t w_{t+1} = w_{t+1}$, all determined at date t. Furthermore, because consumption at date t is determined at date $t-2$, the output gap satisfies

$$x_t = v_{t-2} + s_{t-1}$$

where v_{t-2} is an endogenous variable determined at date $t-2$ and s_{t-1} is an exogenous variable determined at date $t-1$ and unforecastable at date $t-2$.

The objective function (3.1) can then be rewritten as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_p \left(\bar{\pi}_{t-1} - \gamma_p \bar{\pi}_{t-2}\right)^2 + \lambda_w \left(\bar{\pi}^w_{t-1} - \gamma_w \bar{\pi}_{t-2}\right)^2 + \lambda_x \left(v_{t-2} + s_{t-1} - \delta v_{t-3} - \delta s_{t-2} - \hat{x}^*\right)^2\right]$$

$$= \beta E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_p \left(\bar{\pi}_t - \gamma_p \bar{\pi}_{t-1} \right)^2 + \lambda_w \left(\bar{\pi}^w_t - \gamma_w \bar{\pi}_{t-1} \right)^2 + \beta \lambda_x \left(v_t - \delta v_{t-1} - \delta s_t - \hat{x}^* \right)^2 \right] + tip$$

where *tip* represents again terms independent of policy adopted at date 0, such as endogenous variables determined before date 0. Note that to get the second line we also used the fact that s_t is unforecastable, so that $E_0 z_t s_{t+1} = E_0 \left[z_t \left(E_t s_{t+1} \right) \right] = 0$ for any date $t \geq 0$ and any variable z_t determined at date t or earlier.

A policy that is *optimal from a timeless perspective* (Woodford, 2003, chap. 7; Giannoni and Woodford, 2002a) from some date t_0 onward minimizes the expected value of the terms in this objective function that can be affected at date t_0 or later, conditional upon the state of the world at date t_0, subject to the constraints that $\bar{\pi}_{t_0}, \bar{\pi}^w_{t_0}$, and

$$\xi_w (\eta \varphi v_{t_0} + \beta^{-1} \bar{w}_{t_0}) - E_{t_0} (\bar{\pi}^w_{t_0+1} - \gamma_w \bar{\pi}_{t_0})$$

take certain values. These latter constraints are defined in such as way as to result in an optimal policy problem that is recursive in form. This requires that these constraints be of a *self-consistent* form, such that the solution to the constrained optimization problem satisfies relations of the same form (changing only the time subscripts) at all later dates. Thus the initial constraints are of a type that the central bank would *wish to commit itself to satisfy* at all dates later than t_0.

Combining the objective function with the constraints (2.14), (C.1) – (C.2), the Lagrangian for this problem can be written in the form

$$\begin{aligned}
\mathcal{L}_{t_0} = \; & E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2} \left[\lambda_p \left(\bar{\pi}_t - \gamma_p \bar{\pi}_{t-1} \right)^2 + \lambda_w \left(\bar{\pi}^w_t - \gamma_w \bar{\pi}_{t-1} \right)^2 + \beta \lambda_x \left(v_t - \delta v_{t-1} - \delta s_t - \hat{x}^* \right)^2 \right] \right. \\
& + \varphi_{1,t} \left[\bar{\pi}_t - \gamma_p \bar{\pi}_{t-1} - \xi_p \omega_p v_{t-1} - \xi_p \bar{w}_t - \beta \bar{\pi}_{t+1} + \beta \gamma_p \bar{\pi}_t \right] \\
& + \varphi_{2,t} \left[\bar{\pi}^w_{t+1} - \gamma_w \bar{\pi}_t - \xi_w \left(\omega_w v_t + \varphi \left[\left(1 + \beta \eta^2 \right) v_t - \eta v_{t-1} - \beta \eta v_{t+1} \right] \right) + \xi_w \bar{w}_{t+1} \right. \\
& \left. \left. - \beta \bar{\pi}^w_{t+2} + \beta \gamma_w \bar{\pi}_{t+1} \right] + \varphi_{3,t} \left[\bar{w}_t - \bar{w}_{t-1} - \bar{\pi}^w_t + \bar{\pi}_t \right] \right\} \\
& - \varphi_{1,t_0-1} \bar{\pi}_{t_0} + \varphi_{2,t_0-1} \left[\xi_w \eta \varphi v_{t_0} + \beta^{-1} \xi_w \bar{w}_{t_0} - (\bar{\pi}^w_{t_0+1} - \gamma_w \bar{\pi}_{t_0}) \right] + \beta^{-1} (\varphi_{2,t_0-1} - \varphi_{2,t_0-2}) \bar{\pi}^w_{t_0}.
\end{aligned}$$

Here the terms on the final line are added to reflect the additional constraints on initial outcomes mentioned in the previous paragraph. The particular notation used for the Lagrange multipliers associated with these constraints has been chosen so as to result in first-order conditions of a time-invariant form, making clear the recursive character of the optimization problem in the case of a suitable definition of the initial constraint values.

