Separate Appendix–Not for Publication

A Data Appendix

The S&P 500 Index is an unmanaged index of 500 common stocks that is generally considered representative of the U.S. stock market. The Select Sector SPDR Trust consists of nine separate investment portfolios (each a Select Sector SPDR Fund or a Fund and collectively the Select Sector SPDR Funds or the Funds). Each Select Sector SPDR Fund is an index fund that invests in a particular sector or group of industries represented by a specified Select Sector Index. The companies included in each Select Sector Index are selected on the basis of general industry classification from a universe of companies defined by the Standard & Poor's 500 Composite Stock Index (S&P 500). The nine Select Sector Indexes (each a Select Sector Index) upon which the Funds are based together comprise all of the companies in the S&P 500. The investment objective of each Fund is to provide investment results that, before expenses, correspond generally to the price and yield performance of publicly traded equity securities of companies in a particular sector or group of industries, as represented by a specified market sector index. The financial sector's ticker is XLF. Table A reports the XLF holdings before and after the crisis.

B Model Appendix

B.1 Valuing the Consumption Claim

We start by valuing the consumption claim. Consider the investor's Euler equation for the consumption claim $E_t[M_{t+1}R^a_{t+1}] = 1$. This can be decomposed as:

$$1 = (1-p_t)E_t[\exp(\alpha \log\beta - \frac{\alpha}{\psi}\Delta c^{ND}_{t+1} + \alpha r^{ND}_{a,t+1})] + p_t E_t[\exp(\alpha\log\beta - \frac{\alpha}{\psi}\Delta c^{D}_{t+1} + \alpha r^{D}_{a,t+1})],$$

where ND (D) denotes the Gaussian (disaster) component of consumption growth, dividend growth or returns. We define "resilience" for the consumption claim as:

$$H^c_t = 1 + p_t\left(E_t\left[\exp\left\{(\gamma-1)J^c_{t+1}\right\}\right] - 1\right).$$

We log-linearize the total wealth return $R^a_{t+1} = \frac{W_{t+1}}{W_t - C_t}$ as follows: $r_{a,t+1} = \kappa^c_0 + wc_{t+1} - \kappa^c_1 wc_t + \Delta c_{t+1}$ with linearization constants:

$$\kappa^c_1 = \frac{e^{\overline{wc}}}{e^{\overline{wc}} - 1} \quad (3)$$

$$\kappa^c_0 = -\log\left(e^{\overline{wc}} - 1\right) + \kappa^c_1\overline{wc}. \quad (4)$$

The wealth-consumption ratio differs across Markov states. Let wc_i be the log wealth-consumption ratio in Markov state i. The mean log wealth-consumption ratio can be computed using the stationary distribution:

$$\overline{wc} = \sum_{i=1}^{I} \Pi_i wc_i \quad (5)$$

where Π_i is the i^{th} element of vector Π. Note that the linearization constants κ_0^c and κ_1^c depend on $\overline{wc}$. Using the log linearization for the total wealth return, the Euler equation can be restated as follows:

$$1 = \exp(h_t^c) E_t \left[\exp\left\{ \alpha \log\beta - \frac{\alpha}{\psi}(\mu_c + \sigma_{ci}\eta_{t+1}) + \alpha(\kappa_0^c + wc_{t+1} - \kappa_1^c wc_t + \Delta c_{t+1}^{ND}) \right\} \right].$$

Resilience takes a simple form in our setting:

$$\begin{aligned} h_t^c &\equiv \log(H_t^c) = \log\left(1 + p_t \left[\exp\left\{\bar{h}^c\right\} - 1\right]\right), \\ \bar{h}^c &\equiv \log E_t \left[\exp\left\{(\gamma-1)J_{t+1}^c\right\}\right] = \omega\left(\exp\left\{(\gamma-1)\theta_c + .5(\gamma-1)^2\delta_c^2\right\} - 1\right), \end{aligned}$$

where we used the cumulant-generating function to compute $\bar{h}^c$. It is now clear that resilience only varies with the probability of a disaster p_t. Therefore, it too is a Markov chain. Denote by h_i^c the log resilience in Markov state i. Solving the Euler equation for the consumption claim amounts to solving for the log wealth-consumption ratio in each state i. We obtain the following system of I equations, which can be solved for wc_i, $i = 1, \dots I$:

$$1 \;=\; \exp(h_i^c)\exp\left\{\alpha(\log\beta + \kappa_0^c) + (1-\gamma)\mu_c - \alpha\kappa_1^c wc_i + \frac{1}{2}(1-\gamma)^2\sigma_{ci}^2\right\}\sum_{j=1}^{N}\pi_{ij}\exp\left\{\alpha wc_j\right\}$$

where π_{ij} is the transition probability between states i and j. Taking logs on both sides we get the following system of equations which can be solved in conjunction with (3), (4), and (5):

$$0 = h_i^c + \alpha(\log\beta + \kappa_0^c) + (1-\gamma)\mu_c - \alpha\kappa_1^c wc_i + \frac{1}{2}(1-\gamma)^2\sigma_{ci}^2 + \log\sum_{j=1}^{N}\pi_{ij}\exp\left\{\alpha wc_j\right\}.$$

B.2 Valuing the Dividend Claim

The investor's Euler equation for the stock is $E_t[M_{t+1}R_{t+1}^d] = 1$, which can be decomposed as:

$$\begin{aligned} 1 \;=\; & (1-p_t)E_t\left[\exp(\alpha\log\beta - \frac{\alpha}{\psi}\Delta c_{t+1}^{ND} + (\alpha-1)r_{a,t+1}^{ND} + r_{d,t+1}^{ND})\right] \\ & + p_t E_t\left[\exp(\alpha\log\beta - \frac{\alpha}{\psi}\Delta c_{t+1}^{D} + (\alpha-1)r_{a,t+1}^{D} + r_{d,t+1}^{D})\right] \end{aligned}$$

If we define "resilience" for the dividend claim as:

$$H_t^d = 1 + p_t\left(E_t\left[\exp\left\{\gamma J_{t+1}^c - J_{t+1}^d - \lambda_d J_{t+1}^a\right\}\right] - 1\right),$$

then the Euler equation simplifies to:

$$1 = H_t^d E_t\left[\exp\left\{\alpha\log\beta - \frac{\alpha}{\psi}\Delta c_{t+1}^{ND} + (\alpha-1)r_{a,t+1}^{ND} + r_{d,t+1}^{ND}\right\}\right].$$

We log-linearize the stock return on bank i, R_{t+1}^d, as $r_{d,t+1} = \kappa_0^d + \kappa_1^d pd_{t+1} - pd_t + \Delta d_{t+1}$, with the linearization constants:

$$\kappa_1^d \;=\; \frac{e^{\overline{pd}}}{1 + e^{\overline{pd}}}, \tag{6}$$

$$\kappa_0^d \;=\; \log(1 + e^{\overline{pd}}) - \kappa_1^d\overline{pd}. \tag{7}$$

To compute the resilience term, we proceed as before:

$$
\begin{aligned}
h_t^d &\equiv \log\left(1 + p_t\left(\exp\left\{\bar{h}_d\right\} - 1\right)\right), \\
\bar{h}_d &\equiv \log E_t\left[\exp\left\{\gamma J_{t+1}^c - J_{t+1}^d - \lambda_d J_{t+1}^a\right\}\right].
\end{aligned}
$$

By using the independence of the three jump processes conditional on a given number of jumps, we can simplify the last term to:

$$
\begin{aligned}
\bar{h}_d &= \log\left(\sum_{n=0}^{\infty} \frac{e^{-\omega}\omega^n}{n!} e^{n(\gamma\theta_c + .5\gamma^2\delta_c^2)} e^{n(-\theta_d + .5\delta_d^2)} \right. \\
&\quad \left. \times \left\{ e^{n(-\lambda_d\theta_r + .5\lambda_d^2\delta_r^2)} \Phi\left(\frac{\underline{J} - n\theta_r + n\lambda_d\delta_r^2}{\sqrt{n}\delta_r}\right) + e^{-\lambda_d \underline{J}} \Phi\left(\frac{n\theta_r - \underline{J}}{\sqrt{n}\delta_r}\right) \right\} \right).
\end{aligned}
$$

The derivation uses Lemma 1 below. The last expression, while somewhat complicated, is straightforward to compute. In the no-bailout case ($\underline{J} \to +\infty$), the last exponential term reduces to $e^{n(-\lambda_d\theta_r + .5\lambda_d^2\delta_r^2)}$. The dynamics of h_t^d are fully determined by the dynamics of p_t, which follows a Markov chain. Denote by h_i^d the resilience in Markov state i.

