Online Appendix for "A Calibratable Model of Optimal CEO Incentives in Market Equilibrum"

Alex Edmans, Xavier Gabaix and Augustin Landier
August 21, 2007

D Options and General Incentive Contracts

We return to the baseline model with a binary effort decision, and generalize from stocks to a broader range of compensation instruments. The CEO receives fixed pay f, and ν units of a "security"; one unit of the security pays $V(P_1)$. For instance, for an option with strike K, $V(P_1) = \max(0, P_1 - K)$. Total compensation is $c = f + \nu V(P_1)$.

In equilibrium, the CEO should be paid $w \equiv E[cg(0) \mid e = 0]$. If he shirks, the CEO's utility is:

$$\begin{aligned} E[cg(\underline{e}) \mid e = \underline{e}] &= E[f + \nu V(P_1(1+\underline{e}))] g(\underline{e}) = E[f + \nu V(P_1) - \nu\Delta] g(\underline{e}) \\ &= (w - \nu\Delta)/(1-\Lambda), \end{aligned}$$

with $\Delta \equiv E[V(P_1)] - E[V(P_1(1+\underline{e}))]$. Hence, the CEO works if $E[cg(0) \mid e = 0] \geq E[cg(\underline{e}) \mid e = \underline{e}]$, i.e.

$$w \geq (w - \nu\Delta)/(1-\Lambda) \Leftrightarrow \nu \geq \nu^* = w\frac{\Lambda}{\Delta}.$$

This leads to the following generalization of Proposition 3.

Proposition 10 *(General incentive contracts). Using general incentive contracts, the conclusions of Proposition 3 remain the same, with a change of notation. The manager's expected pay is w, which comprise fixed base pay, f^*, and $\nu^* E[V(P_1)]$ worth of securities, with:*

$$\begin{aligned} \text{\$Incentivized pay} &= \nu^* E[V(P_1)] = w\Lambda', \\ \text{\$Fixed pay} &= f^* = w(1-\Lambda'), \end{aligned} \tag{52}$$

with $\Lambda'' = \{E[V(P_1)] - E[V(P_1(1-L))]\} / E[V(P_1)]$.

Realized pay is:

$$c = w + w\Lambda \frac{V(P_1) - E[V(P_1)]}{E[V(P_1)] - E[V(P_1(1-L))]}.$$

For small $P_1/P_0 - 1$ and L, by Taylor expansion: $L' \to E[V'(P_1) P_1] L / E[V(P_1)]$. Hence, regressing the ex post compensation c on the firm return $r = P_1/P - 1$ yields:

$$b^{III} = P\frac{\partial E[c]}{\partial P_1} = w\frac{\Lambda}{\Delta} P E[V'(P_1)] = w\frac{\Lambda}{L}\xi,$$

with

$$\xi = \frac{PE\left[V'\left(P_1\right)\right]}{E\left[V\left(P_1\right)\right] - E\left[V\left(P_1\left(1-L\right)\right)\right]/L}. \tag{53}$$

For instance, if the security is a stock, $V(P) = P$, $E\left[V'\left(P_1\right)\right] = 1$, and $\xi = 1$. For small $P_1/P_0 - 1$ and L, by Taylor expansion, $\xi \to \frac{PE[V'(P)]}{E[V'(P)]PL/L} = 1$. We can therefore think of ξ as approaching 1, and so the broader economics are unchanged.

Proposition 11 *Using general incentive contracts, the conclusions of Proposition 3 remain the same, modified only by the introduction of a parameter ξ. The pay-performance sensitivities are:*

$$\begin{aligned} b_n^I &= \xi \frac{\Lambda}{L} \\ b_n^{II} &= \xi \frac{\Lambda}{L} \frac{w_n}{S_n} \\ b_n^{III} &= \xi \frac{\Lambda}{L} w_n, \end{aligned}$$

with ξ given in (53). In many cases, $\xi \simeq 1$. Proposition 4 remains exactly the same.

E Several Projects or Effort Levels

We generalize to N or a continuum of effort levels, or projects, indexed by $e \in \mathcal{E}$ where $\mathcal{E}$ has an upper bound of 0. Effort level e yields a firm return of $P_1 = P\left(1 + L(0) - L(e)\right)(1+\eta)$, where $L(e)$ is the loss due to project e and and $L'(e) < 0$. We normalize $L(0) = 0$. $e = 0$ maximizes both firm value and, by assumption, total surplus (i.e. the gains to firm value exceed the utility loss to the CEO). CEO utility remains as $E\left[c \cdot g(e)\right]$. We normalize $g(0) = 1$ and use the notation $g(e) = \frac{1}{1-\Lambda(e)}$.

In order to implement $e = 0$, we need to satisfy the following:

$$\begin{aligned} \text{Incentive Compatibility} &: \forall e \in \mathcal{E},\ E_e\left[c \cdot g(e)\right] \le E_0\left[c \cdot g(0)\right], \\ \text{Participation Constraint} &: E_0\left[c \cdot g(0)\right] \ge w\,. \end{aligned}$$

As before, the CEO earns $c = f + \nu P$. The IC condition becomes:

$$w = E\left[c \cdot g(0)\right] \ge E_e\left[c \cdot g(e)\right] = \frac{f + \nu P\left(1 - L(e)\right)}{1 - \Lambda(e)} = \frac{w - \nu P L(e)}{1 - \Lambda(e)}.$$

The required number of shares is given by:

$$\nu^* = \min\ \left\{ v \text{ such that } \forall e,\ \nu P \ge \frac{\Lambda(e)}{L(e)} \right\} = \frac{1}{P} \max_{\substack{e \in \mathcal{E} \\ e \neq 0}} \frac{\Lambda(e)}{L(e)}\ .$$

Proposition 12 *With multiple effort levels, the conclusions of Propositions 1, 2, 3 and 4 are unchanged, where* $\frac{\Lambda}{L}$ *is replaced by*

$$\frac{\Lambda}{L} \equiv \max_{\substack{e \in \mathcal{E} \\ e \neq 0}} \frac{\Lambda(e)}{L(e)}.$$

As in Proposition 7, the restriction $f = w\left(1 - \frac{\Lambda}{L}\right) \geq 0$ may be violated for some levels of e and thus some inefficiency cannot be avoided without paying the manager rents. Without loss of generality, we order the effort levels so that $\frac{\Lambda(e)}{L(e)}$ is decreasing in e (e.g. a higher e represents the manager taking more and more private benefits, starting with the benefit with the greatest ratio of utility gain to value loss). Define $\widehat{e}$ by $\frac{\Lambda(\widehat{e})}{L(\widehat{e})} = 1$. Then, from equation (5), $f \geq 0$ requires $\frac{\Lambda(e)}{L(e)} \leq 1$, and so incentive compensation cannot prevent the manager exerting effort level $\widehat{e}$. As with perk consumption, this gives rise to a role for active corporate governance.