The associated system of first-order conditions is given by

$$\begin{aligned} 0 &= \lambda_p \left[\left(\bar{\pi}_t - \gamma_p \bar{\pi}_{t-1} \right) - \beta \gamma_p \left(E_t \bar{\pi}_{t+1} - \gamma_p \bar{\pi}_t \right) \right] - \beta \lambda_w \gamma_w \left(E_t \bar{\pi}^w_{t+1} - \gamma_w \bar{\pi}_t \right) \\ &\quad + \varphi_{1,t} - \beta \gamma_p E_t \varphi_{1,t+1} - \left(\varphi_{1,t-1} - \beta \gamma_p \varphi_{1,t} \right) - \beta \gamma_w \left(\varphi_{2,t} - \varphi_{2,t-1} \right) + \varphi_{3,t} \quad (C.3) \\ 0 &= \lambda_w \left(\bar{\pi}^w_t - \gamma_w \bar{\pi}_{t-1} \right) + \left(\varphi_{2,t-1} - \varphi_{2,t-2} \right) - \varphi_{3,t} \quad (C.4) \\ 0 &= -\xi_p \varphi_{1,t} + \xi_w \varphi_{2,t-1} + \varphi_{3,t} - \beta E_t \varphi_{3,t+1} \quad (C.5) \\ 0 &= \lambda_x \left[\left(v_t - \delta v_{t-1} - \delta s_t - \hat{x}^* \right) - \beta \delta E_t \left(v_{t+1} - \delta v_t - \delta s_{t+1} - \hat{x}^* \right) \right] \\ &\quad - \xi_p \omega_p E_t \varphi_{1,t+1} + \xi_w E_t \left[B(L) \varphi_{2,t+1} \right] \quad (C.6) \end{aligned}$$

for each $t \geq t_0$, where

$$\begin{aligned} B(L) &\equiv \varphi (\eta\beta - L)(1 - \eta L) - \omega_w L \\ &\equiv B_0 + B_1 L + B_2 L^2. \end{aligned}$$

The optimal plan must in addition satisfy a transversality condition. The latter is however necessarily satisfied as we restrict our attention to bounded solutions of the above first-order conditions. A policy that is optimal from a timeless perspective must result in an equilibrium that satisfies these conditions for all $t \geq t_0$, for some values of the initial Lagrange multipliers $\varphi_{1,t_0-1}, \varphi_{2,t_0-1}$, and φ_{2,t_0-2}. The target criteria that we propose imply particular values for these multipliers as functions of the state of the world in period $t_0 - 1$; the relations that identify the initial multipliers are ones that also hold in each period $t \geq t_0$ in the optimal equilibrium.

As in Giannoni and Woodford (2002a, 2002b), we combine these first-order conditions to obtain conditions for optimality that involve only target variables, *i.e.*, inflation, wage inflation, and the output gap. As mentioned in the text, we find it more convenient to express the target criteria in terms of the real wage rather than wage inflation. Furthermore, to simplify the algebra, we specialize the analysis to the case $\gamma_p = \gamma_w = 1$, in accordance with our estimates (as well as the model of Christiano et al., 2001). In this case, adding (C.3) to (C.4), and using (C.2) to replace $\bar{\pi}^w_t$ with $\bar{\pi}_t + \bar{w}_t - \bar{w}_{t-1}$ yields

$$E_t \left\{ \left(1 - \beta L^{-1}\right) (1 - L) \left(a_t + \xi_{1,t}\right) \right\} = 0 \tag{C.7}$$

for all $t \geq t_0$, where the variable a_t and the new multiplier $\xi_{1,t}$ are defined as

$$\begin{aligned} a_t &\equiv \bar{\pi}_t + \lambda_w \bar{w}_t \\ \xi_{1,t} &\equiv \varphi_{1,t} + \varphi_{2,t-1}, \end{aligned}$$

and we recall that $\lambda_p + \lambda_w = 1$. As a_t and $\xi_{1,t}$ are bounded, (C.7) is equivalent to

$$(1 - L) \left(a_t + \xi_{1,t}\right) = 0 \tag{C.8}$$

in the sense that (C.7) holds for all $t \geq t_0$ if and only if (C.8) holds for all $t \geq t_0$. In addition, if (C.8) holds for all $t \geq t_0$, then we must have

$$a_t + \xi_{1,t} = \bar{a}, \tag{C.9}$$

where $\bar{a}$ is a constant satisfying

$$\begin{aligned} \bar{a} &\equiv a_{t_0-1} + \xi_{1,t_0-1} \\ &= \bar{\pi}_{t_0-1} + \lambda_w \bar{w}_{t_0-1} + \varphi_{1,t_0-1} + \varphi_{2,t_0-2}. \end{aligned} \tag{C.10}$$

Note that the value of $\bar{a}$ is not uniquely determined by the state of the world at date t_0, because of the dependence of the above expression on the value of the initial multipliers. The values of these will depend on our specification of the constraints on initial outcomes, and the requirement of self-consistency alone will not uniquely determine what the initial multipliers will be. (Below, we exhibit a one-parameter family of optimal targeting rules, in which $\bar{a}$ is an arbitrary parameter.)

Applying the linear operator $E_t \left[\left(1 - \beta L^{-1}\right) (\cdot) \right]$ to (C.4), using (C.2) to eliminate $\bar{\pi}^w_t$ and using (C.5),we obtain

$$E_t \left\{ \left(1 - \beta L^{-1}\right) (1 - L) \left[\lambda_w \left(\bar{\pi}_t + \bar{w}_t\right) + \varphi_{2,t-1} \right] \right\} = \xi_{2,t} \tag{C.11}$$

for all dates $t \geq t_0$, where the new multiplier $\xi_{2,t}$ is defined as

$$\xi_{2,t} \equiv \xi_p \varphi_{1,t} - \xi_w \varphi_{2,t-1}.$$

Note that the variable $\left(\xi_{2,t} - \xi_p \xi_{1,t}\right)$ satisfies

$$\xi_{2,t} - \xi_p \xi_{1,t} = E_{t-1}\left(\xi_{2,t} - \xi_p \xi_{1,t}\right). \tag{C.12}$$