Solving the Euler equation for the dividend claim amounts to solving for the log price-dividend ratio in each state i, pd_i. We can solve the following system of N equations for pd_i:

$$
\begin{aligned}
pd_i &= h_i^d + \alpha \log\beta - \gamma\mu_c + (\alpha - 1)\left(\kappa_0^c - \kappa_1^c wc_i\right) + \kappa_0^d + \mu_d + \frac{1}{2}(\phi_d - \gamma)^2\sigma_{ci}^2 + \frac{1}{2}\sigma_{di}^2 \\
&\quad + \log\left(\sum_{j=1}^{N} \pi_{ij} \exp\left\{(\alpha - 1)wc_j + \kappa_1^d pd_j\right\}\right),
\end{aligned}
$$

together with the linearization constants in (6) and (7), and the mean pd ratio:

$$
\overline{pd} = \sum_j \Pi_j pd_j. \tag{8}
$$

B.3 Dividend Growth and Return Variance, Return Covariance, and the Equity Risk Premium

Preliminaries Recall that dividend growth in state i today is

$$
\begin{aligned}
\Delta d_i &= (1 - p_i)\Delta d_i^{ND} + p_i \Delta d_i^D, \\
\Delta d_i^{ND} &= \mu_d + \phi_d\sigma_{ci}\eta + \sigma_{di}\epsilon, \\
\Delta d_i^D &= \mu_d + \phi_d\sigma_{ci}\eta + \sigma_{di}\epsilon - J^d - \lambda_d J^a
\end{aligned}
$$

where the shock $\epsilon = \sqrt{\xi_d}\epsilon^a + \sqrt{1 - \xi_d}\epsilon^i$ is the sum of a common shock and an idiosyncratic shock, both of which are standard normally distributed and i.i.d. over time. Stock returns in state i today and assuming a transition to state j next period are:

$$\begin{aligned} r_i &= (1-p_i) r_i^{ND} + p_i r_i^D, \\ r_i^{ND} &= \mu_{rij} + \phi_d \sigma_{ci} \eta + \sigma_{di} \epsilon, \\ r_i^D &= \mu_{rij} + \phi_d \sigma_{ci} \eta + \sigma_{di} \epsilon - J^d - \lambda_d J^a, \\ \mu_{rij} &= \mu_d + \kappa_0^d + \kappa_1^d pd_j - pd_i, \\ J^a &= \min(J^r, \underline{J}). \end{aligned}$$

We are interested in computing the variance of dividend growth rates, the variance of returns and the covariance between a pair of returns. This will allow us to compute the volatility of returns and the correlation of returns.

Applying Lemma 4 below to the J^a process and conditioning on n jumps, we get that

$$\begin{aligned} E[J^a|n] &= E[\min(J^r, \underline{J})|n] \\ &= E[J^r 1_{(J^r < \underline{J})}|n] + \underline{J} E[1_{(J^r \geq \underline{J})}|n] \\ &= n\theta_r \Phi\left(\frac{\underline{J} - n\theta_r}{\sqrt{n}\delta_r}\right) - \sqrt{n}\delta_r \phi\left(\frac{\underline{J} - n\theta_r}{\sqrt{n}\delta_r}\right) + \underline{J}\Phi\left(\frac{n\theta_r - \underline{J}}{\sqrt{n}\delta_r}\right), \end{aligned}$$

and

$$\begin{aligned} E[J^{a2}|n] &= E[\min(J^r, \underline{J})^2|n] \\ &= E[J^{r2} 1_{(J^r < \underline{J})}|n] + \underline{J}^2 E[1_{(J^r \geq \underline{J})}|n] \\ &= \left(n\delta_r^2 + n^2\theta_r^2\right) \Phi\left(\frac{\underline{J} - n\theta_r}{\sqrt{n}\delta_r}\right) - \sqrt{n}\delta_r(\underline{J} + n\theta_r)\phi\left(\frac{\underline{J} - n\theta_r}{\sqrt{n}\delta_r}\right) + \underline{J}^2\Phi\left(\frac{n\theta_r - \underline{J}}{\sqrt{n}\delta_r}\right). \end{aligned}$$

Note that the corresponding moments for the J^d process are:

$$\begin{aligned} E[J^d|n] &= n\theta_d \\ E[J^{d2}|n] &= n\delta_d^2 + n^2\theta_d^2. \end{aligned}$$

We now average over all possible realizations of the number of jumps n to get:

$$\begin{aligned} E[J^d] &= \sum_{n=1}^{\infty} \frac{e^{-\omega}\omega^n}{n!} E[J^d|n] = \theta_d, \\ E[J^{d2}] &= \sum_{n=1}^{\infty} \frac{e^{-\omega}\omega^n}{n!} E[J^{d2}|n] = \delta_d^2 + 2\theta_d^2, \\ E[J^a] &= \sum_{n=1}^{\infty} \frac{e^{-\omega}\omega^n}{n!} E[J^a|n] \equiv \theta_a, \\ E[J^{a2}] &= \sum_{n=1}^{\infty} \frac{e^{-\omega}\omega^n}{n!} E[J^{a2}|n], \\ E[J^d J^a] &= \sum_{n=1}^{\infty} \frac{e^{-\omega}\omega^n}{n!} n\theta_d E[J^a|n], \\ E[J^{d,1} J^{d,2}] &= \sum_{n=1}^{\infty} \frac{e^{-\omega}\omega^n}{n!} (n\theta_d)(n\theta_d) = 2\theta_d^2 \end{aligned}$$

where we used our assumption that $\omega = 1$, which implies that $\sum_{n=1}^{\infty} \frac{e^{-\omega}\omega^n}{n!} n = 1$ and $\sum_{n=1}^{\infty} \frac{e^{-\omega}\omega^n}{n!} n^2 = 2$. The last but one expression uses the fact that the two jumps are uncorrelated, conditional on a given number of jumps. The last expression computes the expectation of the product of the idiosyncratic jumps for two different stocks. Note that the correlation between these two idiosyncratic jump processes is zero if and only if $\theta_d = 0$, an assumption we make in our calibration.

Dividend Growth and Return Volatility The variance of dividend growth of a firm can be computed as follows

$$
\begin{aligned}
Var[\Delta d_i] &= (1-p_i)E[(\Delta d_i^{ND})^2] + p_i E[(\Delta d_i^{D})^2] - \left[(1-p_i)E[\Delta d_i^{ND}] + p_i E[\Delta d_i^{D}]\right]^2, \\
&= (1-p_i)\left[\mu_d^2 + \phi_d^2\sigma_{ci}^2 + \sigma_{di}^2\right] \\
&\quad + p_i\left[\mu_d^2 + \phi_d^2\sigma_{ci}^2 + \sigma_{di}^2 + E[J^{d2}] + \lambda_d^2 E[J^{a2}] + 2\lambda_d E[J^d J^a] - 2\mu_d(E[J^d] + \lambda_d E[J^a])\right] \\
&\quad - \left[(1-p_i)\mu_d + p_i[\mu_d - E[J^d] - \lambda_d E[J^a]]\right]^2, \\
&= \phi_d^2\sigma_{ci}^2 + \sigma_{di}^2 + p_i(\delta_d^2 + 2\theta_d^2 + \lambda_d^2 E[J^{a2}] + 2\lambda_d E[J^d J^a]) - p_i^2(\theta_d + \lambda_d\theta_a)^2
\end{aligned}
$$

Similarly, mean dividend growth is given by $E[\Delta d_i] = \mu_d - p_i(\theta_d + \lambda_d\theta_a)$. If $\theta_d = 0$, as we assume, mean dividend growth is simply $\mu_d - p_i\lambda_d\theta_a$.

The variance of returns can be derived similarly, with the only added complication that we need to take into account state transitions from i to j that affect the mean return μ_{rij}.

$$
\begin{aligned}
Var[r_i] &= (1-p_i)E[(r_i^{ND})^2] + p_i E[(r_i^{D})^2] - \left[(1-p_i)E[r_i^{ND}] + p_i E[r_i^{D}]\right]^2, \\
&= (1-p_i)\left[\sum_{j=1}^{I}\pi_{ij}\mu_{rij}^2 + \phi_d^2\sigma_{ci}^2 + \sigma_{di}^2\right] \\
&\quad + p_i\left[\sum_{j=1}^{I}\pi_{ij}\mu_{rij}^2 + \phi_d^2\sigma_{ci}^2 + \sigma_{di}^2 + E[J^{d2}] + \lambda_d^2 E[J^{a2}] + 2\lambda_d E[J^d J^a] - 2\sum_{j=1}^{I}\pi_{ij}\mu_{rij}(E[J^d] + \lambda_d E[J^a])\right] \\
&\quad - \left[\sum_{j=1}^{I}\pi_{ij}\mu_{rij} - p_i(E[J^d] + \lambda_d E[J^a])\right]^2, \\
&= \zeta_{ri} + \phi_d^2\sigma_{ci}^2 + \sigma_{di}^2 + p_i(\delta_d^2 + 2\theta_d^2 + \lambda_d^2 E[J^{a2}] + 2\lambda_d E[J^d J^a]) - p_i^2(\theta_d + \lambda_d\theta_a)^2,
\end{aligned}
$$

where

$$
\zeta_{ri} \equiv \sum_{j=1}^{I}\pi_{ij}\mu_{rij}^2 - \left(\sum_{j=1}^{I}\pi_{ij}\mu_{rij}\right)^2,
$$

is an additional variance term that comes from state transitions that affect the price-dividend ratio. The volatility of the stock return is the square root of the variance.

