ADDITIONAL DOCUMENTATION (NOT FOR PUBLICATION)

Small and large price changes and the propagation of monetary shocks

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B Data Appendix

This appendix provides further empirical evidence. Section B.1 describes evidence for the US taken from scanner datasets. Section B.2 offers more detailed statistical information on the French data, including a sectoral split and a robustness-to-trimming exercise. Section B.3 explores the extent to which the statistical protocols used to measure prices are responsible for the small price changes, as suggested by Eichenbaum et al. (2014).

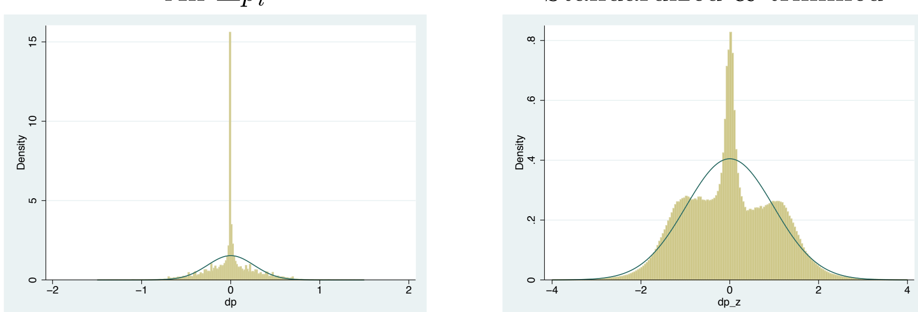
B.1 Evidence from US scanner datasets (weekly prices)

This appendix uses a large scanner data set, also used by Eichenbaum, Jaimovich, and Rebelo (2011), to measure the kurtosis of price changes in a dataset that allows to control for heterogeneity and measurement error in a more precise way than is doable with the typical CPI data. The data contains information on weekly average prices and quantities for over 100 stores (index s), 13,000 goods (index u) over 100 weeks (index t). Price changes are measured by the log changes, i.e. $\Delta p_{t,u,s} = 100(\log(p_{t,u,s}) - \log(p_{t-1,u,s}))$, i.e. the percentage price change (in log points). Since measured prices are weekly averages (measured by total revenues divided by quantity) a change from one week to the other may occur even in the absence of a true price change, if the proportion of customers with discount coupons changes. We think that this is the most prevalent type of measurement error in the scanner type data (where there are no unrecorded product substitutions or transcription errors).

Figure 6: US: Eichenbaum et al (2011) scanner data

Size distribution of non-zero Δp_i (based on average weekly prices)

All Δp_i — Standardized & trimmed



kurtosis = 4.9 ; 11 million obs. — kurtosis = 3.0 ; 8.9 million obs.

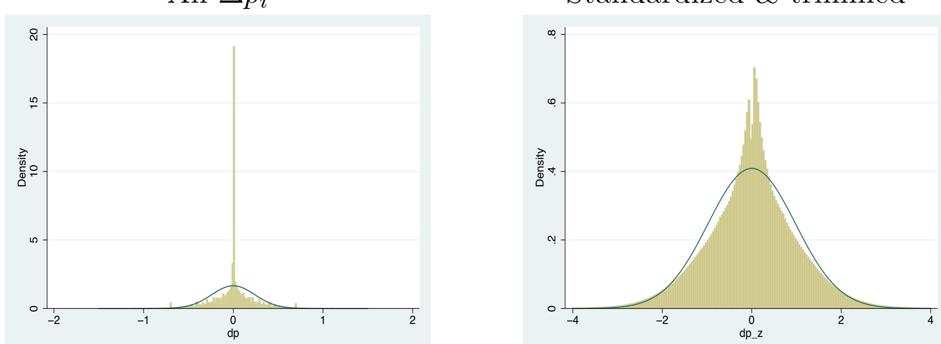
Figure 6 presents the results for the $\Delta p_{i,u,s}$ before any treatment is done to the data (left panel) and after removing the price changes smaller than 1 cent (which for sure reflect a composition effect in the weekly prices, e.g. averaging customers with discount coupons and those without). When removing such price changes, kurtosis drops to 3.8. Standardizing the price changes at the store-good level (and trimming price changes smaller than 1 cent) further reduces kurtosis to around 2.8 (see Alvarez, Lippi, and Pozzi (2014) for a more detailed analysis). These patterns are not very sensitive to the specific trimming that is chosen.

We complement this evidence with a larger dataset covering more stores and more products, the Symphony IRI dataset described in Bronnenberg, Kruger, and Mela (2008). The dataset contains weekly scanner price and quantity data covering a panel of stores in 50 metropolitan areas from January 2001 to December 2011, with multiple chains of retailers for each market. The dataset includes grocery and drug chain data involving approximately 25%-30% of consumer packaged good sales in a grocery store and it contains around 2.4 billion transactions from over 170,000 products and around 3,000 stores. Each outlet has a time invariant identifier and for each retail outlet, weekly data are available at the UPC level. Goods are classified into 31 general product categories such as milk, coffee, beer, etc. Brand information is included but all private-label UPCs have the same brand identification. Detailed information about each good, such as volume and size, is also included. Each retailer reports the total dollar value of weekly sales, inclusive of retail features, displays and retailer coupons but not manufacturer coupons, for each UPC code as well as total units sold. Therefore, the average retail price for each UPC during that week can be recovered. Figure 7 compares the raw data on price changes (no trimming and no standardization) with the data standardized at the store-good level and after discarding price changes outside the interval $0.1\% < |\Delta p_i| < 100\%$.

Figure 7: US: Symphony-IRI scanner data

Size distribution of non-zero Δp_i (based on average weekly prices)

All Δp_i | Standardized & trimmed



kurtosis = 34.3 ; 820 million obs. | kurtosis = 4.3 ; 610 million obs.

B.2 Details on data and further sectoral statistics for France

Some additional features of our data treatment of French CPI price records are as follows.

Dealing with product replacement. The dataset contains flags for product replacement as well as imputed prices which we use as follows to design our dataset. First, we discarded observations with item substitution, as item substitution may result into spurious values for price changes, if quality adjustment is not accounted for or imperfectly measured (Berardi, Gautier, and Le Bihan (2013) investigate the inclusion of information on item substitutions). Second, we replaced any "imputed price" in the dataset, by the previous price of the same item in the same outlet present in the data, i.e. a carry–forward procedure. In the source dataset imputed prices are introduced by the INSEE when prices are missing.[32] Imputed prices are constructed either using the carry–forward procedure, or imputing the average price change of similar goods observed in the close area. The latter procedure makes sense from the aggregate CPI point of view but is obviously ill-suited for characterizing price change at the individual level. We used the flag for imputed prices to locate and replace them by carry-forward prices. This procedure amounts to discarding imputed prices when computing the distribution of (non-zero) price changes.

Computing price changes and dealing with outliers. Price changes were computed as 100 times the log-difference in prices per unit. We compute a consistent price per unit by, when relevant, dividing prices by the indicator of quantity sold (package size). We removed outliers, which in our baseline analysis we define as price changes smaller in absolute value than 0.1 percent, or larger in absolute value than $100 \cdot \log(10/3)$. These thresholds are set as a first crude ways to deal with measurement errors. Some robustness checks are presented in Table 6. The upper threshold for outliers is set with sales in mind, as we informally observe that price rebates as large as 70% are sometime advertised in sales periods. Our threshold allows for a price to decrease by up to 70% and subsequently return to its former level without discarding the observation. Price changes larger than this threshold are discarded as being outliers.[33]

Identifying sales. The flag for sale allows to identify sales. Two kinds of sales-promotion discounts, that have a different status, exist in France: seasonal sales or temporary discounts. Seasonal sales ('soldes') are subject to administrative restrictions: the time period (twice a year) is decided by local authorities and price posting is subject to precise regulations. Temporary discounts are not subject to such restrictions but sales below cost are prohibited by commercial law. By contrast, selling below cost is allowed in the case of seasonal sales. On the sample period, seasonal sales are observed only in some specific categories of goods (mainly clothes). The proportion of price quotes that are flagged as seasonal sales is 0.76% and the proportion of temporary discounts amounts to 1.92%.

Main facts at sectoral level. The different sectors in the CPI have very different pricing patterns, as well documented in recent research. The purpose of this appendix section is to illustrate that the peakedness of the price change distribution is a fact observed in all sectors. Table 4 documents pricing patterns fact using a breakdown into six broad economic sectors.[34] As previous research, we observe many sectoral specificities: prices change less often and rarely decrease in services; the size of price changes is smaller in services; energy prices change frequently and by small amounts; reflecting sales, the variance of price change is huge in clothes. However, noticeably a large kurtosis is observed in all sectors, one exception being clothes for which kurtosis (2.2) is lower than that of the Gaussian distribution. The fraction of small price changes, using one fourth of mean absolute price change as a threshold, ranges between 8% and 27% for all categories other than energy. Using a sector and type

[32] Prices may be missing because of stock-outs, closed outlet due e.g. to holidays or seasonality in product availability, for instance.

[33] An example of outlier is the fee for parking in the street, which is free in some cities in summer.

[34] The breakdown we use (into food; durable goods; clothing & textile; other manufactured goods ; energy; services) is one we deem the most meaningful to capture price-setting idiosyncracies.

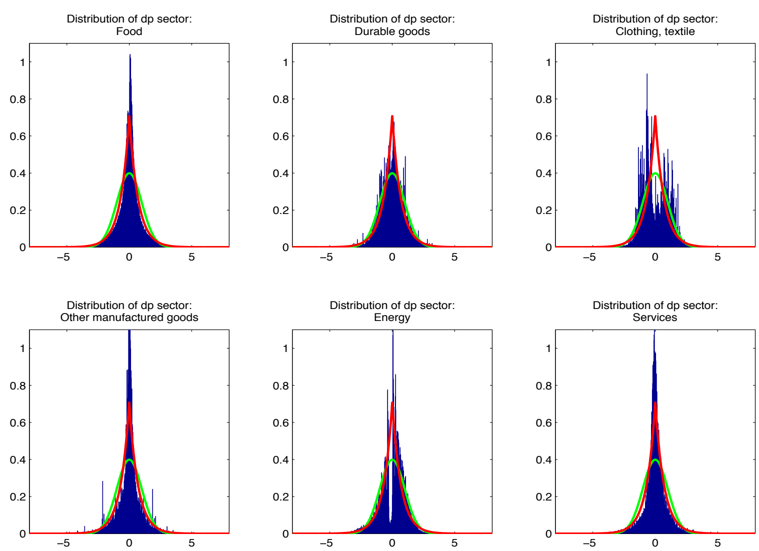
Table 4: Results by type of goods

Good type	Freq	Avg $\lvert\Delta p_i\rvert$	Std (Δp_i)	Kur (Δp)	Frac25
Food	19.4	9.2	8.7	10.8	29.3
Durable goods	15.2	14.7	11.5	6.0	18.1
Clothing	11.0	42.5	19.3	2.2	10.2
Other manufactured goods	11.4	10.3	11.7	9.4	34.0
Energy	77.0	3.8	6.5	6.9	12.1
Services	6.5	7.8	11.4	17.6	21.3

Source is INSEE, monthly price records from French CPI, data from 2003:4 to 2011:4. Coverage is aroud 65% of CPI weight since rents, and prices of fresh food and centrally collected items (e.g. electricity, train and airplane tickets) are not included in the dataset. Freq. denotes monthly frequency of price change in percent. Size of price change Δp are the first-difference in the logarithm of price per unit, expressed in percent. Avg is average, Std standard devation, Frac25 the share of absolute price change that are inferior to 0.25 Avg[$\lvert\Delta p_i\rvert$], Kur denotes kurtosis. Observations with imputed prices or quality change are discarded. Moments are computed aggregating all prices changes using CPI weigths at the product level.

of good partition, Table 5further documents that this fact is consistently observed at higher levels of disaggregation.

Figure 8: Distribution of standardized Price Adjustments by group of goods



Note: The figures uses the elementary CPI data from France 2003-2011 (see the text).

Table 5: Statistics by type of goods and outlet category (un-standardized price changes)

Good type	**Outlet type**	**Freq**	**Avg** $\lvert\Delta p_i\rvert$	**Std** (Δp)	**Kur** (Δp_i)	**Frac25**
Food	Hypermakets	27.56	8.89	8.68	10.25	30.79
Food	Supermarkets	18.84	9.84	9.20	10.57	30.36
Food	Traditional	7.52	7.84	6.66	11.68	15.63
Food	Services	7.14	9.45	8.14	7.52	12.06
Durable goods	Hypermakets	15.82	13.35	11.45	6.36	21.02
Durable goods	Supermarkets	19.11	14.96	11.01	5.52	16.38
Durable goods	Traditional	7.93	14.77	13.15	7.08	22.02
Durable goods	Services	8.02	23.45	17.67	3.36	20.14
Clothing	Hypermakets	8.09	45.13	22.21	1.89	17.41
Clothing	Supermarkets	9.55	43.23	19.19	2.20	10.79
Clothing	Traditional	12.68	41.85	18.77	2.24	7.31
Clothing	Services	10.86	41.20	18.86	1.87	12.53
Other manufactured goods	Hypermakets	15.69	9.40	8.97	11.25	32.71
Other manufactured goods	Supermarkets	12.14	11.87	11.72	7.94	33.99
Other manufactured goods	Traditional	8.22	11.51	15.50	8.16	34.59
Other manufactured goods	Services	11.25	6.59	12.51	12.91	32.85
Energy	Hypermakets	80.89	3.56	6.27	9.23	8.28
Energy	Supermarkets	76.43	3.56	6.30	8.50	8.60
Energy	Traditional	75.55	4.22	6.92	5.39	14.35
Energy	Services	71.93	3.35	6.01	4.69	8.99
Services	Hypermakets	5.13	13.84	15.14	7.71	22.64
Services	Supermarkets	9.99	9.70	10.53	10.33	26.22
Services	Traditional	6.34	7.74	9.04	19.97	19.54
Services	Services	6.41	7.65	11.80	18.30	20.86

Source is INSEE, monthly price records from French CPI, data from 2003:4 to 2011:4. Coverage is around 65% of CPI weight since rents, and prices of fresh food and centrally collected items (e.g. electricity, train and airplane tickets) are not included in the dataset. Freq. denotes monthly frequency of price change in percent. Size of price change Δp are the first-difference in the logarithm of price per unit, expressed in percent. Avg is average, Std standard deviation, Frac25 the share of absolute price change that are inferior to 0.25 Avg$[\lvert\Delta p_i\rvert]$, Kur denotes kurtosis. Observations with imputed prices or quality change are discarded. Moments are computed aggregating all prices changes using CPI weights at the product level.

B.3 Small price changes and measurement error

This appendix examines to what extent the arguments of Eichenbaum et al. (2014) apply to our data and investigates the robustness of our findings to various criteria for trimming the data. Measurement errors may arise for several reasons. Eichenbaum, Jaimovich, and Rebelo (2011) and Eichenbaum et al. (2014) articulate two concerns about the small price change. First they notice that in scanner data studies the price level of an item is typically computed as the ratio of recorded weekly revenues to quantity sold. To the extent that there are temporary or individual specific discounts (say coupons), this will generate spurious small

Table 6: Robustness to trimming

Type of trimming	Flag	Freq.	Avg($\lvert\Delta p_i\rvert$)	Std(Δp)	Frac25	Kur[Δp_i]	Kur[z]
$\lvert\Delta p_i\rvert \leq 100 \cdot \log(2)$	1	17.1	8.5	14.7	28.9	10.2	7.3
Exc. flagged sales	2	14.8	5.0	7.7	18.8	13.6	8.6
$\lvert\Delta p_i\rvert \leq 100 \cdot \log(10/3)$	3	17.2	9.1	16.5	30.3	12.9	9.0
$0.1 \leq \lvert\Delta p_i\rvert \leq 100 \cdot \log(2)$	4	16.9	8.6	14.8	28.5	10.1	7.2
$0.5 \leq \lvert\Delta p_i\rvert \leq 100 \cdot \log(2)$	5	16.5	8.8	15.0	27.1	9.8	6.9
$0.1 \leq \lvert\Delta p_i\rvert \leq 100 \cdot \log(10/3)$ & ex.sales	6	14.7	5.1	8.0	18.2	20.9	10.4
$0.1 \leq \lvert\Delta p_i\rvert \leq 100 \cdot \log(10/3)$	8	17.1	9.2	16.6	29.9	12.8	8.9
$1 \leq \lvert\Delta p_i\rvert \leq 100 \cdot \log(2)$	11	15.3	9.7	15.7	22.5	8.9	6.3

(Table, continued) Moments of standardized price change z (where $m \equiv Avg(|z|)$)

Type of trimming	Flag	Frac($< 0.25m$)	Frac($< 0.5m$)	Frac($> 2m$)	Frac($> 4m$)
$\lvert\Delta p_i\rvert \leq 100 \cdot \log(2)$	1	22.0	39.3	13.1	1.8
Exc. flagged sales	2	20.6	38.6	12.6	2.0
$\lvert\Delta p_i\rvert \leq 100 \cdot \log(10/3)$	3	22.3	39.6	12.9	1.8
$0.1 \leq \lvert\Delta p_i\rvert \leq 100 \cdot \log(2)$	4	21.9	39.1	13.1	1.7
$0.5 \leq \lvert\Delta p_i\rvert \leq 100 \cdot \log(2)$	5	20.9	38.4	12.8	1.6
$0.1 \leq \lvert\Delta p_i\rvert \leq 100 \cdot \log(10/3)$ & ex.sales	6	20.7	38.6	12.5	2.0
$0.1 \leq \lvert\Delta p_i\rvert \leq 100 \cdot \log(10/3)$	8	22.2	39.3	12.9	1.8
$1 \leq \lvert\Delta p_i\rvert \leq 100 \cdot \log(2)$	11	17.7	35.6	12.1	1.3

Source is INSEE, monthly price records from French CPI, data from 2003:4 to 2011:4. Coverage is aroud 65% of CPI weight since rents, and prices of fresh food and centrally collected items (e.g. electricity, train and airplane tickets) are not included in the dataset. Freq. denotes monthly frequency of price change in percent. Size of price change Δp are the first-difference in the logarithm of price per unit, expressed in percent. Avg is average, Std standard devation, Frac25 the share of absolute price change that are inferior to 0.25 Avg[$|\Delta p_i|$], Kur denotes kurtosis. Kur[z] denotes kurtosis of the distribution of standardized price changes. Standardized price changes are computed at the category of good * type of outlet level. Observations with imputed prices or quality change are discarded. Moments are computed aggregating all prices changes using CPI weigths at the product level. Each row describes a sub-sample constructed applying the filter described by the column "type of trimming". "Ex. sales" exclude observations flagged as sales by the INSEE data collectors. The subsample with flag code 8 is taken as the baseline in the main text of the paper.

price changes.[35] Moreover Eichenbaum et al. (2014) highlight a related problem for some CPI items: they spot 27 items (named ELIS in the BLS terminology) that are problematic because these prices are typically computed as a Unit Value Index (a ratio of expenditure to quantity purchased), or they are not consistently recorded in the same outlet, or they are the price of a bundle of goods (for instance the sum of airplane fare and airport tax). We were able to match these items with their counterparts in our French dataset. Out of the 27 problematic items 15 are not present in our data because in the French CPI those

[35] Notice that in principle CPI data are immune from this type of measurement error, as these data are direct transaction prices observed by a field agent. Indeed, in the instance of a temporary discount, the CPI dataset will record either no price change, or the large price change of observed during the discount, if the field agent happens to be collecting data during the temporary discount. Further, the protocol of data collection requires that the field agent records the price faced by a regular customer, not benefiting from individual-specific discounts.

items are not recorded by a field agent but are centrally collected (thus not made available in the subset of CPI we have access to).[36] Concerning the 12 remaining items virtually no price record in the French CPI is computed as a Unit Value Index, which is hypothesized by Eichenbaum et al. (2014) as a major source of small price changes. Inspecting the patterns of price changes over these 12 potentially "problematic" items in our dataset shows that the amount of small price changes is not significantly different from the one detected over the rest of our sample. One exception is the price of "Residential water" where it can be suspected that many small variations in local taxes occur.[37]

A second investigation on measurement error was developed by varying the upper and lower thresholds of small and large price changes used to define outliers. Results are displayed in Table 6. In each of the variants considered in Table 6, both kurtosis and the fraction of small price changes remain large. The lowest level of kurtosis obtains when we use the most stringent thresholds for outliers.