Subtracting (C.11) from (C.7) yields

$$E_t\left\{\left(1 - \beta L^{-1}\right)(1 - L)\left(\lambda_p \bar{\pi}_t + \varphi_{1,t}\right)\right\} = -\xi_{2,t}. \tag{C.13}$$

Then multiplying (C.13) by ξ_p and subtracting from it ξ_w times (C.11), one obtains

$$E_t\left\{A(L)\,\xi_{2,t+1}\right\} = b_t \tag{C.14}$$

for all $t \geq t_0$, where

$$\begin{aligned} A(L) &\equiv (L - \beta)(1 - L) + \left(\xi_p + \xi_w\right) L \\ b_t &\equiv E_t\left\{\left(1 - \beta L^{-1}\right)(1 - L)\left[\xi_w \lambda_w \left(\bar{\pi}_t + \bar{w}_t\right) - \xi_p \lambda_p \bar{\pi}_t\right]\right\}. \end{aligned} \tag{C.15}$$

Because the quadratic polynomial $A(L)$ satisfies $A(0) = -\beta < 0$, $A(1) = \xi_p + \xi_w > 0$ and $A(+\infty) = -\infty$, it must have two positive real roots, one smaller than 1 and one larger than 1. Factoring $A(L) = -\beta(1 - \mu_1 L)(1 - \mu_2 L)$, where $0 < \mu_1 < 1 < \mu_2$ and $\mu_2 = (\beta \mu_1)^{-1}$, we can rewrite equation (C.14) equivalently as

$$E_t\left\{(1 - \mu_1 L)\left(1 - \mu_2^{-1} L^{-1}\right)\beta \mu_2 \xi_{2,t}\right\} = b_t.$$

Given that $\xi_{2,t}$ and b_t are both bounded variables, and that $\left|\mu_2^{-1}\right| < 1$, the previous equation is equivalent to

$$(1 - \mu_1 L)\,\xi_{2,t} = d_t \tag{C.16}$$

for all dates $t \geq t_0$, where

$$d_t \equiv \mu_1 E_t\left\{\left(1 - \mu_2^{-1} L^{-1}\right)^{-1}\left(1 - \beta L^{-1}\right)(1 - L)\left[\left(\xi_w \lambda_w - \xi_p \lambda_p\right)\bar{\pi}_t + \xi_w \lambda_w \bar{w}_t\right]\right\}$$

is a function of target variables.

Note that

$$d_t + \xi_p a_t = E_{t-1}\left(d_t + \xi_p a_t\right), \tag{C.17}$$

as a consequence of (C.9), (C.12), and (C.16). This is a restriction on the path of target variables at all dates $t \geq t_0$.

Furthermore, using the identities

$$\begin{aligned} \varphi_{1,t} &= \left(\xi_p + \xi_w\right)^{-1}\left(\xi_w \xi_{1,t} + \xi_{2,t}\right) \\ \varphi_{2,t-1} &= \left(\xi_p + \xi_w\right)^{-1}\left(\xi_p \xi_{1,t} - \xi_{2,t}\right), \end{aligned}$$

we can eliminate $\varphi_{1,t}$ and $\varphi_{2,t-1}$ from (C.6) and obtain

$$e_t = \left(\xi_p + \xi_w\right)^{-1} E_t\left[\xi_p \xi_w \left(\omega_p L - B\left(L\right)\right)\xi_{1,t+2} + \left(\xi_p \omega_p L + \xi_w B\left(L\right)\right)\xi_{2,t+2}\right] \tag{C.18}$$

where

$$\begin{aligned} e_t &\equiv \lambda_x E_t\left[\left(1 - \beta\delta L^{-1}\right)\left(v_t - \delta v_{t-1} - \delta s_t - \hat{x}^*\right)\right] \\ &= \lambda_x E_t\left[\left(1 - \beta\delta L^{-1}\right)\left(x_{t+2} - \delta x_{t+1} - \hat{x}^*\right)\right]. \end{aligned} \tag{C.19}$$

Using (C.9) and (C.16) to substitute for $E_t\xi_{i,t+j}$ terms in (C.18), we obtain

$$\begin{aligned} & e_t - \left(\xi_p + \xi_w\right)^{-1} E_t\left\{\xi_p \xi_w \left(B\left(L\right) - \omega_p L\right)\left(a_{t+2} - \bar{a}\right)\right\} \\ = & \left(\xi_p + \xi_w\right)^{-1} E_t\left\{\xi_w B_0 d_{t+2} + \left(\xi_p \omega_p + \xi_w B_0 \mu_1 + \xi_w B_1\right)\xi_{2,t+1} + \xi_w B_2 \xi_{2,t}\right\} \\ = & \left(\xi_p + \xi_w\right)^{-1} E_t\left\{\xi_w B_0 d_{t+2} + \left(\xi_p \omega_p + \xi_w B_0 \mu_1 + \xi_w B_1\right) d_{t+1} + \left(\xi_p \omega_p \mu_1 + \xi_w \mu_1^2 B\left(\mu_1^{-1}\right)\right)\xi_{2,t}\right\} \end{aligned}$$

or equivalently

$$h_t = \alpha_2 \xi_{2,t} \tag{C.20}$$

where

$$\begin{aligned} h_t &\equiv e_t - E_t\left\{C\left(L\right)\left(a_{t+2} - \bar{a}\right) + D\left(L\right) d_{t+2}\right\} \\ C\left(L\right) &\equiv \left(\xi_p + \xi_w\right)^{-1} \xi_p \xi_w \left(B\left(L\right) - \omega_p L\right) \equiv C_0 + C_1 L + C_2 L^2 \\ D\left(L\right) &\equiv \left(\xi_p + \xi_w\right)^{-1}\left[\xi_w B_0 + \xi_w\left(B_0 \mu_1 + B_1\right) L + \xi_p \omega_p L\right] \equiv D_0 + D_1 L \end{aligned}$$

and

$$\alpha_2 \equiv \left(\xi_p + \xi_w\right)^{-1} \left(\xi_w \mu_1^2 B\left(\mu_1^{-1}\right) + \xi_p \omega_p \mu_1\right).$$

