Appendices to : Market-based emissions regulation when damages vary across sources: What are the gains from differentiation?

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Appendix 1: Optimal Emissions and Policy Parameters.

We suppose that the abatement cost function is a quadratic in the level of abatement, where abatement is defined to be the difference between the firms' chosen emissions level e_i and uncontrolled, "business as usual" emissions e_i^{BAU}. In most cases, including the policy setting that is of primary interest here, a quadratic functional form provides a reasonable approximation to the true form. We transform this quadratic abatement function so that costs are expressed as a function of emissions:

$$C_i = \gamma_{1i}(e_i^{BAU} - e_i) + \gamma_{21}(e_i^{BAU} - e_i)^2 \tag{1}$$

$$= \gamma_{1i}e_i^{BAU} - \gamma_{1i}e_i + \gamma_{21}e_i^{BAU2} - 2\gamma_{21}e_i^{BAU}e_i + \gamma_{21}e_i^2 \tag{2}$$

$$= \alpha_{0i} - \alpha_{1i}e_i + \beta_i e_i^2 \tag{3}$$

1.1 Optimal Emissions.

Employing the above functional form for abatement costs and the linear and additively separable damage function discussed in Section 2. of the main text, we first solve for the socially optimal emissions levels:

$$\underset{e_h,e_l}{MIN} : \alpha_{0H} - \alpha_{1H}e_H + \beta_H {e_H}^2 + \alpha_{0L} - \alpha_{1L}e_L + \beta_L {e_L}^2 + \delta_H e_H + \delta_L e_L \tag{4}$$

First order conditions imply:

$$e_H^* = \frac{\alpha_{1H} - \delta_H}{2\beta_H} \tag{5}$$

$$e_L^* = \frac{\alpha_L - \delta_L}{2\beta_L} \tag{6}$$

The optimal level of aggregate emissions is thus:

$$
\begin{aligned}
E^* &= \frac{\alpha_{1H} - \delta_H}{2\beta_H} + \frac{\alpha_{1L} - \delta_L}{2\beta_L} \\
&= \frac{(\alpha_{1H}\beta_L + \alpha_{1L}\beta_H - \delta_H\beta_L - \delta_L\beta_H)}{2\beta_L\beta_H}
\end{aligned}
\tag{7}
$$

1.2 Welfare maximizing differentiated tax.

Here we solve for the differentiated tax structure that minimizes total social costs as defined by equation (1) in the text. We assume that both the low and high damage firms' objective is to minimize the sum of their abatement costs and tax payments, where (τ_i) is the emissions tax rate for firm (i):

$$
\min_{e_i} : TC_i = \alpha_{0i} - \alpha_i e_i + \beta_i e_i^2 + \tau_i e_i \tag{8}
$$

Firms set:

$$
\begin{aligned}
-\alpha_i + 2\beta_i e_i + \tau &= 0 \\
e_i &= \frac{\alpha_i - \tau_i}{2\beta_i}
\end{aligned}
$$

The regulator sets the tax τ_i to minimize total social costs. Beginning with the low damage source:

$$
\begin{aligned}
\frac{\partial}{\partial \tau_L}\left(\begin{array}{c} -\alpha_L\left(\frac{\alpha_L-\tau_L}{2\beta_L}\right) + \beta_L\left(\frac{\alpha_L-\tau_L}{2\beta_L}\right)^2 - \alpha_H\left(\frac{\alpha_H-\tau_H}{2\beta_H}\right) \\ +\beta_H\left(\frac{\alpha_H-\tau_H}{2\beta_H}\right)^2 + \delta_L\left(\frac{\alpha_L-\tau_L}{2\beta_L}\right) + \delta_H\left(\frac{\alpha_H-\tau_H}{2\beta_H}\right) \end{array}\right) &= 0 \\
\frac{1}{2\beta_L}\left(\tau_L - \delta_L\right) &= 0 \\
\tau_L &= \delta_L
\end{aligned}
$$

By symmetry:

$$\begin{aligned}
\frac{\partial}{\partial \tau_H}\left(\begin{array}{c} -\alpha_L\left(\frac{\alpha_L-\tau_L}{2\beta_L}\right)+\beta_L\left(\frac{\alpha_L-\tau_L}{2\beta_L}\right)^2-\alpha_H\left(\frac{\alpha_H-\tau_H}{2\beta_H}\right) \\ +\beta_H\left(\frac{\alpha_H-\tau_H}{2\beta_H}\right)^2+\delta_L\left(\frac{\alpha_L-\tau_L}{2\beta_L}\right)+\delta_H\left(\frac{\alpha_H-\tau_H}{2\beta_H}\right) \end{array}\right) &= 0 \\
\tau_H &= \delta_H
\end{aligned}$$

Making the substitution, we can solve for the aggregate emissions under this optimal differentiated tax regime:

$$E = \frac{\alpha_H\beta_L + \beta_H\alpha_L - \beta_H\delta_L - \delta_H\beta_L}{2\beta_H\beta_L}$$

Note that this is equal to the optimal level of aggregate emissions.