Finally, Table 7 compares the fraction of small price changes in US vs the French data. The table uses the same thresholds of Eichenbaum et al. (2014) to measure the fraction of small price changes. The presence of small price changes (in absolute value) is at first sight a more prominent fact in France than in the US. One factor that may contribute to explaining this pattern is the fact that sales are less prevalent in France. Measurement error, as discussed above, may play a role. We nevertheless observe that, if we define small price changes as relative to the mean average price change, rather than with an absolute threshold, the fraction of small price change appears to be lower in France than in the US, as shown in Table 7.

C Power series representation of density $f(y)$

From equation (6) we can write f as the product of a power of y and the sums of two modified Bessel functions of the first and second kind, multiplied by appropriate constants.

Consider then $n \geq 3$ and n odd, so that $\nu = n/2 - 1$ is not an integer. When n is even the expression for K_ν requires to evaluate the limit, so it is more complicated. Thus, we can write:

$$\left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{4}-\frac{1}{2}\right)} I_{\frac{n}{2}-1}\left(2\sqrt{\frac{\lambda y}{2\sigma^2}}\right) = \left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda y}{2\sigma^2}\right)^i$$

where

$$\beta_{i,\frac{n}{2}-1} \equiv \frac{1}{i!\,\Gamma(i+n/2)}$$

[36] These items are Hospital room in-patient; Hospital in-patient services other than room ; Electricity; Utility natural gas service; Telephone services, local charges ; Interstate telephone services ; Community antenna or cable TV ; Cigarettes; Garbage and trash collection; Airline fares; New cars; New trucks; Ship fares; Prescription drugs and medical supplies; Automobile insurance.

[37] Otherwise, on the bulk of consumption items, there are no local taxes in France, and the main, nation-wide, rate of the Value Added Tax rate did not move over the sample period.

Table 7: Fraction of small price changes: US and French CPI

Moments for the absolute value of price changes: $\lvert\Delta p_i\rvert$	France	US	Normal	Laplace
Average $\lvert\Delta p_i\rvert$	9.2	14.0		
Fraction of $\lvert\Delta p_i\rvert$ below 1%	11.8	12.5		
Fraction of $\lvert\Delta p_i\rvert$ below 2.5%	32.5	24.0		
Fraction of $\lvert\Delta p_i\rvert$ below 5%	57.1	40.6		
Fraction of $\lvert\Delta p_i\rvert$ below $(1/14)\cdot\mathbb{E}(\lvert\Delta p_i\rvert)$	2.4	12.5	4.5	6.9
Fraction of $\lvert\Delta p_i\rvert$ below $(2.5/14)\cdot\mathbb{E}(\lvert\Delta p_i\rvert)$	13.5	24.0	11.3	16.4
Fraction of $\lvert\Delta p_i\rvert$ below $(5/14)\cdot\mathbb{E}(\lvert\Delta p_i\rvert)$	28.7	40.6	22.4	30.0
Number of obs	1,542,586	1,047,547		

Note: For France the source is INSEE monthly price records from the French CPI (2003:4 to 2011:4). Coverage is around 65% of CPI weight since rents, and prices of fresh food and centrally collected items (e.g. electricity, train and airplane tickets) are not included in the dataset. Frequency of price change is monthly, in percent. Size of price change are the first-difference in the logarithm of price per unit, expressed in percent. Data are trimmed as in the baseline of Table 1. Observations with imputed prices or quality change are discarded. Moments are computed aggregating all prices changes using CPI weights at the product level. The US data are taken from Eichenbaum et al. (2014) Table 1, and refer to "Posted price changes" from 1998:1 to 2011:6. The mean absolute size of price changes is taken from Klenow and Kryvtsov (2008) table III where data are from 1998:1 to 2005:1. Figures for the US are weighted and cover around 70% of the CPI (US CPI includes owners equivalent rents, while French CPI does not). In the third panel we compute the threshold for defining small price changes as fraction of the mean so as to match the US figures in column 2 of the second panel. The Normal and Laplace distributions used in the last two columns have a zero mean and, without loss of generality, standard deviation equal to one.

and for ν not an integer

$$\begin{aligned}
\left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{4}-\frac{1}{2}\right)} K_{\frac{n}{2}-1}\left(2\sqrt{\frac{\lambda y}{2\sigma^2}}\right) &= \frac{\frac{\pi}{2}}{\sin\left(\left(\frac{n}{2}-1\right)\pi\right)} \sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda y}{2\sigma^2}\right)^i \\
&- \frac{\frac{\pi}{2}}{\sin\left(\left(\frac{n}{2}-1\right)\pi\right)} \left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{4}-\frac{1}{2}\right)} I_{\frac{n}{2}-1}\left(2\sqrt{\frac{\lambda y}{2\sigma^2}}\right)
\end{aligned}$$

where

$$\beta_{i,1-\frac{n}{2}} \equiv \frac{1}{i!\,\Gamma(i+2-n/2)}$$

This means we can write:

$$\begin{aligned}
f(y) &= \left(C_I - \frac{\frac{\pi}{2}}{\sin\left(\left(\frac{n}{2}-1\right)\pi\right)} C_K\right) \left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda y}{2\sigma^2}\right)^i \\
&+ C_K \frac{\frac{\pi}{2}}{\sin\left(\left(\frac{n}{2}-1\right)\pi\right)} \sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda y}{2\sigma^2}\right)^i
\end{aligned}$$

Since $f(0) > 0$ and

$$f(0) = C_K \frac{\frac{\pi}{2}}{\sin\left((\frac{n}{2}-1)\pi\right)} \beta_{0,1-\frac{n}{2}} = C_K \frac{\frac{\pi}{2}}{\sin\left((\frac{n}{2}-1)\pi\right)} \frac{1}{\Gamma(2-n/2)}$$

then $C_K > 0$. Then to set $f(\bar{y}) = 0$ we obtain:

$$\frac{C_I - \frac{\frac{\pi}{2}}{\sin\left((\frac{n}{2}-1)\pi\right)} C_K}{-\frac{\frac{\pi}{2}}{\sin\left((\frac{n}{2}-1)\pi\right)} C_K} = \frac{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i}{\left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i}$$

Using the expressions for $f(0)$ and $f(\bar{y}) = 0$ we can then rewrite f as:

$$\begin{aligned} f(y) &= -f(0)\, \Gamma\left(2 - \frac{n}{2}\right) \left(\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i\right) \times \\ & \left[\frac{\left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda y}{2\sigma^2}\right)^i}{\left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i} - \frac{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda y}{2\sigma^2}\right)^i}{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i}\right] \end{aligned}$$

Using that $1 = \int_0^{\bar{y}} f(y) dy$ we obtain an expression for $f(0)$ and replacing in the previous formula we obtain:

$$\begin{aligned} f(y) &= \left[\frac{\left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda y}{2\sigma^2}\right)^i}{\left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i} - \frac{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda y}{2\sigma^2}\right)^i}{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i}\right] \Bigg/ \\ & \left[\frac{\left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \frac{\bar{y}}{i+n/2} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i}{\left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i} - \frac{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \frac{\bar{y}}{i+1} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i}{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i}\right] \end{aligned} \tag{32}$$

Remark. While this expression was obtained for $n \geq 2$ and n odd, it does work for any real huber $n \geq 2$ different from an even natural. Since it is continuous on n, the expression equation (32) can be used to obtain the values of f in the case of n is even by taking the limit as n approaches any even natural, or by evaluating at a real number very close to the desired even natural number.

D Power series representation of Kurtosis

Given $(\lambda, \sigma^2, \bar{y})$ the kurtosis of the steady state price distribution can be written as:

$$Kur\left(\Delta p_i\right) = \frac{Q(0)}{\frac{\sigma^4}{N(\Delta p_i)^2}} = \frac{\left(\lambda/\sigma^2\right)^2 Q(0)}{\left(\mathcal{L}(\phi, n)\right)^2}$$

where $Q(y)$ is the expected fourth moment at the time of adjustment τ conditional on having today a squared price gap y, i.e.

$$Q(y) = \mathbb{E}\left(\Delta p_i^4(\tau) \,|\, y(0) = y\right) = \frac{3}{(n+2)n} \, \mathbb{E}\left(y^2(\tau) | y(0) = y\right)$$

where $y(\tau)$ is the value of the squared price gap at the stopping time and where, using results from Alvarez and Lippi (2014), we have that $Kur(\Delta p_i | y) = \frac{3n}{(n+2)}$ and the variance is $Var\left(\Delta p_i \,|\, ||p||^2 = y\right) = y/n$. Notice that for $y \in [0, \bar{y}]$ the function $Q(y)$ obeys the o.d.e.:

$$\lambda Q(y) = \lambda \frac{3y^2}{(n+2)n} + Q'(y)\, n\sigma^2 + Q''(y)\, 2\sigma^2 y$$

with boundary condition $Q(\bar{y}) = \frac{3\bar{y}^2}{(n+2)n}$. Assuming that $Q(y) = \sum_{i=0}^{\infty} a_i y^i$, matching coefficients, and writing them as function of a_0 one obtains:

$$\begin{aligned} a_1(a_0) &= \frac{a_0}{\frac{\sigma^2}{\lambda} n} \;,\; a_2(a_0) = \frac{a_1(a_0)}{2\frac{\sigma^2}{\lambda}(n+2)} \;,\; a_3(a_0) = \frac{a_2(a_0) - \frac{3}{(n+2)n}}{3\frac{\sigma^2}{\lambda}(n+4)} \text{ and} \\ a_{i+1}(a_0) &= \frac{a_i(a_0)}{(i+1)\frac{\sigma^2}{\lambda}(n+2i)} \text{ for } i \geq 3 \end{aligned}$$

Thus $Q(0) = a_0$ is determined as the solution to $\sum_{i=0}^{\infty} a_i(a_0)\, \bar{y}^i = \frac{3\,\bar{y}^2}{(n+2)n}$. After tedious but simple algebra this gives:

$$Q(0) = a_0 = \frac{3n}{(n+2)} \left(\frac{\sigma^2}{\lambda}\right)^2 \left[\frac{\phi^2 + \sum_{i=3}^{\infty} \left(\prod_{j=3}^{i} \frac{n}{j[n+2(j-1)]}\right) \phi^i}{1 + \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{n}{j[n+2(j-1)]}\right) \phi^i}\right]$$

where $\phi \equiv \frac{\lambda \bar{y}}{n\sigma^2}$.

Replacing $Q(0)$ into $Kur\left(\Delta p_i\right) = \frac{\left(\lambda/\sigma^2\right)^2 Q(0)}{\mathcal{L}^2}$ and using equation (5) for $\mathcal{L}(\phi, n)$ we get

$$Kur\left(\Delta p_i\right) = \frac{3n}{(n+2)} \left[\frac{\phi^2 + \sum_{i=3}^{\infty} \left(\prod_{j=3}^{i} \frac{n}{j[n+2(j-1)]}\right) \phi^i}{1 + \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{n}{j[n+2(j-1)]}\right) \phi^i}\right] \left(\frac{1 + \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{n}{j[n+2(j-1)]}\right) \phi^i}{\sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{n}{j[n+2(j-1)]}\right) \phi^i}\right)^2$$

Thus

$$Kur\left(\Delta p_i\right) = \frac{3n}{(n+2)} \frac{\left(\phi^2 + \sum_{i=3}^{\infty} \left(\prod_{j=3}^{i} \frac{n}{j[n+2(j-1)]}\right) \phi^i\right) \left(1 + \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{n}{j[n+2(j-1)]}\right) \phi^i\right)}{\left(\sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{n}{j[n+2(j-1)]}\right) \phi^i\right)^2} \tag{33}$$

For future reference note that $Kur\left(\Delta p_i\right)/N\left(\Delta p_i\right) = (1/\lambda)\mathcal{L}(\phi, n)\, Kur\left(\Delta p_i\right)$ so

$$\frac{Kur\left(\Delta p_i\right)}{N\left(\Delta p_i\right)} \quad = \quad \frac{1}{\lambda}\frac{3n}{(n+2)}\, \frac{\left(\phi^2 + \sum_{i=3}^{\infty}\left(\prod_{j=3}^{i}\frac{n}{j\,[\,n+2(j-1)\,]}\right)\phi^i\right)}{\sum_{i=1}^{\infty}\left(\prod_{j=1}^{i}\frac{n}{j\,[\,n+2(j-1)\,]}\right)\phi^i}$$

Using that

$$\prod_{j=1}^{i}\frac{n}{j\left[\,n+2\left(j-1\right)\,\right]} = \frac{(n/2)^i}{i!}\prod_{j=1}^{i}\frac{1}{\left[\,\frac{n}{2}+\left(j-1\right)\,\right]} = \frac{(n/2)^i}{i!}\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}+i\right)}$$

we can write:

$$\begin{aligned}
\frac{Kur\left(\Delta p_i\right)}{N\left(\Delta p_i\right)} \quad &= \quad \frac{1}{\lambda}\frac{3n}{(n+2)}\, \frac{\left(\phi^2 + 2\,\Gamma\left(\frac{n}{2}+2\right)\,\left(\frac{2}{n}\right)^2\sum_{i=3}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i\right)}{\Gamma\left(\frac{n}{2}\right)\,\sum_{i=1}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i} \\
&= \quad \frac{1}{\lambda}\frac{3n}{(n+2)}\,\frac{2\,\Gamma\left(\frac{n}{2}+2\right)\,\left(\frac{2}{n}\right)^2}{\Gamma\left(\frac{n}{2}\right)}\,\frac{\left((1/2)\,\left(1/\Gamma\left(\frac{n}{2}+2\right)\right)\,\left(\frac{n}{2}\right)^2\phi^2 + \sum_{i=3}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i\right)}{\sum_{i=1}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i} \\
&= \quad \frac{1}{\lambda}\frac{12n}{(n+2)}\,\frac{2\,\left(n/2+1\right)\left(n/2\right)}{n^2}\,\frac{\left(\frac{1}{2\Gamma\left(\frac{n}{2}+2\right)}\left(\phi\, n/2\right)^2 + \sum_{i=3}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i\right)}{\sum_{i=1}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i} \\
&= \quad \frac{6}{\lambda}\,\frac{\sum_{i=2}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i}{\sum_{i=1}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i}
\end{aligned}$$

So that

$$\frac{Kur\left(\Delta p_i\right)}{6\,N\left(\Delta p_i\right)} = \frac{1}{\lambda}\,\frac{\sum_{i=2}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}{\sum_{i=1}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i} \tag{34}$$

From there it is immediate that for a fixed n, this ratio is increasing in $\lambda\bar{y}/\sigma^2$ and that for a fixed $\lambda\bar{y}/\sigma^2$, this ratio is increasing in n.

E Proof that $\lim_{\bar{y}\to\infty}\xi(\sigma^2, r+\lambda, n, \bar{y}) = 0$

Note that, by examining the definition of κ_i and the sums in the expression for ξ we have that:

$$\lim_{\bar{y}\to\infty}\xi(\sigma^2, r+\lambda, n, \bar{y}) = \lim_{\bar{y}\to\infty}\xi\left(1, 1, n, \frac{(r+\lambda)\,\bar{y}}{\sigma^2}\right)$$

so this limit cannot depend on $r+\lambda$ or σ^2. Thus we denote it as:

$$\bar{\xi}(n) \equiv \lim_{\bar{y}\to\infty} \xi(1,1,n,\bar{y})$$

So we have:

$$\bar{y} \approx \frac{\psi}{B}\,(r+\lambda)\,\left[1-\bar{\xi}(n)\right] \quad \text{for large } \psi\,.$$

Now we show that $\bar{\xi}(n)=0$. First we notice that the power series:

$$g(x) = \sum_{i=1}^{\infty}\prod_{s=1}^{i}\frac{1}{(s+2)(n+2s+2)}\;x^i$$

converges for all values of x since its coefficients satisfy the Cauchy-Hadamard inequality. Then we can write:

$$\xi(1,1,n,\bar{y}) \equiv \frac{\frac{2(n+2)}{\bar{y}}\frac{1}{g(\bar{y})}+\frac{1}{g(\bar{y})}+\frac{1}{\bar{y}^2}}{\frac{2(n+2)}{\bar{y}}\frac{1}{g(\bar{y})}+2\frac{1}{g(\bar{y})}+\sum_{i=1}^{\infty}\omega(i,\bar{y})\;(2+i)}$$

where the weights $\omega(i,\bar{y})$ are given by:

$$\omega(i,x) = \frac{\frac{x^i}{\prod_{s=1}^{i}(s+2)(n+2s+2)}}{\sum_{j=1}^{\infty}\prod_{s=1}^{j}\frac{1}{(s+2)(n+2s+2)}\;x^j}$$

Note that for higher x the weights of smaller i decrease relative to the ones for higher i. Now since $g(\bar{y})\to\infty$ as $\bar{y}\to\infty$, then:

$$\bar{\xi}(n) = \frac{1}{\lim_{\bar{y}\to\infty}\sum_{i=1}^{\infty}\omega(i,\bar{y})\;(2+i)}$$

To show that $\bar{\xi}(n)=0$, suppose, by contradiction that is finite. Say, without loss of generality that equals $j+2$ for some integer j. Note that, by the form of the $\omega's$ and because $g(\bar{y})$ diverges as $\bar{y}$ gets large enough, then by any j and $\epsilon>0$ there exist a y^* large enough so that $\sum_{i=1}^{j}\omega(i,\bar{y})<\epsilon$ for any $\bar{y}>y^*$. Thus, the expected value must be larger than $2+j$.

