# C Separate Appendix

## C.1 Representative Agent Model

The budget constraint of the representative agent who consumes aggregate consumption $c_t(z^t)$ reads as

$$\begin{aligned} & c_t(z^t) + \sum_{z_{t+1}} a_t(z^t, z_{t+1}) q_t(z^t, z_{t+1}) + \sigma_t(z^t) v_t(z^t) \\ \leq \;& e_t(z^t) + a_{t-1}(z^{t-1}, z_t) + \sigma_{t-1}(z^{t-1}) \left[ v_t(z^t) + \alpha e_t(z_t) \right] \end{aligned}$$

After deflating by the aggregate endowment $e_t(z^t)$, the budget constraint reads as

$$\begin{aligned} & \hat{c}_t(z^t) + \sum_{z_{t+1}} \hat{a}_t(z^t, z_{t+1}) \hat{q}_t(z^t, z_{t+1}) + \sigma_t(z^t) \hat{v}_t(z^t) \\ \leq \;& 1 + \hat{a}_{t-1}(z^{t-1}, z_t) + \sigma_{t-1}(z^{t-1}) \left[ \hat{v}_t(z^t) + \alpha \right], \end{aligned}$$

where $\hat{a}_t(z^t, z_{t+1}) = \frac{a_t(z^t, z_{t+1})}{e_{t+1}(z^{t+1})}$ and $\hat{q}_t(z^t, z_{t+1}) = q_t(z^t, z_{t+1}) \lambda(z_{t+1})$ as well as $\hat{v}_t(z^t) = \frac{v_t(z^t)}{e_t(z^t)}$, precisely as in the Arrow model. Obviously, in an equilibrium of this model the representative agent consumes the aggregate endowment.

**Lemma C.1.** *Equilibrium asset prices are given by*

$$\begin{aligned} \hat{q}_t(z^t, z_{t+1}) &= \hat{\beta} \hat{\phi}(z_{t+1}) = \hat{q}(z_{t+1}) \text{ for all } z_{t+1}. \\ \hat{v}_t(z^t) &= \hat{\beta} \sum_{z_{t+1}} \hat{\phi}(z_{t+1}) \left[ \hat{v}_{t+1}(z^{t+1}) + \alpha \right]. \end{aligned}$$

## C.2 Recursive Utility

We consider the class of preferences due to Epstein and Zin (1989) . Let $V(c^i)$ denote the utility derived from consuming $c^i$ :

$$V(c^i) = \left[ (1-\beta) c_t^{1-\rho} + \beta (\mathcal{R}_t V_1)^{1-\rho} \right]^{\frac{1}{1-\rho}},$$

where the risk-adjusted expectation operator is defined as:

$$\mathcal{R}_t V_{t+1} = \left( E_t V_{t+1}^{1-\alpha} \right)^{1/1-\alpha}.$$

$\alpha$ governs risk aversion and $\rho$ governs the willingness to substitute consumption intertemporally. These preferences impute a concern for the timing of the resolution of uncertainty to agents. In the special case where $\rho = \frac{1}{\alpha}$, these preferences collapse to standard power utility preferences with CRRA coefficient $\alpha$. As before, we can define *growth-adjusted* probabilities and the growth-adjusted discount factor as:

$$\hat{\pi}(s_{t+1}|s_t) = \frac{\pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}}{\sum_{s_{t+1}} \pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}} .$$

$$\text{and } \hat{\beta}(s_t) = \beta \left( \sum_{s_{t+1}} \pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha} \right)^{\frac{1-\rho}{1-\alpha}} .$$

As before, $\hat{\beta}(s_t)$ is stochastic as long as the original Markov process is not *iid* over time. Note that the adjustment of the discount rate is affected by both $\rho$ and $\alpha$. If $\rho = \frac{1}{\alpha}$, this transformation reduces to the case we discussed in section (2).

Finally, let $\hat{V}_t(\hat{c})(s^t)$ denote the lifetime expected continuation utility in node $s^t$, under the new transition probabilities and discount factor, defined over consumption shares $\left\{\hat{c}_t(s^t)\right\}$ :

$$\hat{V}_t(\hat{c})(s^t) = \left[(1-\beta)\hat{c}_t^{1-\rho} + \hat{\beta}(s_t)(\hat{\mathcal{R}}_t\hat{V}_{t+1}(s^{t+1}))^{1-\rho}\right]^{\frac{1}{1-\rho}} ,$$

where $\mathcal{R}$ denotes the following operator:

$$\hat{\mathcal{R}}_t V_{t+1} = \left(\hat{E}_t\hat{V}_{t+1}^{1-\alpha}\right)^{1/1-\alpha} .$$

and $\hat{E}$ denotes the expectation operator under the hatted measure $\hat{\pi}$.