Equation (C.20) is a restriction that must be satisfied by the projected paths of the target variables at all dates $t \geq t_0$, and that depends only on the multiplier $\xi_{2,t}$. Let us suppose, in addition, that (C.20) holds at date $t_0 - 1$. (This can be arranged through a suitable choice of the constraints on initial outcomes; and the constraint that is needed is self-consistent, since relation (C.20) must hold at all later dates in an optimal equilibrium, regardless of the way in which the initial constraints are defined.) Then, quasi-differencing (C.20), and using (C.16) to substitute for the multiplier, we finally obtain

$$(1 - \mu_1 L)\, h_t = \alpha_2 d_t \qquad \text{(C.21)}$$

for every $t \geq t_0$.

In the case of initial constraints of the kind just hypothesized, both (C.17) and (C.21) must be satisfied by the processes $\{a_t, d_t, e_t\}$ at all dates, for some value of $\bar{a}$. We furthermore note that the choice of $\bar{a}$ is arbitrary, since for any value of $\bar{a}$, the assumption of initial Lagrange multipliers such that

$$a_{t_0-1} = \bar{a} \qquad \text{(C.22)}$$

would be an example of a relation between the multipliers and the lagged endogenous variables that also holds at all later dates in the constrained-optimal equilibrium. Nor is there any contradiction between our assumption of initial constraints that imply that (C.20) holds at $t_0 - 1$ and an assumption of initial constraints that imply (C.22) for some arbitrary choice of $\bar{a}$. For the former assumption requires that

$$\xi_p \varphi_{1,t_0-1} - \xi_w \varphi_{2,t_0-2}$$

be a certain function of the lagged endogenous variables, while the latter requires that

$$\varphi_{1,t_0-1} + \varphi_{2,t_0-2}$$

be another function of the lagged variables (that depends on $\bar{a}$). Because these two combinations of the lagged multipliers are linearly independent, it is possible to choose the initial constraints so that both relations are simultaneously satisfied. Thus (C.17) and (C.21) are two criteria to define define optimal policy, and that involve only the projected paths of the target variables, where the choice of the constant $\bar{a}$ in the definition of h_t is arbitrary.

C.1.1 Special case: Flexible wages

To give some intuition about the two target criteria (C.17) and (C.21) it may be useful to consider the special case in which wages are flexible ($\xi_w \rightarrow +\infty$), in addition to maintaining $\gamma_w = \gamma_p = 1$, as the optimal target criteria are simple to characterize analytically. In this case, we have $\lambda_w = 0$, $\lambda_p = 1$, and the roots of (C.15) satisfy $\mu_1 \rightarrow 0$ and $\mu_2 \rightarrow +\infty$. It follows that $a_t = \pi_{t+1}$ and $d_t = 0$.

The short-run optimal target criterion (C.17) reduces thus to

$$\pi_{t+1} = E_{t-1}\pi_{t+1}.$$

This indicates that under optimal policy, the central bank has to make inflation totally predictable two periods in advance.

The long-run optimal target criterion (C.21) reduces in turn to

$$\begin{aligned} 0 &= h_t \\ &= e_t - E_t\left\{C\left(L\right)\left(a_{t+2} - \bar{a}\right)\right\} \\ &= e_t - \xi_p E_t\left\{\left(\varphi\left(\eta\beta - L\right)\left(1 - \eta L\right) - \omega L\right)\left(\pi_{t+3} - \bar{a}\right)\right\} \\ &= e_t - \xi_p E_t\left\{\left[\varphi\eta\beta L^{-1} - \left(\varphi\left(1 + \beta\eta^2\right) + \omega\right) + \eta\varphi L\right]\left(\pi_{t+2} - \bar{a}\right)\right\} \\ &= e_t + \xi_p\varphi\vartheta E_t\left\{\left[-\beta\delta L^{-1} + \left(1 + \beta\delta^2\right) - \delta L\right]\left(\pi_{t+2} - \bar{a}\right)\right\} \end{aligned}$$

where we use (B.17) to obtain the last equality. Using (C.19) to substitute for e_t, we can then write

$$E_t\left\{\left(1 - \beta\delta L^{-1}\right)\left(1 - \delta L\right)\left[\pi_{t+2} + \phi x_{t+2}\right]\right\} = \left(1 - \beta\delta\right)\left[\left(1 - \delta\right)\bar{a} + \phi\hat{x}^*\right], \qquad \text{(C.23)}$$

where

$$\phi = \frac{\lambda_x}{\xi_p\varphi\vartheta} = \theta_p^{-1},$$

when we use the definition of the weight λ_x. As $|\beta\delta| < 1$, a commitment to (C.23) at all dates $t \geq t_0$ is then equivalent to a commitment to

$$E_t \left[(\pi_{t+2} - \delta\pi_{t+1}) + \phi \left(x_{t+2} - \delta x_{t+1} \right) \right] = (1-\delta)\pi^*, \tag{C.24}$$

at all dates $t \geq t_0$, where

$$\pi^* \equiv \bar{a} + (\phi/1-\delta)\hat{x}^*.$$

C.2 Numerical Characterization of the Optimal Target Criteria

We now describe how the optimal target criteria (C.17) and (C.21) derived above can be rewritten as (3.3) – (3.7) in the text.