Finally, we consider the case of $n\to\infty$. In this case we have that, the value function divided by n gives:

$$v = \min_T B\int_0^T \sigma^2\,t\,e^{-(\lambda+r)}\,dt + e^{-(r+\lambda)T}(\Psi+v)$$

where $\Psi=\lim_{n\to\infty}\psi/n$. The first order condition for T gives, for a finite T:

$$0 = \left(B\,\sigma^2\,T-(r+\lambda)\Psi\right)-(r+\lambda)e^{-(r+\lambda)T}v \tag{35}$$

Now consider the case where $\Psi\to\infty$. Note that v is finite since $T=\infty$, a feasible strategy as a finite value. Also let $\bar{Y}=\sigma^2T=\lim_{n\to\infty}\frac{\bar{y}(n)}{n}$. Note that as $\Psi\to\infty$ then $\bar{Y}$ must also diverge towards ∞. Dividing the previous expression by Ψ:

$$\frac{\bar{Y}}{\Psi} = \frac{(r+\lambda)}{B} + (r+\lambda)e^{-(r+\lambda)T}\frac{v}{\Psi}$$

and taking the limits:

$$\lim_{\Psi\to\infty}\frac{\bar{Y}}{\Psi} = \frac{r+\lambda}{B} \quad . \quad \square$$

F Note on Solutions of value function $v(y)$, expected time to adjust $\mathcal{T}(y)$ and invariant density of the squared price gap $f(y)$.

First we state a proposition which gives an explicit closed form solution to the value function $v(y)$ in the inaction region, i.e. for $y \in (0, \bar{y})$ subject to $v(0) < \infty$. The solution is parameterized by $\beta_0 = v(0)$.

Proposition 10 *Let $\sigma > 0$. The ODE in equation (4) is solved by the analytical function: $v(y) = \sum_{i=0}^{\infty} \beta_i\, y^i$, for $y \in [0, \bar{y}]$ where, for any β_0, the coefficients $\{\beta_i\}$ solve: $\beta_0 = \frac{n\sigma^2}{r}\beta_1$, $\beta_2 = \frac{(r+\lambda)\beta_1 - B}{2\sigma^2(n+2)}$, $\beta_{i+1} = \frac{r+\lambda}{(i+1)\sigma^2\ (n+2i)}\ \beta_i$ for $i \geq 2$.*

The function described in this proposition allows to fully characterize the solution of the firm's problem. One can use it to evaluate the two boundary conditions described above, value matching and smooth pasting, and define a system of two equations in two unknowns, namely β_0 and $\bar{y}$.

The alert reader may have noticed that to solve for the invariant density f we have followed a standard procedure, i.e. set a 2nd order ordinary linear difference equation (the Kolmogorov forward equation) and find its solutions in terms of two constant, and using two boundary conditions to find the value of the constants. Instead to solve for v and $\mathcal{T}$ we have followed a different approach, we guess an infinite expansion around $y = 0$ and compute its coefficients. Additionally, it may have looked that we did not provide enough boundary conditions to be able to solve for $\mathcal{T}$ and v. For instance, for $\mathcal{T}$ we gave only one equation as boundary conditions, namely $\mathcal{T}(\bar{y}) = 0$. Here we explain that we could have followed the more standard route, which required an analysis of the behavior close to the $y = 0$ boundary, to set one constant to zero and also would have produced a less informative result, i.e. one in terms of modified Bessel functions. Nevertheless we include it here for completeness.

Note that $v(y)$, $\mathcal{T}(y)$ and $f(y)$ are solutions to a linear ODE on y whose homogeneous component, say $q(\cdot)$, solves :

$$y\, q''(y) + a\, q'(y) + b\, q(y) \;=\; 0 \tag{36}$$

for $y \in [0, \bar{y}]$, for (different) constants a and b, with different particular solution, and different boundary conditions. The general solution of the homogeneous equation (36) is given by:

$$q(y) = |b\, y|^{(1-a)/2}\left[C_1 I_\nu\left(2\sqrt{|b\, y|}\right) + C_2 K_\nu\left(2\sqrt{|b\, y|}\right)\right] \tag{37}$$

provided that $b\,y < 0$, i..e. that $b < 0$, where C_1 and C_2 are arbitrary constants, $\nu = |1-a|$ and where I_ν and K_v are the modified Bessel functions of the first and second kind respectively. The values of $b = -\lambda/(2\sigma^2)$ in the three cases. The value of $a = n/2$ for $\mathcal{T}$ and for v, which are the same Kolmogorov backward equation, and $a = -(n/2-2)$ for f, which is the Kolmogorov forward equation.

It is important to notice the behavior of $I_\nu(z)$ and $K_\nu(z)$ for values of $0 < z$ but very close to zero. We have:

$$I_\nu \backsim \frac{1}{\Gamma(\nu+1)}\left(\frac{z}{2}\right)^\nu \tag{38}$$

and

$$K_\nu \backsim \begin{cases} \frac{\Gamma(\nu+1)}{2}\left(\frac{2}{z}\right)^\nu & \text{if } \nu > 0 \\ -\log(z/2) - \gamma & \text{if } \nu = 0 \end{cases} \tag{39}$$

We thus have that each of the solution will behave as:

$$\begin{aligned} I_{|1-a|}\left(y^{1/2}\right)\; y^{(1-a)/2} \quad &\backsim \quad \frac{1}{\Gamma(|1-a|+1)}\left(\frac{y^{1/2}}{2}\right)^{|1-a|} \quad y^{(1-a)/2} \\ &= \quad \frac{1}{\Gamma(|1-a|+1)}\left(\frac{1}{2}\right)^{|1-a|} \quad y^{(1-a)/2+|1-a|/2} \end{aligned}$$

So if $1-a = -|1-a|$, i.e. if $1-a \leq 0$, the value of this product is finite at $y \downarrow 0$. Otherwise it diverges to ∞. Likewise for $\nu = |1-a| > 0$:

$$\begin{aligned} K_{|1-a|}\left(y^{1/2}\right)\; y^{(1-a)/2} \quad &\backsim \quad \frac{\Gamma(|1-a|+1)}{2}\left(\frac{2}{y^{1/2}}\right)^{|1-a|} \quad y^{(1-a)/2} \\ &= \quad \frac{\Gamma(|1-a|+1)}{2}\left(\frac{2}{1}\right)^{|1-a|} \quad y^{(1-a)/2-|1-a|/2} \end{aligned}$$

So if $1-a = |1-a|$, i.e. if $1-a \geq 0$, the value of this product is finite at $y \downarrow 0$. Otherwise it diverges to ∞. The case of $\nu = 0$ i.e. $a = 1$ is special, but $K_0(z)$ also diverges and $I_0(z)$ converges to a non-zero constant as $z \downarrow 0$.

Note that $v(0)$ and $\mathcal{T}(0)$ are both finite. For these two cases the Kolmogorov backward equation has $a = n/2$ so $1-a \geq 0$ iff $n \geq 2$. In these cases we have that C_2, the constant associated with K_ν must be zero. We can use the constant C_1 to impose the boundary condition $\mathcal{T}(\bar{y}) = 0$ for $\mathcal{T}$ and to have a one dimensional representation of v in the range of inaction given $\bar{y}$. Then we can use smooth pasting and value matching, i.e. two boundary conditions, to find the constants C_1 and $\bar{y}$.

Note that for f we don't require that $f(0)$ be zero, since the density at zero gap can be infinite if the y mean reverts to zero fast enough. Thus in this case we will, in general, have both constants be non-zero.

G The price response to a monetary shock

To compute the IRF of the aggregate price level we find the contribution to the aggregate price level of each firm at the time of the shock. They start with price gaps distributed according to g, the invariant distribution. Then the monetary shock displaces them, by subtracting the monetary shock δ to each of them. After that we divide the firms in two groups. Those that adjust immediately and those that adjust at some future time. Note that, for each firm in the cross section, it suffices to keep track only of the contribution to the aggregate price level of the first adjustment after the shock because after that one the future contributions are all equal to zero in expected value. Now we develop the notation to define the impulse response of the aggregate price level.

Let $g\left(p; n, \lambda/\sigma^2, \bar{y}\right)$ be the density of firms with price gap vector $p = (p_1, ..., p_n)$ at time $t = 0$, just before the monetary shock, which corresponds to the invariant distribution with constant money supply. The density g equals the density f of the steady state square norms of the price gaps given by Lemma 2 evaluated at $y = p_1^2 + \cdots + p_n^2$ times a correction for area of sphere and the different variables.[38] In particular we have

$$g\left(p_1, ..., p_n\,; n, \frac{\lambda}{\sigma^2}, \bar{y}\right) = f\left(p_1^2 + \cdots + p_n^2\,; n, \frac{\lambda}{\sigma^2}, \bar{y}\right) \frac{\Gamma\left(n/2\right)}{\pi^{n/2}\ \left(p_1^2 + \cdots + p_n^2\right)^{(n-2)/2}} \tag{40}$$

To define the impulse response we introduce two extra pieces of notation. First we let $\{(\bar{p}_1(t,p), \ldots, \bar{p}_n(t,p))\}$ the process for n independent BM, each one with variance per unit of time equal to σ^2, which at time $t = 0$ start at p, so $\bar{p}_i(0,p) = p_i$. We also define the stopping time $\tau(p)$, also indexed by the initial value of the price gaps p as the minimum of two stopping times, τ_1 and $\tau_2(p)$. The stopping time τ_1 denotes the first time since $t = 0$ that jump occurs for a Poisson process with arrival rate λ per unit of time. The stopping time $\tau_2(p)$ denotes the first time that $||\bar{p}(t,p)||^2 > \bar{y}$. Thus $\tau(p)$ is the first time a price change occurs for a firm that starts with price gap p at time zero. The stopped process $\bar{p}(\tau(0), p)$ is the vector of price gaps at the time of price change for such a firm.

The impulse response for the aggregate price level, of which Figure 9 displays several cases, can be written as:

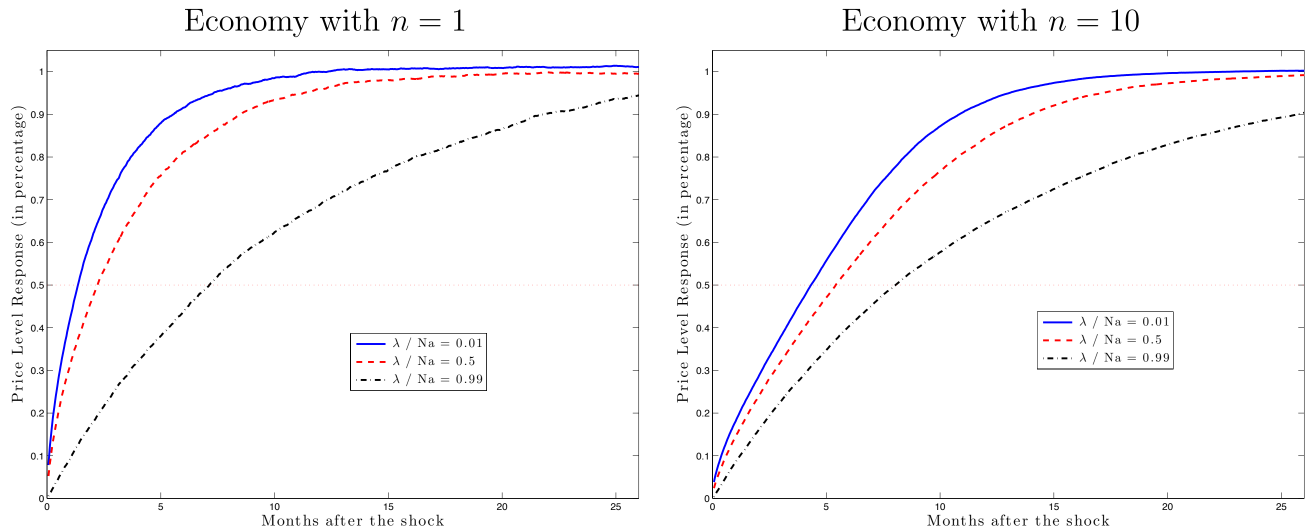
$$\mathcal{P}(t, \delta; \sigma, \lambda, \bar{y}) \;\; = \;\; \Theta(\delta; \sigma, \lambda, \bar{y}) \, + \int_0^t \theta(\delta, s; \sigma, \lambda, \bar{y})\, ds\,, \tag{41}$$

where $\Theta(\delta)$ gives the impact effect, the contribution of the monetary shock δ to the aggregate price level on impact, i.e. at the time of the monetary shock. The integral of the θ's gives the remaining effect of the monetary shock in the aggregate price level up to time t, i.e. $\theta(\delta, s)ds$ is the contribution to the increase in the average price level in the interval of times $(s, s+ds)$ from a monetary shock of size δ. Instead the functions θ and Θ are easily defined in terms of the density g, the process $\{\bar{p}\}$ and the stopping times τ:

$$\Theta(\delta; \sigma, \lambda, \bar{y}) \equiv \int_{||p(0)-\iota\delta|| \geq \bar{y}} \left(\delta - \frac{\sum_{j=0}^n p_j(0)}{n}\right) g\left(p(0); n, \frac{\lambda}{\sigma^2}, \bar{y}\right)\, dp_1(0) \cdots dp_n(0)$$

[38] See Section 5 of Alvarez and Lippi (2014) for this result and the Online appendix for a derivation.

Figure 9: CPI response to a monetary shock of size $\delta = 1\%$



The figures represent an economy with $N(\Delta p_i) = 1.0$ and $std(\Delta p_i) = 0.10$.

and $\theta(\delta, t; \sigma, \lambda, \bar{y})$ is the density, i.e. the derivative with respect to t of the following expression:

$$\int_{||p(0)-\iota\delta||<\bar{y}} \mathbb{E}\left[-\frac{\sum_{j=0}^{n} \bar{p}_j\left(\tau(p), p\right)}{n} \mathbf{1}_{\{\tau(p)\leq t\}} \,\middle|\, p = p(0) - \iota\delta\right] g\left(p(0); n, \frac{\lambda}{\sigma^2}, \bar{y}\right) dp_1(0)\cdots dp_n(0)$$

where ι is a vector of n ones. This expression takes each firm that has not adjusted price on impact, i.e. those with $p(0)$ satisfying $||p(0) - \iota\delta|| < \bar{y}$, weights them by the relevant density g, displaces the initial price gaps by the monetary shock, i.e. sets $p = p(0) - \iota\delta$, and then looks a the (negative) of the average price gap at the time of the first price adjustment, $\tau(p)$, provided that the price adjustment has happened before or at time t. We make a few remarks about this expression. First, price changes equal the negative of the price gaps because price gaps are defined as prices minus the ideal price. Second, we define θ as a density because, strictly speaking, there is no effect on the price level due to price changes at *exactly* time t, since in continuous time there is a zero mass of firms adjusting at any given time. Third, we can disregard the effect of any subsequent adjustment because each of them has an expected zero contribution to the average price level. Fourth, the impulse response is based on the steady-state decision rules, i.e. adjusting only when $y \geq \bar{y}$ even after an aggregate shock occurs.

Given the results in Proposition 3 -Proposition 4 we can parametrize our model either in terms of $(n, \lambda, \sigma^2, \psi/B)$ or instead parametrize it, for each n, in terms of the implied observable statistics $(N(\Delta p_i), Std(\Delta p_i), \ell)$. These propositions show that this mapping is indeed one-to-one and onto. We refer to ℓ as an "observable" statistic, because we have shown that the "shape" of the distribution of price changes depends only on it.

Proposition 11 *Fix an economy whose firms produce n products and with steady state statistics $(N(\Delta p_i), Std(\Delta p_i), \ell)$. The cumulative proportional response of the aggregate price level $t \geq 0$ periods after a once and for all proportional monetary shock of size δ can be obtained from the one of an economy with one price change per period and with unitary standard deviation of price changes as follows:*

$$\mathcal{P}\left(t\,,\,\delta\,;\,N(\Delta p_i)\,,\,Std(\Delta p_i)\right) = Std(\Delta p_i)\;\mathcal{P}\left(t\,N(\Delta p_i)\,,\,\frac{\delta}{Std(\Delta p_i)}\,;\,1\,,\,1\right)\;. \tag{42}$$

This proposition extends the result of Proposition 8 in Alvarez and Lippi (2014) to the case of $\ell \equiv \lambda/N(\Delta p_i) > 0$.[39]

Proof. (of Proposition 11). The proof proceeds by verification. It is made of three parts. First we introduce a discrete-time, discrete-state version of the model. Second we show the scaling of time with respect to N_a , and finally the homogeneity of degree one with respect to $Std(\Delta p_i)$ and δ. The step by step passages are reported in the online Appendix I.

The proposition establishes that the shape of the impulse response is completely determined by 2 parameters: n and ℓ, whose comparative static is explored in Figure 9. Economies sharing these parameters but differing in terms of $N(\Delta p_i)$ or $Std(\Delta p_i)$ are immediately analyzed by rescaling the values of the horizontal and/or vertical axis. In particular, a higher frequency of price adjustments will imply that the economy "travels faster" along the impulse response function (this is the sense of the rescaling the horizontal axis). Instead, the effect of a larger dispersion of price changes is seen by rescaling the monetary shock δ by $Std(\Delta p_i)$ and by a proportional scaling of the vertical axis. A further simplification to the last result is given by next corollary, showing that for small values of the monetary shocks one can overlook the scaling by $Std(\Delta p_i)$ so that, for a given n and ℓ determining the shape, the most important parameter is the frequency of price changes $N(\Delta p_i)$:

Corollary 1 *For small monetary shocks $\delta > 0$, the impulse response is independent of $Std(\Delta p_i)$. Differentiating equation (42) gives:*

$$\mathcal{P}\left(t\,,\,\delta\,;\,N(\Delta p_i)\,,\,Std(\Delta p_i)\,\right) = \delta\,\frac{\partial}{\partial\delta}\,\mathcal{P}\left(t\,N(\Delta p_i)\,,\,0\,;\,1\,,\,1\,\right) + o(\delta)$$

for all $t > 0$ and, since $f(\bar{y}) = 0$, then the initial jump in prices can be neglected, i.e.:

$$\mathcal{P}\left(0\,,\,\delta\,;\,N(\Delta p_i)\,,\,Std(\Delta p_i)\,\right) \equiv \Theta_{n,\ell}\left(\delta; Std\left(\Delta p_i\right)\right) = o(\delta)\;.$$

H Fat-tailed shocks

This appendix compares the baseline multi-product model with random free adjustment opportunities with an otherwise "equivalent" multi-product model with fat-tailed shocks to costs. We present three propositions which show that:

[39] The proof in Alvarez and Lippi is constructive in nature, exploiting results from applied math on the characterization of hitting times for brownian motions in hyper-spheres, which is not longer valid for $\lambda > 0$. Here we use a different strategy which relies on limits of discrete-time, discrete state approximations.