1.3 Welfare maximizing undifferentiated tax.

We now impose the constraint of a uniform tax. We solve for the welfare maximizing undifferentiated tax. Under an undifferentiated tax regime, firms minimize costs:

$$\min_{e_i} : TC_i = \alpha_{0i} - \alpha_{1i}e_i + \alpha_{2_i}e_i^2 + \tau e_i \tag{9}$$

The first order condition for cost minimization:

$$\begin{aligned}
-\alpha_L + 2\beta_L e_L + \tau &= 0 \\
e_L &= \frac{\alpha_L - \tau}{2\beta_L}
\end{aligned}$$

The regulator chooses the uniform tax rate τ to minimize total social costs:

$$\frac{\partial}{\partial\tau}\left(-\alpha_L\left(\frac{\alpha_L-\tau}{2\beta_L}\right)+\beta_L\left(\frac{\alpha_L-\tau}{2\beta_L}\right)^2-\alpha_H\left(\frac{\alpha_H-\tau}{2\beta_H}\right)+\beta_H\left(\frac{\alpha_H-\tau}{2\beta_H}\right)^2+\delta_L\left(\frac{\alpha_L-\tau}{2\beta_L}\right)+\delta_H\left(\frac{\alpha_H-\tau}{2\beta_H}\right)\right)$$

$=$

$$\frac{1}{2\beta_H\beta_L}\left(\tau\beta_H+\tau\beta_L-\beta_H\delta_L-\delta_H\beta_L\right)=0$$

The welfare maximizing undifferentiated tax:

$$\tau=\frac{1}{\beta_H+\beta_L}\left(\beta_H\delta_L+\delta_H\beta_L\right)$$

Making the substitution:

$$\begin{aligned}E &= \frac{\alpha_L-(\frac{1}{\beta_H+\beta_L}\left(\beta_H\delta_L+\delta_H\beta_L\right))}{2\beta_L}+\frac{\alpha_H-(\frac{1}{\beta_H+\beta_L}\left(\beta_H\delta_L+\delta_H\beta_L\right))}{2\beta_H}\\ &= \frac{\beta_H\alpha_L-\beta_H\delta_L-\delta_H\beta_L+\alpha_H\beta_L}{2\beta_H\beta_L}\end{aligned}$$

Note that the aggregate emissions under the social cost minimizing differentiated and undifferentiated tax regimes are equivalent.

1.4 Welfare maximizing undifferentiated emissions trading program.

In the context of an undifferentiated permit system firm (L) faces the following problem, where permit sales for the low damage firm are denoted (A_{sLH}), permit purchases for the low damage firm are denoted (A_{bLH}), and the initial endowment of permits for the low damage firm is denoted (A_L):

$$\min_{e_j} TC_L=\alpha_{0L}-\alpha_{1L}e_L+\beta_L e_L^2+P(A_{bLH}-A_{sLH}) \tag{10}$$

$$s.t\ e_L\leq A_L-A_{sLH}+A_{bLH}$$

The associated Lagrangian with the firm's cost minimization problem is shown in (11).

$$L_L = \alpha_{0L} - \alpha_{1L}e_L + \beta_L e_L^2 + P(A_{bLH} - A_{sLH}) + \lambda_L(e_L - A_L + A_{sLH} - A_{bLH}) \tag{11}$$

The first order conditions with respect to emissions (e_L), permit sales (A_{sLH}), and permit purchases (A_{bLH}) are shown in (12).

$$\begin{aligned} \frac{\partial L_L}{\partial e_L} &= \lambda_L - \alpha_{1L} + 2\beta_L e_L = 0 \\ \frac{\partial L_L}{\partial A_{bLH}} &= P - \lambda_L = 0 \\ \frac{\partial L_L}{\partial A_{sLH}} &= \lambda_L - P = 0 \end{aligned} \tag{12}$$

By substitution among first-order conditions:

$$-\alpha_{1L} + 2\beta_L e_L = P$$

Thus, the cost-minimizing (e_L) equates marginal cost to the extant permit price (P).

