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1 Introduction

We want to capture the following ideas.

- Political bosses support cronies.
- If political bosses are the residual claimants on the cronies' businesses, they have an incentive to support the "best" firms.
- The support that political bosses can provide consists of; (a) help in circumventing onerous rules; (b) protection from competitors; and (c) preferential access to resources. (a) is welfare improving but (b) and (c) are not.
- The cost of (b) is attenuated when there are numerous jurisdictions, and a political boss can only provide protection from competitors in their own jurisdiction.

2 Integrated National Economy without Crony Capitalism

Preferences:

$$U = \left(\int_0^1 C_z^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

The dual price index:

$$P = \left(\int_0^1 p_z^{1-\sigma} dz \right)^{\frac{1}{1-\sigma}}$$

There are two potential technologies for each product, given by $(1-\delta)e^{A(1-z)}$ ("A" technology) and $(1-\delta)e^{Bz}$ ("B" technology) where $0 < \delta < 1$ represents the TFP loss from "bad" institutions. The chosen technology for product is $T_z = \max\{(1-\delta)e^{A(1-z)}, (1-\delta)e^{Bz}\}$. Suppose furthermore that output of product (also equal to consumption because for now this is a closed economy) is given by $Y_z = T_z L_z$. Then, the price is the standard markup over marginal cost $p_z = \frac{\sigma}{\sigma-1}\frac{w}{T_z}$.

Define $\tilde{z}$ as the cutoff where the A technology is chosen for $z < \tilde{z}$ and B is chosen for $z > \tilde{z}$.[1] The real wage is then given by[2]

$$\frac{w}{P} = \frac{\sigma-1}{\sigma} \left(\int_0^{\tilde{z}} \left[(1-\delta)e^{A(1-z)} \right]^{\sigma-1} dz + \int_{\tilde{z}}^1 \left[(1-\delta)e^{Bz} \right]^{\sigma-1} dz \right)^{\frac{1}{\sigma-1}} \tag{1}$$

Labor used for product z, as a share of total employment, is[3]

$$l_z = \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} \left(\frac{P}{w} T_z \right)^{\sigma-1}. \tag{2}$$

[1]The cutoff product is given by $\tilde{z} = \dfrac{A}{A+B}$.

[2]This is derived by noting that

$$\frac{w}{P} = \frac{w}{\left[\int_0^1 p_z^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}}} = \frac{w}{\left[\int_0^1 \left(\frac{\sigma}{\sigma-1}\frac{w}{T_z}\right)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}}} = \frac{\sigma-1}{\sigma} \left[\int_0^1 {T_z}^{\sigma-1} dz \right]^{\frac{1}{\sigma-1}}$$

[3]This is derived as follows. First, the Marshallian demand function for product z is given by $C_z = p_z^{-\sigma} P^{\sigma-1} I$, where I represents the (nominal) income. The nominal income I in turn is given by $I = \int_0^1 p_z Y_z dz = \int_0^1 \frac{\sigma}{\sigma-1}\frac{w}{T_z} T_z L_z dz = \frac{\sigma}{\sigma-1} wL$. Thus, noting that $Y_z = T_z L_z$ and $C_z = Y_z$, we have

$$l_z = \frac{L_z}{L} = \frac{Y_z}{T_z L} = \frac{C_z}{T_z L} = \frac{p_z^{-\sigma} P^{\sigma-1}}{T_z L} \frac{\sigma}{\sigma-1} wL = \left(\frac{\sigma}{\sigma-1}\frac{w}{T_z} \right)^{-\sigma} P^{\sigma-1} \frac{\sigma}{\sigma-1}\frac{w}{T_z} = \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} \left(\frac{P}{w} T_z \right)^{\sigma-1}.$$

Total employment of "A" technology, as a share of total employment, is thus given by

$$L_A = \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma-1} \left(\frac{P}{w}\right)^{\sigma-1} \int_0^{\tilde{z}} \left[(1-\delta)e^{A(1-z)}\right]^{\sigma-1} dz.$$

The corresponding employment share of "B" technology is given by $L_B = 1 - L_A$.