1. If the fat-tailed shocks are sufficiently large, the threshold for adjustment is the same as in the model with random free adjustment opportunities.

2. The distribution of price changes with fat-tailed shock is different, since it includes the large shocks, and thus it contributes to kurtosis by mostly adding large price changes.

3. Since the model has more parameters, mainly the distribution of the fat-tailed shocks, it can capture more behavior, or putt it differently it is hard to identify the parameters with the same observations.

Set-up with fat-tailed shocks. Assume that the price gap for each product evolve as follows:

$$dp_i(t) = \sigma\, dW_i(t) \,+\, \xi_i(t)\, d\mathcal{N}(t) \quad \text{for } i = 1, ..., n$$

where $\mathcal{N}(t)$ is the counter of a Poisson process with intensity $\lambda \geq 0$. When $d\mathcal{N}(t) = 1$, the price gap has a change of size ξ_i. The vector $\xi = (\xi_1, ..., \xi_n)$, is drawn from a joint distribution with cdf Q, assumed to be symmetric around zero. Furthermore we assume that the marginal distribution of each of the coordinates of the vector ξ are identical. Each of the W_i are standard BMs, independent across products i. The realization of the Poisson counter $\mathcal{N}$ is common across the n products. This stylized set-up is meant to capture a generalization (to any $n \geq 1$) of the fat-tailed shocks in Midrigan's (2011) model.

Value function. The state of the problem is $p = (p_1, ..., p_n)$. For all $p \in \mathbb{R}^n$ the value function must solve the variational inequalities:

$$\begin{aligned} r\, v\,(p_1, ..., p_n) \leq {} & B \left[\sum_{i=1}^{n} p_i^2 \right] + \sum_{i=1}^{n} \frac{\sigma^2}{2}\, v_{ii}(p_1, ..., p_n) && (43) \\ & + \lambda \int \cdots \int \min\left\{0\,,\, \psi + v(0) - v\,(p_1 + \xi_1, ..., p_n + \xi_n)\right\} dQ\,(\xi_1, \cdots, \xi_n)\ , \\ v\,(p_1, ..., p_n) \leq {} & v(0) + \psi\ . && (44) \end{aligned}$$

with at least one of the two inequalities holding as equality at each vector $p \in \mathbb{R}^n$.

The variational inequality (43)-(44) has the advantage that it does not presume in the form of the control and of the inaction region, which is a delicate issue for a stopping time with fixed cost and jumps in the state. Nevertheless a function that solves this variational inequality must be the solution of the problem.

This problem has more parameters. As in the problem with free adjustment opportunities we have one integer and four positive scalars: n, B, σ^2, r, ψ. The new element on this formulation is the function $Q : \mathbb{R}^n \to [0, 1]$ for the distribution of the fat-tailed shocks.

No small fat-tailed shock. We assume that the shocks are bounded below. We consider distributions where:

$$0 < \underline{\xi} \equiv \inf ||\xi||\ :\ (\xi_1, ..., \xi_n) \in \text{supp } Q \text{ and } \xi \neq 0\ , \qquad (45)$$

so that the minimization is on the support of Q for which not all the coordinates are equal to zero. The assumption that $\underline{\xi} > 0$ is very natural for the case of one product to capture fat-tails. It is not clear what is the most natural generalization for multi-product case of $n > 1$. Note that the assumption so far allows that the components of ξ_i to be independent or not. In the case of independence, we can either assume that each coordinate of ξ_i as a strictly positive support, or otherwise assume that there is a mass point at zero and that the remaining of the support is strictly positive. In the latter case, the probability of the event in which all the coordinates are simultaneously zero, can be ignored but suitable rescaling λ.[40]

Simple threshold policies. We say that a firm follows a simple threshold policy if there is a threshold $\bar{y} > 0$ for which the firm adjust its price the first time that $||p||^2 \geq \bar{y}$ reverting its price gap to 0 in all components. Moreover, we require that the firm change prices every time a fat-tailed shock arrives.

Note that a possibility is that for the general case of a stopping time with fixed cost and jumps in the state there is the possibility that the optimal policy will be given by a inaction region made of the union of disconnected sets, see Alvarez and Lippi (2013) for a discussion and references on the applied math literature. The next lemma gives an intermediate step to be used for the characterization of simple threshold policies in the proposition below. The lemma finds a lower bound on the support of the shocks, relative to the threshold for the norm of the state, so that the shocks will always take the post-shock state outside the region where it is smaller than the threshold.

Lemma 5 *Let p be a vector satisfying $||p||^2 \leq \bar{y}$, and consider the size of the square norm of the state after the occurrence of a large shock $||p+\xi||^2$. Then*

$$||p+\xi||^2 \geq \bar{y} \text{ for all } \xi \text{ for which } ||\xi|| \geq \left(1+\sqrt{2}\right)\sqrt{\bar{y}}\,.$$

The next proposition gives a straightforward way to characterize simple threshold policies. It says that if the fat-tailed shocks are sufficiently large so that they always trigger an adjustment, then one can use the same formulas than for the model with free adjustment opportunities, for which we have a complete characterization of $\bar{y}$.

Proposition 12 *Optimality of simple threshold policies for any $n \geq 1$. Let $\bar{y} \geq 0$ be the optimal threshold for the problem with free adjustment opportunities at rate $\lambda \geq 0$ but without fat-tailed shocks. Then consider the problem with fat-tailed shocks ξ that occur with Poisson rate $\lambda \geq 0$ but without free adjustment opportunities. Assume that the support of the large shocks satisfies $\underline{\xi} \geq \left(1+\sqrt{2}\right)\sqrt{\bar{y}}$. The optimal policy for this problem is a simple threshold policy with the same value $\bar{y}$ as in the problem with free adjustment opportunities.*

The previous proposition highlights the similarities in the determination of the threshold $\bar{y}$ between random menu cost and fat-tailed shocks. We remark that even if the policy is not

[40] Another case is the one in which only one product is subject to a large shock, a case we refer to a the isolated shock case. This is equivalent to make the Poisson shock independent for the arrival of the large changes to be independent across the n products.

simple, under mild conditions (such as independence across products) it will be of threshold type for the square norm of the vector; the difference is that characterization of the threshold requires a different analysis. The following results explore the difference implications for price changes for a given threshold.

Comparison of price changes. The distribution of price changes of a model with free-adjusment opportunities and one with fat-tailed shocks are different. The main difference is that in the fat tailed shock model considered above, every time that a ξ shock occur there are (some) *large* change in prices. Thus, fat-tailed shocks contribute to kurtosis mostly by having more frequent large shocks. Certainly, relative to the model with free-adjustment opportunities they contribute more to large price changes than to small price changes. In this context to have more frequent small price changes, the fat-tailed model relies on the multi product features, as in Midrigan (2011), as can be easily seen in the version with $n = 1$ in which the fat-tailed model has no small price changes. This implies that the peakeadness of the distribution of price changes around $\Delta p = 0$ is mostly determined by the multi-product feature of the model. For a more thorough analysis we need to add specify more about the distribution Q.

Independent shock case. Assume that each of the component of ξ_i are independently drawn, and that satisfy equation (45). Thus, without loss of generality, assume that $\xi_i > 0$ with probability one for each product $i = 1, ..., n$. Note that in this case the lower bound of the support in each dimension satisfy:

$$\underline{\xi}_i \geq \left(1 + \sqrt{2}\right) \sqrt{\bar{y}/n} \tag{46}$$

Note that in this case it is possible to have, in some component, very small price changes when there is a fat tail shock. Of course, if $\underline{\xi}_i$ is large enough that is not possible. In this case the marginal distribution is simpler to compute because it is the sum between the marginal distribution in the model with free adjustment opportunities (conditional on $||p||^2 < \bar{y}$ plus the (marginal) distribution of ξ_i.)

Proposition 13 *Assume that the fat-tailed shocks are independent across products and that $n \geq 3$. The resulting distribution of price changes is less peaked around small price changes when compared with the model with the same value of λ describing the arrival of free adjustment opportunities. In particular the level of the density is smaller around zero and the second derivative of this density around zero price changes is larger for the model with fat-tailed shocks.*

The proposition deals with the case of $n \geq 3$ because for $n = 2$ the distribution does not have a unique mode at $\Delta p_i = 0$, and indeed has density diverging to ∞ at values discreetly away from $\Delta p_i = 0$.

Lack of identification. The version of the model we wrote has more parameters than the equivalent model with free adjustment opportunities. In particular, there is a whole new function Q. Because of this, without looking at more evidence it allows many more possibilities. To illustrate this we take it to a extreme and show a lack of identification result.

Proposition 14 *Let w be an arbitrary distribution of price changes. There are parameters ψ and a function Q for which the model produces a distribution of price changes arbitrary close to the w.*

The proof of this proposition is trivial. Since we have shown that $\bar{y}$ is decreasing in the cost ψ and that $\bar{y}$ tends to zero as $\psi \downarrow 0$, then we let $\underline{\xi}$ be arbitrarily close to zero and allow almost all the price changes to happen a the time of large shocks, i.e. we can set $\Delta p_i = -\xi$. In other words, in a world with (almost) no menu cost, price changes will occur only because there are cost changes, and they will mirror them. Note that his is consistent with few price changes, because if cost changes happen infrequently so will price changes. This is clearly an extreme result, but highlights the need to think about identification of the objects on this version of the model.

Proofs.

Proof. (of Lemma 5)

$$\begin{aligned}||p+\xi||^2 &= \sum_{i=1}^{n}(p_i+\xi_i)^2 = ||p||^2 + ||\xi||^2 + 2\sum_{i=1}^{n}\xi_i\, p_i \\ &\geq ||p||^2 + ||\xi||^2 - 2\,||p||\,||\xi||\end{aligned}$$

where the inequality follows from the Cauchy-Schwarz inequality: $|\sum_{i=1}^{n} p_i\, \xi_i| \leq ||p||\,||\xi||$. Thus if $||p||^2 \leq \bar{y}$ and $||\xi|| \geq \kappa\sqrt{\bar{y}}$ then

$$\begin{aligned}||p+\xi||^2 &\geq ||p||^2 + ||\xi||^2 - 2\,||p||\,||\xi|| \geq ||\xi||\,(||\xi|| - 2\,||p||) \\ &\geq \kappa\,\sqrt{\bar{y}}\left(\kappa\,\sqrt{\bar{y}} - 2\,\sqrt{\bar{y}}\right) = \bar{y}\,\kappa\,(\kappa\, - 2)\end{aligned}$$

Hence taking $\kappa \geq 1 + \sqrt{2}$ we obtain the desired result. □

Proof. (of Proposition 12)We have shown that the solution of the value function for the problem with free adjustment opportunities but without the large shocks can be obtained by solving the value function $\hat{v}$ and the simple policy given by threshold y so that:

$$(r+\lambda)\,\hat{v}(p) = B\,||p||^2 + \frac{\sigma^2}{2}\sum_{i=1}^{n}\hat{v}_{ii}(p) + \lambda\hat{v}(0) \text{ for all } p : ||p||^2 \leq \bar{y} \tag{47}$$

$$\hat{v}(p) = v(0) + \psi\ , \quad \text{and } \hat{v}_i(p) = 0 \text{ for all } p : ||p||^2 = \bar{y} \tag{48}$$

Now consider the problem without free adjustment opportunities but with fat-tailed shocks. We use a guess and verify strategy. The first part obtains a value function which satisfies the pde in the inaction and the boundary conditions using the same threshold $\bar{y}$. Here we use Lemma 5 which implies that every fat-tailed shocks takes the state out the inaction region and hence leads to an adjustment. Then the value function in inaction and boundary conditions are:

$$(r+\lambda)\,v(p) = B\,||p||^2 + \frac{\sigma^2}{2}\sum_{i=1}^{n} v_{ii}(p) + \lambda\,(v(0)+\psi) \;\text{ for all } p : ||p||^2 \leq \bar{y} \tag{49}$$

$$v(p) = v(0) + \psi\;, \quad \text{and } v_i(p) = 0 \text{ for all } p : ||p||^2 = \bar{y} \tag{50}$$

That v solves equation (49) and equation (50) follows by setting $v(p) = \hat{v}(p) + a$. The only difference is that when the Poisson shock occurs the adjustment cost ψ is paid. Subtracting one equation from the other in the inaction region:

$$(r+\lambda)a = \lambda\psi$$

so that $a = -\lambda\psi/(r+\lambda)$ or $v(p) = \hat{v}(p) + \lambda\psi/(r+\lambda)$. Furthermore, the boundary conditions are also satisfied since the constant either does not affect the derivative or cancel in both sides of the equation. Finally, one can use the shape of the function $\hat{v}$, which is increasing in $||p||^2$, to show that the variational inequalities (43)-(44) are satisfied. $\square$

Proof. (of Proposition 13) The proof proceed by obtain an expression for the marginal distribution of price changes for the case of fat-tailed shocks, and then examining both its second derivative and its level around zero price changes. First, consider the price changes conditional on $||p||^2 = y < \bar{y}$. This price changes have marginal distribution $\tilde{w}(x; y)$. To described this distribution we first introduce the distribution of the price gaps conditional on the norm square just before the large shock. As shown in the body of the paper it is given by

$$\omega(x_i; y) \;=\; \frac{1}{Beta\left(\frac{n-1}{2}, \frac{1}{2}\right)\;\sqrt{y}} \left(1 - \left(\frac{x_i}{\sqrt{\bar{y}}}\right)^2\right)^{(n-3)/2} \tag{51}$$

In the case of fat-tailed shocks the price changes is the sum of the price gap before the shock and the shock, namely $x_i + \xi_i$ and hence its distribution is given by:

$$\tilde{w}(\Delta p_i; y) = \int_{-\infty}^{\infty} \omega(\Delta p_i - x; y)\, q(x)\, dx$$

where $q = Q_i'$ is the density of each of the coordinates of ξ_i. The second derivative of this conditional density evaluated at zero is:

$$\begin{aligned}\tilde{w}''(0; y) &= \int_{-\infty}^{\infty} \omega''(-x; y)\, q(x)\, dx = \int_{-\min\{\underline{\xi}_i, \sqrt{y}\}}^{\min\{\underline{\xi}_i, \sqrt{y}\}} \omega''(-x; y)\, q(x)\, dx \\ &= 2\int_{0}^{\min\{\underline{\xi}_i, \sqrt{y}\}} \omega''(x; y)\, q(x)\, dx > \omega''(0; y)\end{aligned} \tag{52}$$

where the second equation use the symmetry of $\tilde{w}$ and of q around zero, as well as support of p_i and ξ_i. Note that for $n = 3$, the density $\tilde{w}$ is uniform, so its second derivative is zero everywhere. For $n \geq 3$ it has a peak at $\Delta p_i = 0$ with a strictly negative second derivative.

For $n \geq 4$ the distribution is concave and then convex. The last inequality uses the properties described from $\tilde{w}$. The density of the distribution of price changes is given by

$$w(\Delta p_i) = \omega(\Delta p_i; \bar{y})\,(1-\ell) + \left[\int_0^{\bar{y}} \tilde{w}(\Delta p_i; y) f(y) dy\right] \ell \qquad \text{for } n \geq 2 \ . \tag{53}$$

where we use the density of the price gaps f is independent of the fat-tailed shocks to price gaps and where ℓ has the same definition as in the body of the paper. Thus

$$\begin{aligned} w''(0) &= \omega''(0; \bar{y})\,(1-\ell) + \left[\int_0^{\bar{y}} \tilde{w}''(0; y) f(y) dy\right] \ell \\ &> \omega''(0; \bar{y})\,(1-\ell) + \left[\int_0^{\bar{y}} \omega''(0; y) f(y) dy\right] \ell \,, \end{aligned} \tag{54}$$

and thus the second derivative is larger for the model with fat-tailed shocks.

Finally, the same steps imply that the level of the density at zero is smaller with fat-tailed shocks, i.e.:

$$\begin{aligned} w(0) &= \omega(0; \bar{y})\,(1-\ell) + \left[\int_0^{\bar{y}} \tilde{w}(0; y) f(y) dy\right] \ell \\ &< \omega(0; \bar{y})\,(1-\ell) + \left[\int_0^{\bar{y}} \omega(0; y) f(y) dy\right] \ell \,, \end{aligned} \tag{55}$$

since

$$\begin{aligned} \tilde{w}(0; y) &= \int_{-\infty}^{\infty} \omega(-x; y)\, q(x)\, dx = \int_{-\min\{\underline{\xi}_i, \sqrt{y}\}}^{\min\{\underline{\xi}_i, \sqrt{y}\}} \omega(-x; y)\, q(x)\, dx \\ &= 2 \int_0^{\min\{\underline{\xi}_i, \sqrt{y}\}} \omega(x; y)\, q(x)\, dx < \omega(0; y) \end{aligned} \tag{56}$$

where we use that $\omega(\cdot; y)$ is single peaked for $n \geq 3$. $\square$

I Discrete Time Formulation for Proposition 11.