We next consider the policy design problem faced by a regulator who is constrained to implement an undifferentiated emissions trading program. We begin by setting up a standard abatement cost minimization problem subject to the constraint that the sum of emissions between the two firms is less than or equal to the cap ($\overline{E}$). We use this set-up to derive general expressions for cost-minimizing emission levels for the high and low damage firms that hold for any given aggregate cap. Then, we conduct a search among all possible values of the cap ($\overline{E}$) for that which minimizes the welfare loss relative to the first best allocation at $\{e_L^*, e_H^*\}$. The steps involved in this procedure are shown below, beginning with the derivation of the first-order conditions for cost minimization.

$$\min_{e_L, e_H} \left(\alpha_{0L} - \alpha_{1L} e_L + \beta_L e_L^2\right) + \left(\alpha_{0H} - \alpha_{1H} e_H + \beta_H e_H^2\right) \tag{13}$$

$$s.t. \overline{E} \geq e_L + e_H$$

The associated Lagrangian is given in (14), where (ϕ) is a Lagrange multiplier .

$$L = \left(\alpha_{0L} - \alpha_{1L} e_L + \beta_L e_L^2\right) + \left(\alpha_{0H} - \alpha_{1H} e_H + \beta_H e_H^2\right) - \phi(\bar{e} - e_L - e_H) \tag{14}$$

The first-order conditions for cost-minimization with respect to $\{e_L, e_H\}$ are:

$$\begin{aligned} \frac{\partial L_L}{\partial e_L} &= \phi - \alpha_L + 2\beta_L e_L \\ \frac{\partial L_L}{\partial e_H} &= \phi - \alpha_H + 2\beta_H e_H \end{aligned} \tag{15}$$

We next derive expressions for the cost-minimizing emissions levels, which are denoted $\{e_L^U, e_H^U\}$, for the two regulated firms by setting their respective first-order conditions equal to zero and solving for $\{e_L, e_H\}$. This yields:

$$\begin{aligned} e_L^U &= \frac{\alpha_L - \phi}{2\beta_{1L}} \\ e_H^U &= \frac{\alpha_H - \phi}{2\beta_{1H}} \end{aligned} \tag{16}$$

We evaluate (TSC) at the cost-minimizing emission levels $\{e_L^U, e_H^U\}$ and then at the first-best emission levels $\{e_L^*, e_H^*\}$. The goal is to find the aggregate cap which minimizes (ΔTSC): the difference in welfare between the first-best allocation and that resulting from the undifferentiated policy. Since (ϕ) is the shadow value of a small change to the aggregate cap, it is determined by the stringency of the cap. As such, rather than minimizing (ΔTSC) over all combinations of $\{e_L^U, e_H^U\}$, we focus on determining the optimal value of (ϕ). To do this we take $\frac{\partial \Delta TSC}{\partial \phi}$, set it equal to zero, and solve for (ϕ). The solution is: $\phi^* = \frac{(\beta_L\delta_H + \beta_H\delta_L)}{\beta_L+\beta_H}$. Importantly, this suggests that welfare loss is minimized when (ϕ^*) is equal to the optimal tax (τ) in the undifferentiated tax policy.

Further, we know from (12) that firms minimize total compliance costs by equating their marginal costs to the permit price. The permit price, through market forces, reflects the shadow value of relaxing the aggregate cap, in an undifferentiated design. Therefore, firms equate marginal costs to (ϕ). Making the substitution:

$$\begin{aligned} E &= \frac{\alpha_L - (\frac{1}{\beta_H+\beta_L}(\beta_H\delta_L + \delta_H\beta_L))}{2\beta_L} + \frac{\alpha_H - (\frac{1}{\beta_H+\beta_L}(\beta_H\delta_L + \delta_H\beta_L))}{2\beta_H} \\ &= \frac{\beta_H\alpha_L - \beta_H\delta_L - \delta_H\beta_L + \alpha_H\beta_L}{2\beta_H\beta_L} \end{aligned}$$

The emissions level that corresponds is precisely the aggregate emissions level from the case of the second-best undifferentiated tax. This is also the optimal quantity of permits allocated to firms.