Covariance of Returns The covariance of a pair of returns (r^1, r^2) in state i is:

$$
\begin{aligned}
Cov[r_i^1, r_i^2] &= (1-p_i)E[r_i^{1,ND} r_i^{2,ND}] + p_i E[r_i^{1,D} r_i^{2,D}] \\
&\quad - \left[(1-p_i)E[r_i^{1,ND}] + p_i E[r_i^{1,D}]\right]\left[(1-p_i)E[r_i^{2,ND}] + p_i E[r_i^{2,D}]\right], \\
&= (1-p_i)\left[\sum_{j=1}^{I} \pi_{ij}\mu_{rij}^2 + \phi_d^2\sigma_{ci}^2 + \sigma_{di}^2\xi_d\right] \\
&\quad + p_i\left[\sum_{j=1}^{I} \pi_{ij}\mu_{rij}^2 + \phi_d^2\sigma_{ci}^2 + \sigma_{di}^2\xi_d + E[J^{d,1}J^{d,2}] + \lambda_d^2 E[J^{a2}] + 2\lambda_d E[J^d J^a] - 2\sum_{j=1}^{I}\pi_{ij}\mu_{rij}(\theta_d + \lambda_d\theta_a)\right] \\
&\quad - \left(\sum_{j=1}^{I}\pi_{ij}\mu_{rij}\right)^2 - p_i^2(\theta_d + \lambda_d\theta_a)^2 + 2\sum_{j=1}^{I}\pi_{ij}\mu_{rij}(\theta_d + \lambda_d\theta_a), \\
&= \zeta_{ri} + \phi_d^2\sigma_{ci}^2 + \sigma_{di}^2\xi_d + p_i(2\theta_d^2 + \lambda_d^2 E[J^{a2}] + 2\lambda_d E[J^d J^a]) - p_i^2(\theta_d + \lambda_d\theta_a)^2,
\end{aligned}
$$

where we recall that ξ_d is the fraction of the variance of the Gaussian ϵ shock that is common across all stocks. The correlation between two stocks is the ratio of the covariance to the variance (given symmetry).

Equity Risk premium By analogy with the derivations above, we have

$$
\begin{aligned}
E[J^c] &= \sum_{n=1}^{\infty} \frac{e^{-\omega}\omega^n}{n!} E[J^c|n] = \theta_c, \\
E[J^d J^c] &= \sum_{n=1}^{\infty} \frac{e^{-\omega}\omega^n}{n!} (n\theta_d)(n\theta_d) = 2\theta_c\theta_d, \\
E[J^a J^c] &= \sum_{n=1}^{\infty} \frac{e^{-\omega}\omega^n}{n!} n\theta_c E[J^a|n]
\end{aligned}
$$

We also have

$$
\begin{aligned}
m^{ND} &= \mu_{mij} - \gamma\sigma_{ci}\eta, \\
m^D &= \mu_{mij} - \gamma\sigma_{ci}\eta + \gamma J^c, \\
\mu_{mij} &= \alpha\log\beta + (\alpha-1)(\kappa_0^c + wc_j - \kappa_1^c wc_i) - \gamma\mu_c,
\end{aligned}
$$

The equity risk premium is $-Cov(m, r)$, which can be derived similarly to the covariance between two returns. In particular:

$$
\begin{aligned}
Cov[m_i, r_i] &= (1-p_i)E[m_i^{ND} r_i^{ND}] + p_i E[m_i^D r_i^D] \\
&\quad - \left[(1-p_i)E[m_i^{ND}] + p_i E[m_i^D]\right] \left[(1-p_i)E[r_i^{ND}] + p_i E[r_i^D]\right], \\
&= (1-p_i)\left[\sum_{j=1}^{I} \pi_{ij}\mu_{rij}\mu_{mij} - \gamma\phi_d\sigma_{ci}^2\right] \\
&\quad + p_i \left[\sum_{j=1}^{I} \pi_{ij}\mu_{rij}\mu_{mij} - \gamma\phi_d\sigma_{ci}^2 - \gamma E[J^d J^c] - \gamma\lambda_d E[J^a J^c] + \gamma \sum_{j=1}^{I}\pi_{ij}\mu_{rij}\theta_c - \sum_{j=1}^{I}\pi_{ij}\mu_{mij}(\theta_d + \lambda_d\theta_a)\right] \\
&\quad - \left[\sum_{j=1}^{I}\pi_{ij}\mu_{mij} + p_i\gamma\theta_c\right]\left[\sum_{j=1}^{I}\pi_{ij}\mu_{rij} - p_i(\theta_d + \lambda_d\theta_a)\right] \\
&= \zeta_{mi} - \gamma\phi_d\sigma_{ci}^2 - p_i\gamma(2\theta_d\theta_c + \lambda_d E[J^c J^a]) + p_i^2\gamma\theta_c(\theta_d + \lambda_d\theta_a),
\end{aligned}
$$

where

$$
\zeta_{mi} \equiv \sum_{j=1}^{I}\pi_{ij}\mu_{rij}\mu_{mij} - \left(\sum_{j=1}^{I}\pi_{ij}\mu_{rij}\right)\left(\sum_{j=1}^{I}\pi_{ij}\mu_{mij}\right).
$$

B.4 Valuing Options

The main technical contribution of the paper is to price options in the presence of a bailout guarantee. We are interested in the price per dollar invested in a put option (cost per dollar insured) on a bank stock. For simplicity, we assume that the option has a one-period maturity and is of the European type. We denote the put price by Put:

$$
Put_t = E_t\left[M_{t+1}\left(K - R_{t+1}\right)^+\right] = (1-p_t)Put_t^{ND} + p_t Put_t^D,
$$

where the strike price K is expressed as a fraction of a dollar (that is, $K = 1$ is the ATM option). The put price is the sum of a disaster component and a non-disaster component. We derive both components below. But first, we state and prove two important lemmas which are invoked repeatedly to derive the option prices.

B.4.1 Auxiliary Lemmas

Lemma 1. *Let $x \sim N(\mu_x, \sigma_x^2)$ and $y \sim N(\mu_y, \sigma_y^2)$ with $Corr(x, y) = \rho_{xy}$. Then*

$$
E[\exp(ax + by)1_{c>y}] = \Psi(a, b; x, y)\Phi\left(\frac{c - \mu_y - b\sigma_y^2 - a\rho_{xy}\sigma_x\sigma_y}{\sigma_y}\right) \tag{9}
$$

where $\Psi(a, b; x, y) = \exp\left(a\mu_x + b\mu_y + \frac{a^2\sigma_x^2}{2} + \frac{b^2\sigma_y^2}{2} + ab\rho_{xy}\sigma_x\sigma_y\right)$ is the bivariate normal moment-generating function of x and y evaluated at (a, b).

Proof. Lemma 1 First, note that $x|y \sim N\left(\mu_x + \frac{\rho_{xy}\sigma_x}{\sigma_y}[y - \mu_y], \sigma_x^2(1-\rho_{xy}^2)\right)$, therefore

$$
E[\exp(ax)|y] = Q\exp\left(\frac{a\rho_{xy}\sigma_x}{\sigma_y}y\right)
$$

where $Q = \exp\left(a\mu_x - \frac{a\rho_{xy}\sigma_x\mu_y}{\sigma_y} + \frac{a^2\sigma_x^2(1-\rho_{xy}^2)}{2}\right)$. Denote $\Gamma = E[\exp(ax+by)1_{c>y}]$, then:

$$
\begin{aligned}
\Gamma &= E[E\{\exp(ax)|y\}\exp(by)1_{c>y}] \\
&= QE\left[\exp\left(y\left\{\frac{a\rho_{xy}\sigma_x}{\sigma_y}+b\right\}\right)1_{c>y}\right] \\
&= Q\int_{-\infty}^{c}\exp\left(y\left\{\frac{a\rho_{xy}\sigma_x}{\sigma_y}+b\right\}\right)dF(y) \\
&= Q\int_{-\infty}^{c}\exp\left(y\left\{\frac{a\rho_{xy}\sigma_x}{\sigma_y}+b+\frac{\mu_y}{\sigma_y^2}\right\}-\frac{y^2}{2\sigma_y^2}-\frac{\mu_y^2}{2\sigma_y^2}\right)\frac{dy}{\sigma_y\sqrt{2\pi}} \\
&\quad \text{Complete the square} \\
&= Q\exp\left(\frac{\sigma_y^2}{2}\sigma_y\left\{\frac{a\rho_{xy}\sigma_x}{\sigma_y}+b\right\}^2+\mu_y\left\{\frac{a\rho_{xy}\sigma_x}{\sigma_y}+b\right\}\right)\int_{-\infty}^{c}\exp\left(-\frac{\left[y-\sigma_y^2\left\{\frac{a\rho_{xy}\sigma_x}{\sigma_y}+b+\frac{\mu_y}{\sigma_y^2}\right\}\right]^2}{2\sigma_y^2}\right)\frac{dy}{\sigma_y\sqrt{2\pi}} \\
&\quad \text{Substitute } u = \frac{y-\sigma_y^2\left\{\frac{a\rho_{xy}\sigma_x}{\sigma_y}+b+\frac{\mu_y}{\sigma_y^2}\right\}}{\sigma_y},\ du\sigma_y = dy \\
&= \exp\left(a\mu_x+\frac{a^2\sigma_x^2(1-\rho_{xy}^2)}{2}+\frac{\sigma_y^2}{2}\left\{\frac{a\rho_{xy}\sigma_x}{\sigma_y}+b\right\}^2+b\mu_y\right)\Phi\left(\frac{c-b\sigma_y^2-a\rho_{xy}\sigma_x\sigma_y-\mu_y}{\sigma_y}\right)
\end{aligned}
$$

□

Lemma 2. *Let $x \sim N(\mu_x, \sigma_x^2)$, then*

$$
E\left[\Phi\left(b_0+b_1x\right)\exp\left(ax\right)1_{x<c}\right] = \Phi\left(\frac{b_0-t_1}{\sqrt{1+b_1^2\sigma_x^2}}, \frac{c-t_2}{\sigma_x}; \rho\right)\exp(z_1) \tag{10}
$$

where $t_1 = -b_1t_2$, $t_2 = a\sigma_x^2+\mu_x$, $z_1 = \frac{a^2\sigma_x^2}{2}+a\mu_x$, $\rho = \frac{-b_1\sigma_x}{\sqrt{1+b_1^2\sigma_x^2}}$, and $\Phi(\cdot\ ,\cdot\ ;\rho)$ is the cumulative density function (CDF) of a bivariate standard normal with correlation parameter ρ.