We start with discrete time version of the process for price gaps, with length of the time period Δ, which makes some of the arguments more accessible. Let N be

$$N(t+\Delta) = \begin{cases} N(t) & \text{with probability } (1-\lambda\Delta) \\ N(t)+1 & \text{with probability } \lambda\Delta \end{cases} \tag{57}$$

Thus, as $\Delta \downarrow 0$ this process converges to a continuous time Poisson counter with instantaneous intensity rate λ per unit of time. Let $\bar{p}_i$ follow n drift-less random walks

$$\bar{p}_i(t+\Delta, p) = \begin{cases} \bar{p}_i(t,p) \; + \; \sigma\sqrt{\Delta} & \text{with probability } 1/2 \\ \bar{p}_i(t,p) \; - \; \sigma\sqrt{\Delta} & \text{with probability } 1/2 \end{cases} \tag{58}$$

where the initial condition satisfies:

$$\bar{p}_i(0) = p_i \text{ for } i = 1, .., n \ ,$$

and where the n random walks are independent of each other and of the Poisson counter. As $\Delta \downarrow 0$ the process for $\bar{p}$ converges to a Brownian motion whose changes have variance σ^2 per unit of time. We define the stopping time of the first price adjustment $\tau(p)$, conditional on the starting at price gap vector p at time zero, as:

$$\begin{aligned}
\tau_1 &\equiv \min \{t = 0, \Delta, 2\Delta, ... \ : N(j\Delta + \Delta) - N(j\Delta) = 1\} , \\
\tau_2(p) &\equiv \min \left\{ t = 0, \Delta, 2\Delta, ... \ : \sum_{i=1}^{n} (\bar{p}_i(j\Delta + \Delta, p))^2 \geq \bar{y} \right\} \text{ and} \\
\tau(p) &\equiv \min \{\tau_1 \ , \ \tau_2(p)\} .
\end{aligned}$$

The function g is the density for the continuous time limit, i.e. the case where $\Delta \downarrow 0$. For small Δ, we can approximate the distribution of the *fraction* of firms with price gap vector p as the product of the density g and a correction to convert it into a probability, i.e a fraction. This gives:

$$g\left(p_1, ..., p_; n, \lambda/\sigma^2, \bar{y}\right) \ \left(\sigma\sqrt{\Delta}\right)^n$$

where the last term uses that in each dimension price gaps vary discretely in steps of size $\sigma\sqrt{\Delta}$. We can write the discrete time impulse response function as:

$$\mathcal{P}(t, \delta; \sigma, \lambda, \bar{y}, \Delta) \ = \ \Theta(\delta; \sigma, \lambda, \bar{y}, \Delta) + \sum_{s=\Delta}^{t} \theta(\delta, s; \sigma, \lambda, \bar{y}, , \Delta) \, \Delta \, ,$$

In this expression we can, without loss of generality, restrict t to be an integer multiple of Δ. We have divided the expression for θ by Δ, and hence multiplied its contribution back by Δ in $\mathcal{P}$, so that it has the interpretation of the contribution per unit of time to the IRF of price changes at time t, i.e. it has the units of a density. Moreover, in this manner the term has a non-zero limit, and the expression in $\mathcal{P}$ converges to an integral. Thus we get the $\mathcal{P} = \lim \mathcal{P}(\Delta)$ as $\Delta \downarrow \infty$. The functions θ and Θ are given by:

$$\Theta(\delta; \sigma, \lambda, \bar{y}, \Delta) \equiv \sum_{||p(0) - \iota\delta|| \geq \bar{y}} \left(\delta - \frac{\sum_{j=0}^{n} p_j(0)}{n} \right) g\left(p(0); n, \frac{\lambda}{\sigma^2}, \bar{y}\right) \ \left(\sigma\sqrt{\Delta}\right)^n \ , \text{ and}$$

$$\theta(\delta, t; \sigma, \lambda, \bar{y}, \Delta) \equiv$$
$$-\frac{1}{\Delta} \sum_{||p(0) - \iota\delta|| < \bar{y}} \mathbb{E}\left[\frac{\sum_{j=0}^{n} \bar{p}_j(t, p)}{n} \, \mathbf{1}_{\{\tau(p) = t\}} \ \middle| \ p = p(0) - \iota\delta \right] g\left(p(0); n, \frac{\lambda}{\sigma^2}, \bar{y}\right) \ \left(\sigma\sqrt{\Delta}\right)^n$$

Time scaling of the IRF with $N(\Delta p_i)$. For this (i) Note that if multiply the parameters σ^2 and λ by a constant $k > 0$, leaving $\bar{y}$ unaltered, then $N(\Delta p_i)' = k \, N(\Delta p_i)$, where primes are used to denote the values that correspond to the scaled parameters. This follows directly from the expression we derive for $N(\Delta p_i) = 1/T(0)$ in Proposition 3. (ii) By Proposition 4 with these changes the distribution of price changes implied by $(\sigma^2, \lambda, \bar{y})$ is exactly the same as the one implied by $(k\sigma^2, k\lambda, \bar{y})$. (iii) we change notation and write $(\sigma^2, \lambda, \bar{y})$ instead of $(\lambda, \sigma^2, \psi/B)$ and omit n. We establish that

$$\mathcal{P}_n\left(\frac{t}{k}, \delta; k\sigma^2, k\lambda, \bar{y}\right) = \mathcal{P}_n\left(t, \delta; \sigma^2, \lambda, \bar{y}\right)$$

We will do so by establishing this proposition for the discrete time version of the IRF. Yet the result is immediate, since λ and σ^2 are the only two parameters which are rates per unit of time (the other parameters are n and $\bar{y}$), so by multiplying them by k we just scale time. The details can be found in the discrete time formulation, whose notation we develop below. We show that

$$\mathcal{P}(t, \delta; k\sigma^2, k\lambda, \bar{y}, \Delta/k) = \mathcal{P}(t/k, \delta; \sigma^2, \lambda, \bar{y}, \Delta) \tag{59}$$

We will do so by establishing this proposition for the discrete time version of the IRF. Let $\Delta' = \Delta/k$, $\sigma'^2 = \sigma^2 k$ and $\lambda' = \lambda k$. Note that, by construction $\sigma' \sqrt{\Delta'} = \sigma \sqrt{\Delta}$ and $\lambda'/(\sigma')^2 = \lambda/(\sigma)^2$. To establish this we first note that, for a given shock δ, Θ depends only on n, $\bar{y}$, $\sigma\sqrt{\Delta}$, and λ/σ^2. This is because the invariant density g and the scaling factor to convert it into probabilities depends only on those parameters. Second we show that

$$\sum_{s=\Delta/k}^{t/k} \frac{\Delta}{k}\, \theta\left(s, \delta; k\sigma^2, k\lambda, \bar{y}, \frac{\Delta}{k}\right) = \sum_{s=\Delta}^{t} \Delta\, \theta\left(s, \delta; \sigma, \lambda, \bar{y}, \Delta\right)$$

This follows because for each s and $p(0)$

$$\begin{aligned} & \mathbb{E}\left[\frac{\sum_{j=0}^{n} \bar{p}_j\left(s, p\right)}{n}\, \mathbf{1}_{\{\tau(p)=s\}} \;\Big|\; p = p(0) - \iota\delta\,;\, \sigma, \lambda, \Delta\right] \\ = \;& \mathbb{E}\left[\frac{\sum_{j=0}^{n} \bar{p}_j\left(\frac{s}{k}, p\right)}{n}\, \mathbf{1}_{\left\{\tau(p)=\frac{s}{k}\right\}} \;\Big|\; p = p(0) - \iota\delta\,;\, \sigma', \lambda', \Delta'\right] \end{aligned}$$

where we include the parameters $(\lambda, \sigma^2, \Delta)$ as argument of the expected values. This itself follows because, using equation (57) and equation (58) then the processes for $\{\bar{p}_i\}$ are the same in the original time and in the time time scales by k since the probabilities of the counter to go up $\lambda'\Delta' = \lambda\Delta$ and the steps of the symmetric random walks $\sigma'\sqrt{\Delta'} = \sigma\sqrt{\Delta}$ are the same in the original time and the time scaled by k. In particular we have that

$$\bar{p}_j\left(\frac{s}{k}, p; \lambda', \sigma'^2, \Delta'\right) \equiv \bar{p}_j\left(\frac{s}{k}, p; k\lambda, k\sigma^2, \frac{\Delta}{k}\right) = \bar{p}_j\left(s, p; \lambda, \sigma^2, \Delta\right) = \hat{p}$$

with exactly the same probabilities for each price gap $\hat{p} \in \mathbb{R}$ and each time $s \geq 0$. Also, repeating the arguments used for Θ, we have $g\left(p(0); n, \frac{\lambda}{\sigma^2}, \bar{y}\right)\left(\sigma\sqrt{\Delta}\right)^n = g\left(p(0); n, \frac{\lambda'}{\sigma'^2}, \bar{y}\right)\left(\sigma'\sqrt{\Delta'}\right)^n$.

Thus, since equation (59) holds for all $\Delta > 0$, taking limits

$$\mathcal{P}\left(\frac{t}{k}, \delta; k\sigma^2, k\lambda, \bar{y}\right) = \lim_{\Delta \downarrow 0} \mathcal{P}\left(\frac{t}{k}, \delta; k\sigma^2, k\lambda, \bar{y}, \frac{\Delta}{k}\right) = \lim_{\Delta \downarrow 0} \mathcal{P}\left(t, \delta; \sigma^2, \lambda, \bar{y}, \Delta\right) = \mathcal{P}\left(t, \delta; \sigma^2, \lambda, \bar{y}\right)$$

Scaling of the IRF in the monetary shock with $Std(\Delta p_i)$. For this we use properties of the invariant distribution f, which are then inherited by g. In particular, we will compare the IRF with parameters $(\lambda, \sigma^2, \bar{y})$ with one with parameters $(\lambda', \sigma'^2, \bar{y})$ where $\lambda' = \lambda$, $\sigma'^2 = k\,\sigma^2$ and $\bar{y}' = k\,\bar{y}$. With this choice we have $N(\Delta p_i)' = N(\Delta p_i)$ and thus $\ell = \lambda'/N(\Delta p_i)'$ since $\lambda\bar{y}/(n\sigma^2) = \lambda'\bar{y}'/(n\sigma'^2)$ (see Proposition 3). Then by Proposition 1 we have that the standard deviation of price changes scales up with k, i.e.: $Std(\Delta p_i)' = \sqrt{k}\,Std(\Delta p_i)$. The main idea is that the invariant distribution corresponding to the $'$ parameters is a radial expansion of the original, so that $\int_0^y f(x; \lambda, \sigma^2, \bar{y})dx = \int_0^{yk} f(x; \lambda', \sigma'^2, \bar{y}')dx$ and thus $f(y, \lambda, \sigma^2, \bar{y}) = kf(yk, \lambda', \sigma'^2, \bar{y}')$. Indeed using Lemma 2 we have:

$$f\left(y; \frac{\lambda}{\sigma^2}, \bar{y}\right) = k\,f\left(yk; \frac{\lambda}{k\sigma^2}, k\bar{y}\right) \equiv k\,f\left(yk; \frac{\lambda'}{\sigma'^2}, \bar{y}'\right) \quad . \tag{60}$$

Thus we have:

$$\begin{aligned}
g\left(p_1, ..., p_n, \frac{\lambda}{\sigma^2}, \bar{y}\right) &= f\left(p_1^2 + \cdots + p_n^2; n, \frac{\lambda}{\sigma^2}, \bar{y}\right) \frac{\Gamma\left(n/2\right)}{2\,\pi^{n/2}\;\left(p_1^2 + \cdots + p_n^2\right)^{(n-2)/2}} = \\
&= kf\left(k(p_1^2 + \cdots + p_n^2); n, \frac{\lambda'}{\sigma'^2}, \bar{y}'\right) \frac{\Gamma\left(n/2\right) k^{(n-1)/2}}{2\,\pi^{n/2}\;\left(k(p_1^2 + \cdots + p_n^2)\right)^{(n-2)/2}} \\
&= g\left(\sqrt{k}(p_1, ..., p_n); n, \frac{\lambda'}{\sigma'^2}, \bar{y}'\right) k^{(n-2)/2}\,k
\end{aligned}$$

Using this for the discrete time formulation we have:

$$\begin{aligned}
g\left(p; n, \frac{\lambda}{\sigma^2}, \bar{y}\right)\left(\sigma\sqrt{\Delta}\right)^n &= g\left(\sqrt{k}\,p; n, \frac{\lambda'}{\sigma'^2}, \bar{y}'\right)\left(\sigma'\sqrt{\Delta}\right)^n k^{(n-2)/2}\,k\,k^{-n/2} \\
&= g\left(\sqrt{k}\,p; n, \frac{\lambda'}{\sigma'^2}, \bar{y}'\right)\left(\sigma'\sqrt{\Delta}\right)^n
\end{aligned}$$

Note that $\{||p(0) - \iota\delta|| \geq \bar{y}\} = \{||\sqrt{k}\,p(0) - \iota\sqrt{k}\,\delta|| \geq \sqrt{k}\,\bar{y}\} = \{||\sqrt{k}\,p(0) - \iota\delta'|| \geq \bar{y}'\}$. Also

$$\left(\delta - \frac{\sum_{j=0}^n p_j(0)}{n}\right)\sqrt{k} = \left(\delta' - \frac{\sum_{j=0}^n \sqrt{k}\,p_j(0)}{n}\right)$$

Thus

$$\begin{aligned}
&\sqrt{k} \sum_{||p(0)-\iota\delta|| \geq \bar{y}} \left(\delta - \frac{\sum_{j=0}^{n} p_j(0)}{n}\right) g\left(p(0); n, \frac{\lambda}{\sigma^2}, \bar{y}\right) \left(\sigma\sqrt{\Delta}\right)^n \\
= &\sum_{||\sqrt{k}p(0)-\iota\delta'|| \geq \bar{y}'} \left(\delta' - \frac{\sum_{j=0}^{n} \sqrt{k}p_j(0)}{n}\right) g\left(\sqrt{k}\, p(0); n, \frac{\lambda'}{\sigma'^2}, \bar{y}'\right) \left(\sigma'\sqrt{\Delta}\right)^n
\end{aligned}$$

Using the definition of $\Theta(\cdot, \Delta)$:

$$\sqrt{k}\, \Theta(\delta; \sigma, \lambda, \bar{y}, \Delta) = \Theta\left(\sqrt{k}\, \delta; k\sigma^2, \lambda, k\bar{y}, \Delta\right) \equiv \Theta\left(\delta'; \sigma'^2, \lambda', \bar{y}'\Delta\right) .$$

Since this holds for all Δ, by taking limits as $\Delta \downarrow 0$, we have shown the desired result for Θ. The result for θ follows the steps for g. We set $\Delta' = \Delta$ and note that for all $p(0) \in \mathbb{R}^n$, scaling factor $k > 0$ and time horizon $s > 0$:

$$\begin{aligned}
&\sqrt{k}\, \mathbb{E}\left[\frac{\sum_{j=0}^{n} \bar{p}_j\,(s, p)}{n}\, \mathbf{1}_{\{\tau(p)=s\}} \,\middle|\, p = p(0) - \iota\delta\,;\, \sigma, \lambda, \Delta\right] \\
= \;\; &\mathbb{E}\left[\frac{\sum_{j=0}^{n} \bar{p}_j\,(s, p)}{n}\, \mathbf{1}_{\{\tau(p)=s\}} \,\middle|\, p = \sqrt{k}\, p(0) - \iota\delta'\,;\, \sigma', \lambda', \Delta\right] .
\end{aligned}$$

This follows because $\lambda' = \lambda$ and $\sigma'\sqrt{\Delta'} = \sqrt{k}\, \sigma\, \sqrt{\Delta}$, thus the each $p \in \mathbb{R}^n$ the paths $\sqrt{k}\, \bar{p}(s, p; \sigma, \lambda) = \bar{p}(s, \sqrt{k}p; \sigma', \lambda')$ occur with the same probabilities.

J Detailed Proof. of Proposition 9.

Proof. (of Proposition 9.) In general we have $\underline{\delta} = 2\sqrt{\bar{y}/n}$, since for a shock of this size every single firm for which $||p||^2 = y \leq \bar{y}$ before the shock will find that $||p - \iota\delta||^2 \geq \bar{y}$, where ι is a vector of ones. In particular we want to find out the smallest value of δ for which

$$||p - \iota\delta||^2 = ||p||^2 - 2\,\delta \sum_i p_i + n\delta^2 \geq \bar{y}$$

for any $||p||^2 \leq \bar{y}$. Using that $\sum_i p_i \leq n\sqrt{y/n}$ for $y = ||p||^2$ it is easy to establish the desired result.

We can rewrite it as $\underline{\delta} = 2\sqrt{\bar{y}/n} = 2\sqrt{\sigma^2/\lambda}\,\sqrt{\phi}$, which gives an equivalent way to write the expression for $\underline{\delta}$ as

$$\underline{\delta} = Std(\Delta p_i)\, 2\,\sqrt{\frac{\phi}{\mathcal{L}(\phi, n)}} \text{ where } \phi \equiv \bar{y}\,\lambda/(n\sigma^2) .$$

where $\phi(n, \ell) \equiv \bar{y}\lambda/(n\sigma^2)$ a function that depends only on ℓ and n, as shown in Proposition 3.

Using Proposition 1 we have:

$$(N(\Delta p_i)/\lambda)\ Var(\Delta p_i) = \sigma^2/\lambda \text{ or } \sigma^2/\lambda = Var(\Delta p_i)/\ell$$

Combining the two equations we obtain the desired result.

Note that $\phi(\ell, n)/\ell = \phi/\mathcal{L}(\phi, n)$. Since $\mathcal{L}(\phi, n)$ is increasing in ϕ with $\lim_{\phi\to\infty} \mathcal{L}(\phi, n) = 1$, then $\lim_{\ell\to 1} \phi(\ell, n)/\ell = \infty$. To study the limit as $\ell \to 0$, using the functional form of $\mathcal{L}$, and taking a Taylor expansion of $\mathcal{L}(\phi, n) = \phi + o(\phi)$, thus

$$\frac{\phi}{\mathcal{L}(\phi, n)} = \frac{\phi}{\phi + o(\phi)} = \frac{1}{1 + o(\phi)/\phi},$$

and hence

$$\lim_{\ell\to 0} \frac{\phi(\ell, n)}{\ell} = \lim_{\phi\to 0} \frac{\phi}{\mathcal{L}(\phi, n)} = 1\,.$$

Omitting n to simplify the notation we have:

$$\frac{\partial}{\partial\phi}\left[\frac{\phi}{\mathcal{L}(\phi)}\right] = \frac{1}{\mathcal{L}(\phi)}\left[1 - \frac{\mathcal{L}'(\phi)\phi}{\mathcal{L}(\phi)}\right]$$

and rewriting $\mathcal{L}(\phi) = \frac{g(\phi)}{1+g(\phi)}$ we obtain: $\mathcal{L}'(\phi) = \frac{g'(\phi)}{[1+g(\phi)]^2}$ and thus

$$\frac{\mathcal{L}'(\phi)\phi}{\mathcal{L}(\phi)} = \frac{g'(\phi)}{(1+g(\phi))^2}\frac{(1+g(\phi))}{g(\phi)}\phi = \frac{g'(\phi)}{(1+g(\phi))}\frac{\phi}{g(\phi)}$$

since $g(\cdot)$ is convex and $g(0) = 0$ then $0 = g(0) \geq g(\phi) + g'(\phi)(0 - \phi)$ or $g(\phi) \leq g'(\phi)\phi$ $\frac{\mathcal{L}'(\phi)\phi}{\mathcal{L}(\phi)} \leq \frac{1}{1+g(\phi)} \leq 1$ and thus $\phi(\ell, n)/\ell$ is strictly increasing in ℓ for all $\ell \in (0, 1)$. $\square$

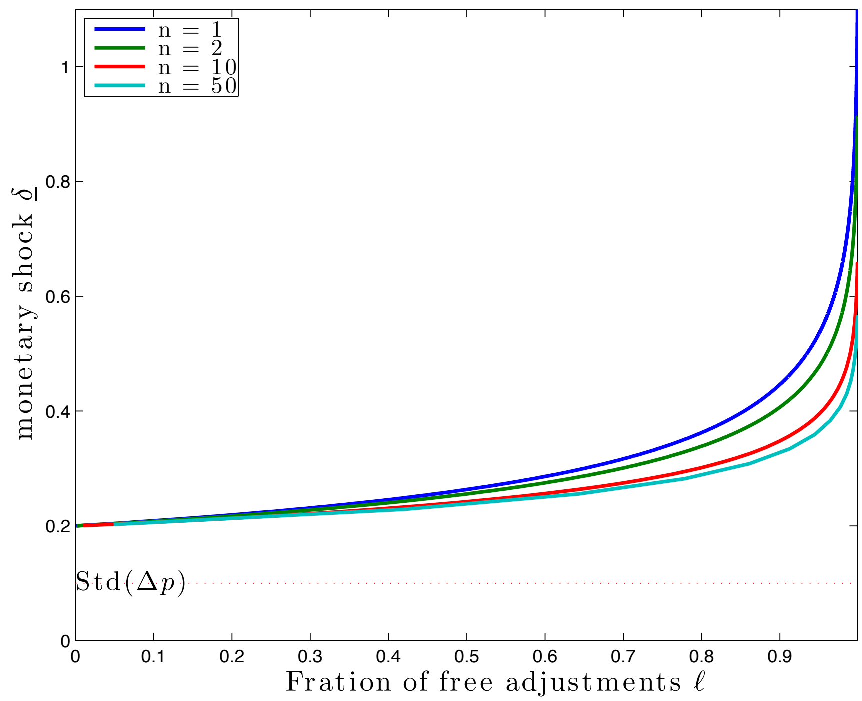
K Proof of Lemma 3

Proof. (of Lemma 3.) We use the property of the n independent BM's to write m as a function of a pair (z, y), where $z = \sum_i p_i$, as well as to write g as a function of (z, y) only. If each price gap follows an independent BM with common variance per unit of time σ^2, then, applying Ito's Lemma one can show that the pair (y, z) follows:

$$\begin{aligned}
\mathrm{d}y(t) &= n\sigma^2\mathrm{d}t + 2\sigma\sqrt{y(t)}\ \mathrm{d}\mathcal{W}^a(t) \\
\mathrm{d}z(t) &= \sqrt{n}\sigma\left[\frac{z(t)}{\sqrt{n\,y(t)}}\ \mathrm{d}\mathcal{W}^a(t) + \sqrt{1 - \left(\frac{z(t)}{\sqrt{n\,y(t)}}\right)^2}\ \mathrm{d}\mathcal{W}^b(t)\right]
\end{aligned}$$

where $\mathcal{W}^a, \mathcal{W}^b$ are 2 standard (univariate) independent BM's. So that $\mathbb{E}(\mathrm{d}y)^2 = 4\sigma^2 y\,\mathrm{d}t$, $\mathbb{E}(\mathrm{d}z)^2 = n\sigma^2\,\mathrm{d}t$, and $\mathbb{E}(\mathrm{d}z\,\mathrm{d}y) = 2\sigma^2 z\,\mathrm{d}t$.

