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Appendix A: Model

The model builds off of Barsky et al's (2007) model of sticky prices and durable goods but is different in the following ways. Rather than specifying a process for money growth, we model monetary policy using a Taylor rule; we allow for the production technology to differ across sectors (i.e., differing capital intensities), as in Eusepi et al (2011); we assume that the allocation of the capital stock *across* sectors at time t is determined at time $t-1$; we incorporate capital accumulation and time-varying rates of capital utilization; we allow for adjustment costs for investment in capital *and* consumption expenditures on durables; we allow for sticky (nominal) wages, as in Erceg et al (2000); and we incorporate neutral technology shocks, an investment-specific technology shock and a government spending shock.

Firms

Durable ($Y_{d,t}$) and nondurable ($Y_{n,t}$) final goods are produced from intermediate goods, which are produced by two types of firms: those with high- and low-capital-intensive technologies. Competitive final goods producers take prices as given (both output ($P_{j,t}$) and input ($P_{jf,t}$) and solve

$$\max P_{j,t}Y_{j,t} - \sum_{f=h,l} P_{jf,t}Y_{jf,t}$$

subject to

$$Y_{j,t} = \left[\sum_{f=h,l} Y_{jf,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $Y_{jf,t}$ are composite intermediate goods. These composites are are produced by competitiatve firms (i.e., take prices as given), which solve the following problem

$$\max P_{jf,t}Y_{jf,t} - \int_0^1 P_{jf,t}(l)y_{jf,t}(l)dl$$

subject to

$$Y_{jf,t} = \left[\int_0^1 y_{jf,t}(l)^{\frac{\varepsilon-1}{\varepsilon}}dl\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $y_{jf,t}(l)$ are individual varieties of the goods, $\varepsilon > 1$ is the elasticity of substitution among the varieties, and $P_{jf,t}(l)$ is the price of variety l.

Cost minimization by the intermediate-bundling firms gives the following demand for individual varieties

$$y_{jf,t}(l) = \left(\frac{P_{jf,t}(l)}{P_{jf,t}}\right)^{-\varepsilon} Y_{jf,t}, \tag{1}$$

and implies that the price indices $P_{jf,t}$ can be written as

$$P_{jf,t} = \left[\int_0^1 p_{jf,t}(l)^{1-\varepsilon}dl\right]^{\frac{1}{1-\varepsilon}}.$$

Likewise, cost minimization by the producers of final goods yields

$$Y_{jf,t} = \left(\frac{P_{jf,t}}{P_{j,t}}\right)^{-\varepsilon} Y_{j,t} \tag{2}$$

and implies that

$$P_{j,t} = \left[\sum_{f=h,l} P_{jf,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.$$

Turning to the production of the individual varieties of the intermediate goods, we assume a continuum of monopolistically competitive firms that rent capital and labor services from households in sector-wide markets. The production function for firm l that produces good j with technology f is

$$y_{jf,t}(l) = A_{jf,t} k_{jf,t}(l)^{\alpha_f} n_{jf,t}(l)^{1-\alpha_f},$$

where $A_{jf,t}$ is sector-specific total factor productivity (TFP), and $k_{jf,t}(l)$ and $n_{jf,t}(l)$ are capital and labor services employed. Sectoral TFP follows the process:

$$ln(A_{jf,t}) = (1-\rho_{a_{jf}}) ln\bar{A}_{jf} + \rho_{a_{jf}} ln(A_{jf,t-1}) + \epsilon_t^{a_{jf}}, \tag{3}$$

where $\bar{A}_{jf}$ is the steady-state sector TFP level and the shocks $\epsilon^{a_{jf}}$ have iid Normal distributions: $\epsilon^{a_{jf}} \sim N(0, \sigma_{\epsilon^{a_{jf}}})$.[1]

Unit-cost minimization determines the generic firm's demand functions for capital and labor. Formally,

$$\min R_{jf,t}^k k_{jf,t}(l) + W_{jf,t} n_{jf,t}(l)$$

subject to

$$A_{jf,t} k_{jf,t}(l)^{\alpha_f} n_{jf,t}(l)^{1-\alpha_f} \geq 1,$$

where $W_{jf,t}$ and $R_{jf,t}^k$ are, respectively, the nominal wage and capital rental rate paid by firms in sector jf.

First-order conditions are

$$R_{jf,t}^k = MC_{jf,t} \alpha_f A_{jf,t} \left(\frac{k_{jf,t}(l)}{n_{jf,t}(l)}\right)^{\alpha_f - 1}, \tag{4}$$

$$W_{jf,t} = MC_{jf,t}(1-\alpha_f) A_{jf,t} \left(\frac{k_{jf,t}(l)}{n_{jf,t}(l)}\right)^{\alpha_f}, \tag{5}$$

where $MC_{jf,t}$ is the Lagrange multiplier associated with the constraint and is equal to the nominal marginal cost in sector jf. Note that because the production function has constant returns to scale, and because capital and labor can flow freely across firms within the same sector, firms in the same sector will have the same nominal marginal cost and identical capital-labor ratios. Thus, $k_{jf,t}(l)/n_{jf,t}(l) = K_{jf,t}/N_{jf,t}$.

[1] This distributional assumption will apply to all shocks in the model.

Then nominal marginal cost $MC_{jf,t}$ for all firms in sector jf is given by

$$MC_{jf,t} = \frac{W_{jf,t}}{(1-\alpha_f)A_{jf,t}\left(\frac{K_{jf,t}}{N_{jf,t}}\right)^{\alpha_f}} = \frac{W_{jf,t}^{1-\alpha_f} R_{jf,t}^{k\ \ \alpha_f}}{A_{jf,t}\alpha_f^{\alpha_f}(1-\alpha_f)^{1-\alpha_f}}.$$

Note that since the technology shocks are at the sector level and factors are hired in sector-wide markets, marginal cost is a function of sector-specific variables. As mentioned above, constant returns to scale at the firm level implies that all firms *within* the sector have the same marginal cost. The predetermination of sectoral capital and imperfect mobility of capital (and possibly labor) means that marginal costs can vary across sectors, and the different production technologies imply that the slope of the sectoral marginal cost curve will also vary across sectors.

Goods prices are sticky. We use the Calvo (1983) assumption whereby monopolistically competitive firms change prices with a constant probability of $(1-\theta_{jf})$, regardless of the history of price changes. When a firm gets a chance to choose prices, it sets the optimal price $P^*_{jf,t}(l)$ by maximizing expected discounted profits

$$E_t \sum_{s=0}^{\infty} \rho_{t,t+s} (\theta_{jf})^s \left(P^*_{jf,t}(l) - MC_{jf,t+s}\right) \left(\frac{P^*_{jf,t}(l)}{P_{jf,t+s}}\right)^{-\varepsilon} Y_{jf,t+s},$$

where the (stochastic) discount factor $\rho_{t,t+s}$ will reflect the households' valuation of profits. The first-order condition for this problem is given by

$$E_t \sum_{s=0}^{\infty} \rho_{t,t+s} (\theta_{jf})^s \left(P^*_{jf,t}(l) - \mu MC_{jf,t+s}\right) \left(\frac{P^*_{jf,t}(l)}{P_{jf,t+s}}\right)^{-\varepsilon} Y_{jf,t+s} = 0,$$

where $\mu = \frac{\varepsilon}{\varepsilon-1}$ is the constant desired markup. Since all updating firms choose the same price $P^*_{jf,t}(l)$, which we now denote simply by $P^*_{jf,t}$, and are randomly chosen, the law of motion for the aggregate price index $P_{jf,t}$ is given by

$$P_{jf,t} = \left[(1-\theta_{jf})\left(P^*_{jf,t}\right)^{1-\varepsilon} + \theta_{jf}\left(P_{jf,t-1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.$$