Proof. Lemma 2 Denote $\Omega = E\left[\Phi\left(b_0 + b_1 x\right)\exp\left(ax\right)1_{x<c}\right]$, then:

$$
\begin{aligned}
\Omega &= \int_{-\infty}^{c}\int_{-\infty}^{b_0+b_1x} \exp\left(ax\right) dF(v)dF(x) \\
&= \int_{-\infty}^{c}\int_{-\infty}^{b_0+b_1x} \exp\left(ax - \frac{v^2}{2} - \frac{\left[x-\mu_x\right]^2}{2\sigma_x^2}\right)\frac{dv\ dx}{\sigma_x 2\pi} \\
&\quad \text{Substitute } v = u + b_1 x,\ dv = du \\
&= \int_{-\infty}^{c}\int_{-\infty}^{b_0} \exp\left(ax - \frac{(u+b_1x)^2}{2} - \frac{\left[x-\mu_x\right]^2}{2\sigma_x^2}\right)\frac{du\ dx}{\sigma_x 2\pi} \\
&= \int_{-\infty}^{c}\int_{-\infty}^{b_0} \exp\left(-\frac{u^2}{2} - x^2\left(\frac{1}{2\sigma_x^2} + \frac{b_1^2}{2}\right) - b_1 ux + 0u + x\left(a + \frac{\mu_x}{\sigma_x^2}\right) - \frac{\mu_x^2}{2\sigma_x^2}\right)\frac{du\ dx}{\sigma_x 2\pi} \\
&\quad \text{Complete the square in two variables using Lemma 3} \\
&= \int_{-\infty}^{c}\int_{-\infty}^{b_0} \exp\left\{\begin{pmatrix} u-t_1 \\ x-t_2\end{pmatrix}'\begin{pmatrix} s1 & s2 \\ s2 & s3\end{pmatrix}\begin{pmatrix} u-t_1 \\ x-t_2\end{pmatrix} + z_1\right\}\frac{du\ dx}{\sigma_x 2\pi} \\
&= \int_{-\infty}^{c}\int_{-\infty}^{b_0} \exp\left(-\frac{1}{2}(U-T)'(-2S)(U-T) + z_1\right)\frac{du\ dx}{\sigma_x 2\pi}
\end{aligned}
$$

where $U = (u,x), T = (t_1,t_2), -2S = \begin{pmatrix} 1 & b_1 \\ b_1 & b_1^2 + \frac{1}{\sigma_x^2}\end{pmatrix}, (-2S)^{-1} = \begin{pmatrix} 1 + b_1^2\sigma_x^2 & -b_1\sigma_x^2 \\ -b_1\sigma_x^2 & \sigma_x^2\end{pmatrix}$. This is the CDF for $U \sim N(T,(-2S)^{-1})$. Let $w_1 = \frac{u-t_1}{\sqrt{1+b_1^2\sigma_x^2}}$, $w_2 = \frac{x-t_2}{\sigma_x}$, and $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1\end{pmatrix}$ with $\rho = \frac{-b_1\sigma_x}{\sqrt{1+b_1^2\sigma_x^2}}$. We have that $W' = (w_1,w_2) \sim N(0,\Sigma)$. Also, $du = dw_1\sqrt{1+b_1^2\sigma_x^2}$ and $dx = dw_2\sigma_x$.

$$
\begin{aligned}
\Omega &= \exp(z_1)\left\{\int_{-\infty}^{\frac{c-t_2}{\sigma_x}}\int_{-\infty}^{\frac{b_0-t_1}{\sqrt{1+b_1^2\sigma_x^2}}} \exp\left(-\frac{1}{2}W'\Sigma^{-1}W\right)\frac{dw_1\ dw_2}{2\pi\sqrt{1-\rho^2}}\right\}\sqrt{1+b_1^2\sigma_x^2}\sqrt{1-\rho^2} \\
&= \Phi\left(\frac{b_0-t_1}{\sqrt{1+b_1^2\sigma_x^2}}, \frac{c-t_2}{\sigma_x}; \rho\right)\exp(z_1)
\end{aligned}
$$

where we used that $\sqrt{1+b_1^2\sigma_x^2}\sqrt{1-\rho^2} = 1$, and where completing the square implies $t_1 = -b_1 t_2$, $t_2 = a\sigma_x^2 + \mu_x$, $s_1 = -.5$, $s_2 = -.5b_1$, $s_3 = -.5b_1^2 - \frac{1}{2\sigma_x^2}$, and $z_1 = \frac{a^2\sigma_x^2}{2} + a\mu_x$ by application of Lemma 3. □

Lemma 3. *Bivariate Complete Square*

$$
Ax^2 + By^2 + Cxy + Dx + Ey + F = \begin{pmatrix} x-t_1 \\ y-t_2\end{pmatrix}'\begin{pmatrix} s_1 & s_2 \\ s_2 & s_3\end{pmatrix}\begin{pmatrix} x-t_1 \\ y-t_2\end{pmatrix} + z_1
$$

where

$$
\begin{aligned}
t_1 &= -(2BD - CE)/(4AB - C^2) && s_1 = A \\
t_2 &= -(2AE - CD)/(4AB - C^2) && s_2 = C/2 \\
z_1 &= F - \frac{BD^2 - CDE + AE^2}{4AB - C^2} && s_3 = B.
\end{aligned}
$$

The following lemma will be useful in deriving the variance and covariances of stock returns.

Lemma 4. *Let* $Z \sim N(\mu, \sigma^2)$ *and define* $\phi = \phi\left(\frac{b-\mu}{\sigma}\right)$ *and* $\Phi = \Phi\left(\frac{b-\mu}{\sigma}\right)$. *Then*

$$\begin{aligned} E[Z 1_{Z<b}] &= \mu\Phi - \sigma\phi, &(11)\\ E[Z^2 1_{Z<b}] &= \left(\sigma^2+\mu^2\right)\Phi - \sigma(b+\mu)\phi &(12)\end{aligned}$$

Proof.

$$E[Z1_{Z<b}] = E[Z|Z<b]Pr(Z<b) = \left(\mu - \frac{\sigma\phi}{\Phi}\right)\Phi = \mu\Phi - \sigma\phi$$

The second result is shown similarly:

$$\begin{aligned} E[Z^2 1_{Z<b}] &= E[Z^2|Z<b]Pr(Z<b)\\ &= (Var[Z^2|Z<b] + E[Z|Z<b]^2)Pr(Z<b)\\ &= \left(\sigma^2 - \frac{\sigma(b-\mu)\phi}{\Phi} - \sigma^2\frac{\phi^2}{\Phi^2} + \left[\mu - \frac{\sigma\phi}{\Phi}\right]^2\right)\Phi\\ &= \left(\sigma^2+\mu^2\right)\Phi - \sigma(b+\mu)\phi.\end{aligned}$$

□

B.4.2 Option Prices Conditional on No Disaster

Conditional on no disaster in the next period, we are back to the familiar Black-Scholes world (with Epstein-Zin preferences). The option value in state i is:

$$\begin{aligned} Put_i^{ND} &= E\left[M^{ND}(K-R^{ND})^+\right]\\ &= -E\left[\exp\left(m^{ND}+r^{ND}\right)1_{k>r^{ND}}\right] + KE\left[\exp\left(m^{ND}\right)1_{k>r^{ND}}\right]\end{aligned}$$

We condition on a Markov state transition from state i in the current period to state j in the next one. Then, the log SDF and log return are bivariate normally distributed; see Appendices B.3 and B.3. Application of Lemma 1 in Appendix B.4.1 leads to the familiar Black-Scholes value of a put option:

$$Put_{ij}^{ND} = -\Psi(1,1;m^{ND},r^{ND})\Phi(d_{ij}-\sigma_{ri}) + Ke^{-r_{ij}^{f,ND}}\Phi(d_{ij}), \tag{13}$$

where $d_{ij}^{ND} = \frac{k-\mu_{rij}-\sigma_{m,r}}{\sigma_{ri}}$, where $k=\log(K)$, μ_{rij} is the mean log stock return conditional on a transition from i to j and no disaster, σ_{ri} is the volatility of the log stock return in state i, σ_{mr} is the covariance of the log return and log SDF, and where $\Psi(a,b;x,y) = \exp\left(a\mu_x + b\mu_y + \frac{a^2\sigma_x^2}{2} + \frac{b^2\sigma_y^2}{2} + ab\rho_{xy}\sigma_x\sigma_y\right)$ is the bivariate normal moment-generating function of x and y evaluated at (a,b). We have used the fact that $\Psi(1,0;m^{ND},r^{ND}) = \exp(\mu_{mj}+.5\sigma_m^2) = \exp(-r_{ij}^{f,ND})$, where $r_{ij}^{f,ND}$ is the risk-free rate in Markov state i, conditional on a transition to state j and conditional on no disaster. As an aside, if there were no disaster state, then $\Psi(1,1;m^{ND},r^{ND}) = 1$.[21] Since we conditioned on a particular transition to state j, we still