1.5 Welfare maximizing differentiated emissions trading program.

Facing a differentiated permit system, the objective of firm (L) is cost minimization subject to a constraint that is slightly modified with respect to (10). As above, we denote the initial allocation of permits (A_L), purchases of permits (A_{bLH}), sales of permits (A_{sLH}), and the extant price of permits (P). We add (r_L) as defined in Section 2.1.4 of the paper.

$$\min_{e_j} TC_L = \alpha_{0L} - \alpha_{1L}e_L + \beta_L e_L^2 + P(A_{bLH} - A_{sLH}) \tag{17}$$

$$s.t.\ r_L e_L \leq A_L - A_{sLH} + A_{bLH}$$

The associated Lagrangian expression is:

$$L_j = \alpha_{0L} - \alpha_{1L}e_L + \beta_L e_L^2 + P(A_{bLH} - A_{sLH}) - \lambda_L(r_L e_L - A_L + A_{sLH} - A_{bLH}) \tag{18}$$

The first order conditions with respect to emissions (e_L), purchases of permits (A_{bLH}), and sales of permits (A_{sLH}) is given by:

$$\begin{aligned} \frac{\partial L_L}{\partial e_L} &= -\lambda_L r_L - \alpha_{1L} + 2\beta_L e_L \\ \frac{\partial L_L}{\partial A_{bLH}} &= P - \lambda_L \\ \frac{\partial L_L}{\partial A_{sLH}} &= \lambda_L - P \end{aligned} \tag{19}$$

By substitution among first-order conditions, the cost-minimizing emission level (e_L) equates marginal cost to the extant permit price (P) weighted by the firm-specific trading ratio (r_L).

The regulator's problem

Now let's approach the design of a differentiated permit system from the regulators' perspective. To do this, we follow Muller and Mendelsohn (2009) and set up the regulator's problem with a cap on damage ($\bar{D}$). Within this framing we then derive expressions for optimal emissions and then show that aggregate emissions are the same as in the case of the undifferentiated permits and, by extension, the tax policies.

$$\min_{e_L, e_H} \; \left(\alpha_{0L} - \alpha_{1L}e_L + \beta_L e_L^2\right) + \left(\alpha_{0H} - \alpha_{1H}e_H + \beta_H e_H^2\right) \tag{20}$$

$$s.t. \; \bar{D} \geq \delta_L e_L + \delta_H e_H$$

The Lagrangian corresponding to the regulator's cost-minimization problem is:

$$L=\left(\alpha_{0L}-\alpha_{1L}e_L+\beta_L e_L^2\right)+\left(\alpha_{0H}-\alpha_{1H}e_H+\beta_H e_H^2\right)-\psi(\bar{D}-\delta_L e_L-\delta_H e_H) \tag{21}$$

The first-order conditions with respect to the emissions vector $\{e_L, e_H\}$ are:

$$\begin{aligned} \frac{\partial L}{\partial e_L} &= -\alpha_{1L}+2\beta_L e_L+\psi\delta_L \\ \frac{\partial L}{\partial e_H} &= -\alpha_{1H}+2\beta_H e_H+\psi\delta_H \end{aligned} \tag{22}$$

Following the exercise from the undifferentiated permit system, we first solve for optimal emission levels: $e_L^D=\frac{\alpha_{1L}-\psi\delta_L}{2\beta_L}$ and $e_H^D=\frac{\alpha_{1H}-\psi\delta_H}{2\beta_H}$.Next, we evaluate (TSC) at $\{e_L^D, e_H^D\}$ and $\{e_L^*, e_H^*\}$.Taking the difference (ΔTSC) and minimizing with respect to (ψ), yields $\psi^*=1$. From $\left(e_L^D\right)$ and $\left(e_H^D\right)$ above, conditional on $\psi^*=1$, the resulting, firm-specific, emission levels are equal to the first best. Furthermore, total emissions under the differentiated policy design, with an optimal cap are: $e_L^D+e_H^D=\frac{\alpha_{1L}-\delta_L}{2\beta_L}+\frac{\alpha_{1H}-\delta_H}{2\beta_H}$. This is equal to the aggregate emission levels in both of the undifferentiated policy designs, provided: $\tau=\phi=\frac{(\beta_H\delta_L+\delta_H\beta_L)}{\beta_H+\beta_L}$.

Finally, it is straightforward to solve for the welfare maximizing permit allocation. The total quantity of permits allocated is given by:

$$\begin{aligned} A &= \frac{2\delta_H}{\delta_H+\delta_L}\left(\frac{\alpha_{1H}-\delta_H}{2\beta_H}\right)+\frac{2\delta_L}{\delta_H+\delta_L}\left(\frac{\alpha_L-\delta_L}{2\beta_L}\right) \\ &= \frac{\beta_H\alpha_L\delta_L+\delta_H\beta_L\alpha_H-\beta_H\delta_L^2-\delta_H^2\beta_L}{\beta_H\beta_L\left(\delta_H+\delta_L\right)} \end{aligned}$$

Appendix 2: The benefits from differentiation.