Households

There is a unit mass of households indexed by h. Household h derives utility from consumption of a non-durable good and from the service flow of a durable good, incurs disutility from hours worked, and owns physical capital and state-contingent debt securities. The objective of the household is

$$\max\ E_t \sum_{s=0}^{\infty} \beta^s \left[u\left(C_{n,t+s}(h), D^c_{t+s}(h)\right) - v(N_{t+s}(h))\right]$$

subject to the flow budget constraint

$$\sum_j P_{j,t}C_{j,t}(h) + P_t^{d'}\Delta_t^d(h) + E_t\{\rho_{t,t+1}B_{t+1}(h)\} + P_{d,t}\sum_{jf}\tilde{I}_{jf,t}(h) + \sum_{jf}P^k_{jf,t}\Delta_{jf,t}(h) =$$
$$\sum_{jf} W^h_{jf,t}N_{jf,t}(h) + Div_t + \sum_{jf} R^k_{jf,t}u_{jf,t}(h)K^s_{jf,t}(h) + \Pi_t + B_t(h) - T_t, \qquad (6)$$

the law of motion for the durable consumption stock

$$D_t^c(h) = (1-\delta)D_{t-1}^c(h) + C_{d,t}(h) - \frac{S^{''}}{2}\left(\frac{C_{d,t}(h)}{D_{t-1}^c(h)} - \delta\right)^2 D_{t-1}^c(h) + \Delta_t^d, \tag{7}$$

the law of motion for sector-specific capital stocks

$$K_{jf,t+1}^s(h) = (1-\delta)K_{jf,t}^s(h) + I_{jf,t}(h) - \frac{S^{''}}{2}\left(\frac{I_{jf,t}(h)}{K_{jf,t}^s(h)} - \delta\right)^2 K_{jf,t}^s(h) + \Delta_{jf,t}(h), \tag{8}$$

an expression defining investment expenditures

$$\tilde{I}_{jf,t} = \frac{I_{jf,t}(h) + a(u_{jf,t}(h))K_{jf,t}^s(h)}{\epsilon_t^i}, \tag{9}$$

and the aggregation of hours worked

$$N_t(h) = \left(\sum_{j=n,d} (N_{j,t}(h))^{(\zeta_N+1)/\zeta_N}\right)^{\zeta_N/(\zeta_N+1)}, \tag{10}$$

where $C_{n,t}(h)$ is the amount of non-durable consumption, $C_{d,t}(h)$ is the amount of newly purchased durables, $D_t^c(h)$ is the stock of the durable good, $N_t(h)$ is labor supply, $\tilde{I}_{jf,t}$ denotes investment expenditures in sector-jf, Δ_t^d is installed durables purchased from other housholds at price $P_t^{d'}$, $\Delta_{jf,t}$ is installed capital purchased from other housholds at price $P_{jf,t}^k$, $W_{jf,t}^h$ is wage received by the household for labor supplied to sector jf, $N_{jf,t}(h)$ is the number of hours worked in sector jf, $u_{jf,t}(h)$ is the utilization rate of the capital stock $K_{jf,t}^s$ in sector jf (i.e., $K_{jf,t} = u_{jf,t}K_{jf,t}^s$) and $a(\cdot)$ are maintenance costs that depend on capital utilization, Div_t denotes profits form labor unions that are returned lump-sum to the households, Π_t denotes profits from firms that are returned lump-sum to the households, and T_t denotes lump-sum taxes paid to the government.[2] Moreover, $B_{t+1}(h)$ is the nominal value of the complete set of state-contingent securities at the beginning of period $t+1$, and $\rho_{t,t+1}$ is the unique stochastic discount factor that prices the securities in period t.[3] ϵ_t^i is an investment-specific technology shock; greater productivity means that fewer expenditures are needed to achieve a given increase in the capital stock or cover maintenance costs. Finally, β is the discount factor, δ is the rate of depreciation of the durable good, $S^{''}$ governs the strength of investment adjustment costs, and ζ_N indexes the mobility of labor across sectors.

Note that the cross-sector distribution of the capital stock at time t is determined at time $t-1$. The flexibility of capital *services* in the short-run will thus be determined by the adjustment costs on capital utilization $a(\cdot)$. Labor services are perfectly flexible if $\zeta_N = \infty$, but imperfectly mobile otherwise.

In equilibrium, households will make the same choices for consumption, hours worked, bonds, and the supply of capital, so we will drop the h index except for when it would be clarifying. Let

[2]Here we are implicitly assuming that the service flow from the durable good is proportional to the stock of the good. Without loss of generality, the coefficient of proportionality is normalized to 1.

[3]To avoid cluttering, we do not use an explicit state-contingent notation.

λ_t be the Lagrange multiplier on eqn (6), γ_t be the Lagrange multiplier on eqn (7), and ϕ_t be the Lagrange multiplier on eqn (8). The first-order necessary conditions for the optimal choice of consumption/saving, labor supply and investment/utilization are then given by

$$u_C\left(C_{n,t}, D_t^c\right) = \lambda_t P_{n,t} \tag{11}$$

$$\lambda_t P_{d,t} = \gamma_t \left[1 - S^{''}\left(\frac{C_{d,t}}{D_{t-1}^c} - \delta\right)\right] \tag{12}$$

$$u_D\left(C_{n,t}, D_t^c\right) = \gamma_t - \beta E_t \gamma_{t+1}\left[1 - \delta + \frac{S^{''}}{2}\left(\frac{C_{d,t+1}}{D_t^c} - \delta\right)\left(\frac{C_{d,t+1}}{D_t^c} + \delta\right)\right] \tag{13}$$

$$\lambda_t P_t^{d'} = \gamma_t \tag{14}$$

$$v'(\tilde{N}_t)\left(\frac{\tilde{N}_{jf,t}}{\tilde{N}_t}\right)^{1/\zeta_N} = \lambda_t W_{jf,t}^h,\ \forall jf \tag{15}$$

$$\lambda_t P_{d,t} = \epsilon_t^i \phi_t \left[1 - S^{''}\left(\frac{I_{jf,t}}{K_{jf,t}^s} - \delta\right)\right],\ \forall jf \tag{16}$$

$$\begin{aligned} \phi_t = {} & \beta E_t \lambda_{t+1}\left[R_{jf,t+1}^k u_{jf,t+1} - \frac{P_{d,t+1} a(u_{jf,t+1})}{\epsilon_{t+1}^i}\right] \\ & + \beta E_t \phi_{t+1}\left[1 - \delta + \frac{S^{''}}{2}\left(\frac{I_{jf,t+1}}{K_{jf,t}^s} - \delta\right)\left(\frac{I_{jf,t+1}}{K_t^s} + \delta\right)\right],\ \forall jf \end{aligned} \tag{17}$$

$$\lambda_t P_{jf,t}^k = \phi_t,\ \forall jf \tag{18}$$

$$\frac{P_{d,t} a'(u_{jf,t})}{\epsilon_t^i} = R_{jf,t}^k,\ \forall jf \tag{19}$$

$$\rho_{t,t+s} = \beta^s \frac{\lambda_{t+s}}{\lambda_t} \tag{20}$$

where $\tilde{N}_t \equiv \int_0^1 N_t(h)dh$[4] and eqn (20) holds for each state of nature.[5] The gross nominal interest rate R_t is then defined as

$$R_t = \frac{1}{E_t \rho_{t,t+1}} = \frac{\lambda_t}{\beta E_t \lambda_{t+1}}. \tag{21}$$

[4]We use the notation $\tilde{N}$ to distinguish between labor supplied by households and labor that is employed by firms. The two need not be equal because of wage dispersion that results from labor unions that set wages and intermediate the exchange. We will clarify the relationship when we discuss equilibrium constraints for the model.

[5]We rule out ponzi schemes and the first-order conditions also include a transversality condition.

Labor Unions

Following Erceg et al (2000) and Smets and Wouters (2007), households supply their labor to intermediate labor unions (in each sector) which differentiate the labor services and set wages according to a Calvo process.

Sector-jf firms demand labor $N_{jf,t} \equiv \int_0^1 n_{jf,t}(l)dl$, where $N_{jf,t}$ are aggregates of the differentiated labor produced by the unions. In particular,

$$N_{jf,t} = \left[\int_0^1 N_{jf,t}(i)^{\frac{\varepsilon^w-1}{\varepsilon^w}} di\right]^{\frac{\varepsilon^w}{\varepsilon^w-1}},$$

where $N_{jf,t}(i)$ are individual varieties and $\varepsilon^w > 1$ is the elasticity of substitution among the varieties. Cost minimization by firms gives the following demand for an individual variety

$$N_{jf,t}(i) = \left(\frac{W_{jf,t}(i)}{W_{jf,t}}\right)^{-\varepsilon^w} N_{jf,t}, \tag{22}$$

where $W_{jf,t}(i)$ is the price of labor service i and the price indices $W_{jf,t}$ are defined as

$$W_{jf,t} = \left[\int_0^1 W_{jf,t}(i)^{1-\varepsilon^w} di\right]^{\frac{1}{1-\varepsilon^w}}.$$

The unions are intermediates between the households and firms. The unions allocate and differentiate the labor services from the households and have market power (i.e., they can choose the wage subject to labor demand, eqn (22)). Recall that household sectoral labor supply is given by equation (15), which can be written as

$$\frac{v'(\tilde{N}_t)}{u_C\left(C_{n,t}, D_t^c\right)}\left(\frac{\tilde{N}_{jf,t}}{\tilde{N}_t}\right)^{1/\zeta_N} = \frac{W^h_{jf,t}}{P_{n,t}}.$$

The real wage desired by the households here reflects the marginal rate of substitution between leisure and consumption. Labor unions take this marginal rate of substitution as the cost of the labor services. The markup above this cost will be distributed to households in the form of dividends.