3 Integrated National Economy with Crony Capitalism (One City Case)

Now suppose that the political leader favors A firms and chooses the best A firms as her cronies. Her cronies get the following benefits:

- Exemption from onerous rules: We model this as a level shift in the TFP of the crony firms, from $(1-\delta)e^{Az}$ to $(1-\delta+\gamma)e^{Az}$ where $\gamma \geq 0$.
- Block competitors: Potential competitors of the favored firms are blocked from the market. For firms $z < \tilde{z}$, this has no effect as they already were the highest productivity firms. But for firms $z > \tilde{z}$, this means that the less productive firm prevails.

Suppose that all firms $z \in [0, z_c]$ are cronies. We can endogenize this cutoff as a function of the fixed costs of helping a crony, the assistance the political leader is able to provide her cronies, and the benefit the political leader gets from helping her cronies. Profits of the cutoff firm are given by $\left[(1-\delta+\gamma)\frac{e^{A(1-z_c)}}{w}\right]^{\sigma-1}$ and the political leader's share of the profits is given by $\beta^{\sigma-1}$. Suppose furthermore that the fixed cost of assisting a crony is given by $(\frac{Fe^A}{w})^{\sigma-1}$. Then, a firm qualifies as a crony if and only if

$$\left[\beta(1-\delta+\gamma)\frac{e^{A(1-z)}}{w}\right]^{\sigma-1} > \left(\frac{Fe^A}{w}\right)^{\sigma-1} \quad \Leftrightarrow \quad \beta(1-\delta+\gamma)e^{A(1-z)} > Fe^A. \tag{3}$$

The cutoff z_c is in fact given by

$$z_c = \ln[\beta(1-\delta+\gamma)/F]. \tag{4}$$

The political leader has more cronies–z_c is larger–when she gets a larger share of the firm's profits, can provide more assistance, and when the fixed cost is low.

The key question is whether $z_c < \tilde{z}$. When this is the case, then crony capitalism is unambiguously welfare improving. The real wage is now given by:

$$\frac{w}{P} = \frac{\sigma-1}{\sigma}\left(\int_0^{z_c}\left[(1-\delta+\gamma)e^{A(1-z)}\right]^{\sigma-1}dz + \int_{z_c}^{\tilde{z}}\left[(1-\delta)e^{A(1-z)}\right]^{\sigma-1}dz + \int_{\tilde{z}}^{1}\left[(1-\delta)e^{Bz}\right]^{\sigma-1}dz\right)^{\frac{1}{\sigma-1}}. \tag{5}$$

The cutoff product $\tilde{z}$ is still given by $\frac{A}{A+B}$ because the TFP of the marginal product does not change. Cronies benefit from the improvement in TFP. This effect among the crony firms $z < z_c$ (the first term in the equation) increases aggregate TFP. Since these firms already had the highest productivity for their product, blocking has no effect. Thus, crony capitalism is unambiguously welfare improving.

However, if $z_c > \tilde{z}$, then the effect is ambiguous.[4] The real wage is given by:

$$\frac{w}{P} = \frac{\sigma-1}{\sigma}\left(\int_0^{\tilde{z}}\left[(1-\delta+\gamma)e^{A(1-z)}\right]^{\sigma-1}dz + \int_{\tilde{z}}^{z_c}\left[(1-\delta+\gamma)e^{A(1-z)}\right]^{\sigma-1}dz + \int_{z_c}^{1}\left[(1-\delta)e^{Bz}\right]^{\sigma-1}dz\right)^{\frac{1}{\sigma-1}}. \tag{6}$$

Comparing with (1), the first term shows the effect of TFP improvement for cronies $z \in [0, \tilde{z}]$. The second term, however, shows a welfare loss because the less productive firms (i.e. cronies) replace the more productive firms that are blocked out. (Note that $(1-\delta+\gamma)e^{A(1-z)} < (1-\delta)e^{Bz}$ for $z \in [\tilde{z}, z_c]$.) There is no change for products $z \in [z_c, 1]$ (the third term). Thus, the welfare-impact of crony capitalism depends on the net effect of the first two terms.

The share of labor used by "A" technology is now

$$L_A = \begin{cases} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(\frac{P}{w}\right)^{\sigma-1}\left[\int_0^{z_c} e^{\gamma A(1-z)(\sigma-1)}dz + \int_{z_c}^{\tilde{z}} e^{A(1-z)(\sigma-1)}dz\right], & \text{if } z_c < \tilde{z}, \\ \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(\frac{P}{w}\right)^{\sigma-1}\int_0^{z_c} e^{\gamma A(1-z)(\sigma-1)}dz, & \text{if } z_c \geq \tilde{z}, \end{cases}$$

which is higher relative to the scenario without "cronies."