Figure 10: Minimum size of monetary shock for full price flexibility



Hence we can write $\tilde{m}(p_1, ..., p_n) = \tilde{m}\left(||p||^2, \sum_{i=1}^{n} p_i\right)$, in which case $\tilde{m}$ solves the PDE :

$$\lambda \tilde{m}(z,y) = -z + \tilde{m}_y(z,y) n\sigma^2 + \tilde{m}_{zz}(z,y)\frac{n\sigma^2}{2} + \tilde{m}_{yy}(z,y)\frac{4\sigma^2 y}{2} + \tilde{m}_{zy}(z,y)\, 2\sigma^2 z$$

with boundary conditions: $\tilde{m}(z,\bar{y}) = 0$. We guess, and verify, that $\tilde{m}(z,y) = z\,\kappa_n(z)$ for some function $\kappa_n(\cdot)$ and where for emphasis we include the subindex n indicating the number of products. We then obtain:

$$\lambda \kappa_n(y) = -1 + \kappa_n'(y)\,(n+2)\sigma^2 + \kappa_n''(y)\, 2\,\sigma^2\, y$$

for all $0 \le y \le \bar{y}$ and $\kappa_n(\bar{y}) = 0$. Note that, except of the sign, this function obeys the same ODE and boundary conditions than the one for the time until adjustment $\mathcal{T}_{n+2}(y)$, which we solved to obtain $\mathcal{L}$ as if there were $n+2$ products instead of n products, and hence we get:

$$\kappa_n(y) = -\mathcal{T}_{n+2}(y) \tag{61}$$

The joint density of the invariant distribution $h(z,y)$ can be written as:

$$h(z,y) = s(z|y)\, f(y)$$

where f is the invariant distribution of y and $s(z|y)$ is the density distribution of the sum of the coordinates of a uniform distribution on an n dimensional hypersphere with square norm equal to y. In Alvarez and Lippi (2014) we have shown that this distribution is given by

$$s(z|y) = \frac{1}{Beta\left(\frac{n-1}{2}, \frac{1}{2}\right)\ \sqrt{n\,y}} \left(1 - \left(\frac{z}{\sqrt{y\,n}}\right)^2\right)^{(n-3)/2} \quad \text{for} \quad z \in (-\sqrt{y\,n}\,,\ \sqrt{y\,n}) \tag{62}$$

Thus we can write

$$\mathcal{M}(\delta) \ = \ \frac{1}{\epsilon}\int_0^{\bar{y}}\int_{-\sqrt{ny}}^{\sqrt{ny}} \frac{1}{n}\,\tilde{m}(z - n\delta, y - 2z\delta + n\delta^2)\ h(z,y)\ dz\ dy \tag{63}$$

and where we can express the invariant distribution of (z, y) with density h. Differentiating this expression w.r.t. δ and evaluating it at $\delta = 0$:

$$\begin{aligned}
\mathcal{M}'(0) \ &= \ -\frac{1}{n\epsilon}\int_0^{\bar{y}}\int_{-\sqrt{ny}}^{\sqrt{ny}} \left[n\,\frac{\partial\tilde{m}(z,y)}{\partial z} \ + \ \frac{\partial\tilde{m}(z,y)}{\partial y}\,2\,z\right]\ h(z,y)\ dz\ dy \\
&= \ -\frac{1}{n\epsilon}\left[\int_0^{\bar{y}} n\,\kappa_n(y)\,f(y)\,dy + 2\int_0^{\bar{y}}\kappa_n'(y)\int_{-\sqrt{ny}}^{\sqrt{ny}}\ z^2\ s(z|y)dz\,f(y)\ dy\right].
\end{aligned}$$

Integrating z^2 w.r.t. s gives $\int_{-\sqrt{ny}}^{\sqrt{ny}} z^2\ s(z|y)dz = y$ so

$$\begin{aligned}
\mathcal{M}'(0) \ &= \ -\frac{1}{n\epsilon}\int_0^{\bar{y}}\left[\ n\,\kappa_n(y) + 2\,\kappa_n'(y)y\ \right] f(y)\ dy \\
&= \ \frac{1}{\epsilon}\int_0^{\bar{y}}\left[\mathcal{T}_{n+2}(y) + \frac{2}{n}\,\mathcal{T}_{n+2}'(y)\,y\ \right] f(y)\ dy
\end{aligned}$$

where the last equality uses equation (61). □

L Proof of Lemma 4

Proof. (of Lemma 4) We can rewrite this expression as

$$\frac{\lambda\,Kur\,(\Delta p_i)}{6\,N\,(\Delta p_i)} = \frac{\sum_{i=1}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+1+i\right)}\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i\ \frac{1}{1+i}}{\sum_{i=0}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+1+i\right)}\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i\ \frac{1}{1+i}} = \frac{\sum_{i=1}^{\infty}\ \gamma_i\ \frac{1}{1+i}}{\sum_{i=0}^{\infty}\ \gamma_i\ \frac{1}{1+i}} \tag{64}$$

Thus the equation

$$\frac{\lambda\,Kur\,(\Delta p_i)}{6\,N\,(\Delta p_i)} = \int_0^{\bar{y}}\left[\lambda\left(\mathcal{T}_{n+2}(y) + \mathcal{T}_{n+2}'(y)\,y\,\frac{2}{n}\right)\right] f(y)\,dy \tag{65}$$

is equivalent to:

$$\frac{\sum_{i=1}^{\infty}\,\gamma_i\,\frac{1}{1+i}}{\sum_{i=0}^{\infty}\,\gamma_i\,\frac{1}{1+i}} - \frac{\sum_{i=1}^{\infty}\,\gamma_i}{\sum_{i=0}^{\infty}\,\gamma_i} = -\frac{\sum_{i=1}^{\infty}\,\gamma_i\,\left(1+\frac{2\,i}{n}\right)}{\sum_{i=0}^{\infty}\,\gamma_i}\int_0^{\bar{y}}\,\left(\frac{y}{\bar{y}}\right)^i\,f(y)\,dy$$

We can write this equation as:

$$\begin{aligned}&\frac{\left(\sum_{i=0}^{\infty}\,\gamma_i\right)\left(\sum_{i=1}^{\infty}\,\gamma_i\,\frac{1}{1+i}\right) - \left(\sum_{i=1}^{\infty}\,\gamma_i\right)\left(\sum_{i=0}^{\infty}\,\gamma_i\,\frac{1}{1+i}\right)}{\left(\sum_{i=0}^{\infty}\,\gamma_i\,\frac{1}{1+i}\right)\left(\sum_{i=0}^{\infty}\,\gamma_i\right)}\\ &= -\frac{\sum_{i=1}^{\infty}\,\gamma_i\,\left(1+\frac{2\,i}{n}\right)}{\sum_{i=0}^{\infty}\,\gamma_i}\int_0^{\bar{y}}\,\left(\frac{y}{\bar{y}}\right)^i\,f(y)\,dy\end{aligned}$$

or

$$\begin{aligned}&\frac{\left(\gamma_0+\sum_{i=1}^{\infty}\,\gamma_i\right)\left(\sum_{i=1}^{\infty}\,\gamma_i\,\frac{1}{1+i}\right) - \left(\sum_{i=1}^{\infty}\,\gamma_i\right)\left(\gamma_0+\sum_{i=1}^{\infty}\,\gamma_i\,\frac{1}{1+i}\right)}{\sum_{i=0}^{\infty}\,\gamma_i\,\frac{1}{1+i}}\\ &= -\left[\sum_{i=1}^{\infty}\,\gamma_i\,\left(1+\frac{2\,i}{n}\right)\right]\int_0^{\bar{y}}\,\left(\frac{y}{\bar{y}}\right)^i\,f(y)\,dy\end{aligned}$$

or

$$\frac{\gamma_0\left(\sum_{i=1}^{\infty}\,\gamma_i\,\frac{1}{1+i}\right) - \left(\sum_{i=1}^{\infty}\,\gamma_i\right)\gamma_0}{\sum_{i=0}^{\infty}\,\gamma_i\,\frac{1}{1+i}} = -\left[\sum_{i=1}^{\infty}\,\gamma_i\,\left(1+\frac{2\,i}{n}\right)\right]\int_0^{\bar{y}}\,\left(\frac{y}{\bar{y}}\right)^i\,f(y)\,dy$$

and using that $\gamma_0 = 1$ and rearranging:

$$\sum_{i=1}^{\infty}\frac{\gamma_i\,\frac{1}{1+i}}{\sum_{j=0}^{\infty}\,\gamma_j\,\frac{1}{1+j}}\,i = \left[\sum_{i=1}^{\infty}\,\gamma_i\,\left(1+\frac{2\,i}{n}\right)\right]\int_0^{\bar{y}}\,\left(\frac{y}{\bar{y}}\right)^i\,f(y)\,dy \tag{66}$$

Using the expression for f, and solving the integrals of terms by term we have:

$$\begin{aligned}&\sum_{j=1}^{\infty}\frac{\gamma_j\,\frac{1}{1+j}}{\sum_{s=0}^{\infty}\,\gamma_s\,\frac{1}{1+s}}\,j = \sum_{j=1}^{\infty}\,\gamma_j\,\left(1+\frac{2\,j}{n}\right)\times \qquad (67)\\ &\left(\left[\frac{\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)}\,\sum_{i=0}^{\infty}\beta_{i,\frac{n}{2}-1}\,\frac{\bar{y}}{\frac{n}{2}+i+j}\,\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}{\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)}\,\sum_{i=0}^{\infty}\beta_{i,\frac{n}{2}-1}\,\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i} - \frac{\sum_{i=0}^{\infty}\beta_{i,1-\frac{n}{2}}\,\frac{\bar{y}}{i+1+j}\,\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}{\sum_{i=0}^{\infty}\beta_{i,1-\frac{n}{2}}\,\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}\right]\right/\\ &\left.\left[\frac{\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)}\,\sum_{i=0}^{\infty}\beta_{i,\frac{n}{2}-1}\,\frac{\bar{y}}{\frac{n}{2}+i}\,\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}{\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)}\,\sum_{i=0}^{\infty}\beta_{i,\frac{n}{2}-1}\,\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i} - \frac{\sum_{i=0}^{\infty}\beta_{i,1-\frac{n}{2}}\,\frac{\bar{y}}{i+1}\,\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}{\sum_{i=0}^{\infty}\beta_{i,1-\frac{n}{2}}\,\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}\right]\right)\end{aligned}$$

canceling the values of $\bar{y}$, and defining

$$\xi_i = \frac{1}{i!\,\Gamma\left(i+\frac{n}{2}\right)}\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}+i-1\right)} \quad \text{and} \quad \rho_i = \frac{1}{i!\,\Gamma\left(i+2-\frac{n}{2}\right)}\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^{i}$$

$$\sum_{j=1}^{\infty} \frac{\gamma_j \, \frac{1}{1+j}}{\sum_{s=0}^{\infty} \gamma_s \, \frac{1}{1+s}}\, j = \sum_{j=1}^{\infty} \gamma_j \left(1+\frac{2\,j}{n}\right) \times \tag{68}$$
$$\left(\left[\frac{\sum_{i=0}^{\infty} \xi_i \, \frac{1}{\frac{n}{2}+i+j}}{\sum_{i=0}^{\infty} \xi_i} - \frac{\sum_{i=0}^{\infty} \rho_i \, \frac{1}{i+1+j}}{\sum_{i=0}^{\infty} \rho_i}\right] \Big/ \left[\frac{\sum_{i=0}^{\infty} \xi_i \, \frac{1}{\frac{n}{2}+i}}{\sum_{i=0}^{\infty} \xi_i} - \frac{\sum_{i=0}^{\infty} \rho_i \, \frac{1}{i+1}}{\sum_{i=0}^{\infty} \rho_i}\right]\right)$$

M Detailed Proof. of Proposition 8.

First we turn to the steady state firm's problem considered in Section 3.2. In that firm's problem we use the same discount rate r for any inflation rate μ. The reason for this is that the period return function is itself normalized by nominal wages which we assume that growth at a constant rate μ and that the nominal rate is equal to $r+\mu$, so that these two effect cancel. The price gap p_i is a real quantity, the difference between the ideal markup and the current markup, and has drift equal to minus the inflation rate due to the increase in the nominal wages. The period return is still $B||p||^2 \equiv B\,y$, but each of the product's price gap evolve as $\mathrm{d}p_i(t) = -\mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_i(t)$. In this problem it is not longer true that y is sufficient to index the state of the firm's problem, since the distribution of $y(t+dt)$ cannot be computed only knowing $y(t)$. While in Alvarez and Lippi (2014) we show that one can take the state to be (y,z) where z is the sum of the price gaps: $z=\sum_{i=1}^{n} p_i$, for the arguments here we keep the entire price gap vector $p \in \mathbb{R}^n$ as the state. In this case the inaction set is no longer a hyper-sphere, nor is the optimal return point to set a zero price gap for each of the products. We let $\mathcal{I}(\mu) \subset \mathbb{R}^n$ be the inaction set –so the firm adjust only if it receives a free adjustment opportunity or if it exist the inaction set. We regard $\mathcal{I}(z)$ as a correspondence parametrized by μ, and let $\hat{p}(\mu) \in \mathbb{R}^n$ be the optimal return point -which is identical across all products- a function parametrized by μ. Note that for any rectangle $\subset \mathbb{R}^n$ the uncontrolled price gaps satisfy that $\Pr\{p(t)-p(0) \in \mathfrak{p} \,|\, \mu\} = \Pr\{-(p(t)-p(0)) \in \mathfrak{p} \,|\, -\mu\}$. This equality uses that the increments of a standard brownian motion are normally distributed. Using this property, and the symmetry around zero of the period return function, it is easy to show that $\hat{p}(\mu) = -\hat{p}(\mu)$. Also, one can see that if $p \in \mathcal{I}(\mu)$ then it must be the case that $-p \in \mathcal{I}(-\mu)$. From these two properties of the decision rules one concludes that $N(\Delta p_i)(\mu)$ and that any even centered moment of the distribution of the price changes, and hence its ratio such as kurtosis $Kur(\Delta p_i;\mu)$, is symmetric around $\mu=0$. The same property is shown in Alvarez, Lippi, and Paciello (2011) for a closely related model. Likewise, the (negative) symmetry of $\mathcal{M})(\delta,\mu)$ follows by considering first the invariant distribution of price gaps, and then the dynamics of each one. For the invariant distribution of price gaps as defined in Section G, whose density is denoted by $g(p;\mu)$, we note that $g(p;\mu) = g(-p;-\mu)$ –where we now indexed the density only by the inflation rate μ, allowing the optimal decision rule to change with it. Following the same steps we can construct the impulse response of prices $\mathcal{P}(t,\delta;\mu)$ which we index in the same way as the density. We define this impulse response as the change in price level t periods after a once and for all shock δ to the path of the level of money that has occurred to an economy starting at the steady state distribution of price gaps. The price level is in $\mathcal{P}(\delta, t; \mu)$ is measured relative to what the prices would have been absence of a shock, where they would have been rising at a constant rate μ. Using the results previously established we have: $\mathcal{P}(t, -\delta; -\mu) = -\mathcal{P}(t, \delta; \mu)$. Using this property of the impulse response of the price level into definition of $\mathcal{M}$ in equation (15), we obtain the desired (negative) symmetry of this function.

Second, we sketch the differences in the GE set-up when $\mu \neq 0$. In this case the same arguments yields that both nominal interest rates and wages growth at a constant rate μ independently of the distribution of prices at time zero. Additionally, the nominal profit function of the firm, once we replace the first order condition for the households for consumption, labor, and money, can be written as a function of the price gap (i.e. the deviation relative to the markup that maximizes static profits) and the period nominal wages. Hence, one can approximate the real profits (deflated by the money supply) in the same way as with zero inflation, obtaining the same second order approximation. Finally, the result in Proposition 7 in Alvarez and Lippi (2014) which states that GE feedback effects are of order higher than second order in the firm's problem applies almost with no changes.