Nominal wages are sticky. We use the Calvo (1983) assumption whereby monopolistically competitive unions change wages with a constant probability of $(1-\theta^w_{jf})$, regardless of the history of price changes. When a union gets a chance to choose prices, it sets the optimal price $W^*_{jf,t}(i)$ by maximizing expected discounted dividends

$$E_t \sum_{s=0}^{\infty} \rho_{t,t+s} \left(\theta^w_{jf}\right)^s \left(W^*_{jf,t}(i) - W^h_{jf,t+s}\right) \left(\frac{W^*_{jf,t}(i)}{W_{jf,t+s}}\right)^{-\varepsilon^w} N_{jf,t+s}.$$

The first-order condition for this problem is given by

$$E_t \sum_{s=0}^{\infty} \rho_{t,t+s} \left(\theta^w_{jf}\right)^s \left(W^*_{jf,t}(i) - \mu^w W^h_{jf,t+s}\right) \left(\frac{W^*_{jf,t}(i)}{W_{jf,t+s}}\right)^{-\varepsilon^w} N_{jf,t+s} = 0,$$

where $\mu^w = \frac{\varepsilon^w}{\varepsilon^w - 1}$ is the constant desired markup. Since all updating unions choose the same price $W^*_{jf,t}(i)$, which we now denote simply by $W^*_{jf,t}$, and they are randomly chosen, the law of motion for the aggregate price index $W_{jf,t}$ is given by

$$W_{jf,t} = \left[\left(1 - \theta^w_{jf}\right) \left(W^*_{jf,t}\right)^{1-\varepsilon^w} + \theta^w_{jf} \left(W_{jf,t-1}\right)^{1-\varepsilon^w} \right]^{\frac{1}{1-\varepsilon^w}} .$$

Government

The central bank conducts monetary policy using a Taylor rule of the form

$$\ln\left(\frac{R_t}{\bar{R}}\right) = b_r \ln\left(\frac{R_{t-1}}{\bar{R}}\right) + (1 - b_r)\left[b_\pi \ln\left(\frac{\pi_t}{\bar{\pi}}\right) + b_y \ln\left(\frac{Y_t}{Y^f_t}\right)\right] + r_t \tag{23}$$

where b_r is the interest rate smoothing parameter and is strictly bounded between 0 and 1, b_π and b_y are non-negative parameters, $\bar{x}$ denotes the steady-state value of variable x, $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, Y_t is total output, Y^f_t is defined as the output in the flexible price and wage economy, and r_t is a monetary policy shock that follows the process:

$$r_t = \rho_r r_{t-1} + \epsilon^r_t. \tag{24}$$

(We define the aggregate price level and aggregate output Y_t in the following subsection.)

Note that to follow Barsky et al (2007) by specifying monetary policy using a process for money growth rather than a Taylor rule is straight-forward. First, we would assume the money demand is proportional to nominal GDP,

$$M_t = P_t Y_t. \tag{25}$$

Then, we would *replace* the Taylor rule (eqn (23) and eqn (24)) with a process for money supply, e.g., $ln(M_t) = ln(M_{t-1}) + \epsilon^M_t$. Note that in the model with a money-supply process, M_t evolves exogenously while the nominal interest rate is determined in equilibrium by eqn (21). Conversely, when monetary policy follows a Taylor rule, the nominal interest rate is set by eqn (23) and the stock of money will then be determined in equilibrium by eqn (25). Although it is straight-forward to interchange these two specifications for monetary policy, they do represent different policies so that model results can differ substantially.

Turning to fiscal policy, the government budget constraint is of the form

$$P_t G_t + B_t = T_t + \frac{B_{t+1}}{R_t},$$

where

$$P_t G_t = \sum_{j=n,d} P_{j,t} G_{j,t}, \tag{26}$$

T_t are nominal lump-sum taxes (or subsidies) that also appear in the household's budget constraint, and G_t and $G_{j,t}$ are aggregate and sector-specific government (real) expenditures, respectively.

To investigate the effects of fiscal policy, we specify a process for aggregate government spending expressed relative to steady-state output $g_t = G_t/\bar{Y}$:

$$ln(g_t) = (1 - \rho_g) ln\bar{g} + \rho_g ln(g_{t-1}) + \epsilon^g_t. \tag{27}$$

The government will then choose how to allocate this spending between nondurable and durable goods by solving the following (nominal) expenditure-minimization problem:

$$\min \ E_t \sum_{s=0}^{\infty} \left(\frac{1}{\prod_{\tau=t+1}^{t+s} R_{\tau-1}} \right) \sum_j P_{j,t+s} G_{j,t+s}$$

subject to an aggregator function for government expenditures

$$H(G_{n,t}, D_t^g) \geq G_t,$$

and the law of motion for the government durable good

$$D_t^g = (1-\delta) D_{t-1}^g + G_{d,t} - \frac{S''}{2} \left(\frac{G_{d,t}}{D_{t-1}^g} - \delta \right)^2 D_{t-1}^g. \tag{28}$$

Equilibrium Constraints

Finally, goods, labor, capital and bond markets must all be in equilibrium. Bonds are in zero net supply ($B_t = B_{t+1} = 0$). For goods markets,

$$C_{n,t} + G_{n,t} = Y_{n,t}, \tag{29}$$

$$C_{d,t} + G_{d,t} + \sum_{j,f} \tilde{I}_{jf,t} = Y_{d,t}. \tag{30}$$

Market clearing for capital and labor are given by

$$K_{jf,t} = \int_0^1 k_{jf,t}(l) dl = \int_0^1 K_{jf,t}(h) dh,$$

$$N_{jf,t} = \int_0^1 n_{jf,t}(l) dl = \left[\int_0^1 N_{jf,t}(i)^{\frac{\varepsilon^w - 1}{\varepsilon^w}} di \right]^{\frac{\varepsilon^w}{\varepsilon^w - 1}},$$

where l indexes different firms, h indexes households, and i indexes unions/households.

Note that there are not sectoral production functions in the New Keynesian model; that is, given information only about sectoral inputs and technology, it is not possible to say what sectoral output $Y_{jf,t}$ is. This is because $Y_{jf,t}$ depends upon how inputs are distributed among various producers. But, Yun (1996) has shown how to derive a relationship between inputs, output and price dispersion.

Let $\tilde{Y}_{jf,t}$ denote the unweighted integral of gross output across sector-jf producers:

$$\tilde{Y}_{jf,t} \equiv \int_0^1 y_{jf,t}(l) dl = \int_0^1 A_{jf,t} \left(\frac{k_{jf,t}(l)}{n_{jf,t}(l)} \right)^{\alpha_f} n_{jf,t}(l) dl = A_{jf,t} \left(\frac{K_{jf,t}}{N_{jf,t}} \right)^{\alpha_f} N_{jf,t} = A_{jf,t} K_{jf,t}^{\alpha_f} N_{jf,t}^{1-\alpha_f}.$$

An alternative representation of $\tilde{Y}_{jf,t}$ makes use of the demand curve, eqn (1):

$$\tilde{Y}_{jf,t} = Y_{jf,t} \int_0^1 \left(\frac{P_{jf,t}(l)}{P_{jf,t}} \right)^{-\varepsilon} dl = Y_{jf,t} \tilde{p}_{jf,t}^{-1},$$

where

$$\tilde{p}_{jf,t} = \left(\frac{\int_0^1 P_{jf,t}(l)^{-\varepsilon} dl}{P_{jf,t}^{-\varepsilon}} \right)^{-1} = \left(\frac{\left[\int_0^1 P_{jf,t}(l)^{-\varepsilon} dl \right]^{\frac{-1}{\varepsilon}}}{P_{jf,t}} \right)^{\varepsilon} \equiv \left(\frac{\tilde{P}_{jf,t}}{P_{jf,t}} \right)^{\varepsilon}. \tag{31}$$

Thus,

$$Y_{jf,t} = \tilde{p}_{jf,t} A_{jf,t} K_{jf,t}^{\alpha_f} N_{jf,t}^{1-\alpha_f}. \tag{32}$$

Similarly, for labor inputs, recall that $\tilde{N}_{jf,t}$ denotes the unweighted integral of labor supply across sector-jf unions (and households that supply labor to sector jf):

$$\tilde{N}_{jf,t} \equiv \int_0^1 N_{jf,t}(i) di = N_{jf,t} \int_0^1 \left(\frac{W_{jf,t}(i)}{W_{jf,t}} \right)^{-\varepsilon^w} di = N_{jf,t} \tilde{w}_{jf,t}^{-1},$$

where

$$\tilde{w}_{jf,t} = \left(\frac{\int_0^1 W_{jf,t}(i)^{-\varepsilon^w} di}{W_{jf,t}^{-\varepsilon^w}} \right)^{-1} = \left(\frac{\left[\int_0^1 W_{jf,t}(i)^{-\varepsilon^w} di \right]^{\frac{-1}{\varepsilon^w}}}{W_{jf,t}} \right)^{\varepsilon^w} \equiv \left(\frac{\tilde{W}_{jf,t}}{W_{jf,t}} \right)^{\varepsilon^w}. \tag{33}$$