[21] This would follow immediately from the fact that the no-disaster return would satisfy the Euler equation in this case. We would then have that $\mu_r = r^{f,ND} - \sigma_{m,r} - .5\sigma_r^2$ with $-\sigma_{m,r} = \gamma\phi\sigma_{ci}^2$ as the familiar Gaussian equity risk premium. Equation (13) would therefore collapse to the standard Black-Scholes formula, with $d^{ND} = \frac{k-r^{f,ND}+.5\sigma_r^2}{\sigma_r}$.

have to average over all such transitions to obtain the no-disaster option price in state i:

$$Put_i^{ND} = \sum_{j=1}^{I} \pi_{i,j} Put_{ij}^{ND}.$$

B.4.3 Option Prices Conditional on a Disaster

Conditional on having a disaster, the formulae become substantially more involved due to the presence of a bailout option. Backus, Chernov, and Martin (2011) derive option prices in a setting similar to ours, but one that does not have the bailout option. In their setting, Black-Scholes can be applied because log returns are a Poisson mixtures of normals, so that they are normally distributed conditional on a given number of jumps. Option prices are then weighted-averages of Black-Scholes values, weighted by the Poisson probability of a given number of jumps. In the presence of the bailout option, log stock returns are no longer normally distributed; They contain a term $J^a = \min(J^r, \underline{J})$, where J^r is normal conditional on a given number of jumps so that J^a is not normal. A technical contribution of the paper is to show that we can still obtain closed-form expressions for the put option price. The result hinges on repeated application of Lemmas 1 and 2, stated in Appendix B.4.1. The details of the derivation are relegated to Appendix B.4.3.

We start by conditioning on a Markov state transition from state i to state j and we condition on n jumps to the three jump processes (J^c, J^i, J^r). The option value is

$$\begin{aligned} Put_{ijn}^D &= E\left[M^D(K - R^D)^+\right] \\ &= -E\left[\exp\left(m^D + r^D\right) 1_{k>r^D}\right] + KE\left[\exp\left(m^D\right) 1_{k>r^D}\right], \\ &= -Put_{ijn1}^D + Put_{ijn2}^D. \end{aligned}$$

We define the random variable $\tilde{r} = r^{ND} - J^d$. Log returns in the disaster state are $r^D = \tilde{r} - \lambda_d J^a$. The appendix derives the following expressions for the two terms in the put price:

$$\begin{aligned} Put_{ijn1}^D &= \Bigg\{ e^{n(-\lambda_d\theta_r + .5\lambda_d^2\delta_r^2)} \Phi\left(\frac{k - \mu_{rij} + n\theta_i - \sigma_{\tilde{r}}^2 - \sigma_{m^D,\tilde{r}} + n(\lambda_d\theta_r - \lambda_d^2\delta_r^2)}{\sqrt{\sigma_{\tilde{r}}^2 + n\lambda_d\delta_r^2}}, \frac{\underline{J} - n\theta_r + n\lambda_d\delta_r^2}{\sqrt{n}\delta_r}; \rho\right) \\ &\quad + e^{-\lambda_d\underline{J}} \Phi\left(\frac{\lambda_d\underline{J} + k - \mu_{rij} + n\theta_i - \sigma_{\tilde{r}}^2 - \sigma_{m^D,\tilde{r}}}{\sigma_{\tilde{r}}}\right) \Phi\left(\frac{n\theta_r - \underline{J}}{\delta_r\sqrt{n}}\right) \Bigg\} \Psi(1, 1; m^D, \tilde{r}) \qquad (14) \\ Put_{ijn2}^D &= Ke^{-r_{ijn}^{f,D}} \Bigg\{ \Phi\left(\frac{k - \mu_{rij} + n\theta_i - \sigma_{m^D,\tilde{r}} + n\lambda_d\theta_r}{\sqrt{\sigma_{\tilde{r}}^2 + n\lambda_d^2\delta_r^2}}, \frac{\underline{J} - n\theta_r}{\sqrt{n}\delta_r}; \rho\right) \\ &\quad + \Phi\left(\frac{\lambda_d\underline{J} + k - \mu_{rij} + n\theta_i - \sigma_{m^D,\tilde{r}}}{\sigma_{\tilde{r}}}\right) \Phi\left(\frac{n\theta_r - \underline{J}}{\delta_r\sqrt{n}}\right) \Bigg\} \qquad (15) \end{aligned}$$

We note that $\Psi(1, 0; m^D, \tilde{r}) = e^{-r_{ijn}^{f,D}}$, where $r_{ijn}^{f,D}$ is the risk-free rate conditional on a disaster realization, n jumps, and a Markov transition from state i to j. The correlation coefficient is:

$$\rho = -\frac{\sqrt{n}\lambda_d\delta_r}{\sqrt{\sigma_r^2 + n\delta_i^2 + n\lambda_d^2\delta_r^2}}.$$

Note that equations (14) and (15) are entirely in terms of the structural parameters of the model. Thus, we essentially obtain closed-form solutions for the option prices.

Finally, we sum over the various jump events and Markov states j to obtain the disaster option price in state i:

$$Put_i^D = \sum_{j=1}^{I} \pi_{i,j} \sum_{n=1}^{\infty} \frac{e^{-\omega}\omega^n}{n!} \left(-Put_{ijn1}^D + Put_{ijn2}^D\right). \tag{16}$$

Derivation We condition on the disaster state occurring in the next period, on a transition from state i to state j and on a known number of jumps n for the jump variables. Later we will average over the possible values for each. The put option value in this state is:

$$\begin{aligned} Put_{ijn}^D &= E\left[M^D(K - R^D)1_{K>R^D}\right] \\ &= -E\left[\exp\left(m^D + r^D\right)1_{k>r^D}\right] + KE\left[\exp\left(m^D\right)1_{k>r^D}\right] \\ &= -Put_{ijn1}^D + Put_{ijn2}^D. \end{aligned}$$

We now develop the two terms. For ease of notation, let $V_1^D = Put_{ijn1}^D$ and $V_2^D = Put_{ijn2}^D$.

Recall that $\tilde{r} = r^{ND} - J_i$ and $r^D = \tilde{r} - \lambda_d \min(J^r, \underline{J})$. Our derivation below exploits the normality of the following two random variables:

$$\begin{aligned} m^D &= \mu_{mij} - \gamma\sigma_{ci}\eta + \gamma J_c \sim N(\mu_m + \gamma n\theta_c, \sigma_m^2 + \gamma^2 n\delta_c^2) \\ \tilde{r} &= \mu_{rij} + \phi\sigma_{ci}\eta + \sigma_{di}\epsilon - J^i \sim N(\mu_{rj} - n\theta_i, \sigma_{\tilde{r}}^2) \\ \sigma_{\tilde{r}}^2 &= \sigma_r^2 + n\delta_i^2, \quad \sigma_{m^D,\tilde{r}} = \sigma_{m,r} = -\gamma\phi\sigma_{ci}^2 \end{aligned}$$

First term V_1^D

$$\begin{aligned} V_1^D &= E\left[\exp\left(m^D + r^D\right)1_{k>r^D}1_{J^r<\underline{J}}\right] + E\left[\exp\left(m^D + r^D\right)1_{k>r^D}1_{J^r>\underline{J}}\right] \\ &= E\left[\exp\left(m^D + r^{ND} - J_i - \lambda_d J^r\right)1_{k>r^D}1_{J^r<\underline{J}}\right] + E\left[\exp\left(m^D + r^{ND} - J_i - \lambda_d \underline{J}\right)1_{k>r^D}1_{J^r>\underline{J}}\right] \\ &= V_{11}^D + V_{12}^D \end{aligned}$$

The first term V_{11}^D can be solved as follows:

$$\begin{aligned} V_{11}^D &= E\left[\exp\left(m^D + \tilde{r} - \lambda_d J^r\right)1_{k>r^D}1_{J^r<\underline{J}}\right] \\ &= E\left[E\left\{\exp\left(m^D + \tilde{r} - \lambda_d J^r\right)1_{k+\lambda_d J^r>\tilde{r}}|J^r\right\} \mid 1_{J^r<\underline{J}}\right] \\ &= E\left[E\left\{\exp\left(m^D + \tilde{r}\right)1_{k+\lambda_d J^r>\tilde{r}}|J^r\right\}\exp\left(-\lambda_d J^r\right)1_{J^r<\underline{J}}\right] \\ &= \Psi(1,1;m^D,\tilde{r})E\left[\Phi\left(\phi_0 + \phi_1 J^r\right)\exp\left(-\lambda_d J^r\right)1_{J^r<\underline{J}}\right] \quad \text{by Lemma 1} \\ &= \Psi(1,1;m^D,\tilde{r})\exp(z_1)\Phi\left(\frac{\phi_0 - t_1}{\sqrt{1+\phi_1^2 n\delta_r^2}}, \frac{\underline{J} - t_2}{\sqrt{n}\delta_r}; \rho\right) \quad \text{by Lemma 2} \end{aligned}$$

where $\phi_1 = \frac{\lambda_d}{\sigma_{\tilde{r}}}$, $\phi_0 = \frac{\phi_1}{\lambda_d}\left(k - \mu_{rij} + n\theta_i - \sigma_{\tilde{r}}^2 - \sigma_{m^D,\tilde{r}}\right)$, $t_2 = n(\theta_r - \lambda_d\delta_r^2)$, $t_1 = -\phi_1 t_2$, $\rho = \frac{-\phi_1\sqrt{n}\delta_r}{\sqrt{1+\phi_1^2 n\delta_r^2}}$, and $z_1 = \frac{n\lambda_d^2\delta_r^2}{2} - n\lambda_d\theta_r$.