N Power series representation of $\mathcal{T}_{n+2} + \mathcal{T}'_{n+2}\, y\,(2/n)$

Lemma 3 shows that $\partial m/\partial\delta$ can be written in terms of $\mathcal{T}_{n+2}$, the expected time until a price adjustment, as characterized in Proposition 3. In that proof we obtain the power series representation

$$\mathcal{T}_{n+2}(y) = \sum_{i=0}^{\infty} \alpha_{i,\,n+2}\, y^i$$

with

$$\alpha_{1,n+2} = \frac{1}{(\sigma^2/\lambda)(n+2)}\alpha_{0,n+2} - \frac{1}{\sigma^2(n+2)} = \frac{1}{(\sigma^2/\lambda)(n+2)}\left[\alpha_{0,n+2} - \frac{1}{\lambda}\right]$$

and for $i \geq 1$:

$$\alpha_{i+1,\,n+2} = \frac{\alpha_{i,\,n+2}}{(i+1)\,(\sigma^2/\lambda)\,(n+2+2i)} = \frac{\alpha_{i,\,n+2}}{(i+1)\,(\sigma^2/\lambda)\,(n/2+1+i)}\frac{1}{2}\left[\alpha_{0,n+2} - \frac{1}{\lambda}\right].$$

and using the properties of the Γ function:

$$\alpha_{i,\,n+2} = \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left(\frac{\lambda}{2\sigma^2}\right)^i\left[\alpha_{0,n+2} - \frac{1}{\lambda}\right]$$

Note that $\mathcal{T}_{n+2}(0) = \alpha_{0,\,n+2}$

Given the power series representation we have for all $y \in [0, \bar{y}]$:

$$\mathcal{T}_{n+2}(y) + \mathcal{T}'_{n+2}(y)\, y\, \frac{2}{n} = \sum_{i=0}^{\infty} \alpha_{i,\,n+2} \left[1 + i\frac{2}{n}\right] y^i$$
$$= \alpha_{0,n+2} + \left[\alpha_{0,n+2} - \frac{1}{\lambda}\right] \sum_{i=1}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)} \left[1 + i\frac{2}{n}\right] \left(\frac{\lambda y}{2\sigma^2}\right)^i$$

Note that $\alpha_{0,n+2} = \mathcal{T}_{n+2}(0)$ with

$$\lambda\, \alpha_{0,n+2} = \ell = \frac{\sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{1}{j[n+2+2(j-1)]}\right) \left(\frac{\lambda \bar{y}}{\sigma^2}\right)^i}{1 + \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{1}{j[n+2+2(j-1)]}\right) \left(\frac{\lambda \bar{y}}{\sigma^2}\right)^i} = \frac{\sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{1}{j\left[\frac{n}{2}+j\right]}\right) \left(\frac{\lambda \bar{y}}{2\,\sigma^2}\right)^i}{1 + \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{1}{j\left[\frac{n}{2}+j\right]}\right) \left(\frac{\lambda \bar{y}}{2\,\sigma^2}\right)^i}$$
$$= \frac{\sum_{i=1}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+1+i\right)} \left(\frac{\lambda \bar{y}}{2\,\sigma^2}\right)^i}{\sum_{i=0}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+1+i\right)} \left(\frac{\lambda \bar{y}}{2\,\sigma^2}\right)^i}$$

Thus we have:

$$\lambda \left(\mathcal{T}_{n+2}(y) + \mathcal{T}'_{n+2}(y)\, y\, \frac{2}{n}\right) = \frac{\sum_{i=1}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)} \left(\frac{\lambda \bar{y}}{2\,\sigma^2}\right)^i}{\sum_{i=0}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)} \left(\frac{\lambda \bar{y}}{2\,\sigma^2}\right)^i}$$
$$+ \left[\frac{\sum_{i=1}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)} \left(\frac{\lambda \bar{y}}{2\,\sigma^2}\right)^i}{\sum_{i=0}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)} \left(\frac{\lambda \bar{y}}{2\,\sigma^2}\right)^i} - 1\right] \left[\sum_{i=1}^{\infty} \left[1 + i\frac{2}{n}\right] \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)} \left(\frac{\lambda y}{2\sigma^2}\right)^i\right]$$
$$= \frac{\sum_{i=1}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)} \left(\frac{1}{2}\right)^i \left(\frac{\lambda \bar{y}}{\sigma^2}\right)^i - \sum_{i=1}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)} \left[1 + i\frac{2}{n}\right] \left(\frac{\lambda y}{2\,\sigma^2}\right)^i}{\sum_{i=0}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)} \left(\frac{1}{2}\right)^i \left(\frac{\lambda \bar{y}}{\sigma^2}\right)^i}$$
$$= \frac{\sum_{i=1}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)} \left[\left(\frac{\lambda \bar{y}}{2\,\sigma^2}\right)^i - \left(1 + \frac{2i}{n}\right) \left(\frac{\lambda y}{2\sigma^2}\right)^i\right]}{\sum_{i=0}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)} \left(\frac{\lambda \bar{y}}{2\,\sigma^2}\right)^i} \tag{69}$$

We can write this as:

$$\lambda \left(\mathcal{T}_{n+2}(y) + \mathcal{T}'_{n+2}(y)\, y\, \frac{2}{n}\right) = \frac{\sum_{i=1}^{\infty} \gamma_i}{\sum_{i=0}^{\infty} \gamma_i} - \frac{\sum_{i=1}^{\infty} \gamma_i \left(1 + \frac{2\,i}{n}\right) \left(\frac{y}{\bar{y}}\right)^i}{\sum_{i=0}^{\infty} \gamma_i} \tag{70}$$

where

$$\gamma_i = \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+1+i\right)} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i \tag{71}$$

O Details of the solution for the model with $n = 1$

Integrating the Bellman equation gives the following value function

$$V(p) = \frac{Bp^2 + \lambda V(0)}{\lambda + r} + \frac{B\sigma^2}{(\lambda + r)^2} + C\left(e^{p\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-p\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} \right)$$

where we already used that $V(p) = V(-p)$. Notice that the value function has a minimum (and zero derivative) at $p = 0$, which is the optimal return point. The constant C and the threshold value $\bar{p}$ are the values that solve the 2 equation system given by the value matching condition and the smooth pasting conditions.

The expected time to adjustment, $T(p)$ obeys the differential equation $\lambda T(p) = 1 + \frac{\sigma^2}{2} T''(p)$ with boundary condition $T(\bar{p}) = 0$. Given the symmetry of the law of motion for p, the function is symmetric, i.e. $T(p) = T(-p)$. Integrating gives $T(p) = \frac{1}{\lambda}\left(1 - \frac{e^{\sqrt{\frac{2\lambda}{\sigma^2}}p} + e^{-\sqrt{\frac{2\lambda}{\sigma^2}}p}}{e^{\sqrt{\frac{2\lambda}{\sigma^2}}\bar{p}} + e^{-\sqrt{\frac{2\lambda}{\sigma^2}}\bar{p}}}\right)$.

The distribution of price gaps $g(p)$ satisfies the Kolmogorov forward equation $0 = -\frac{2\lambda}{\sigma^2} g(p) + g''(p) \quad \text{for} \quad 0 < |p| \leq \bar{p}$. The density is symmetric, $g(p) = g(-p)$, and satisfies the boundary conditions: $g(\bar{p}) = 0$ and it integrates to one i.e. $2\int_0^{\bar{p}} g(p)\, dp = 1$ where we used that it is symmetric.[41]

P $\mathcal{M}$ and $kur(\Delta p_i)$ for $n = 1$ and $n = \infty$

This section discusses two limiting cases for which a tractable closed form expression can be derived which bracket the possible range of output effects. For each case we derive the implications for the cumulative output effect while considering the full range of values for $\ell \in (0, 1)$ and keeping the frequency and variance of price changes constant. We begin by stating the main results:

$$\mathcal{M}'(0) = \begin{cases} \frac{1}{\epsilon\, N(\Delta p_i)} \frac{\left(e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}}\right)\left(e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}} - 2(1+\phi)\right)}{\left(e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}} - 2\right)^2} & \text{where} \quad \ell = \frac{e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}} - 2}{e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}}} \quad \text{and} \quad n = 1 \\ \\ \frac{1}{\epsilon\, N(\Delta p_i)} \left(\frac{1 - (1+\phi)e^{-\phi}}{\left(1 - e^{-\phi}\right)^2}\right) & \text{where} \quad \ell = 1 - e^{-\phi} \quad \text{and} \quad n \to \infty \end{cases}$$

where different values of ϕ map monotonically into the fraction of free adjustments ℓ as shown in the display. We find the $n \to \infty$ case interesting because of its tractability and because, as shown in Figure 5, it provides a benchmark for the cases where n is high (e.g. $n \cong 10$).

[41] The first boundary can be derived as the limit of the discrete time, discrete state, low of motion where each period is of length Δ and where p increases or decreases with probability $1/2$, so that $g(p) = \frac{1}{2}g(p + \Delta) + \frac{1}{2}g(p - \Delta)$. At the boundary $\bar{p}$ this law of motion is $g(\bar{p}) = \frac{1}{2}g(\bar{p} - \Delta)$, which shows that $g(\bar{p}) \downarrow 0$ as $\Delta \downarrow 0$.

P.1 Analytical computation of $\mathcal{M}$ in the case of $n = 1$

We give an analytical summary expression for the effect of monetary shocks in two interesting cases, those for one product, i.e. $n = 1$, and those for the large number of product, i.e. $n = \infty$. The summary expression is the area under the impulse response for output, i.e. the sum of the output above steady state after a monetary shock of size $\delta > 0$, which we denote as:

$$\mathcal{M}_n(\delta) = (1/\epsilon) \int_0^{\infty} [\delta - \mathcal{P}_n(\delta, t)] \, dt \tag{72}$$

where ϵ is a the reciprocal of intertemporal elasticity of substitution, and where $\mathcal{P}_n(\delta, t)$ is the cumulative effect of monetary shock δ in the (log) of the price level after t periods. For large enough shocks, given the fixed cost of changing prices, the model display more price flexibility. Because of their preminence in the literature, and because of realism, we consider the case of small shocks δ by taking the first order approximation to equation (72), so we consider $\mathcal{M}_n(\delta) \approx \mathcal{M}_n'(0)\delta$.

For the case of $n = 1$ we obtain an analytical expression which, after normalizing by $N(\Delta p_i)$ depends only on $\lambda/N(\Delta p_i)$. Thus as $\lambda/N(\Delta p_i)$ ranges from 0 to 1 the model ranges from a version of the menu cost model of *Golosov and Lucas* to a version using *Calvo* pricing. The analytical expression is based upon the following characterization:

$$\mathcal{M}_1(\delta) = (1/\epsilon) \int_{-\bar{p}}^{\bar{p}-\delta} m(p_0) \, g(p_0 + \delta) \, dp_0 \tag{73}$$

where p_0 is the price gap *after* the monetary shocks and where $m(p)$ gives the contribution to the area under the IRF of firms that start with price gap, after the shock, equal to p_0. Since the monetary shock happens when the economy is in steady state, the distribution right after the shock has the steady state density h displaced by δ. Immediately after the shock the firms with the highest price gap have price gap $\bar{p} - \delta$. Note that the integral in equation (73) does not include the firms that adjust on impact, those that before the shock have price gaps in the interval $[-\bar{p}, \bar{p} - \delta)$, whose adjustment does not contribute to the IRF. The definition of m is:

$$m(p) = -\mathbb{E}\left[\int_0^{\tau} p(t) \, dt \;\middle|\; p(0) = p \right] \tag{74}$$

where τ is the stopping time denoting the first time that the firm adjusts its price. This function gives the integral of the negative of the price gap until the first price adjustment. This expression is based on the fact that those firms with negative price gaps, i.e. low markups, contribute positively to output being in excess of its steady state value, and those with high markups contribute negatively. Given a decision rule summarized by $\bar{p}$ we can characterize m as the solution to the following ODE and boundary conditions:

$$\lambda m(p) = -p + \frac{\sigma^2}{2} m''(p) \;\; \text{for all } p \in [-\bar{p}, \bar{p}] \;\; \text{and} \;\; m(p) = 0 \;\; \text{otherwise .} \tag{75}$$

The solution for the function m is:

$$m(p) = -\frac{p}{\lambda} + \frac{\bar{p}}{\lambda}\left(\frac{e^{\sqrt{2\phi}\frac{p}{\bar{p}}} - e^{-\sqrt{2\phi}\frac{p}{\bar{p}}}}{e^{\sqrt{2\phi}} - e^{-\sqrt{2\phi}}}\right) \quad \text{for all} \quad p \in [-\bar{p}\,,\,\bar{p}] \ . \tag{76}$$

$\phi \equiv \lambda \bar{p}^2/\sigma^2$. We have then:

$$\mathcal{M}(\delta) \approx \mathcal{M}'(0)\delta = (\delta/\epsilon)\int_{\bar{p}}^{\bar{p}} m(p)\, g'(p)\, dp = (\delta/\epsilon)\, 2\int_{0}^{\bar{p}} m(p)\, g'(p)\, dp \tag{77}$$

since $m(\bar{p})g(\bar{p}) = 0$. The last equality uses that m is negative symmetric, i.e. $m(p) = -m(-p)$, and that g is symmetric around zero. Using the expression for g in Section 3.1

$$g'(p) = -\frac{2\phi}{2\bar{p}^2\left(e^{\sqrt{2\phi}} - 1\right)^2}\left(e^{\sqrt{2\phi}\left(2-\frac{p}{\bar{p}}\right)} + e^{\sqrt{2\phi}\frac{p}{\bar{p}}}\right) \quad \text{for} \quad p \in [0, \bar{p}] \ .$$

we obtain:

$$\begin{aligned}
\mathcal{M}'(0)\delta &= \left(\frac{\delta}{\epsilon}\right)\frac{-2\phi}{\lambda\left(e^{\sqrt{2\phi}} - 1\right)^2}\left(\frac{e^{\sqrt{2\phi}}\left(2 + 2\phi - 2\cosh\left(\sqrt{2\phi}\right)\right)}{2\phi}\right) \\
&= \left(\frac{\delta}{\epsilon}\right)\frac{-2}{\lambda\left(e^{\sqrt{2\phi}} - 1\right)^2}\left(e^{\sqrt{2\phi}}\left(1 + \phi - \frac{e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}}}{2}\right)\right)
\end{aligned}$$

Using the expression for $N(\Delta p_i)$ for the $n = 1$ and simple algebra we can rewrite it as:

$$\mathcal{M}'(0)\delta = \left(\frac{\delta}{\epsilon}\right)\frac{1}{N(\Delta p_i)}\frac{e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}}}{\left(e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}} - 2\right)^2}\left(e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}} - 2 - 2\phi\right) \tag{78}$$

which yields the cumulative output effect of a small monetary shock of size δ.[42]

Kurtosis. We now verify that the expression can be equivalently obtained by computing the kurtosis, as stated in Proposition 7. For notation convenience let $x \equiv \sqrt{2\phi}$. Using the distribution of price changes derived in Section 3.1 and the definition of kurtosis we get

$$Kur\left(\Delta p_i\right) = \frac{2\ell\left(\frac{12}{x^4} - \frac{12+x^2}{x^2\left(e^{x/2}-e^{-x/2}\right)^2}\right) + 1 - \ell}{\left(2\ell\left(\frac{1}{x^2} + \frac{1}{2-e^{-x}-e^{x}}\right) + 1 - \ell\right)^2} = \frac{12 - \frac{12x^2+x^4}{\left(e^{x/2}-e^{-x/2}\right)^2} + x^4\frac{1-\ell}{2\ell}}{2\ell\left(1 + \frac{x^2}{2-e^{-x}-e^{x}} + x^2\frac{1-\ell}{2\ell}\right)^2}$$

[42] As a check of this formula compute the case for $\phi = 0$, i.e. the cumulative output for the Golosov-Lucas model. In this case we let $\lambda = 0$ and $\bar{p} > 0$. In this case we have: $m(p) = -\frac{\bar{p}^2 p}{3\sigma^2} + \frac{p^3}{3\sigma^2}$. Also $g'(p) = -1/\bar{p}^2$ for $p \in (0, \bar{p}]$, so we have:

$$\mathcal{M}'(0)\delta = \left(\frac{\delta}{\epsilon}\right)\frac{2}{-3\sigma^2\bar{p}^2}\int_0^{\bar{p}}\left[-\bar{p}^2 p + p^3\right]dp = \left(\frac{\delta}{\epsilon}\right)\frac{-2}{3\sigma^2\bar{p}^2}\left[-\frac{\bar{p}^4}{2} + \frac{\bar{p}^4}{4}\right] = \left(\frac{\delta}{\epsilon}\right)\frac{2\bar{p}^2}{3\sigma^2}\frac{2}{8} = \left(\frac{\delta}{\epsilon}\right)\frac{1}{N(\Delta p_i)}\frac{1}{6}$$

which is the same value obtained by taking the limit for $\phi \to 0$ in the general expression above.

Recall from Section 3.1 that $\ell = \frac{e^x+e^{-x}-2}{e^x+e^{-x}}$ so that , after some algebra

$$Kur\left(\Delta p_i\right) \quad = \quad 6 \ \frac{e^x+e^{-x}}{\left(e^x+e^{-x}-2\right)^2} \quad \left(e^x+e^{-x}-2-x^2\right)$$

It is immediate that the kurtosis and the cumulative effect on output shown in equation (78) satisfy Proposition 7.

P.2 Analytical computation of $\mathcal{M}$ in the case of $n = \infty$

Define

$$Y_n(t,\delta) \equiv \frac{1}{n}\sum_{i=1}^{n}[p_i(t)-\delta] = Y_n(t,0) - 2\delta\frac{\sum_{i=1}^{n} p_i(t)}{n} + \delta^2 \ .$$

where the $p_i(t)$ are independent of each other, start at $p_i(0) = 0$ and have normal distribution with $\mathbb{E}\left[p_i(t)\right] = 0$ and $Var\left[p_i(t)\right] = \sigma^2 t$. Then, by an application of the law of large numbers, we have:

$$Y_\infty(t,\delta) = Y_\infty(t,0) + \delta^2 = t\sigma^2 + \delta^2$$

Letting $\bar{Y} \equiv \lim_{n\to\infty} \bar{y}(n)/n$ we can represent the steady state optimal decision rule as adjusting prices when t, the time elapsed since last adjustment, attains $T = \bar{Y}/\sigma^2$. We compute the density of the distribution of products indexed by the time elapsed since the last adjustment t and, abusing notation, we denote it by f. This distribution is a truncated exponential with decay rate λ and with truncation T, thus the density is:

$$f(t) = \lambda\frac{e^{-\lambda t}}{1-e^{-\lambda T}} \ \text{ for all } \ t \in [0,T] \ .$$

The (expected) number of price changes per unit of time is given by the sum of the free adjustments and the ones that reach T, so

$$N(\Delta p_i) = \lambda + f(T) = \lambda\left[1+\frac{e^{-\lambda T}}{1-e^{-\lambda T}}\right] = \frac{\lambda}{1-e^{-\lambda T}}$$

Note that, using the definition of T given above, $\lambda T = \bar{Y}\lambda/\sigma^2$ the parameter which indexes the shape of f and of the distribution of price changes. Since this figures prominently in this expressions we define:

$$\phi \equiv \lambda\, T = \frac{\bar{Y}\,\lambda}{\sigma^2} \ .$$

which is consistent with the definition of ϕ in Proposition 3. Using this definition we get:

$$\ell = \frac{\lambda}{N(\Delta p_i)} = 1 - e^{-\phi} \ \text{ and thus } \ N(\Delta p_i) = \frac{\lambda}{1-e^{-\phi}}$$

Impulse Response of Prices to a monetary Shock. We can now define the impulse response. Note that after the monetary shock firms that have adjusted their prices t periods ago, in average will adjust their price up by δ. This highlights that as $n \to \infty$ there is no selection.