**Proposition C.1.** *Households rank consumption share allocations in the de-trended economy in exactly the same way as they rank the corresponding consumption allocations in the original growing economy: for any $s^t$ and any two consumption allocations $c, c'$*

$$V(c)(s^t) \geq V(c')(s^t) \Longleftrightarrow \hat{V}(\hat{c})(s^t) \geq \hat{V}(\hat{c}')(s^t),$$

*where the transformation of consumption into consumption shares is given by* (4).

**Detrended Arrow Economy** We proceed as before, by conjecturing that the equilibrium consumption shares only depend on $y^t$. Our first result states that if the consumption shares in the de-trended economy do not depend on the aggregate history $z^t$, then it follows that the interest rates in this economy are deterministic.

**Proposition C.2.** *In the de-trended Arrow economy, if there exists a competitive equilibrium with equilibrium consumption allocations $\left\{\hat{c}_t(\theta_0, y^t)\right\}$, then there is a deterministic interest rate process $\left\{\hat{R}_t^A\right\}$ and equilibrium prices $\left\{\hat{q}_t(z^t, z_{t+1})\right\}$, that satisfy:*

$$\hat{q}_t(z^t, z_{t+1}) = \frac{\hat{\phi}(z_{t+1})}{\hat{R}_t^A}. \tag{58}$$

All the results basically go through. We can map an equilibrium of the Bewley economy into an equilibrium of the detrended Arrow economy.

**Theorem C.1.** *An equilibrium of the Bewley model $\{\hat{c}_t(\theta_0, y^t), \hat{a}_t(\theta_0, y^t), \hat{\sigma}_t(\theta_0, y^t)\}$ and $\{\hat{R}_t, \hat{v}_t\}$ can be made into an equilibrium for the Arrow economy with growth, $\left\{a_t(\theta_0, s^t, z_{t+1})\right\}$, $\{\sigma_t(\theta_0, s^t)\}$,*

$\{c_t(\theta_0, s^t)\}$ *and* $\{q_t(z^t, z_{t+1})\}$, $\{v_t(z^t)\}$, *with*

$$\begin{aligned}
c_t(\theta_0, s^t) &= \hat{c}_t(\theta_0, y^t) e_t(z^t) \\
\sigma_t(\theta_0, s^t) &= \hat{\sigma}_t(\theta_0, y^t) \\
a_t(\theta_0, s^t, z_{t+1}) &= \hat{a}_t(\theta_0, y^t) e_{t+1}(z^{t+1}) \\
v_t(z^t) &= \hat{v}_t e_t(z^t) \\
q_t(z^t, z_{t+1}) &= \frac{1}{\hat{R}_t} * \frac{\phi(z_{t+1})\lambda(z_{t+1})^{-\alpha}}{\sum_{z_{t+1}} \phi(z_{t+1})\lambda(z_{t+1})^{1-\alpha}}
\end{aligned}$$

As a result, even for an economy with agents who have these Epstein-Zin preferences, the risk premium is not affected.

## C.3 Proofs

- Proof of Lemma C.1

*Proof.* The first order conditions for the representative agent are given by:

$$\begin{aligned}
1 &= \frac{\hat{\beta}\hat{\phi}(z_{t+1})}{\hat{q}_t(z^t, z_{t+1})} \frac{u'(\hat{c}_{t+1}(z^t, z_{t+1}))}{u'(\hat{c}_t(z^t))} \; \forall z_{t+1}. \\
1 &= \hat{\beta} \sum_{z_{t+1}} \hat{\phi}(z_{t+1}) \left[ \frac{\hat{v}_{t+1}(z^{t+1}) + \alpha}{\hat{v}_t(z^t)} \right] \frac{u'(\hat{c}_{t+1}(z^t, z_{t+1}))}{u'(\hat{c}_t(z^t))}. \qquad (59)
\end{aligned}$$

□

- Proof of Proposition C.1:

*Proof.* First, we divided through by $e_t(z^t)$ on both sides in equation (C.2):

$$\begin{aligned}
\frac{V_t(s^t)}{e_t(z^t)} &= \left[ (1-\beta) \frac{c_t^{1-\rho}}{e_t^{1-\rho}} + \beta \frac{(\mathcal{R}_t V_{t+1})^{1-\rho}}{e_t^{1-\rho}} \right]^{\frac{1}{1-\rho}} \\
\hat{V}_t(s^t) &= \left[ (1-\beta)\hat{c}_t^{1-\rho} + \beta (\frac{\mathcal{R}_t V_{t+1}}{e_t})^{1-\rho} \right]^{\frac{1}{1-\rho}}. \qquad (60)
\end{aligned}$$