Finally, we need to derive an expression for the aggregate price index. Since we want to compare model properties to data on real GDP and the GDP deflator from the BEA, we follow the BEA in using chain-weighting to construct model statistics. Specifically, the growth rate of the aggregate price level is given by

$$\pi_t = \left(\frac{P_{n,t} Y_{n,t-1} + P_{d,t} Y_{d,t-1}}{P_{n,t-1} Y_{n,t-1} + P_{d,t-1} Y_{d,t-1}} \right)^{0.5} \left(\frac{P_{n,t} Y_{n,t} + P_{d,t} Y_{d,t}}{P_{n,t-1} Y_{n,t} + P_{d,t-1} Y_{d,t}} \right)^{0.5},$$

or equivalently, in relative prices (noting that $\pi_t = \frac{P_t}{P_{t-1}}$):

$$1 = \left(\frac{p_{n,t} Y_{n,t-1} + p_{d,t} Y_{d,t-1}}{p_{n,t-1} Y_{n,t-1} + p_{d,t-1} Y_{d,t-1}} \right)^{0.5} \left(\frac{p_{n,t} Y_{n,t} + p_{d,t} Y_{d,t}}{p_{n,t-1} Y_{n,t} + p_{d,t-1} Y_{d,t}} \right)^{0.5}. \tag{34}$$

Nominal output is simply given by

$$P_t Y_t = \sum_{j=n,d} P_{j,t} Y_{j,t}, \tag{35}$$

and we use chain-weighting, equation (34), to convert the nominal output in one period into real output in the units of another period. For example, combining equations (34) and (35), we have

$$Y_t = Y_{t-1} \frac{p_{n,t-1} Y_{n,t} + p_{d,t-1} Y_{d,t}}{p_{n,t} Y_{n,t-1} + p_{d,t} Y_{d,t-1}}. \tag{36}$$

Appendix B: Consumer Expenditure Survey Sample

We estimate Engel curves for 60 of the 70 NIPA goods based on the U.S. Consumer Expenditure Survey (CE) Interview Surveys. These correspond to all our consumption categories except postage, which is not collected in the CE Interview Survey. We also estimate a unique elasticity for all food at home categories, as the Interview Survey does not provide separate categories for food at home. In addition, we estimate an Engel curve for housing services, which we employ for the category of investment in residential structures. (Housing services are measured by rent for renters. For home owners it is measured by household's estimate of the home's rental value.) The estimation is described in the text. Here we focus on describing our CE sample.

We pool the 1982 to 2010 CE surveys. The survey became annual in 1980, but home owner estimates of their home's rental value become available only in 1982. The CE is fairly large, with samples of 5,000 or more households in most years. Each household is assigned a "replicate" weight that maps the CE sample into a representative sample of the national population. We use that weight in all calculations. A household is interviewed about their expenditures for up to four consecutive quarters. Each interview records a household's expenditures by category over the previous three months. In the first and fourth interviews, the household is asked its income over the preceding 12 months, including all transfer income received. As stated in the text, we use these responses, as well as spending information from the first interview, to instrument for the sum of a household's expenditures from surveys two through four.

We restrict our sample to households that complete all four quarterly interview surveys. We also restrict the sample to those with household heads between ages 20 and 64. We exclude households that report annual before-tax income (including transfers) of less than $100, in 1983 dollars, in either the first or fourth interviews. We exclude households that report less than $100 of spending on nondurables for their first quarterly interview, or less than $300 over the subsequent three quarters (again in 1983 dollars). Our resulting sample includes 70,518 households.

Appendix C: Calibrating Cyclical Movements in Capital Utilization

We calibrate movements in capital's utilization rate to utilization rates constructed by Gorodnichenko and Shapiro (2011), largely from the U.S. Census Survey of Plant Capacity. The Gorodnichenko-Shapiro series are available annually for manufacturing series for the years 1974 to 2004, with the exception of 1998. We match their series at the two-digit SIC level (available on Shapiro's web page) to annual series on employment and hours for production and non-supervisor workers drawn from the NBER Productivity Database for manufacturing. For each manufacturing industry, we construct measures of the labor to capital stock ratio from the Productivity Database. We then regress movements in the Gorodnicheko and Shapiro utilization rates on industry movements in labor to capital. All series are HP-filtered with industry-specific filters.

We find that a one-percent increase in the labor to capital stock ratio is associated with a one-third percent increase in the utilization rate of capital. When the labor-capital ratio is measured by production hours to capital, the precise estimate is 0.34 with a standard error of 0.04. When labor is measured by all worker hours, the estimate is 0.33, with standard error of 0.04. (This is true regardless of whether we assume a constant 40-hour workweek for supervisory/non-production workers or assume nonproduction workers exhibit the same workweek movements as production workers.)

Figure A1

Relative **Durables/Nondurables** Response, Sticky Prices

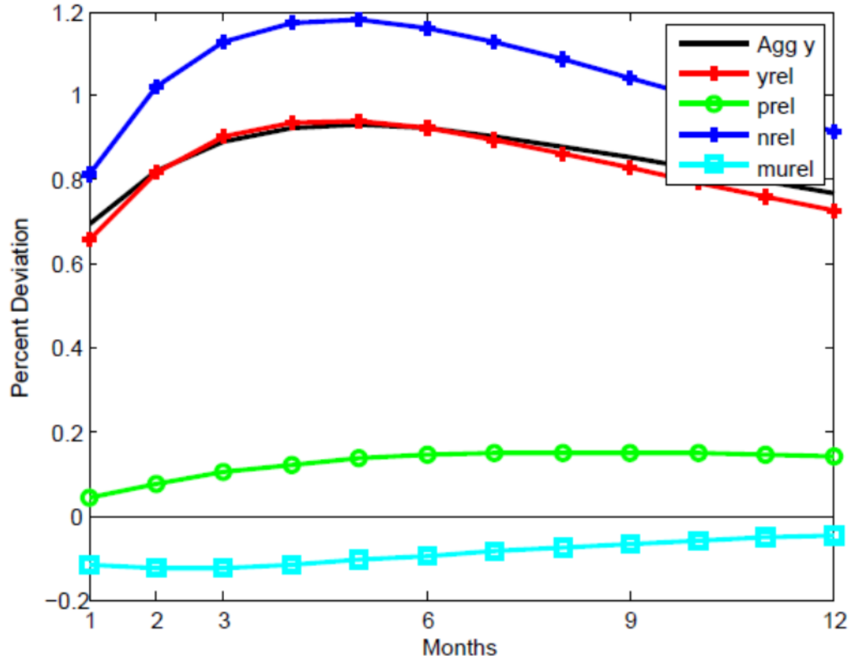
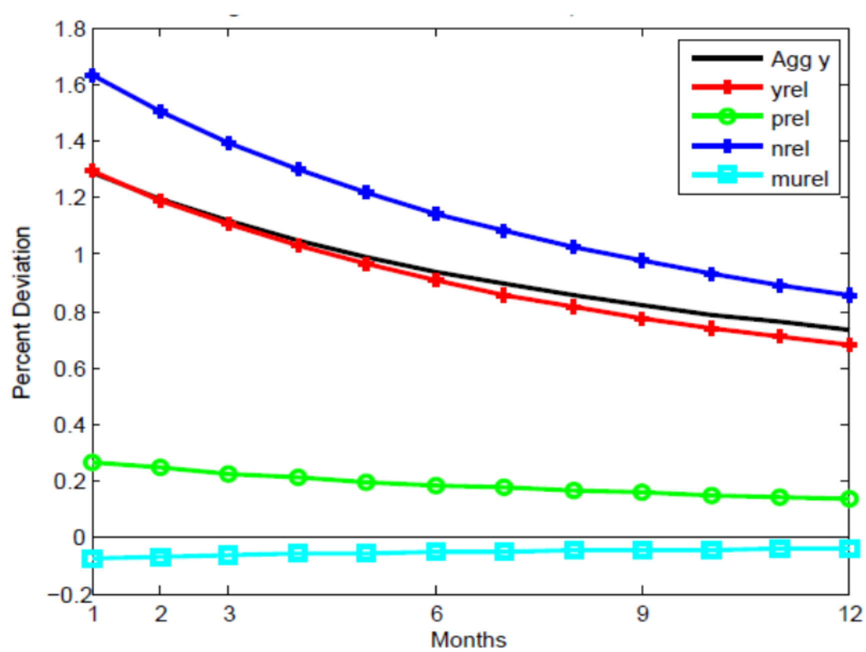


Figure A2

Relative **Durables/Nondurables** Response, Flexible Prices



Notes for Figures A1 & A2: Impulse responses to an **aggregate TFP** shock. Y is output, N labor, P prices and μ markups.