4 Crony Capitalism with Competition (Two City Case)

Now suppose there are two political jurisdictions A and B. Jurisdiction A is where A firms are located and B where B firms are located. Suppose that the political boss in A supports her firms, but the one in B does nothing (B firms cannot sell anything that A firms produce in jurisdiction A). We assume $z_c > \tilde{z}$ (otherwise crony capitalism increases aggregate output, and competition makes no difference).

The key difference is that the political boss in A can only block entrants in her jurisdiction, but has no power in B. Prices can now differ between A and B. We assume workers freely move between A and B so the difference in the nominal wage W is simply given by the difference in the price index (we normalize the dual price in A to 1):

$$\frac{w_A}{w_B} = \frac{P_A}{P_B} = \frac{1}{P_B}. \tag{7}$$

For product z in jurisdiction in A, price is given by:

$$p_A(z) = \begin{cases} \frac{\sigma}{\sigma-1}\cdot\frac{w_A}{(1-\delta+\gamma)e^{A(1-z)}}, & \text{if } z \in [0, z_c] \\ \frac{\sigma}{\sigma-1}\cdot\frac{w_B}{(1-\delta)e^{Bz}}, & \text{if } z \in [z_c, 1] \end{cases}.$$

Note that while $p_B(z)$ is similar, its breakpoint is $\tilde{z}$ instead of z_c. For simplicity, we use the following notation instead of piecewise functions:

$$p_{A,z} = \frac{\sigma}{\sigma-1}\cdot\frac{w_A}{(1-\delta+\gamma)e^{\gamma A(1-z)}},\ p_{B,z} = \frac{\sigma}{\sigma-1}\cdot\frac{w_B}{(1-\delta)e^{Bz}}.$$

[4] In this case, $\tilde{z}$ is now given by:

$$\tilde{z} = \frac{A}{A+B} + \frac{1}{A+B}\ln\left[\frac{1-\delta+\gamma}{1-\delta}\right].$$

Note that this is larger than it was in the previous case.

Dual price in A now has the form of

$$P_A = \left(\int_0^{\tilde{z}} p_{A,z}^{1-\sigma} dz + \int_{\tilde{z}}^{z_c} p_{A,z}^{1-\sigma} dz + \int_{z_c}^{1} p_{B,z}^{1-\sigma} dz \right)^{\frac{1}{1-\sigma}},$$

and the real wage in A is given by

$$\frac{w_A}{P_A} = \frac{\sigma-1}{\sigma} \left(\int_0^{z_c} \left[(1-\delta+\gamma) e^{A(1-z)} \right]^{\sigma-1} dz + P_B^{1-\sigma} \int_{z_c}^{1} \left[(1-\delta) e^{Bz} \right]^{\sigma-1} \right)^{\frac{1}{\sigma-1}}. \tag{8}$$

Remember that workers are mobile so this is also the real wage in B. Comparing this equation with (6), the key change is that the real wage now also depends on P_B. A decline in prices in B increases the real wage in B and A. Compared to the case with one political jurisdiction (and prices are the same in the two locations), prices in B (the location without cronies) fall relative to prices in A. This is because while the crony can block the most productive firms for $z \in [\tilde{z}, z_c]$ in A, she cannot do so in B. Therefore, while the less productive firm prevails in A, the more productive firm wins the market in B. But the perfect mobility assumption implies that workers in A also benefit from lower prices in B, even if they do not have direct access to these prices.

Similarly, the real wage in B is given by

$$\frac{w_B}{P_B} = \frac{\sigma-1}{\sigma} \left(P_B^{\sigma-1} \int_0^{\tilde{z}} \left[(1-\delta+\gamma) e^{A(1-z)} \right]^{\sigma-1} dz + \int_{\tilde{z}}^{1} \left[(1-\delta) e^{Bz} \right]^{\sigma-1} \right)^{\frac{1}{\sigma-1}}. \tag{9}$$

Since the real wage must equal across the two cities, P_B can be solved from

$$\begin{aligned} &\int_0^{z_c} \left[(1-\delta+\gamma) e^{A(1-z)} \right]^{\sigma-1} dz + P_B^{1-\sigma} \int_{z_c}^{1} \left[(1-\delta) e^{Bz} \right]^{\sigma-1} dz \\ &\qquad = P_B^{\sigma-1} \int_0^{\tilde{z}} \left[(1-\delta+\gamma) e^{A(1-z)} \right]^{\sigma-1} dz + \int_{\tilde{z}}^{1} \left[(1-\delta) e^{Bz} \right]^{\sigma-1}. \end{aligned} \tag{10}$$