We are interested in quantifying the gross and net benefits associated with moving from an undifferentiated policy regime to one that incorporates differentiation. To do this, we subtract any increases in costs from decreases in damages to come up with a measure of net benefits. First, we derive an expression for the change in emissions at each source:

$$e_H^D - e_H^U = \frac{\delta_L - \delta_H}{2\left(\beta_L + \beta_H\right)} \tag{23}$$

$$e_L^D - e_L^U = \frac{\delta_H - \delta_L}{2\left(\beta_L + \beta_H\right)} \tag{24}$$

Intuitively, emissions are higher under exposure-based trading at the low damage firm.

To compute the change in abatement costs at each firm we simply substitute the emissions expressions into the corresponding abatement cost functions. The change in abatement costs at the low damage firm:

$$C(e_L^D) - C(e_L^U) = \frac{\left(\delta_H\beta_L + 2\delta_L\beta_H + \delta_L\beta_L\right)\left(\delta_L - \delta_H\right)}{4\left(\beta_H + \beta_L\right)^2} \tag{25}$$

Cost change at high damage firm (abatement costs increase under exposure based trading):

$$C(e_H^D) - C(e_H^U) = \frac{\left(\delta_H\beta_H + 2\delta_H\beta_L + \delta_L\beta_H\right)\left(\delta_H - \delta_L\right)}{4\left(\beta_H + \beta_L\right)^2} \tag{26}$$

Taken together, moving from an emissions-based policy design to one that incorporates trading ratios increases total abatement costs by:

$$\frac{\left(\delta_L - \delta_H\right)^2}{4(\beta_H + \beta_L)} \tag{27}$$

These costs must be compared against the benefits associated with shifting some of the permitted emissions from the high-damage source to the low-damage source:

$$D(e^D) - C(e^U) = \delta_H \frac{\delta_L - \delta_H}{2(\beta_H + \beta_L)} + \delta_L \frac{\delta_H - \delta_L}{2(\beta_H + \beta_L)} \quad (28)$$

$$= -\frac{(\delta_L - \delta_H)^2}{2(\beta_H + \beta_L)} \quad (29)$$

The net welfare change is thus

$$TSC^D - TSC^U = \frac{(\delta_L - \delta_H)^2}{2(\beta_H + \beta_L)} - \frac{(\delta_L - \delta_H)^2}{4(\beta_H + \beta_L)} \quad (30)$$

$$= \frac{(\delta_L - \delta_H)^2}{4(\beta_H + \beta_L)} \geq 0 \quad (31)$$

.

2.1 Uncertain marginal damages.

Here we derive an expression for the difference in total social costs incurred under the differentiated tax versus the undifferentiated tax when damages are uncertain. In the presence of uncertainty in estimates of marginal damages, tax rates are defined in terms of expected damages. Let τ denote the optimal undifferentiated tax: $\tau = \frac{(\beta_H E[\delta_L] + \beta_L E[\delta_H])}{\beta_H + \beta_L}$. We define τ_L to be the expected value of marginal damages for the low-damage firm $E(\delta_L)$.We define τ_H to be the expected value of marginal damages for the high-damage firm $E(\delta_H)$.

First, we derive an expression for the emissions at each source under the differentiated and the undifferentiated taxes.

$$
\begin{aligned}
e_L^D &= \frac{\alpha_{1L} - \tau_L}{2\beta_L} \\
e_L^U &= \frac{\alpha_{1L} - \tau}{2\beta_L} \\
e_H^D &= \frac{\alpha_{1H} - \tau_H}{2\beta_H} \\
e_H^U &= \frac{\alpha_{1H} - \tau}{2\beta_H}
\end{aligned}
$$

We next characterize the change in emissions at each source:

$$
\begin{aligned}
\Delta E_L &= e_L^D - e_L^U \\
&= \frac{\alpha_{1L} - \tau_L}{2\beta_L} - \frac{\alpha_{1L} - \frac{(\beta_H E[\delta_L] + \beta_L E[\delta_H])}{\beta_H + \beta_L}}{2\beta_L} \\
&= \frac{\frac{(\beta_H E[\delta_L] + \beta_L E[\delta_H])}{\beta_H + \beta_L}}{2\beta_L} - \frac{\tau_L}{2\beta_L} \\
&= \frac{(\beta_H E[\delta_L] + \beta_L E[\delta_H])}{2\beta_L(\beta_H + \beta_L)} - \frac{(\beta_H E[\delta_L] + \beta_L E[\delta_L])}{2\beta_L(\beta_H + \beta_L)} \\
&= \frac{E[\delta_H] - E[\delta_L]}{2(\beta_H + \beta_L)}
\end{aligned}
$$