Figure A3

Relative **Low/High Price Flexibility** Response

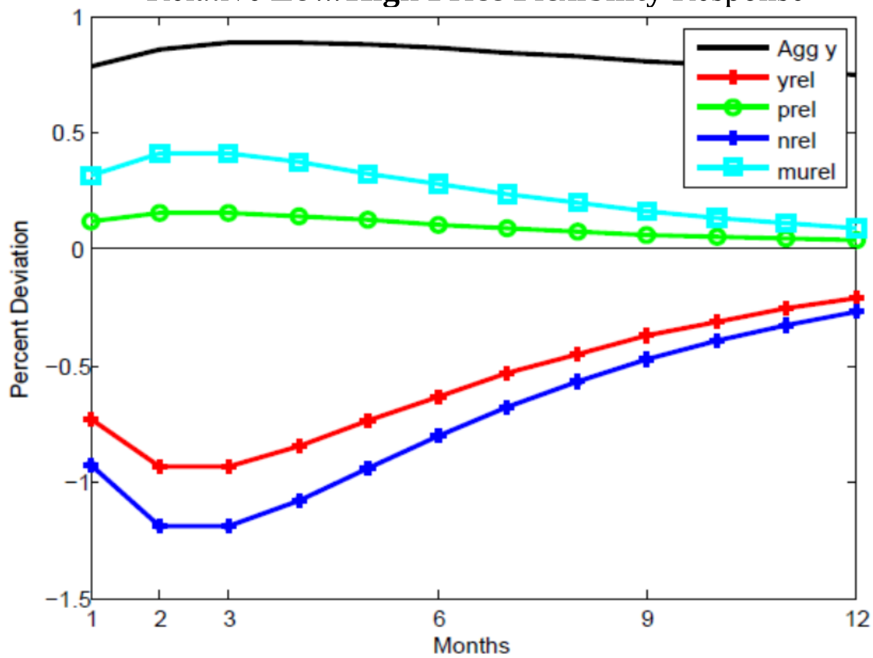
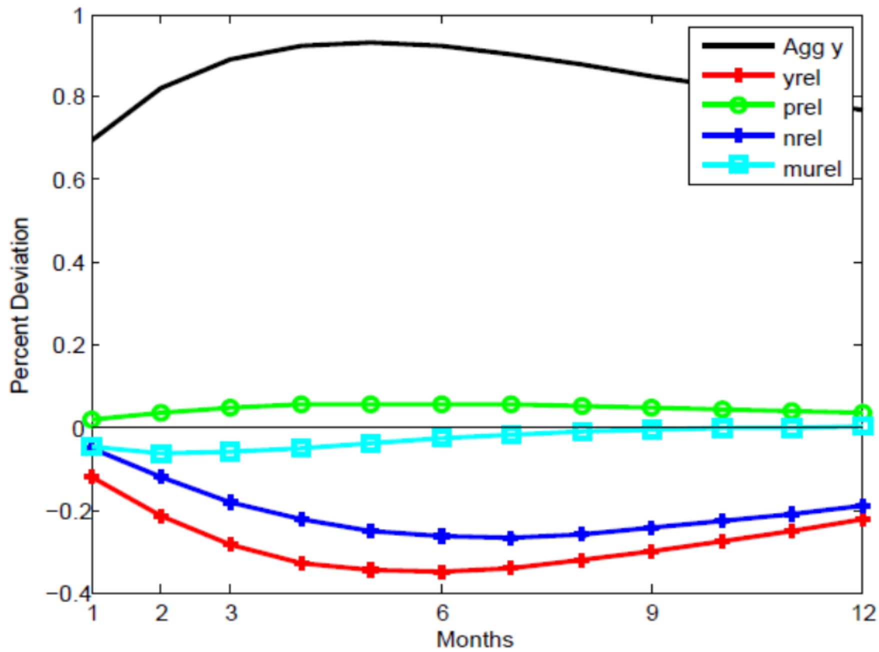


Figure A4

Relative **High/Low Capital Intensity** Response, Sticky Prices



<u>Notes for Figures A3 & A4</u>: Impulse responses to an **aggregate TFP** shock. Y is output, N labor, P prices and μ markups.

Figure A5

Relative **Durables/Nondurables** Response, Sticky Prices

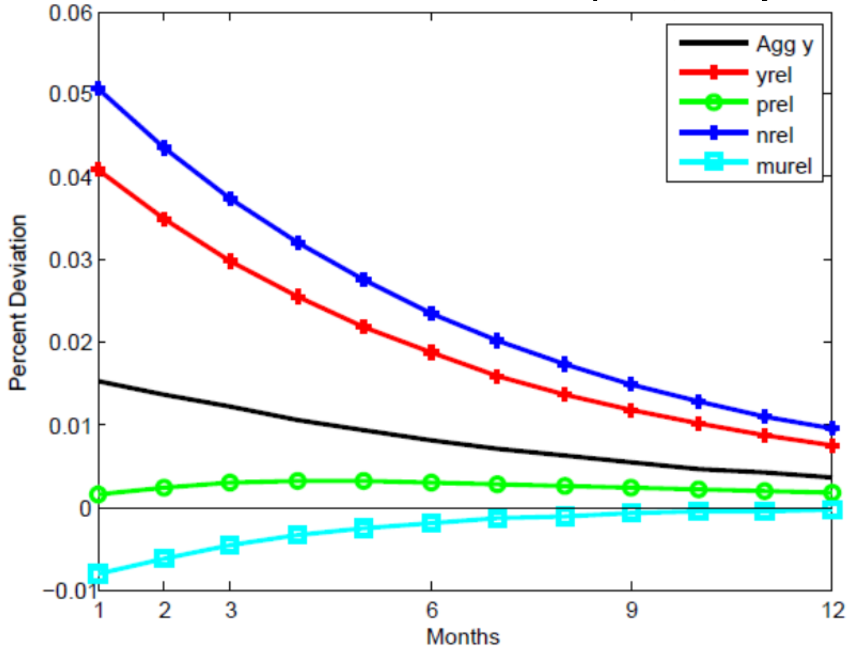
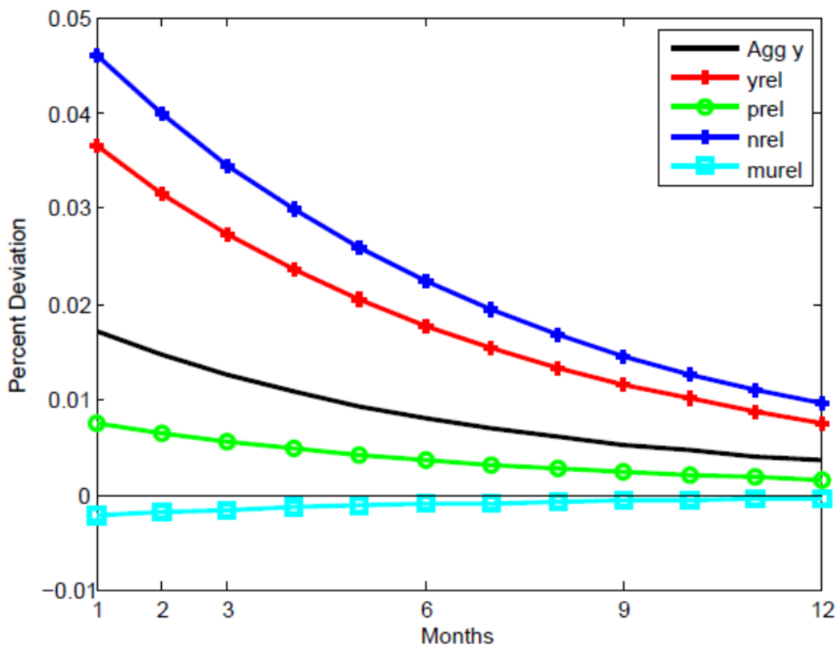


Figure A6

Relative **Durables/Nondurables** Response, Flexible Prices



Notes for Figures A5 & A6: Impulse responses to an **investment-specific technology** shock. Y is output, N labor, P prices and μ markups.