Next, we turn to V_{12}^D:

$$
\begin{aligned}
V_{12}^D &= E\left[\exp\left(m^D + r^{ND} - J_i - \lambda_d \underline{J}\right) 1_{k>r^D} 1_{J^r > \underline{J}}\right] \\
&= \exp(-\lambda_d \underline{J}) E\left[\exp\left(m^D + \tilde{r}\right) 1_{k+\lambda_d \underline{J} > \tilde{r}}\right] \Phi\left(\frac{n\theta_r - \underline{J}}{\delta_r \sqrt{n}}\right) \\
&= \Psi(1,1; m^D, \tilde{r}) \exp(-\lambda_d \underline{J}) \Phi\left(\frac{\lambda_d \underline{J} + k - \mu_{rij} + n\theta_i - \sigma_{\tilde{r}}^2 - \sigma_{m^D,\tilde{r}}}{\sigma_{\tilde{r}}}\right) \Phi\left(\frac{n\theta_r - \underline{J}}{\delta_r \sqrt{n}}\right) \quad \text{by Lemma 1.}
\end{aligned}
$$

Second term V_2^D

$$
\begin{aligned}
V_2^D &= KE\left[\exp\left(m^D\right) 1_{k>r^D}\right] \\
&= KE\left[\exp\left(m^D\right) 1_{k>r^D} 1_{J^r < \underline{J}}\right] + KE\left[\exp\left(m^D\right) 1_{k>r^D} 1_{J^r > \underline{J}}\right] \\
&= V_{21}^D + V_{22}^D.
\end{aligned}
$$

The first term V_{21}^D can be solved as follows:

$$
\begin{aligned}
V_{21}^D &= KE\left[\exp\left(m^D\right) 1_{k>r^D} 1_{J^r < \underline{J}}\right] \\
&= KE\left[E\left\{\exp\left(m^D\right) 1_{k+\lambda_d J^r > \tilde{r}} | J^r\right\} 1_{J^r < \underline{J}}\right] \\
&= K\Psi(1,0; m^D, \tilde{r}) E\left[\Phi\left(\phi_0 + \phi_1 J^r\right) 1_{J^r < \underline{J}}\right] \quad \text{by Lemma 1} \\
&= K\Psi(1,0; m^D, \tilde{r}) \Phi\left(\frac{\phi_0 - t_1}{\sqrt{1 + \phi_1^2 n \delta_r^2}}, \frac{\underline{J} - t_2}{\sqrt{n}\delta_r}; \rho\right) \quad \text{by Lemma 2}
\end{aligned}
$$

where $\phi_1 = \frac{\lambda_d}{\sigma_{\tilde{r}}}$, $\phi_0 = \frac{\phi_1}{\lambda_d}\left(k - \mu_{rij} + n\theta_i - \sigma_{m^D,\tilde{r}}\right)$, $t_2 = n\theta_r$, $t_1 = -\phi_1 t_2$, $\rho = \frac{-\phi_1 \sqrt{n}\delta_r}{\sqrt{1+\phi_1^2 n \delta_r^2}}$, and $z_1 = 0$. Because $z_1 = 0$, $\exp(z_1) = 1$, and we have dropped that term from the expression.

Finally, we turn to V_{22}^D:

$$
\begin{aligned}
V_{22}^D &= KE\left[\exp\left(m^D\right) 1_{k>r^D} 1_{J^r > \underline{J}}\right] \\
&= KE\left[\exp\left(m^D\right) 1_{k+\lambda_d \underline{J} > \tilde{r}} 1_{J^r > \underline{J}}\right] \\
&= KE\left[\exp\left(m^D\right) 1_{k+\lambda_d \underline{J} > \tilde{r}}\right] \Phi\left(\frac{n\theta_r - \underline{J}}{\delta_r \sqrt{n}}\right) \\
&= K\Psi(1,0; m^D, \tilde{r}) \Phi\left(\frac{\lambda_d \underline{J} + k - \mu_{rij} + n\theta_i - \sigma_{m^D,\tilde{r}}}{\sigma_{\tilde{r}}}\right) \Phi\left(\frac{n\theta_r - \underline{J}}{\delta_r \sqrt{n}}\right) \quad \text{by Lemma 1.}
\end{aligned}
$$

B.4.4 Option Pricing Absent Bailout Guarantees

Absent bailout options (NB), $J^a = J^r$, and we obtain substantial simplification to the general formula. This special case arises as $\underline{J} \to +\infty$. In that case, the second terms of equations (14) and (15) are zero.

In both first terms, the bivariate CDF simplifies to a univariate CDF.

$$\begin{aligned}
Put_{ijn}^{D,NB} &= -\Psi(1,1;m^D,\tilde{r})e^{n(-\theta_r+.5\delta_r^2)}\Phi\left(d_{jn}^{NB}-\sqrt{\sigma_r^2+n(\delta_i^2+\lambda_d^2\delta_r^2)}\right)+K\Psi(1,0;m^D,\tilde{r})\Phi\left(d_{jn}^{NB}\right)\\
&= -\exp\left(\mu_{mj}+\mu_{rij}+.5\sigma_m^2+.5\sigma_r^2+\sigma_{m,r}+n(\gamma\delta_c-\theta_i-\theta_r)+.5n(\gamma^2\delta_c^2+\delta_i^2+\delta_r^2)\right)\\
&\quad\times\Phi\left(d_{jn}^{NB}-\sqrt{\sigma_r^2+n(\delta_i^2+\delta_r^2)}\right)+K\exp\left(\mu_{mj}+.5\sigma_m^2+n\gamma\delta_c+.5n\gamma^2\delta_c^2\right)\Phi\left(d_{jn}^{NB}\right)\\
&= \exp\left(n\gamma\delta_c+.5n\gamma^2\delta_c^2\right)\left\{-\Psi(1,1;m^{ND},r^{ND})\exp\left(n(-\theta_i-\theta_r)+.5n(\delta_i^2+\delta_r^2)\right)\right.\\
&\quad\left.\times\Phi\left(d_{jn}^{NB}-\sqrt{\sigma_r^2+n(\delta_i^2+\delta_r^2)}\right)+Ke^{-r_{ij}^{f,ND}}\Phi\left(d_{jn}^{NB}\right)\right\}
\end{aligned}$$

with

$$d_{jn}^{NB}=\frac{k-\mu_{rij}+n(\theta_i+\lambda_d\theta_r)-\sigma_{m^D,\tilde{r}}}{\sqrt{\sigma_r^2+n\delta_i^2+n\lambda_d^2\delta_r^2}}$$

This equation is the counter-part of the Black-Scholes formula in equation (13), except that the mean and volatility of returns are adjusted for the jumps. Indeed, absent bailout options, log returns are normally distributed conditional on a given number of jumps n. We note that the expression for d^{NB} is in terms of the moments of the risk-neutral distribution of log returns. In particular, the risk-neutral mean is

$$\mu_{rij}^*=\mu_{rij}-n(\theta_i+\lambda_d\theta_r)-(-\sigma_{m^D,\tilde{r}}).$$

Thus the risk-neutral mean of the jump size equals the physical mean ($\theta_i^*=\theta_i$ and $\theta_r^*=\theta_r$), which follows from the fact that the jump sizes of the J^r and the J^i processes are independent of those of aggregate consumption J^c. The risk-neutral variance of log returns is equal to the physical variance, as usual ($\sigma_r^*=\sigma_r$, $\delta_i^*=\delta_i$ and $\delta_r^*=\delta_r$). The risk-neutral jump intensity is increased from the physical one as follows: $\omega^*=\omega\exp\left(\gamma\theta_c+.5\gamma^2\delta_c^2\right)$. To see this, note that the term $\exp\left(n\gamma\delta_c+.5n\gamma^2\delta_c^2\right)$, which factors out of the put price, can be folded into the Poisson weights when we sum over all possible number of jumps as in equation (2):

$$\sum_{n=1}^{\infty}\frac{e^{-\omega}\omega^n}{n!}\exp\left(n(\gamma\theta_c+.5\gamma^2\delta_c^2)\right)\cdots=\sum_{n=1}^{\infty}\frac{e^{-\omega^*}\omega^{*n}}{n!}\cdots$$

We recover the formulae of Backus, Chernov, and Martin (2011).

C Robustness Appendix

C.1 Return Correlation Fit

While it avoids the decline in correlation of the model without bailout guarantees, our benchmark calibration does not generate enough of an increase in return correlation from the pre-crisis to the crisis period. Depending on whether one interprets the crisis as an elevated probability of a disaster or as the actual realization of a disaster, the model's return correlation in state 2 is 51.1% or 40.7%. Both are below the observed 57.6%. To improve on this, we estimate the four key parameters ($\underline{J},\theta_r,\delta_r,\delta_d$) so as to best match the put and call basket, index, and spread prices in pre-crisis and crisis (12 moments), as well as the volatility of individual and index returns and return correlations in pre-crisis and crisis (6 moments). We give the return correlation moment a higher weight in the optimization and interpret the crisis data as the actual realization of a disaster. Our best fitting calibration generates a correlation that matches the 45.8% in the pre-crisis period and that increases to 58.7% or 51.2% in state 2 depending on whether a disaster is more likely or actually realized, respectively. They straddle the observed 57.6%; see Table C. The option pricing fit deteriorates slightly, but the model is still able to capture the observed patterns in put and call spreads reasonably well; see Table B. Interestingly, the parameters in this calibration imply that 50% of the value of the financial sector is attributable to the bailout guarantee, just as in the benchmark calibration.