$$
\begin{aligned}
\Delta E_H &= e_H^D - e_H^U \\
&= \frac{\alpha_{1H} - \tau_H}{2\beta_H} - \frac{\alpha_{1H} - \frac{(\beta_H E[\delta_L] + \beta_L E[\delta_H])}{\beta_H + \beta_L}}{2\beta_H} \\
&= \frac{\frac{(\beta_H E[\delta_L] + \beta_L E[\delta_H])}{\beta_H + \beta_L}}{2\beta_H} - \frac{\tau_H}{2\beta_H} \\
&= \frac{(\beta_H E[\delta_L] + \beta_L E[\delta_H])}{2\beta_H(\beta_H + \beta_L)} - \frac{\beta_H E[\delta_H] + \beta_L E[\delta_H]}{2\beta_H(\beta_H + \beta_L)} \\
&= \frac{E[\delta_L] - E[\delta_H]}{2(\beta_H + \beta_L)}
\end{aligned}
$$

We then evaluate the social welfare function given the particular realization of δ'_H and δ'_L and derive the difference in TSC. The increase in abatement costs is the same as above. The reduction in damages is given by:

$$\begin{aligned}
\Delta D &= \delta'_H \cdot e_H^D + \delta'_L \cdot e_L^D + C_L(e_L^D)) - (\delta'_H \cdot e_H^U + \delta'_L \cdot e_L^U) \\
&= \delta'_H(e_H^D - e_H^U) + \delta'_L(e_L^D - e_L^U) \\
&= \delta'_H \frac{E[\delta_L] - E[\delta_H]}{2\,(\beta_H + \beta_L)} + \delta'_L \frac{E[\delta_H] - E[\delta_L]}{2\,(\beta_H + \beta_L)} \\
&= (\delta'_L - \delta'_H) \frac{E[\delta_H] - E[\delta_L]}{2\,(\beta_H + \beta_L)}
\end{aligned}$$

Thus, the change in total social cost (avoided damages less the increase in costs) is:

$$\Delta TSC = (\delta'_L - \delta'_H) \frac{E[\delta_L] - E[\delta_H]}{2\,(\beta_H + \beta_L)} - \frac{(E[\delta_L] - E[\delta_H])^2}{4(\beta_H + \beta_L)}$$

2.2 Uncertain abatement costs.

With no damage uncertainty, the comparison focuses on the difference in emission levels resulting from second-best tax and first-best taxes. In the case of the second best tax, the regulator has imperfect information regarding the slope of each firm's marginal cost function. With perfect information on costs, τ is the optimal undifferentiated tax: $\tau = \frac{(\beta_H \delta_L + \delta_H \beta_L)}{\beta_H + \beta_L}$. In the current case: $\tau = \frac{(b_H \delta_L + \delta_H b_L)}{b_H + b_L}$, where $b_H = E[\beta_H]$, $b_L = E[\beta_L]$. We denote the realization of the cost parameters: $\{\beta'_H, \beta'_L\}$. The first best comparative case features the following $\tau_L = \delta_L$ and $\tau_H = \delta_H$.

We begin by deriving expressions for emissions under both the first-best and second-best tax policies:

$$
\begin{aligned}
e_L^D &= \frac{\alpha_{1L} - (\tau_L)}{2(\beta_{1L})} \\
e_L^U &= \frac{\alpha_{1L} - (\tau)}{2(\beta_{1L})} \\
e_H^D &= \frac{\alpha_{1H} - (\tau_H)}{2(\beta_{1H})} \\
e_H^U &= \frac{\alpha_{1H} - (\tau)}{2(\beta_{1H})}
\end{aligned}
$$

Then we evaluate the social welfare function at these emission levels and determine the change in TSC.

$$
\Delta TSC = (\delta_H(e_H^U) + C_H(e_H^U) - \delta_L(e_L^U) + C_L(e_L^U)) - (\delta_H(e_H^D) + C_H(e_H^D) + \delta_L(e_L^D) + C_L(e_L^D)) \quad (32)
$$

$$
\Delta TSC = \frac{(\delta_L - \delta_H)^2 \left(\beta'_{1H} E[\beta_{1L}^2] + \beta'_{1L} E[\beta_{1H}^2]\right)}{4\beta'_{1L}\beta'_{1H} \left(E[\beta_{1L}] + E[\beta_{1H}]\right)^2} \geq 0 \quad (33)
$$