Figure A7

Relative **Low/High Price Flexibility** Response

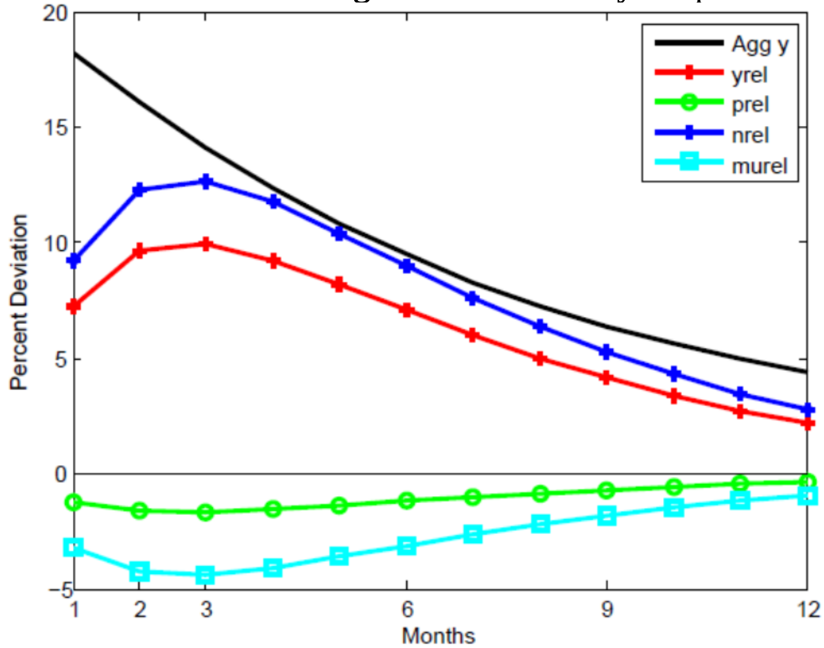
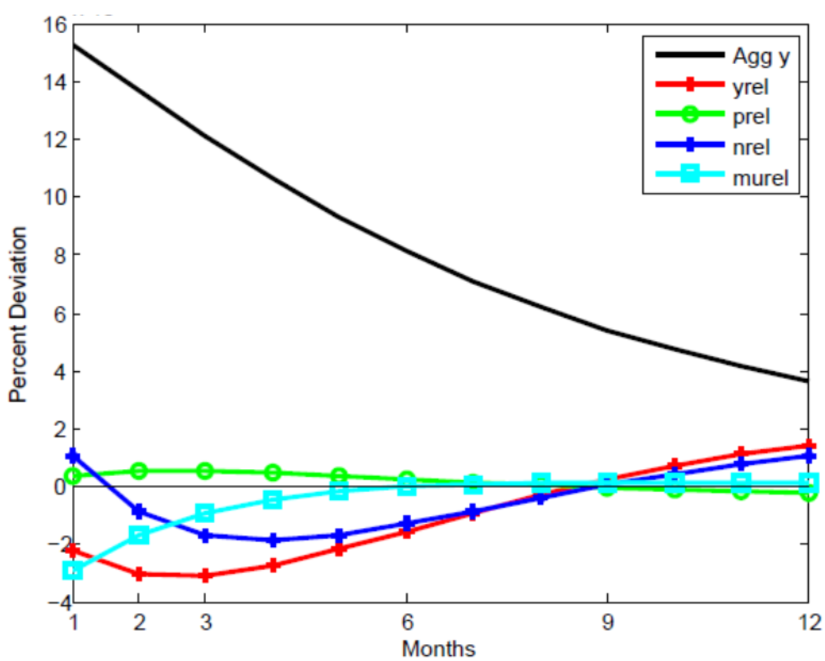


Figure A8

Relative **High/Low Capital Intensity** Response, Sticky Prices



<u>Notes for Figures A7 & A8</u>: Impulse responses to an **investment-specific technology** shock. Y is output, N labor, P prices and μ markups. Units are 10^-3.

Figure A9

Relative **Durables/Nondurables** Response, Sticky Prices

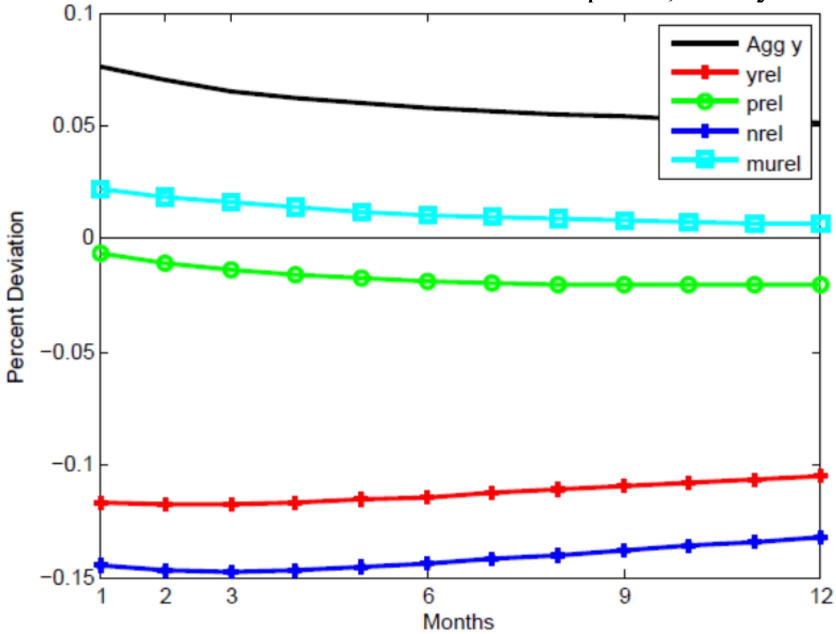
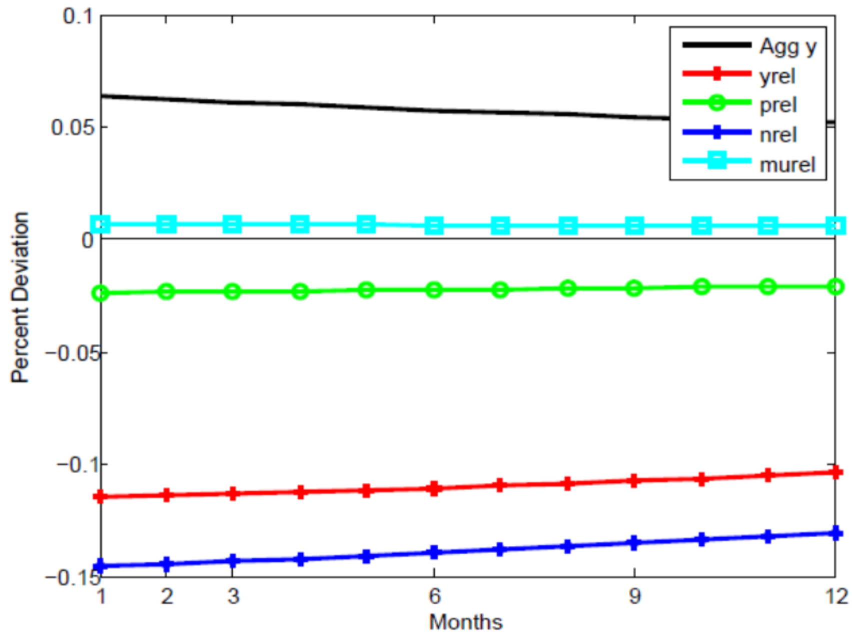


Figure A10

Relative **Durables/Nondurables** Response, Flexible Prices



<u>Notes for Figures A9 & A10</u>: Impulse responses to a **government-spending** shock. Y is output, N labor, P prices and μ markups.