C.2.1 Short-run target criterion

Noting that the variable d_t satisfies

$$\begin{aligned} d_t &= \mu_1 E_t \left\{ \left[-L + \left(1+\beta-\mu_2^{-1}\right) + \left(\mu_2^{-1}-\beta\right)\left(1-\mu_2^{-1}\right) L^{-1} \left(1-\mu_2^{-1}L^{-1}\right)^{-1} \right] \right. \\ &\quad \left. \times \left[\left(\xi_w\lambda_w - \xi_p\lambda_p\right)\bar{\pi}_t + \xi_w\lambda_w\bar{w}_t \right] \right\}, \end{aligned}$$

we can rewrite the short-run target criterion (C.17) as

$$m_t = E_{t-1} m_t \tag{C.25}$$

where

$$\begin{aligned} m_t &\equiv \mu_1 E_t \left\{ \left[\left(1+\beta-\mu_2^{-1}\right) + \left(\mu_2^{-1}-\beta\right)\left(1-\mu_2^{-1}\right) L^{-1} \left(1-\mu_2^{-1}L^{-1}\right)^{-1} \right] \right. \\ &\quad \left. \times \left[\left(\xi_w\lambda_w - \xi_p\lambda_p\right)\bar{\pi}_t + \xi_w\lambda_w\bar{w}_t \right] \right\} + \xi_p \left(\bar{\pi}_t + \lambda_w \bar{w}_t \right). \end{aligned}$$

Here we note that the terms at date $t-1$ cancel out on both sides of equation (C.25). We can then rewrite m_t as

$$m_t = \left[\xi_p + \mu_1 \left(1+\beta-\mu_2^{-1}\right)\left(\xi_w\lambda_w - \xi_p\lambda_p\right) \right] \bar{\pi}_t$$

$$+\mu_1\left(\mu_2^{-1}-\beta\right)\left(1-\mu_2^{-1}\right)\left(\xi_w\lambda_w-\xi_p\lambda_p\right)\sum_{k=0}^{\infty}\mu_2^{-k}E_t\bar{\pi}_{t+k+1}$$

$$+\left[\xi_p\lambda_w+\mu_1\left(1+\beta-\mu_2^{-1}\right)\xi_w\lambda_w\right]\bar{w}_t+\mu_1\left(\mu_2^{-1}-\beta\right)\left(1-\mu_2^{-1}\right)\xi_w\lambda_w\sum_{k=0}^{\infty}\mu_2^{-k}E_t\bar{w}_{t+k+1}$$

or as

$$m_t = S_\pi \sum_{k=1}^{\infty}\alpha_k^\pi E_t\pi_{t+k} + S_w\sum_{k=1}^{\infty}\alpha_k^w E_t w_{t+k} \tag{C.26}$$

where S_π and S_w are the sums of coefficients and α_k^π, α_k^w are the weights defined by

$$\begin{aligned}
S_\pi &= \xi_p+\mu_1\left(1+\beta-\mu_2^{-1}\right)\left(\xi_w\lambda_w-\xi_p\lambda_p\right)+\mu_1\left(\mu_2^{-1}-\beta\right)\left(1-\mu_2^{-1}\right)\left(\xi_w\lambda_w-\xi_p\lambda_p\right)\sum_{k=0}^{\infty}\mu_2^{-k} \\
&= \xi_p+\mu_1\left(\xi_w\lambda_w-\xi_p\lambda_p\right) \\
\alpha_1^\pi &= \left[\xi_p+\mu_1\left(1+\beta-\mu_2^{-1}\right)\left(\xi_w\lambda_w-\xi_p\lambda_p\right)\right]/S_\pi \\
\alpha_k^\pi &= \mu_1\left(\mu_2^{-1}-\beta\right)\left(1-\mu_2^{-1}\right)\left(\xi_w\lambda_w-\xi_p\lambda_p\right)\mu_2^{-k+2}/S_\pi, \qquad \text{for } k\geq 2
\end{aligned}$$

and

$$\begin{aligned}
S_w &= \xi_p\lambda_w+\mu_1\left(1+\beta-\mu_2^{-1}\right)\xi_w\lambda_w+\mu_1\left(\mu_2^{-1}-\beta\right)\left(1-\mu_2^{-1}\right)\xi_w\lambda_w\sum_{k=0}^{\infty}\mu_2^{-k} \\
&= \lambda_w\left(\xi_p+\xi_w\mu_1\right) \\
\alpha_1^w &= \lambda_w\left[\xi_p+\xi_w\mu_1\left(1+\beta-\mu_2^{-1}\right)\right]/S_w \\
\alpha_k^w &= \mu_1\lambda_w\xi_w\left(\mu_2^{-1}-\beta\right)\left(1-\mu_2^{-1}\right)\mu_2^{-k+2}/S_w, \qquad \text{for } k\geq 2.
\end{aligned}$$

Finally, we may rewrite (C.25) – (C.26) more compactly as

$$F_t(\pi)+\phi_w\left[F_t(w)-w_t\right]=E_{t-1}\left\{F_t(\pi)+\phi_w\left[F_t(w)-w_t\right]\right\} \tag{C.27}$$

which corresponds to the target criterion given by (3.3), (3.5). The expression $F_t(z)$ refers to the weighted average of forecasts of the variable z given by

$$F_t(z)\equiv\sum_{k=1}^{\infty}\alpha_k^z E_t z_{t+k} \tag{C.28}$$

where the sums $\sum_{k=1}^{\infty}\alpha_k^\pi=\sum_{k=1}^{\infty}\alpha_k^w=1$, and

$$\phi_w=\frac{S_w}{S_\pi}=\frac{\lambda_w\left(\xi_p+\xi_w\mu_1\right)}{\xi_p+\mu_1\left(\xi_w\lambda_w-\xi_p\lambda_p\right)}$$

lies between 0 and 1.