To build some intuition for how these gains from differentiation vary with the difference between ex ante expected and ex post realized costs, we reformulate the problem. Let $E[\beta_i] = \beta_i$. Let $\beta'_i = \Delta_i + \beta_i,\ i = L, H$. It is now straightforward to investigate how a discrepancy between expected and realized abatement costs affects the gains from policy differentiation in the context of a price-based policy:

$$
\begin{aligned}
&\frac{d}{d\Delta_L}\left((\delta_L - \delta_H)^2 \frac{(\beta_H + \Delta_H)(\beta_L^2) + (\beta_L + \Delta_L)(\beta_H^2)}{4(\beta_L + \Delta_L)(\beta_H + \Delta_H)\left(\beta_L + \beta_H\right)^2}\right) \\
= &-\frac{1}{4}\frac{E[\beta_L]\left(\delta_L - \delta_H\right)^2}{\left(\beta_H + \Delta_H\right)^2 \left(\beta_H + \beta_L\right)^2} < 0
\end{aligned}
$$

$$
\begin{aligned}
& \frac{d}{d\Delta_H}\left((\delta_L-\delta_H)^2\,\frac{(\beta_H+\Delta_H)(\beta_L^2)+(\beta_L+\Delta_L)(\beta_H^2)}{4(\beta_L+\Delta_L)(\beta_H+\Delta_H)\,(\beta_L+\beta_H)^2}\right) \\
= \quad & -\frac{1}{4}\frac{E[\beta_H]\,(\delta_L-\delta_H)^2}{(\beta_L+\Delta_L)^2\,(\beta_H+\beta_L)^2}<0
\end{aligned}
$$

In sum, we find that the gains from policy differentiation are decreasing with the discrepancy Δ_i.

Appendix 3: Marginal Damage Functional Form.

In order to evaluate the plausibility of our assumption that the NOx damage function is linear and additively separable, we conduct a series of auxiliary simulations with the AP2 model. Specifically, the simulations consist of systematically reducing total NOx emissions from EGUs and recomputing the source-specific marginal damage ($/ton) estimates for 565 of the 632 EGUs covered in the analysis. The intent is to test whether the marginal damage is sensitive to the level of emissions and, therefore, ambient concentrations. This design is executed both in the PM2.5 and the O3 modules of AP2 and the resulting marginal damages are then combined to form the estimate of the marginal damage function.

Figure A1 displays the average marginal damage function over the 565 EGUs encompassed in these auxiliary simulations. At observed baseline emission levels, the marginal damage if $1,705/ton NOx. With emissions from these EGUs eliminated (100% reduction), the marginal damage is $1,840/ton NOx. This is an 8% increase.