C.2 Moneyness

Options with different moneyness may be informative about the degree of Gaussian versus tail aggregate and idiosyncratic risk. To investigate this possibility, we recalibrate our model to best fit financial sector basket and index put option prices with moneyness $\Delta = 20$, 30, 40, and 50, and their basket-index spread in pre-crisis and crisis (4 moments each), alongside the return volatility and return correlation moments, for a total of 30 moments. Keeping the Gaussian volatility σ_d constant across states at 15% and keeping the fraction of it that is common at 0%, Panel B of Table D shows a reasonably good fit for the various put prices. However, the model overstates the basket put price in the pre-crisis and understates it in the crisis for at-the-money options. A much better fit is obtained when we allow the Gaussian volatility to rise from 14.5% in state 1 to 30% in state 2 while simultaneously increasing the fraction of Gaussian shocks that are common from 0% in state 1 to 30% in state 2. This implies more Gaussian dividend (and return) risk during the crisis and more of it common across firms. We then reoptimize over the other four structural parameters to best fit the 30 moments under consideration. While the loss rate in a disaster θ_a of 42.0% in logs or 34.3% in levels is similar to that of our benchmark model (46.5% in logs and 37.2% in levels), the parameters $\theta_r = 1.28$ and $\delta_r = .95$ are substantially higher while the bailout parameter $\underline{J} = .79$ is substantially lower. The amount of idiosyncratic tail risk, governed by $\delta_d = .36$, is also lower because there is now more idiosyncratic Gaussian risk. As a result of the higher aggregate tail risk parameters, our estimates of the cost-of-capital savings from the bailout guarantee go up substantially. Removing the bailout option would result in an increase of the equity risk premium by a factor of 3.3-3.5 (from 4.0% to 13.1% in state 1 and from 12.1% to 42.9% in state 2), as opposed to a factor 2 in our benchmark calibration. That suggests our benchmark numbers are conservative.

C.3 Three-state Model

We also consider a model with somewhat richer dynamics for the probability of a disaster. In particular we want to differentiate between the relatively mild crisis of the August 2007-August 2008 and April 2009-June 2009 and the sharp crisis of September 2008-March 2009. A 3-state Markov model allows us to capture the idea that, conditional on being in a mild crisis there is a chance of a substantial deterioration in the health of the financial sector. We leave the disaster probability in state 1 at 7% and set the disaster probability in state 2 to 14% and to 60% in state 3. The 3-state model has the same 13% unconditional disaster probability. The transition probability matrix is $\Pi =$[0.85, 0.15, 0; 0.506, 0.286, 0.208; 0, 0.5, 0.5]. Consumption volatility is 0.35% in state 1, 0.75% in state 2, and 1.5% in state 3. As in the benchmark 2-state model, we hold $\sigma_d = .15$ and $\xi_d = 0$ constant across states. We choose the remaining four parameters to best fit the usual put and call price, and return moments (27 moments). The model generates a large increase in put spread from 0.6 in state 1 to 1.2 in state 2 to 8.3 in state 3. In the data, the put spread increases from 0.8 pre-crisis to 2.7 in the mild crisis subsamples, and to 6.4 in the severe crisis. The model generates a decline in the call spread from 0.2 to -0.2 from pre-crisis to severe crisis, compared to 0.3 to -0.1 in the data. The model is also broadly consistent with the sharp increases in individual and index volatility during the severe crisis, and with the increase in return correlations in both crisis subsamples. Detailed results are available upon request. The model implies an equity risk premium of 5.6% pre-crisis, 21.4% in the mild crisis, and 29.2% in the severe crisis. Absent the bailout option, the risk premium would be 12.3, 39.2, and 73.2%; the value of the financial sector would be 45% lower.

C.4 Heterogeneity across Large and Small Banks

So far, we have considered models were all banks are ex-ante identical. One might think that large banks are more systemically risky and may therefore enjoy larger government guarantees. All else equal, that would result in comparatively lower costs of capital for large banks. To investigate this hypothesis, we consider two groups of banks. The first group consists of the largest ten banks by market capitalization as of the end of July 2007 (see right column of Table A) plus Fannie Mae (number 11) and Freddie Mac (number 14). We refer to this group as the "big 12." The second group contains all other banks in the financial sector index. When we loose a member of the big 12 in our option data set, we replace it the next-largest bank as of the end of July 2007. There are four such replacements (for Fannie and Freddie on September 8, 2008 and for Wachovia and Merrill Lynch on January 1, 2009) so that BNY-Mellon, US Bancorp, Metlife and Prudential join the big 12, in that order. The resulting big 12 group has a stable market share between 45 and 55% of the total market capitalization of all firms in the financial sector index over our sample. A sample without replacement would have a declining market share during the crisis. For these two groups of banks, we hold fixed all aggregate risk parameters $(\underline{J}, \theta_r, \delta_r, \sigma_c)$ at their values from the calibration discussed in Section C.1. We continue to set $\xi_d = 0$ so that all non-priced Gaussian dividend shocks are idiosyncratic. We allow for heterogeneity across the groups in the parameters $(\lambda_d, \delta_d, \sigma_d(1), \sigma_d(2))$. The first parameter governs how much exposure a bank has to the aggregate tail process J^a, the second its idiosyncratic tail risk, and the last two the Gaussian idiosyncratic risk. We set the parameter $\lambda_d = 1.208$ for large banks and $\lambda_d = 0.936$ for small banks in order to match the (within-group average) regression coefficients of individual stock returns on a constant and the financial sector index return using only the most extreme 10% of index returns on the downside. We recall that we normalized $\lambda_d = 1$ for the full sample of banks. Thus, the data suggest that large banks have more aggregate tail risk exposure than small banks. We choose the remaining three parameters for each group so that they are on opposite sides of the common parameter choice of Section C.1, and so that they best fit the return correlation and volatility and the put and call prices of the options for each group.

Panel A of Table E shows the observed put and call prices for the big 12 (Panel A.1) and the other banks (Panel A.2). They are the value-weighted averages within each group, taken over the two pre-crisis and crisis subsamples. They also indicate the put and call spreads, which subtract from the option basket the (common) index option price. Finally, the table reports the (value-weighted) average individual return volatility and pairwise correlation among the stocks within a group. From pre-crisis to crisis, the increase in return volatility and put spread are much larger for the big 12 than for the smaller banks while the increase in return correlation is much smaller. Panel B shows that our model can match these facts for both groups. In addition to a higher aggregate tail risk exposure, large banks have more idiosyncratic tail risk, which is needed to explain their high return volatility during the crisis, and less Gaussian idiosyncratic risk, which is needed to explain their high pre-crisis return correlation which increases only modestly during the crisis. The opposite is true for small banks; the parameter choices are listed in the table caption. Having shown that we can account for the heterogeneity in option price and return features of each group, we can ask how much higher the cost of capital would be for each group absent a bailout guarantee, holding fixed the other group-specific parameters. We find that the cost of capital for large banks would increase by 12% points, 1.5 times the 9% point increase for the small banks. This suggests that large banks' options were "cheap" because they disproportionately enjoyed the government guarantee.