The left hand side is the real wage in A and the right hand side the real wage in B. It is easy to verify that when $\tilde{z} = z_c$, then $P_B = 1$, wages are the same in the two locations, and the real wage is the highest. Furthermore, P_B decreases when the gap between z_c and $\tilde{z}$ increases. Note that equation (10) determines $\tilde{z}$ jointly with P_B.

Finally, the share of employment in the two cities is pinned down by the balanced trade condition. Formally, let s_j^i be the market share of a firm in i in market j. Then, the balanced trade condition can be stated as

$$s_A^B I_A = s_B^A I_B$$

where I_A and I_B are nominal income in A and B, respectively. In words, it says that city B's exports to city A (the left-hand side) must equal city A's exports to B (the right-hand side). But since $I_A = \frac{\sigma}{\sigma-1} w_A L_A$ and $I_B = \frac{\sigma-1}{\sigma} w_B L_B$ (c.f. footnote 3), the expression can be re-written as:

$$s_A^B w_A L_A = s_B^A w_B L_B. \tag{11}$$

The expressions for the shares are given by

$$s_A^B = \int_{z_c}^{1} p_{B,z}^{1-\sigma} P_A^{\sigma-1} dz = \left(\frac{\sigma-1}{\sigma} \frac{1}{w_B} \right)^{\sigma-1} \int_{z_c}^{1} \left[(1-\delta) e^{Bz} \right]^{\sigma-1} dz$$

and

$$s_B^A = \int_0^{\tilde{z}} p_{A,z}^{1-\sigma} P_B^{\sigma-1} dz = \left(\frac{\sigma-1}{\sigma} \frac{P_B}{w_A} \right)^{\sigma-1} \int_0^{\tilde{z}} \left[(1-\delta+\gamma) e^{A(1-z)} \right]^{\sigma-1} dz$$

Suppose the total supply of labor is given by 1. Then, equation (11), together with the condition $L_A + L_B = 1$, pins down L_A and L_B.

5 Crony Capitalism with Competition and Foreign Market

Now suppose there is a foreign market. We shall assume that the foreign country only produces M (which is endogenously given) at a fixed price of P_M and imports all of differentiated variety from either A or B. To allow for exports to this foreign market, and imports from the foreign market, recast utility as

$$U = \left(\int_0^1 C_z^{\frac{\sigma-1}{\sigma}} dz \right)^{\alpha \frac{\sigma}{\sigma-1}} M^{1-\alpha}$$

The utility function implies that the foreign country would have a share of $(1-\alpha)$ while markets A and B in aggregate would have a share of α in each location.

What is interesting is the relationship between domestic sales and exports for the firms in location B. Remember that the productivity of Bs products is increasing in z, so the products location B produces that are blocked in market A ($z \in [\tilde{z}, z_c]$) are Bs lowest quality products. However, these products are still of higher quality compared to the corresponding producer in A. So, for products $z \in [\tilde{z}, z_c]$, B will take over the foreign market even if they cannot sell in domestic market A. The prediction then is that the least productive firms will sell in the foreign market, and not as much in the domestic market. The elasticity of export sales to domestic sales should be high for these firms, because their domestic sales are blocked in the other local market.

Once we introduce a foreign market to the model, we can no longer use the equation (11) for pinning down L_A and L_B. This is because the trade between markets A and B does not have to balance when they can have a trade deficit / surplus with the foreign country. Nonetheless, each market's overall trade balance (i.e. trade balance vis-a-vis the rest of the world) must equal zero. This implies the following:

$$\begin{aligned} s_B^A \, I_B + s_{FN}^A \, I_{FN} &= (1 - s_A^A) \, I_A \\ s_A^B \, I_A + s_{FN}^B \, I_{FN} &= (1 - s_B^B) \, I_B \\ \alpha I_{FN} &= (1-\alpha)(I_A + I_B), \end{aligned}$$

where s_j^i stands for the market share of a firm in i in market j, and $I_A = \frac{\sigma}{\sigma-1} w_A L_A$, $I_B = \frac{\sigma}{\sigma-1} w_B L_B$, and $I_{FN} = P_M M$. Hence, this can be re-written as:

$$\begin{aligned} s_B^A \left(\frac{\sigma}{\sigma-1} w_B L_B \right) + s_{FN}^A P_M M &= (1 - s_A^A) \left(\frac{\sigma}{\sigma-1} w_A L_A \right) \\ s_A^B \left(\frac{\sigma}{\sigma-1} w_A L_A \right) + s_{FN}^B P_M M &= (1 - s_B^A) \left(\frac{\sigma}{\sigma-1} w_B L_B \right) \\ \alpha P_M M &= (1-\alpha) \left(\frac{\sigma}{\sigma-1} w_A L_A + \frac{\sigma}{\sigma-1} w_B L_B \right). \end{aligned}$$

The above three equations are linearly dependent. However, by picking two of the equations above and recalling that $L_A + L_B = 1$, we have three linearly independent equations for three unknowns, L_A, L_B, and P_M. This means that we can construct a matrix to solve for these three unknowns:

$$\begin{bmatrix} -\frac{\sigma}{\sigma-1}(1-s_A^A) w_A & \frac{\sigma}{\sigma-1} s_B^A \, w_B & s_{FN}^A M \\ \frac{\sigma}{\sigma-1}(1-\alpha) w_A & \frac{\sigma}{\sigma-1}(1-\alpha) w_B & -\alpha M \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} L_A \\ L_B \\ P_M \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

where

$$s_A^A = \alpha \int_0^{z_c} p_{A,z}^{1-\sigma} P_A^{\sigma-1} dz = \alpha \left(\frac{\sigma-1}{\sigma} \frac{P_A}{w_A} \right)^{\sigma-1} \int_0^{z_c} \left[(1-\delta+\gamma) e^{A(1-z)} \right]^{\sigma-1} dz,$$

and

$$s_B^A = s_{FN}^A = \alpha \int_0^{\tilde{z}} p_{A,z}^{1-\sigma} P_B^{\sigma-1} dz = \alpha \left(\frac{\sigma-1}{\sigma} \frac{P_B}{w_A} \right)^{\sigma-1} \int_0^{\tilde{z}} \left[(1-\delta+\gamma) e^{A(1-z)} \right]^{\sigma-1} dz.$$

(Note that the dual price in the foreign market, P_{FN}, is the same as the dual price in P_B, where there is no crony capitalism; hence $s_B^A = s_{FN}^A$.)

Once we find L_A, L_B, and P_M (and hence I_A, I_B, and I_{FN}), we can easily compute the total sales and exports by firms located in each market. (Note: s_A^B, s_B^B, and s_{FN}^B can be computed either by noting that $s_A^A + s_A^B = s_B^A + s_B^B = s_{FN}^A + s_{FN}^B = \alpha$, or directly from

$$s_A^B = \alpha \int_{z_c}^1 p_{B,z}^{1-\sigma} P_A^{\sigma-1} dz = \alpha \left(\frac{\sigma-1}{\sigma} \frac{P_A}{w_B} \right)^{\sigma-1} \int_{z_c}^1 \left[(1-\delta) e^{Bz} \right]^{\sigma-1} dz,$$

and

$$s_B^B = s_F^B N = \alpha \int_{\tilde{z}}^1 p_{B,z}^{1-\sigma} P_B^{\sigma-1} dz = \alpha \left(\frac{\sigma-1}{\sigma} \frac{P_B}{w_B} \right)^{\sigma-1} \int_{\tilde{z}}^1 \left[(1-\delta) e^{Bz} \right]^{\sigma-1} dz.)$$

5.1 Crony Capitalism in Both Cities

Next, we shall consider the case where political leaders in both cities A and B support their respective firms and they compete against each other. Thus, suppose that firms $z \in [z_d, 1]$ are now cronies in B. Like their counterparts in A, they receive the following benefits:
i) The TFP of the cronies is raised from e^{Bz} to $e^{\gamma_B Bz}$, ii) Firms in A are blocked from entering the market in B for products $z \in [z_d, 1]$.
To make the above assumptions meaningful, we assume $z_d < \tilde{z} < z_c$. (Otherwise it differs little from the cases considered in previous sections). Then, city A imports $z \in [z_c, 1]$ while city B imports $z \in [0, z_d]$ from city A. The foreign country imports $z \in [0, \tilde{z}]$ from city A and $z \in [\tilde{z}, 1]$ from city B.