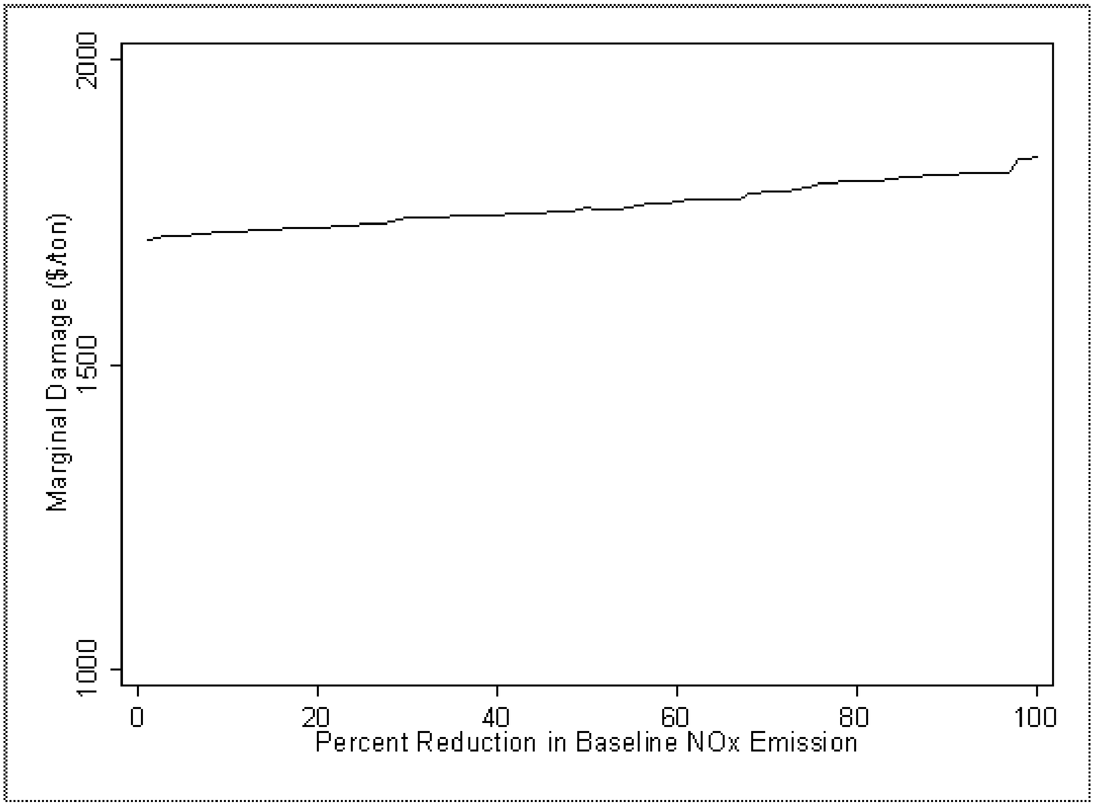

Figure A2 shows the results for Mount Storm Power Station located in West Virginia. This site was chosen because the marginal damages are most sensitive to changes in baseline emission levels. Specifically, the marginal damage increases from $1,000/ton to $1,170/ton over the range of simulated emission levels. This is a 17% increase.

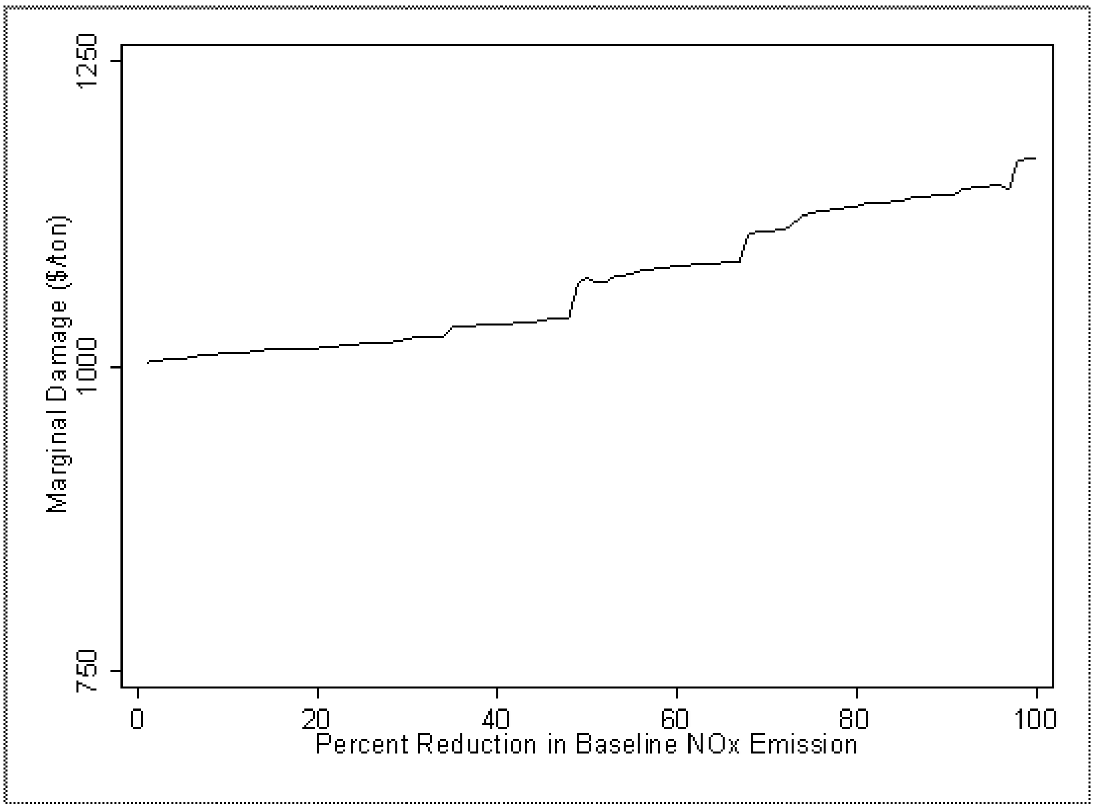

Figure 1: Figure A2: NOx Marginal Damage Function at Mount Storm Power Station.