C.2.2 Long-run target criterion

To express the long-run target criterion (C.21) as in (3.6) – (3.7), we rewrite a_t, d_t, and e_t as follows

$$a_t = \alpha^a q_t$$

where $\alpha^a = [1, \quad \lambda_w]$ and

$$q_t \equiv \begin{bmatrix} \bar{\pi}_t \\ \bar{w}_t \end{bmatrix} = E_t \begin{bmatrix} \pi_{t+1} \\ w_{t+1} \end{bmatrix}.$$

Similarly,

$$d_t = \alpha^d q_{t-1} + E_t \left\{ \sum_{k=0}^{\infty} \alpha_k^d q_{t+k} \right\}$$

where

$$\begin{aligned} \alpha^d &= -\mu_1 \left[\left(\xi_w \lambda_w - \xi_p \lambda_p \right), \quad \xi_w \lambda_w \right] \\ \alpha_0^d &= -\left(1 + \beta - \mu_2^{-1}\right) \alpha^d \\ \alpha_k^d &= \left(\beta - \mu_2^{-1}\right) \left(1 - \mu_2^{-1}\right) \mu_2^{1-k} \alpha^d, \qquad \text{for all } k \geq 1. \end{aligned}$$

Next, it is convenient to write

$$h_t \equiv e_t - E_t \left\{ C(L)(a_{t+2} - \bar{a}) + D(L) d_{t+2} \right\} = e_t - E_t \left\{ \sum_{k=0}^{\infty} \alpha_k^h q_{t+k} \right\} + C(1) \bar{a}$$

where

$$\begin{aligned} \alpha_0^h &= C_2 \alpha^a + D_1 \alpha^d \\ \alpha_1^h &= C_1 \alpha^a + D_1 \alpha_0^d + D_0 \alpha^d \\ \alpha_2^h &= C_0 \alpha^a + D_1 \alpha_1^d + D_0 \alpha_0^d \\ \alpha_k^h &= D_1 \alpha_{k-1}^d + D_0 \alpha_{k-2}^d, \qquad \text{for all } k \geq 3. \end{aligned}$$

In addition, the variable e_t defined in (C.19) may be expressed as

$$e_t = \lambda_x \left[S_x F_t(x) - (1 - \beta\delta) \hat{x}^* \right] \tag{C.29}$$

where $F_t(x)$ is again of the form (C.28) and the weights are given by

$$\begin{aligned} S_x &= 1+\beta\delta^2-\delta-\beta\delta \\ \alpha_1^x &= -\delta/S_x \\ \alpha_2^x &= \left(1+\beta\delta^2\right)/S_x \\ \alpha_3^x &= -\beta\delta/S_x \\ \alpha_k^x &= 0, \qquad \text{for all } k\geq 4. \end{aligned}$$

Using this, we can rewrite the long-run target criterion (C.21) as

$$\begin{aligned} \sum_{k=0}^{\infty}\left(\alpha_k^h+\alpha_2\alpha_k^d\right)E_t q_{t+k}-\lambda_x S_x F_t(x) &= (1-\mu_1)\left[C(1)\bar{a}-\lambda_x(1-\beta\delta)\hat{x}^*\right]-\alpha_2\alpha^d q_{t-1} \\ &\quad +\mu_1\sum_{k=0}^{\infty}\alpha_k^h E_{t-1}q_{t+k-1}-\mu_1\lambda_x S_x F_{t-1}(x). \end{aligned}$$

Premultiplying each of the infinite sums by the sum of coefficients

$$\begin{aligned} [S_{\pi 0}, S_{w0}] &= \sum_{k=0}^{\infty}\left(\alpha_k^h+\alpha_2\alpha_k^d\right) \\ [S_{\pi 1}, S_{w1}] &= -\alpha_2\alpha^d+\mu_1\sum_{k=0}^{\infty}\alpha_k^h, \end{aligned}$$

and dividing on both sides by $S_{\pi 0}$, we can equivalently the rewrite the above long-run target criterion as in (3.6) – (3.7), i.e., as

$$F_t^*(\pi)+\phi_w^* F_t^*(w)+\phi_x^* F_t(x)=(1-\theta_\pi^*)\pi^*+\theta_\pi^* F_{t-1}^1(\pi)+\theta_w^* F_{t-1}^1(w)+\theta_x^* F_{t-1}(x) \quad \text{(C.30)}$$

where

$$\begin{aligned} \phi_w^* &= \frac{S_{w0}}{S_{\pi 0}}, \qquad \phi_x^*=-\frac{\lambda_x S_x}{S_{\pi 0}} \\ \theta_\pi^* &= \frac{S_{\pi 1}}{S_{\pi 0}}, \qquad \theta_w^*=\frac{S_{w1}}{S_{\pi 0}}, \qquad \theta_x^*=\phi_x^*\mu_1 \end{aligned}$$

and the constant π^* is given by

$$\pi^*=(1-\mu_1)\left[C(1)\bar{a}-\lambda_x(1-\beta\delta)\hat{x}^*\right]/(S_{\pi 0}-S_{\pi 1}).$$

The constant π^*. As explained in section C.1, the constant $\bar{a}$ is arbitrary. It follows from this that the constant π^* is similarly arbitrary: rules with different values of π^* bring about equilibria that are each optimal, under a suitable choice of the initial constraints. As noted in the text, in this model there is no welfare significance to any absolute rate of inflation, only to the rate of change of inflation (and to wage inflation relative to price inflation). However, we find nonetheless that an optimal policy rule must involve *some* long-run inflation target π^*, that remains invariant over time. For purposes of our comparison between historical policy and the optimal target criteria, we assume a long-run inflation target equal to the long-run value for inflation under historical policy, as implied by our estimated VAR model of the historical data.