The real wages in A and B are thus given (respectively) by:

$$\frac{w_A}{P_A} = \frac{\sigma-1}{\sigma} \left(\int_0^{\tilde{z_c}} \left[(1-\delta+\gamma_A) e^{A(1-z)} \right]^{\sigma-1} dz + P_B^{1-\sigma} \int_{z_c}^1 \left[(1-\delta+\gamma_B) e^{Bz} \right]^{\sigma-1} dz \right)^{\frac{1}{\sigma-1}}, \tag{12}$$

and

$$\frac{w_B}{P_B} = \frac{\sigma-1}{\sigma} \left(P_B^{\sigma-1} \int_0^{\tilde{z_d}} \left[(1-\delta+\gamma_A) e^{A(1-z)} \right]^{\sigma-1} dz + \int_{z_d}^1 \left[(1-\delta+\gamma_B) e^{Bz} \right]^{\sigma-1} dz \right)^{\frac{1}{\sigma-1}}. \tag{13}$$

As before, we can set the two real wages equal to solve for P_B. That is, P_B can be solved from:

$$\begin{aligned} &\int_0^{z_c} \left[(1-\delta+\gamma_A) e^{A(1-z)} \right]^{\sigma-1} dz + P_B^{1-\sigma} \int_{z_c}^1 \left[(1-\delta+\gamma_B) e^{Bz} \right]^{\sigma-1} dz \\ &\qquad = P_B^{\sigma-1} \int_0^{z_d} \left[(1-\delta+\gamma_A) e^{A(1-z)} \right]^{\sigma-1} dz + \int_{z_d}^1 \left[(1-\delta+\gamma_B) e^{Bz} \right]^{\sigma-1} dz. \end{aligned} \tag{14}$$

The dual price in the foreign market, P_{FN}, no longer equals P_B (for market B is now also distorted by crony capitalism), and is instead given by the following:

$$\begin{aligned} P_{FN} &= \left[\int_0^{\tilde{z}} \left(\frac{\sigma}{\sigma-1} \frac{w_A}{(1-\delta+\gamma_A) e^{A(1-z)}} \right)^{1-\sigma} dz + \int_{\tilde{z}}^1 \left(\frac{\sigma}{\sigma-1} \frac{w_B}{(1-\delta+\gamma_B) e^{Bz}} \right)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}} \\ &= \frac{\sigma}{\sigma-1} \left(w_A^{1-\sigma} \int_0^{\tilde{z}} \left[(1-\delta+\gamma_A) e^{A(1-z)} \right]^{\sigma-1} dz + w_B^{1-\sigma} \int_{\tilde{z}}^1 \left[(1-\delta+\gamma_B) e^{Bz} \right]^{\sigma-1} dz \right)^{\frac{1}{1-\sigma}} \end{aligned} \tag{15}$$

Once we find w_A, w_B, P_B, and $P_F N$, we can proceed to solve for L_A, L_B and P_M in the same way as before. The only differences from the previous section are the expressions for the market shares, which are now given by the following:

$$s_A^A = \alpha \int_0^{z_c} p_{A,z}^{1-\sigma} P_A^{\sigma-1} dz = \alpha (\frac{\sigma-1}{\sigma} \frac{P_A}{w_A})^{\sigma-1} \int_0^{z_c} \left[(1-\delta+\gamma_A) e^{A(1-z)} \right]^{\sigma-1} dz,$$

$$s_B^A = \alpha \int_0^{z_d} p_{A,z}^{1-\sigma} P_B^{\sigma-1} dz = \alpha(\frac{\sigma-1}{\sigma}\frac{P_B}{w_A})^{\sigma-1} \int_0^{z_d} \left[(1-\delta+\gamma_A)e^{A(1-z)}\right]^{\sigma-1} dz,$$

and

$$s_{FN}^A = \alpha \int_0^{\tilde{z}} p_{A,z}^{1-\sigma} P_{FN}^{\sigma-1} dz = \alpha(\frac{\sigma-1}{\sigma}\frac{P_{FN}}{w_A})^{\sigma-1} \int_0^{\tilde{z}} \left[(1-\delta+\gamma)e^{A(1-z)}\right]^{\sigma-1} dz.$$

As before, other market shares can be computed easily from the above three.