Table A: Top 40 Holdings of the Financial Sector Index XLF

	12/30/2010		07/30/2007	
	Name	Weighting	Name	Weighting
1	JPMorgan Chase & Co.	9.01	CITIGROUP INC	11.1
2	Wells Fargo & Co.	8.86	BANK OF AMERICA CORP	10.14
3	Citigroup Inc.	7.54	AMERICAN INTERNATIONAL GROUP I	8.02
4	BERKSHIRE HATHAWAY B	7.52	JPMORGAN CHASE & Co	7.25
5	Bank of America Corp.	7.3	WELLS FARGO & Co NEW	5.44
6	Goldman Sachs Group Inc.	4.66	WACHOVIA CORP 2ND NEW	4.35
7	U.S. BANCORP	2.82	GOLDMAN SACHS GROUP INC	3.71
8	American Express Co.	2.44	AMERICAN EXPRESS CO	3.35
9	MORGAN STANLEY	2.25	MORGAN STANLEY DEAN WITTER & C	3.25
10	MetLife Inc.	2.21	MERRILL LYNCH & Co INC	3.11
11	Bank of New York Mellon Corp.	2.04	FEDERAL NATIONAL MORTGAGE ASSN	2.81
12	PNC Financial Services Group Inc.	1.75	U S BANCORP DEL	2.51
13	Simon Property Group Inc.	1.6	BANK OF NEW YORK MELLON CORP	2.32
14	Prudential Financial Inc.	1.56	METLIFE INC	2.15
15	AFLAC Inc.	1.45	PRUDENTIAL FINANCIAL INC	2
16	Travelers Cos. Inc.	1.39	FEDERAL HOME LOAN MORTGAGE COR	1.83
17	State Street Corp.	1.27	TRAVELERS COMPANIES INC	1.63
18	CME Group Inc. Cl A	1.18	WASHINGTON MUTUAL INC	1.61
19	ACE Ltd.	1.15	LEHMAN BROTHERS HOLDINGS INC	1.59
20	Capital One Financial Corp.	1.06	ALLSTATE CORP	1.56
21	BB&T Corp.	1	C M E GROUP INC	1.46
22	Chubb Corp.	0.99	CAPITAL ONE FINANCIAL CORP	1.41
23	Allstate Corp.	0.93	HARTFORD FINANCIAL SVCS GROUP	1.4
24	Charles Schwab Corp.	0.93	SUNTRUST BANKS INC	1.35
25	T. Rowe Price Group Inc.	0.89	STATE STREET CORP	1.28
26	Franklin Resources Inc.	0.87	A F L A C INC	1.23
27	AON Corp.	0.82	P N C FINANCIAL SERVICES GRP I	1.11
28	EQUITY RESIDENTIAL	0.81	REGIONS FINANCIAL CORP NEW	1.02
29	Marsh & McLennan Cos.	0.81	LOEWS CORP	1.02
30	SunTrust Banks Inc.	0.8	FRANKLIN RESOURCES INC	1.01
31	Ameriprise Financial Inc.	0.78	SCHWAB CHARLES CORP NEW	0.98
32	PUBLIC STORAGE	0.77	B B & T CORP	0.98
33	Vornado Realty Trust	0.74	FIFTH THIRD BANCORP	0.98
34	Northern Trust Corp.	0.73	CHUBB CORP	0.97
35	HCP Inc.	0.73	S L M CORP	0.97
36	Progressive Corp.	0.71	SIMON PROPERTY GROUP INC NEW	0.93
37	Loews Corp.	0.67	ACE LTD	0.91
38	Boston Properties Inc.	0.66	NATIONAL CITY CORP	0.82
39	Host Hotels & Resorts Inc.	0.64	COUNTRYWIDE FINANCIAL CORP	0.81
40	FIFTH THIRD BANCORP	0.64	LINCOLN NATIONAL CORP IN	0.79

This table reports the XLF weights on 12/30/2010 and 07/30/2007. On 12/30/2010, there were 81 companies in XLF; on 07/30/2007, there were 96 companies. This table reports the relative market capitalizations of the top 40 holdings of the index.

Table B: Option Prices in Economy Calibrated to Match Correlations

The table reports option prices and implied volatility for the financial sector index, for its constituents, and pairwise correlations between the stocks in the financial sector index. Panel A is for the January 2003-June 2009 data. Panel B is for a model with parameters $\sigma_d(1) = \sigma_d(2) = 0.15$, $\xi_d(1) = \xi_d(2) = 0$, $\delta_d = 0.39$, $\underline{J} = 0.84$, $\theta_r = 0.95$, and $\delta_r = 0.71$.

	Put Prices			Call Prices		
	Basket	Index	Spread	Basket	Index	Spread
			Panel I: Data			
pre-crisis	4.0	3.2	0.8	1.6	1.3	0.3
crisis	13.7	9.9	3.8	2.4	2.3	0.1
			Panel II: Model with Bailout			
pre-crisis	3.9	3.7	0.2	1.4	1.0	0.4
crisis	11.7	8.8	2.9	2.3	2.1	0.2

Table C: Returns in in Economy Calibrated to Match Correlations

The table reports realized volatility for the financial sector index, for its constituents, and pairwise correlations between the stocks in the non-financial sector index. The crisis numbers for the model represent the unconditional moment in state 2, taking disasters into account probabilistically. The number *in italic* for the model report the moments in state 2 of the model *conditional* on a disaster realization. Panel A is for the January 2003-June 2009 data. Panel B is for a model with parameters $\sigma_d(1) = \sigma_d(2) = 0.15$, $\xi_d(1) = \xi_d(2) = 0$, $\delta_d = 0.39$, $\underline{J} = 0.84$, $\theta_r = 0.95$, and $\delta_r = 0.71$.

	Index	**Individual Stocks**	
	Volatility	Volatility	Correlations
		Panel I: Data	
pre-crisis	11.9	18.1	45.8
crisis	43.8	72.9	57.6
		Panel II: Model without Bailout	
pre-crisis	17.9	24.7	45.8
crisis	31.5	39.7	58.7
	44.2	*59.8*	*51.2*

Table D: Option Prices and Returns by Option Moneyness

The table reports basket and index put option prices for puts with moneyness $\Delta = 20$, 30, 40, and 50. It also reports realized volatility for the financial sector index, for its constituents, and pairwise correlations between the stocks in the non-financial sector index. Panel A is for the January 2003-June 2009 data. Panel B is for a model with parameters $\sigma_d(1) = \sigma_d(2) = 0.15$, $\xi_d(1) = \xi_d(2) = 0$, $\delta_d = 0.47$, $\underline{J} = 0.82$, $\theta_r = 1.2$, and $\delta_r = 0.95$. Panel C is for a model with parameters $\sigma_d(1) = 0.145$, $\sigma_d(2) = 0.30$, $\xi_d(1) = 0$, $\xi_d(2) = 0.30$, $\delta_d = 0.36$, $\underline{J} = 0.79$, $\theta_r = 1.28$, and $\delta_r = 0.95$. The crisis numbers for the model represent the unconditional moment in state 2, taking disasters into account probabilistically. The number *in italics* for the model report the moments in state 2 of the model *conditional* on a disaster realization.

	Puts Delta = 20			**Puts Delta = 30**			**Puts Delta = 40**			**Puts Delta = 50**			**Return moments**		
Panel A: Moments in Data															
	Basket	Index	Spread	Basket	Index	Spread	Basket	Index	Spread	Basket	Index	Spread	Index vol	Indiv vol	Indiv Correl
pre-crisis	4.0	3.2	0.8	5.8	4.6	1.2	7.7	6.1	1.6	9.8	7.7	2.1	11.9	18.1	45.8
crisis	13.7	9.9	3.8	17.8	13.4	4.4	21.6	16.7	4.9	25.5	20.1	5.4	43.8	72.9	57.5
Panel B: Moments in Model with Bailout; fix Gaussian risk															
	Basket	Index	Spread	Basket	Index	Spread	Basket	Index	Spread	Basket	Index	Spread	Index vol	Indiv vol	Indiv Correl
pre-crisis	4.0	3.8	0.2	5.7	5.1	0.5	8.3	6.4	1.9	12.4	8.7	3.7	18.0	25.4	41.5
crisis	12.8	9.3	3.5	16.0	13.6	2.4	18.8	16.6	2.1	21.8	18.7	3.0	31.9/*46.0*	42.1/*66.2*	52.3/*44.4*
Panel C: Moments in Model with Bailout; change Gaussian risk															
	Basket	Index	Spread	Basket	Index	Spread	Basket	Index	Spread	Basket	Index	Spread	Index vol	Indiv vol	Indiv Correl
pre-crisis	3.7	3.6	0.1	5.3	4.9	0.3	8.0	6.1	1.8	12.8	8.2	4.6	17.2	23.5	45.6
crisis	12.3	8.9	3.4	16.4	13.0	3.4	20.4	16.3	4.1	24.4	19.1	5.3	35.1/*46.6*	46.2/*62.9*	53.4/*51.4*

Table E: Heterogeneity: Option and Return Moments for Large and Small Banks

The table reports basket put and call prices for options with moneyness $\Delta = 20$ and maturity of one year, as well as the spread over the corresponding index option price with the same *Delta* and maturity. It also reports individual stock return volatility and pairwise return correlations for the firms within each group. The two groups of firms are discussed in the main text. Panel A is for the January 2003-June 2009 data. Panel B is for the model with common parameters $\underline{J} = 0.84$, $\theta_r = 0.95$, $\delta_r = 0.71$, and $\xi_d(1) = \xi_d(2) = 0$. The big 12 group of large banks has parameters $\lambda_d = 1.208$, $\sigma_d(1) = 0.11$, $\sigma_d(2) = 0.09$, $\delta_d = 0.50$. The group of all other banks has parameters $\lambda_d = 0.936$, $\sigma_d(1) = 0.18$, $\sigma_d(2) = 0.20$, $\delta_d = 0.32$. Within each group, all firms are ex-ante identical. The crisis numbers for the model represent the unconditional moment in state 2, taking disasters into account probabilistically. The number *in italics* for the model report the moments in state 2 of the model *conditional* on a disaster realization.

	Panel A: Data											
	Panel A.1: Big 12						Panel A.2: All other banks					
	Put prices		Call prices		Returns		Put prices		Call prices		Returns	
	basket	spread	basket	spread	indiv vol	correl	basket	spread	basket	spread	indiv vol	correl
pre-crisis	4.0	0.8	1.6	0.3	17.0	57.0	4.0	0.9	1.7	0.3	24.6	38.7
crisis	14.5	4.6	2.4	0.1	84.7	59.4	12.8	2.9	2.4	0.0	44.9	57.6
	Panel B: Model											
	Panel B.1: Big 12						Panel B.2: All other banks					
	Put prices		Call prices		Returns		Put prices		Call prices		Returns	
	basket	spread	basket	spread	indiv vol	correl	basket	spread	basket	spread	indiv vol	correl
pre-crisis	4.6	0.9	1.3	0.2	26.3	57.1	3.7	0.0	1.5	0.5	25.4	38.7
crisis	14.5	5.7	2.4	0.3	45.9	63.0	10.6	1.9	2.3	0.2	38.8	54.4
					72.3	*50.6*					*55.1*	*53.1*