Figure A11

Relative **Low/High Price Flexibility** Response

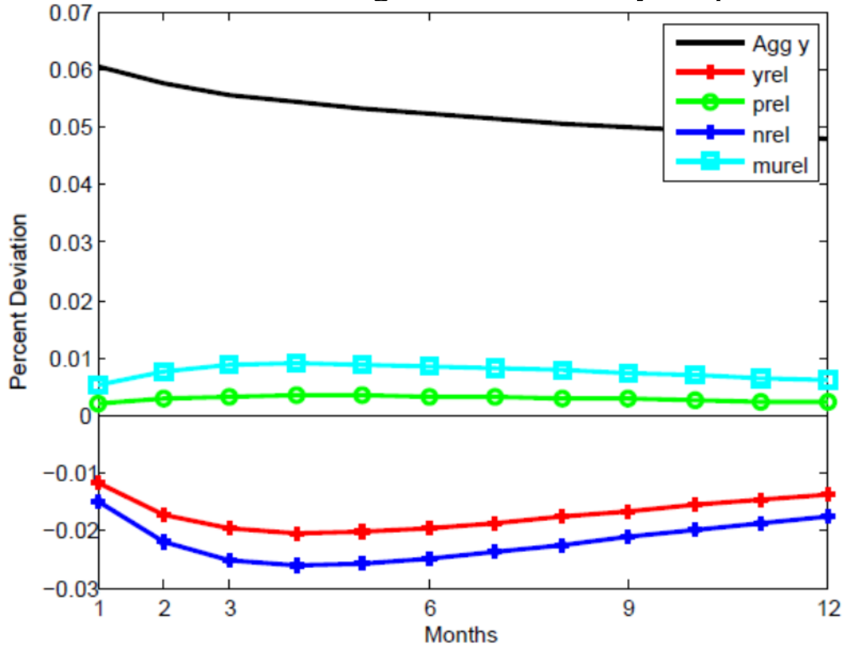
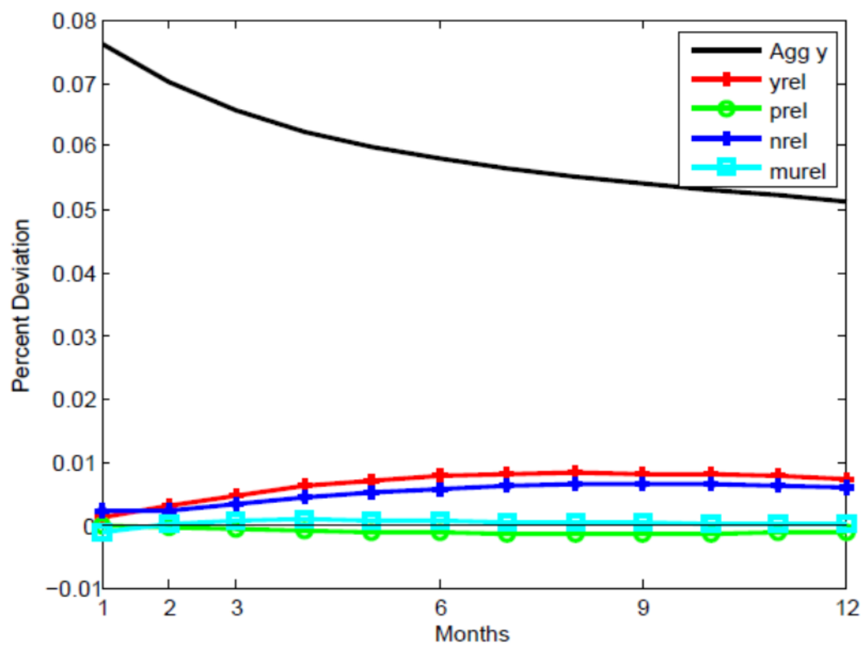


Figure A12

Relative **High/Low Capital Intensity** Response, Sticky Prices



Notes for Figures A11 & A12: Impulse responses to a **government-spending** shock. Y is output, N labor, P prices and μ markups.

Figure A13

Relative **Durables/Nondurables** Response, Sticky Prices

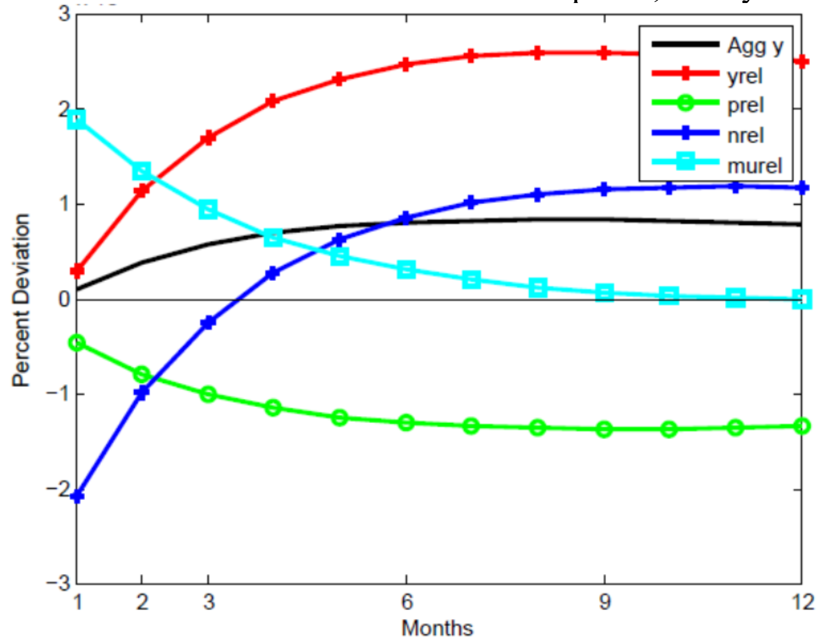
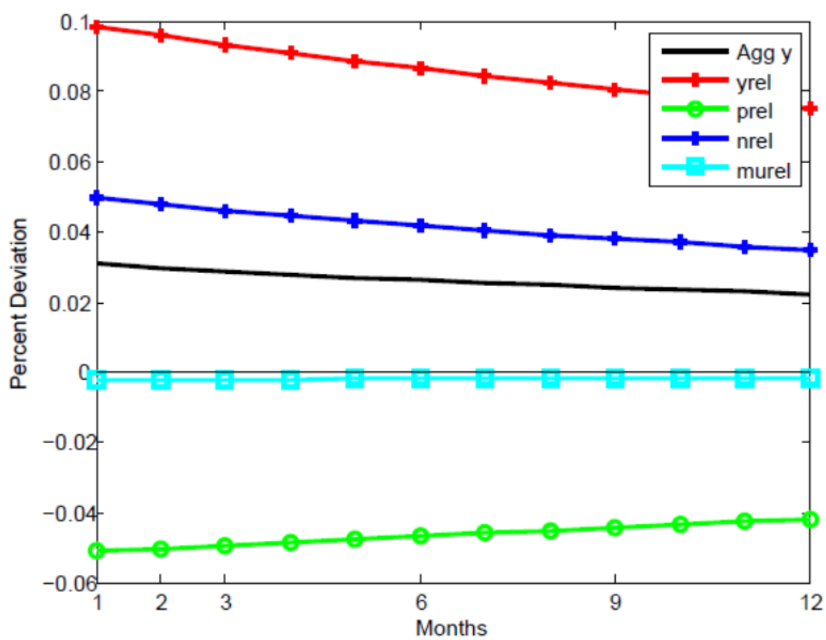


Figure A14

Relative **Durables/Nondurables** Response, Flexible Prices



<u>Notes for Figures A13 & A14</u>: Impulse responses to a **durables TFP** shock. Y is output, N labor, P prices and μ markups. Units for Figure A13 are 10^-3.

Appendix Table 1

NIPA Expenditure Categories (Goods)

Good	Dur. (years)	Engel Curve	Price Freq.	Capital Share	Exp. Share	Emp. Share
Men's and Boys' Apparel	2.78	1.090	0.0841	0.2371	1.468%	0.832%
Women's and Girls' Apparel	2.54	1.216	0.1262	0.2324	2.339%	1.370%
Footwear	2.56	0.945	0.0841	0.2311	0.809%	0.362%
Infants’ and Toddlers' Apparel	2.30	0.554	0.1096	0.2321	0.197%	0.161%
Jewelry and Watches	6.90	1.590	0.0921	0.2631	0.834%	0.398%
Educational Books and Supplies	11.00	1.173	0.1105	0.3229	0.159%	
Tuition and Childcare	0.00	1.742	0.0879	0.3786	1.755%	2.358%
Postage and Delivery Services	0.00	1.000	0.0560	0.2199	0.167%	0.807%
Telephone Services	0.00	0.584	0.2525	0.5513	2.067%	2.239%
Information and Info. Processing	7.10	1.325	0.2812	0.2353	0.603%	0.543%
Cereals and Cereal Products	0.00	0.410	0.1462	0.3227	0.519%	0.411%
Bakery Products	0.00	0.410	0.1234	0.3790	0.891%	0.988%
Beef and Veal	0.00	0.410	0.2383	0.3492	0.518%	0.507%
Pork	0.00	0.410	0.2182	0.3492	0.362%	0.355%
Other Meats	0.00	0.410	0.1533	0.3492	0.310%	0.308%
Poultry	0.00	0.410	0.1990	0.3850	0.570%	0.721%
Fish and Seafood	0.00	0.410	0.1911	0.3643	0.168%	0.181%
Eggs	0.00	0.410	0.3723	0.3499	0.089%	0.081%
Dairy and Related Products	0.00	0.410	0.1942	0.3499	0.378%	0.344%
Fresh Fruit	0.00	0.410	0.3974	0.2402	0.313%	0.215%
Fresh Vegetables	0.00	0.410	0.4368	0.2402	0.467%	0.322%
Processed Fruits and Vegetables	0.00	0.410	0.1334	0.4383	0.330%	0.722%
Juice and Nonalcoholic Drinks	0.00	0.410	0.1196	0.3156	0.967%	0.740%
Beverages Including Coffee and Tea	0.00	0.410	0.1472	0.2418	0.140%	0.084%
Sugar and Sweets	0.00	0.410	0.0932	0.3357	0.558%	0.470%
Fats and Oils	0.00	0.410	0.1342	0.3499	0.199%	0.181%
Other Foods	0.00	0.410	0.1108	0.3136	1.422%	1.089%
Food Away From Home	0.00	1.206	0.0714	0.2030	5.557%	12.035%
Alcoholic Beverages	0.00	1.295	0.1019	0.2614	2.099%	1.067%
Tobacco and Smoking Products	0.00	0.083	0.2128	0.4872	1.130%	0.051%
Personal Care Services	0.00	1.038	0.0363	0.1184	1.094%	0.667%
Miscellaneous Personal Services	0.00	1.439	0.0540	0.3277	3.266%	7.932%
Lodging Away from Home	0.00	1.804	0.3689	0.3360	0.947%	2.629%
Tenants’ and Household Insurance	0.00	1.105	0.0950	0.2126	0.075%	0.893%
Fuel Oil and Other Fuels	0.00	0.777	0.4290	0.4136	0.289%	0.181%