Note that the risk-adjusted continuation utility can be stated as:

$$\begin{aligned}
\frac{\mathcal{R}_t V_{t+1}}{e_t(z^t)} &= \left( E_t \left( \frac{e_{t+1}}{e_t} \right)^{1-\alpha} \frac{V_{t+1}^{1-\alpha}}{e_{t+1}^{1-\alpha}} \right)^{1/1-\alpha} \\
&= \left( \sum_{s_{t+1}} \pi(s_{t+1}|s_t) \lambda(z_{t+1})^{1-\alpha} \hat{V}_{t+1}^{1-\alpha}(s_{t+1}) \right)^{1/1-\alpha}
\end{aligned}$$

Next, we define growth-adjusted probabilities and the growth-adjusted discount factor as:

$$\hat{\pi}(s_{t+1}|s_t) = \frac{\pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}}{\sum_{s_{t+1}} \pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}} \text{ and } \hat{\beta}(s_t) = \beta \sum_{s_{t+1}} \pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}.$$

and note that:

$$\begin{aligned}
\frac{\mathcal{R}_t V_{t+1}}{e_t(z^t)} &= \left(\sum_{s_{t+1}} \pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}\hat{V}_{t+1}^{1-\alpha}(s_{t+1})\right)^{1/1-\alpha} \\
&= \left(\sum_{s_{t+1}} \pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}\right)^{1/1-\alpha} \hat{\mathcal{R}}_t\hat{V}_{t+1}(s_{t+1})
\end{aligned}$$

Using the definition of $\hat{\beta}(s_t)$ :

$$\hat{\beta}(s_t) = \beta\left(\sum_{s_{t+1}} \pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}},$$

we finally obtain the desired result:

$$\hat{V}_t(s^t) = \left[(1-\beta)\hat{c}_t^{1-\rho} + \hat{\beta}(s_t)(\hat{\mathcal{R}}_t\hat{V}_{t+1}(s^{t+1}))^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

As before, if the $z$ shocks are i.i.d, then $\hat{\beta}$ is constant. □

- Proof of Proposition C.2:

*Proof.* First, we suppose the borrowing constraints are not binding, which is the easiest case. Assume the equilibrium allocations only depend on $y^t$, not on $z^t$. Then conditions 2.2 and 2.3 imply that the Euler equations of the Arrow economy, for the contingent claim and the stock respectively, read as follows:

$$\begin{aligned}
1 &= \frac{\hat{\beta}\hat{\phi}(z_{t+1})}{\hat{q}_t(z^t, z_{t+1})}\sum_{y_{t+1}} \varphi(y_{t+1}|y_t)\left(\frac{\hat{c}_{t+1}(y^t, y_{t+1})}{\hat{c}_t(y^t)}\right)^{-\rho}\left(\frac{\hat{V}_{t+1}(y^{t+1})}{\hat{V}_t(y^t)}\right)^{\rho-\alpha} \quad \forall z_{t+1} \\
1 &= \hat{\beta}\sum_{z_{t+1}} \hat{\phi}(z_{t+1})\left[\frac{\hat{v}_{t+1}(z^{t+1})+\alpha}{\hat{v}_t(z^t)}\right] && (61) \\
&\quad * \sum_{y_{t+1}} \varphi(y_{t+1}|y_t)\left(\frac{\hat{c}_{t+1}(y^t, y_{t+1})}{\hat{c}_t(y^t)}\right)^{-\rho}\left(\frac{\hat{V}_{t+1}(y^{t+1})}{\hat{V}_t(y^t)}\right)^{\rho-\alpha} \textit{for all } z_{t+1}. && (62)
\end{aligned}$$

In the first Euler equation, the only part that depends on $z_{t+1}$ is $\frac{\hat{\phi}(z_{t+1})}{\hat{q}_t(z^t,z_{t+1})}$ which therefore implies that $\frac{\hat{\phi}(z_{t+1})}{\hat{q}_t(z^t,z_{t+1})}$ cannot depend on $z_{t+1}$ : $\hat{q}_t(z^t, z_{t+1})$ is proportional to $\hat{\phi}(z_{t+1})$. Thus define $\hat{R}_t^A(z^t)$ by

$$\hat{q}_t(z^t, z_{t+1}) = \frac{\hat{\phi}(z_{t+1})}{\hat{R}_t^A(z^t)} \qquad (63)$$

as the risk-free interest rate in the stationary Arrow economy. Using this condition, the Euler equation simplifies to the following expression:

$$1 = \hat{\beta}\hat{R}_t^A(z^t)\sum_{y_{t+1}}\varphi(y_{t+1}|y_t)\left(\frac{\hat{c}_{t+1}(y^t,y_{t+1})}{\hat{c}_t(y^t)}\right)^{-\rho} \tag{64}$$

$$\left(\frac{\hat{V}_{t+1}(y^{t+1})}{\hat{V}_t(y^t)}\right)^{\rho-\alpha} \tag{65}$$

Apart from $\hat{R}_t^A(z^t)$ noting in this condition depends on $z^t$, so we can choose $\hat{R}_t^A(z^t) = \hat{R}_t^A$.
□