C.3 Historical Time Series for the Target Criteria

This section describes the calculations underlying section 3.3 of the text in which we assess to what extent, under actual policy, the evolution of projections of inflation, the real wage and the output gap have satisfied the optimal target criteria. To perform the projections of future variables, we use the structural VAR (2.2) which we can rewrite in terms of deviations from a long-run steady-state as

$$\hat{Z}_t = B\hat{Z}_{t-1} + U\bar{e}_t$$

where $\hat{Z}_t \equiv \bar{Z}_t - Z^{lr}$ and $B = T^{-1}A$, $U = T^{-1}$. The vector $\bar{Z}_t$ is given by

$$\bar{Z}_t = \left[i_t, \hat{w}_{t+1}, \pi_{t+1}, \hat{Y}_{t+1}, i_{t-1}, \hat{w}_t, \pi_t, \hat{Y}_t, i_{t-2}, \hat{w}_{t-1}, \pi_{t-1}, \hat{Y}_{t-1}\right]',$$

and its long-run value satisfies $Z^{lr} = (I - B)^{-1} T^{-1} a$. Because we assume that the errors $\bar{e}_t$ are unforecastable, the VAR has the property that $E_t\hat{Z}_{t+k} = B^k\hat{Z}_t$ for all $k > 0$.

Using this, we can compute for each date t the weighted average of future inflation forecasts as follows

$$\begin{aligned} F_t(\pi) &= \sum_{k=1}^{\infty} \alpha_k^\pi E_t \pi_{t+k} = \sum_{k=1}^{\infty} \alpha_k^\pi \tilde{P} E_t \bar{Z}_{t+k-1} \\ &= \pi^{lr} + P\hat{Z}_t, \end{aligned}$$

where $\tilde{P}$ is a (1×12) vector with a 1 in the third element and zeros elsewhere, $\pi^{lr} \equiv \tilde{P} Z^{lr}$, and $P \equiv \tilde{P} \sum_{k=1}^{\infty} \alpha_k^{\pi} B^{k-1}$. Similarly, we can compute for each date t the weighted average of real wage forecasts

$$F_t(w) = \sum_{k=1}^{\infty} \alpha_k^w E_t w_{t+k} = W \hat{Z}_t,$$

where $W \equiv \tilde{W} \sum_{k=1}^{\infty} \alpha_k^w B^{k-1}$ and $\tilde{W}$ is a (1×12) vector with a 1 in the second element and zeros elsewhere. (Note that the long-run value of the variable $\hat{w}$, i.e., the percent deviation in the real wage from its trend is zero).

C.3.1 Short-run target criterion

A historical time series for the adjusted inflation projection (3.3) is obtained by computing for each date t:

$$F_t(\pi) + \phi_w [F_t(w) - w_t] = \left(\pi^{lr} + P\hat{Z}_t\right) + \phi_w \left(W\hat{Z}_t - \tilde{W}\hat{Z}_{t-1}\right)$$

A historical time series for the optimal target (3.5) is then obtained by computing for each date t:

$$E_{t-1}\{F_t(\pi) + \phi_w [F_t(w) - w_t]\} = \left(\pi^{lr} + PB\hat{Z}_{t-1}\right) + \phi_w \left(WB - \tilde{W}\right)\hat{Z}_{t-1}.$$

C.3.2 Output gap projections

In addition to inflation projections and real wage projections described above, the long-run target criterion (3.6) – (3.7) involves also projections of the output gap. This raises some difficulties that we address in this subsection.

Let us first consider the simple case in which the natural rate of output displays only negligible fluctuations. In this case, the output gap considered in the target criterion (3.6) – (3.7) corresponds to the deviation of (log) real output from a linear trend (as is the case in Figures 12 and 13 of the text), i.e., to the time series $\hat{Y}_t$ used in the VAR. The weighted average of future output gap forecasts with the weights used in (C.29) is then simply obtained by computing

$$F_t\left(\hat{Y}\right) = E_t \sum_{k=1}^{\infty} \alpha_k^x \hat{Y}_{t+k} = E_t R \bar{Z}_{t+2} = RB^2 \hat{Z}_t$$

where

$$R = [0, 0, 0, \alpha_3^x, 0, 0, 0, \alpha_2^x, 0, 0, 0, \alpha_1^x].$$

Again, we note that the long-run value of the variable $\hat{Y}_t$ is zero and that $\alpha_k^x = 0$ for all $k \geq 4$.

We now turn to the alternative case in which fluctuations in the natural rate of output are recovered from the residuals to the estimated equations of the model. First, we note that the weighted average of projection of future output gaps relevant for the target criterion (3.6) – (3.7), i.e., with the weights used in (C.29) satisfies

$$\begin{aligned} F_t(x) &= E_t \sum_{k=1}^{\infty} \alpha_k^x x_{t+k} = S_x^{-1} E_t \left[-\delta x_{+1} + \left(1 + \beta\delta^2\right) x_{t+2} - \beta\delta x_{t+3}\right] \\ &= S_x^{-1} E_t \left\{\left(1 - \beta\delta L^{-1}\right)\left(x_{t+2} - \delta x_{t+1}\right)\right\}. \end{aligned} \tag{C.31}$$