Appendix Table 1 continued

NIPA Expenditure Categories (Goods)

Good	Dur. (years)	Engel Curve	Price Freq.	Capital Share	Exp. Share	Emp. Share
Gas (piped) and Electricity	0.00	0.456	0.6613	0.7253	2.680%	1.002%
Water and Sewer and Trash Collections	0.00	0.688	0.1045	0.5519	0.910%	0.231%
Window and Floor Coverings and Linens	8.68	1.617	0.0785	0.2294	0.297%	0.143%
Furniture and Bedding	8.97	1.257	0.0969	0.2207	1.182%	1.004%
Appliances	12.09	0.964	0.1310	0.3143	0.596%	0.274%
Other Household Equipment, Furnishings	7.65	1.668	0.0857	0.2411	0.478%	0.480%
Tools, Hardware, Outdoor Equipment	7.50	1.085	0.0823	0.2469	0.293%	0.722%
Household Operations	0.00	2.018	0.0856	0.2472	0.771%	1.126%
Drugs and Medical Supplies	0.00	0.904	0.1388	0.3447	1.558%	1.698%
Professional Services	0.00	1.248	0.0472	0.1535	7.211%	6.939%
Hospital and Related Services	0.00	0.881	0.0991	0.2156	9.268%	10.289%
Health Insurance	0.00	0.919	0.0833	0.2126	1.540%	0.508%
Video and Audio	10.39	0.791	0.1330	0.5777	1.131%	1.042%
Pets, Pet Products and Services	0.00	1.454	0.0862	0.2819	0.380%	1.593%
Sporting Goods	9.60	1.587	0.0989	0.2611	0.602%	0.553%
Photography	2.93	1.401	0.0897	0.1842	0.092%	0.404%
Other Recreational Goods	6.15	1.119	0.0867	0.2454	0.501%	0.459%
Recreation Services	0.00	1.787	0.0894	0.2533	1.611%	2.452%
Recreational Reading Material	2.38	1.305	0.0780	0.2294	0.482%	0.215%
New and Used Motor Vehicles	9.00	0.846	0.3814	0.4133	2.155%	2.501%
Motor Fuel	0.00	0.650	0.8626	0.4428	3.215%	1.625%
Motor Vehicle Parts and Equipment	2.55	0.797	0.1383	0.2572	0.586%	1.821%
Motor Vehicle Maintenance and Repair	0.00	1.094	0.1571	0.1184	2.121%	1.240%
Motor Vehicle Insurance	0.00	0.895	0.1330	0.2126	0.768%	
Motor Vehicle Fees	0.00	1.127	0.0241	0.1184	0.197%	0.140%
Public Transportation	0.00	1.414	0.3651	0.2093	1.117%	0.560%
Commercial and health care structures	42.15	1.145	1.0000	0.1554	1.994%	1.967%
Manufacturing structures	32.01	1.154	1.0000	0.1654	0.651%	0.514%
Power and communication structures	45.22	0.540	1.0000	0.1509	0.824%	1.029%
Mining exploration, shafts, and wells	13.78	0.971	1.0000	0.1655	0.755%	0.584%
Information equipment and software	4.27	1.032	0.0749	0.2699	6.562%	2.647%
Industrial equipment	10.73	0.938	0.0840	0.2768	2.542%	2.114%
Transportation equipment	7.03	0.983	0.1707	0.2588	2.353%	2.758%
Other equipment	6.83	0.987	0.0752	0.2956	2.291%	2.045%
Residential structures	68.15	0.817	0.7340	0.1535	7.237%	5.081%

Notes to Appendix Table 1:

Dur. = durability (years of expected life).

Engel Curve = the cross-household elasticity of expenditures on the good with respect to overall nondurables and services expenditures. Estimated using the U.S. Consumer Expenditure Survey from 1982-2010.

Price Freq. = the monthly frequency of regular price changes in the CPI for consumption goods from 1988-2009, and in the PPI for investment goods.

Capital Share = capital's share of value added in producing industries (1 minus labor's share) from 1987-2009.

Exp. Share = NIPA expenditures on the good relative to expenditures on all 70 goods, averaged over 1990-2011.

Emp. Share = CES employment in producing industries relative to employment for all 68 categories, averaged over 1990-2011.

Appendix Table 2

KLEMS Industries

INDUSTRY	NAICS Code	Log Dur.	Engel Curve	Price Freq.	Cap. Sh.	VA Wt.
Oil and Gas Extraction	211	0.00	0.57	0.75	0.76	1.26
Utilities	22	0.00	0.49	0.59	0.72	2.91
Construction	23	3.90	0.89	0.89	0.14	7.15
Food, Beverage, and Tobacco	311,312	0.00	0.44	0.15	0.49	2.36
Apparel and Leather products	315,316	1.28	1.10	0.10	0.23	0.40
Wood products	321	4.24	0.82	0.73	0.22	0.44
Petroleum and Coal products	324	0.00	0.66	0.83	0.78	1.07
Chemical products	325	0.00	0.90	0.14	0.54	2.67
Plastics and Rubber products	326	1.27	0.80	0.14	0.36	0.98
Fabricated Metal products	332	2.97	1.04	0.62	0.30	1.84
Machinery	333	2.27	0.96	0.08	0.27	1.76
Computer and Electronic products	334	1.80	1.03	0.10	0.24	2.44
Electrical Equipment and Appliances	335	2.28	0.98	0.10	0.37	0.77
Transportation Equipment	336	1.63	0.89	0.18	0.26	2.82
Furniture and related products	337	2.23	1.18	0.09	0.21	0.48
Miscellaneous Manufacturing	339	1.82	1.19	0.08	0.34	0.88
Wholesale Trade	42	0.96	0.85	0.14	0.28	7.64
Retail Trade	44,45	0.95	0.83	0.26	0.23	9.08
Truck Transportation	484	0.00	2.02	0.09	0.21	1.50
Transit and Ground Passenger Transportation	485	0.00	1.41	0.37	0.21	0.28
Other Transportation	487,488, 492	0.00	1.00	0.06	0.22	1.12
Publishing Industries	511,516	0.48	1.35	0.09	0.41	1.81
Broadcasting and Telecommunications	515,517	0.20	0.60	0.24	0.59	3.60
Information and Data Processing Services	518,519	0.00	0.58	0.25	0.33	0.70
Credit Intermediation and Related Activities	521,522	0.00	1.44	0.05	0.45	4.29
Securities, Commodity Contracts, and Investments	523	0.00	1.44	0.05	0.16	2.47

Insurance Carriers and Related Activities	524	0.00	1.04	0.09	0.21	3.05
Real Estate	531	0.00	2.02	0.09	0.83	5.62
Rental and Leasing Services	532,533	2.36	0.82	0.27	0.79	2.01
Legal Services	5411	0.00	1.44	0.05	0.09	2.23
Miscellaneous Professional, Scientific and Technical Services	5412-5414, 5416-5419	0.83	1.35	0.19	0.15	6.58
Administrative and Support Services	561	0.00	2.02	0.09	0.11	3.78
Waste Management and Remediation Services	562	0.00	0.69	0.10	0.41	0.43
Ambulatory Health Care Services	621	0.00	1.25	0.05	0.15	4.52
Hospitals and Nursing and Residential Care Facilities	622,623	0.00	0.88	0.10	0.22	1.61
Performing Arts, Spectator Sports, Museums, and Related Activities	711,712	0.00	1.79	0.09	0.15	0.50
Amusements, Gambling, and Recreation Industries	713	0.00	1.79	0.09	0.28	0.55
Accommodation	721	0.00	1.80	0.37	0.34	1.06
Food Services and Drinking Places	722	0.00	1.22	0.08	0.20	2.35
Other Services, except Government	81	0.10	1.18	0.11	0.12	2.98