Second, we multiply the price inflation equation (2.14) by ξ_w and add it to the wage inflation equation (2.11) multiplied by ξ_p to obtain

$$\begin{aligned} &\xi_w \xi_p E_{t-1}\left\{\left[\omega_p + \omega_w + \varphi\left(1 + \beta\eta^2\right) - \varphi\eta L - \varphi\beta\eta L^{-1}\right] x_t\right\} = \left[\xi_w + \beta\left(\xi_w + \xi_p\right)\gamma_p\right]\pi_t \\ &- \left(\xi_w\gamma_p + \xi_p\gamma_w\right)\pi_{t-1} - \beta\xi_w E_{t-1}\pi_{t+1} + \xi_p \pi_t^w - \beta\xi_p E_{t-1}\pi_{t+1}^w + \xi_w\xi_p E_{t-1}\mu_t \end{aligned}$$

It is convenient to note, using (B.17), that the left-hand-side is in fact equal to

$$\xi_w \xi_p \vartheta \varphi E_{t-1}\left\{\left(1 - \beta\delta L^{-1}\right)\left(x_t - \delta x_{t-1}\right)\right\}$$

where $0 \leq \delta < \eta$ is the same value as the one entering the policymaker's objective function, and where $\vartheta \equiv \eta/\delta > 1$. Next, using the wage identity (1.31) to substitute for π_t^w, we obtain

$$\begin{aligned} \xi_w \xi_p \vartheta \varphi E_{t-1}\left\{\left(1 - \beta\delta L^{-1}\right)\left(x_t - \delta x_{t-1}\right)\right\} &= \left(\xi_w + \xi_p\right)\left(1 + \beta\gamma_p\right)\pi_t - \left(\xi_w\gamma_p + \xi_p\gamma_w\right)\pi_{t-1} \\ &\quad - \beta\left(\xi_w + \xi_p\right) E_{t-1}\pi_{t+1} - \xi_p w_{t-1} + \xi_p (1 + \beta) w_t \\ &\quad - \beta\xi_p E_{t-1} w_{t+1} + \xi_w \xi_p E_{t-1}\mu_t. \end{aligned} \tag{C.32}$$

Thus, by combining (C.31), (C.32) and noting that $E_t\mu_{t+2} = 0$, we obtain a historical time series for projections of future output gaps

$$\begin{aligned} F_t(x) &= S_x^{-1} Q E_t \bar{Z}_{t+2} \\ &= S_x^{-1} Q B^2 \hat{Z}_t. \end{aligned}$$

where

$$Q = \left[0, -\frac{\beta}{\xi_w \vartheta\varphi}, -\frac{\beta\left(\xi_w + \xi_p\right)}{\xi_w \xi_p \vartheta\varphi}, 0, 0, \frac{1+\beta}{\xi_w \vartheta\varphi}, \frac{\left(\xi_w + \xi_p\right)\left(1 + \beta\gamma_p\right)}{\xi_w \xi_p \vartheta\varphi}, 0, 0, -\frac{1}{\xi_w \vartheta\varphi}, -\frac{\left(\xi_w \gamma_p + \xi_p \gamma_w\right)}{\xi_w \xi_p \vartheta\varphi}, 0\right].$$

Again, it turns out that the constant $QB^2 Z^{lr}$ is equal to 0 when $\gamma_p = \gamma_w = 1$, which is the case that we consider here.

C.3.3 Long-run target criterion

A historical time series for the projections (3.6) is obtained by computing for each date t:

$$F_t^*(\pi) + \phi_w^* F_t^*(w) + \phi_x^* F_t(x) = \left(\pi^{lr} + P^* \hat{Z}_t\right) + \phi_w^* W^* \hat{Z}_t + \phi_x^* S_x^{-1} Q B^2 \hat{Z}_t$$

where the weights $\alpha_k^{\pi*}$ and α_k^{w*} are those underlying (C.30), and $P^* \equiv \tilde{P} \sum_{k=1}^{\infty} \alpha_k^{\pi*} B^{k-1}$, $W^* \equiv \tilde{W} \sum_{k=1}^{\infty} \alpha_k^{w*} B^{k-1}$. Similarly, we can compute a historical time series for the optimal target (3.7)

$$\begin{aligned} \pi_t^* &\equiv \pi^* + \theta_\pi^* F_{t-1}^1(\pi) + \theta_w^* F_{t-1}^1(w) + \theta_x^* F_{t-1}(x) \\ &= \pi^* + \theta_\pi^* \left(\pi^{lr} + P^1 \hat{Z}_{t-1}\right) + \theta_w^* W^1 \hat{Z}_{t-1} + \theta_x^* S_x^{-1} Q B^2 \hat{Z}_{t-1}, \end{aligned}$$

where the weights $\alpha_k^{\pi 1}$ and α_k^{w1} are those underlying the weighted sums on the right-hand side of (C.30) and $P^1 \equiv \tilde{P} \sum_{k=1}^{\infty} \alpha_k^{\pi 1} B^{k-1}$, $W^1 \equiv \tilde{W} \sum_{k=1}^{\infty} \alpha_k^{w1} B^{k-1}$. Note that the weighted averages $F_t(x)$ and $F_{t-1}(x)$ are identical.

In the case that the natural rate of output displays only negligible fluctuations so that the output gap considered is $\hat{Y}_t$, the contribution to the projections due to output gap fluctuations is given by

$$\phi_x^* F_t\left(\hat{Y}\right) = \phi_x^* R B^2 \hat{Z}_t.$$

The contribution to the optimal target due to output gap fluctuations is then given by

$$\theta_x^* F_{t-1}\left(\hat{Y}\right) = \theta_x^* R B^2 \hat{Z}_{t-1}.$$