Now we turn to the characterization of the impact effect Θ. In this case we have

$$Y_\infty(t,\delta) = Y_\infty(t,0) + \delta^2 = t\sigma^2 + \delta^2 \geq \bar{Y} = \sigma^2 T \iff t \geq T - \delta^2/\sigma^2 \ .$$

Thus the impact effect is:

$$\Theta(\delta) = \delta \int_{T-\delta^2/\sigma^2}^{T} f(t)dt = \delta\, \frac{e^{-\lambda T + \frac{\lambda}{\sigma^2}\delta^2} - e^{-\lambda T}}{1-e^{-\kappa}} = \delta\, \frac{e^{-\kappa + \frac{\lambda}{\sigma^2}\delta^2} - e^{-\kappa}}{1-e^{-\kappa}}$$

Using that $N(\Delta p_i) Var(\Delta p_i) = \sigma^2$ we can write:

$$\Theta(\delta) = \delta + \delta\, \frac{e^{-\kappa + \frac{\lambda}{N(\Delta p_i)} \frac{\delta^2}{Var(\Delta p_i)}} - 1}{1-e^{-\kappa}} = \delta + \delta\, \frac{\left(1 - \frac{\lambda}{N(\Delta p_i)}\right) e^{\frac{\lambda}{N(\Delta p_i)} \frac{\delta^2}{Var(\Delta p_i)}} - 1}{\lambda/N(\Delta p_i)}$$

Note that

$$\lim \Theta(\delta) = \begin{cases} \delta \left(\frac{\delta}{Std(\Delta p_i)} \right)^2 & \text{as } \lambda/N(\Delta p_i) \to 0 \\ 0 & \text{as } \lambda/N(\Delta p_i) \to 1 \end{cases}$$

and in general

$$\frac{\Theta(\delta)}{\partial(\lambda/N(\Delta p_i))} = \delta\, \frac{e^{\frac{\lambda}{N(\Delta p_i)} \frac{\delta^2}{Var(\Delta p_i)}} \left(\frac{\delta^2}{Var(\Delta p_i)} \frac{\lambda}{N(\Delta p_i)} \left(1 - \frac{\lambda}{N(\Delta p_i)}\right) - 1 \right) + 1}{(\lambda/N(\Delta p_i))^2} < 0$$

whenever $\delta < 2\, Std(\Delta p_i)$.

$$\begin{aligned}
\theta(t) &= \delta e^{-\lambda t} \left[f\left(T - \delta^2/\sigma^2 - t\right) + \lambda \int_0^{T-\delta^2/\sigma^2 - t} f(s)ds \right] \\
&= \delta e^{-\lambda t} \left[\lambda \frac{e^{-\lambda\left(T-\delta^2/\sigma^2 - t\right)}}{1-e^{-\lambda T}} + \lambda \int_0^{T-\delta^2/\sigma^2 - t} \lambda \frac{e^{-\lambda s}}{1-e^{-\lambda T}} ds \right] \\
&= \delta e^{-\lambda t} \left[\lambda \frac{e^{-\lambda\left(T-\delta^2/\sigma^2 - t\right)}}{1-e^{-\lambda T}} + \lambda \frac{1 - e^{-\lambda(T-\delta^2/\sigma^2 - t)}}{1-e^{-\lambda T}} \right] \\
&= \delta \frac{\lambda e^{-\lambda t}}{1-e^{-\lambda T}} \left[e^{-\lambda\left(T-\delta^2/\sigma^2 - t\right)} + 1 - e^{-\lambda(T-\delta^2/\sigma^2 - t)} \right] \\
&= \delta \frac{\lambda e^{-\lambda t}}{1-e^{-\lambda T}}
\end{aligned}$$

We can interpret $\theta(t)dt$ as $\theta(t)$ times the number of firms that adjust its price at times (t, dt). This is the sum of two terms. The first term is the fraction that adjust because they hit the boundary between t and $t + dt$. The second term is the fraction that have not yet adjusted times the fraction that adjust, λdt due to a free opportunity. Both terms are multiplied by

$e^{-\lambda t}$ to take into account those firms that have received a free adjustment opportunity before after the monetary shock but before t.

Thus we have:

$$
\begin{aligned}
\mathcal{P}_\infty(t,\delta) &= \Theta(\delta)+\delta\int_0^t \frac{\lambda e^{-\lambda s}}{1-e^{-\lambda T}}ds = \Theta(\delta)+\delta\frac{1-e^{-\lambda t}}{1-e^{-\lambda T}} = \Theta(\delta)+\delta\frac{1-e^{-\frac{\lambda}{N(\Delta p_i)}\,t\,N(\Delta p_i)}}{1-e^{-\kappa}} \\
&= \Theta(\delta)+\delta\frac{1-e^{-\frac{\lambda}{N(\Delta p_i)}\,t\,N(\Delta p_i)}}{\lambda/N(\Delta p_i)}
\end{aligned}
$$

Using $\mathcal{P}_\infty$ we can compute the IRF for output, and a summary measure for it, namely the area below it:

$$
\begin{aligned}
\mathcal{M}_\infty(\delta) &= \frac{1}{\epsilon}\int_0^T \left[\delta-\mathcal{P}_\infty(\delta,t)\right]dt \approx \delta\frac{1}{\epsilon}\int_0^T\left[1-\frac{1-e^{-\lambda t}}{1-e^{-\lambda T}}\right]dt \\
&= \frac{\delta}{\epsilon}\left[T-\frac{T}{1-e^{-\lambda T}}+\frac{1}{\lambda}\right] = \frac{\delta}{\epsilon}\left[-T\frac{e^{-\lambda T}}{1-e^{-\lambda T}}+\frac{1}{\lambda}\right] \\
&= \frac{\delta}{\epsilon}\frac{1-e^{-\lambda T}}{\lambda}\frac{1}{1-e^{-\lambda T}}\left[-\lambda T\frac{e^{-\lambda T}}{1-e^{-\lambda T}}+1\right] \\
&= \frac{\delta}{\epsilon\,N(\Delta p_i)}\,\frac{1}{1-e^{-\phi}}\,\left[1-\phi\,\frac{e^{-\phi}}{1-e^{-\phi}}\right] = \frac{\delta}{\epsilon\,N(\Delta p_i)}\,\left[\frac{1-(1+\phi)\,e^{-\phi}}{\left(1-e^{-\phi}\right)^2}\right]
\end{aligned}
$$

where the approximation uses the expression for small δ, i.e. its first order Taylor's expansion.

Kurtosis. For completeness we also include here an expression for the kurtosis of the distribution of price changes in the case of $n=\infty$. Price changes are distributed as:

$$
\mathbb{E}\left[(\Delta p_i)^2\right] = \sigma^2/N(\Delta p_i) = \frac{\sigma^2}{\lambda}\frac{\lambda}{N(\Delta p_i)} = \frac{T\sigma^2}{T\lambda}\frac{\lambda}{N(\Delta p_i)} = T\sigma^2\,\frac{1}{T\lambda}\frac{\lambda}{N(\Delta p_i)}
$$

$$
\begin{aligned}
\mathbb{E}\left[(\Delta p_i)^4\right] &= 3\frac{\lambda}{N(\Delta p_i)}\int_0^T \frac{(\sigma^2 t)^2\lambda e^{-\lambda t}}{1-e^{-\lambda T}}dt + \left(1-\frac{\lambda}{N(\Delta p_i)}\right)3\left(\sigma^2 T\right)^2 \\
&= 3\sigma^4\left[\lambda\int_0^T t^2 e^{-\lambda t}dt+\left(1-\frac{\lambda}{N(\Delta p_i)}\right)T^2\right] \\
&= 3\sigma^4 T^2\left[\frac{2-e^{-\lambda T}\left(\lambda T\left(\lambda T+2\right)+2\right)}{\left(T\lambda\right)^2}+\left(1-\frac{\lambda}{N(\Delta p_i)}\right)\right]
\end{aligned}
$$

Kurtosis is then given by:

$$
\begin{aligned}
\frac{\mathbb{E}\left[(\Delta p_i)^4\right]}{\left(\mathbb{E}\left[(\Delta p_i)^2\right]\right)^2} &= 3\,\frac{\frac{2-e^{-\lambda T}(\lambda T(\lambda T+2))}{(T\lambda)^2}+\left(1-\frac{\lambda}{N(\Delta p_i)}\right)}{\left(\frac{1}{T\lambda}\right)^2\left(\frac{\lambda}{N(\Delta p_i)}\right)^2} = 3\,\frac{2-e^{-\lambda T}\left(\lambda T\left(\lambda T+2\right)+2\right)+(T\lambda)^2\left(1-\frac{\lambda}{N(\Delta p_i)}\right)}{\left(\frac{\lambda}{N(\Delta p_i)}\right)^2} \\
&= 3\,\frac{\left(2-e^{-\lambda T}2\lambda T-e^{-\lambda T}2\right)}{\left(\frac{\lambda}{N(\Delta p_i)}\right)^2} = 6\,\frac{\left(1-e^{-\lambda T}\left(1+\lambda T\right)\right)}{\left(\frac{\lambda}{N(\Delta p_i)}\right)^2} = 6\,\frac{1-e^{-\lambda T}\left(1+\lambda T\right)}{\left(1-e^{-\lambda T}\right)^2} \\
&= 6\,\frac{1-e^{-\phi}\left(1+\phi\right)}{\left(1-e^{-\phi}\right)^2}
\end{aligned}
$$

It is immediate to use the expression for kurtosis and the one above for $\mathcal{M}_\infty(\delta)$ to verify Proposition 7.

Q Special case of Proposition 7 for $\ell = 0$.

For $\ell = 0$, or equivalently $\lambda = 0$, we use the result in Alvarez and Lippi (2014) for

$$\mathcal{T}_{n+2}(y) = \frac{\bar{y}-y}{(n+2)\sigma^2}$$

gives:

$$\mathcal{M}'(0) = \frac{1}{n\epsilon}\int_0^{\bar{y}}\left[\frac{n(\bar{y}-y)-2y}{(n+2)\sigma^2}\right] f(y)\,dy$$

and using the following expression for f from Alvarez and Lippi (2014) :

$$
\begin{aligned}
f(y) &= \frac{1}{\bar{y}}\left[\log(\bar{y})-\log(y)\right] \text{ if } n=2, \text{ and} \\
f(y) &= (\bar{y})^{-\frac{n}{2}}\left(\frac{n}{n-2}\right)\left[(\bar{y})^{\frac{n}{2}-1}-(y)^{\frac{n}{2}-1}\right] \text{ otherwise}
\end{aligned}
\tag{79}
$$

gives that:

$$
\begin{aligned}
\mathcal{M}'(0) &= \frac{1}{n\epsilon}\frac{2\,\bar{y}\,n(n-2)}{(n^2-4)\sigma^2} = \frac{1}{\epsilon}\frac{n(n-2)}{2\,(n^2-4)}\frac{1}{\left[\sigma^2/(\bar{y}/n)\right]} = \frac{1}{\epsilon}\frac{n(n-2)}{2\,(n-2)(n+2))}\frac{1}{\left[\sigma^2/(\bar{y}/n)\right]} \\
&= \frac{1}{\epsilon}\frac{n}{2\,(n+2))}\frac{1}{N(\Delta p_i)} = \frac{1}{\epsilon}\frac{3n}{(n+2)}\frac{1}{6N(\Delta p_i)} = \frac{1}{\epsilon}\frac{Kurt(\Delta p_i)}{6\,N(\Delta p_i)}
\end{aligned}
$$

since for $\lambda = 0$ then $Kurt(\Delta p_i) = 3n/(n+2)$ and $N(\Delta p_i) = \sigma^2/(\bar{y}/n)$ we verified the equality in Proposition 7.

R Aggregation of heterogenous sectors

Assume that there are S sectors, each with an expenditure weight $e(s) > 0$, and with different parameters so that each have $N(s)$ price changes per unit of time, and a distribution of price changes with kurtosis $Kur(s)$. In this case, after repeating the arguments above for each sector and aggregating, we obtain that the area under the IRF of *aggregate* output for a small monetary shock δ is

$$\mathcal{M}(\delta) \cong \delta \mathcal{M}'(0) = \frac{\delta}{6\,\epsilon} \sum_{s \in S} \frac{e(s)}{N(s)} Kur(s) = \frac{\delta}{6\,\epsilon} D \sum_{s \in S} d(s)\, Kur(s) \tag{80}$$

where D is the expenditure-weighted average duration of prices $D \equiv \sum_{s \in S} \frac{e(s)}{N(s)}$ and the $d(s) \equiv \frac{e(s)}{N(s)D}$ are weights taking into account both relative expenditures and durations. In the case in which all sectors have the same durations then $d(s) = e(s)$ and $\mathcal{M}$ is proportional to the kurtosis of the standardized data. Likewise, the same result applies if all sectors have the same kurtosis.[43] In general, if sectors are heterogenous in the durations (or expenditures), then the kurtosis of the sectors with longer duration (or expenditures) receive a higher weight in the computation of $\mathcal{M}$. For the French data, computation of the duration weighted kurtosis in equation (80) results in an increase of the order 15%, reflecting a correlation between kurtosis and duration of the same magnitude.

S A model with random cheap adjustment

This version of the model assumes that with probability λ per unit of time the menu cost is smaller than the regular adjustment, namely that it costs $b\,\psi$ with $b \in (0, 1)$. For simplicity we kept the analysis of the model with one product, i.e. $n = 1$.

Firm's problem. The firm's optimal policy now involves two thresholds: $0 < \underline{p} < \bar{p}$. If the price gap is small, i.e. if $|p| \in [0, \underline{p}]$ the firm optimally decides not to adjust the price, even if an opportunity for cheap adjustment occurs. If the price gap is large, i.e. if $|p| \in [\underline{p}, \bar{p})$, the firm adjusts the price only if a cheap adjustment opportunity arises. As in the case where $b = 0$, the firm adjust its price the first time that $|p|$ reaches $\bar{p}$.

Given the values of two thresholds $\underline{p}, \bar{p}$, the value function v can be describe as two functions holding in each segment, as follows:

$$\begin{aligned} r\ v_0(p) &= B\,p^2 + \frac{\sigma^2}{2}\ v_0''(p), \quad \text{for } p \in [0, \underline{p}]\ , \\ r\ v_1(p) &= B\,p^2 + \lambda\,[v_0(0) + b\psi - v_1(p)] + \frac{\sigma^2}{2}\ v_1''(p), \quad \text{for } p \in [\underline{p}, \bar{p}] \end{aligned}$$

where we use that the optimal return point upon adjustment is $v_0(0)$ and where used that by symmetry $v_i(p) = v_i(-p)$ for $i = 0, 1$.

[43]The effect of heterogeneity in $N(\Delta p_i)$ on aggregation is well known, so that D is different from the average of $N(\Delta p_i)$'s, see for example Carvalho (2006) and Nakamura and Steinsson (2010).

The value function can be expressed as the sum of a particular solution and two solutions multiplied by constants K_0 and K_1 and the two parameters $0 < \underline{p}, \bar{p}$. The value function has the following boundary conditions $v_0(\underline{p}) = v_1(\underline{p})$ and $v_0(0) + \psi = v_1(\bar{p})$, as well as the smooth pasting conditions $v_0'(\underline{p}) = v_1'(\underline{p})$ and $0 = v_1'(\bar{p})$. Using the four boundary conditions one solve for both the value function (i.e. the constants K_i) and the thresholds $\underline{p}, \bar{p}$. We give the details in Appendix S.1 and Appendix S.2.

Frequency of price changes. To find the frequency of price changes we first introduce the expected time to adjustment function $T(p)$. This function obeys the following ODE:

$$0 = 1 + \frac{\sigma^2}{2} T_0''(p) \quad \text{for } \ 0 < |p| \le \underline{p} \quad \text{and} \quad \lambda T_1(p) = 1 + \frac{\sigma^2}{2} T_1''(p) \quad \text{for } \underline{p} < |p| \le \bar{p}$$

with $T_i(p) = T_i(-p)$, and boundary conditions $T_0(\underline{p}) = T_1(\underline{p})$, $T_0'(\underline{p}) = T_1'(\underline{p})$ and $T_1(\bar{p}) = 0$. Thus

$$T_0(p) = J - \frac{p^2}{\sigma^2} \quad \text{and} \quad T_1(p) = \frac{1}{\lambda} + K e^{\varphi |p|} + L e^{-\varphi |p|}$$

where the J, K, L are constant to be determined using the boundary conditions, and where $\varphi = \sqrt{2\,\lambda/\sigma^2}$. Thus, given thresholds $\underline{p}, \bar{p}$, solving for the function T boils down to solve three linear equations in three unknowns as detailed in Appendix S.4. In particular the average number of adjustment per period is simply:

$$N\left(\Delta p_i\right) = \frac{1}{T_0(0)} = \frac{1}{J}\,, \tag{81}$$

Kurtosis of price changes. To measure the steady state kurtosis of price changes, we first solve for the density function for the price gaps $g(p) \in [0, \bar{p}]$. This density solves

$$\begin{aligned} 0 &= g_0''(p) \ \text{ for } \ 0 \le |p| \le \underline{p} \quad \text{and} \quad 0 = -\frac{2\lambda}{\sigma^2} g_1(p) + g_1''(p) \ \text{ for } \underline{p} < |p| \le \bar{p} \qquad \text{or} \\ g_0(p) &= C_1 + C_2\,|p| \ \text{ for } \ 0 \le |p| \le \underline{p} \quad \text{and} \quad g_1(p) = C_3 e^{\varphi |p|} + C_4 e^{-\varphi |p|} \ \text{ for } \ \underline{p} \le |p| \le \bar{p} \end{aligned}$$

where the 4 constants solve the 4 equations $g_0(\underline{p}) = g_1(\underline{p})$, $g_0'(\underline{p}) = g_1'(\underline{p})$, $g_1(\bar{p}) = 0$ and $1/2 = \int_0^{\underline{p}} g_0(p)\ \mathrm{d}p + \int_{\underline{p}}^{\bar{p}} g_1(p)\ \mathrm{d}p$ which use that the density is differentiable. Given $\underline{p}, \bar{p}$ the solution boils down to solve four linear equations in four unknowns, as detailed in Appendix S.5.

Then using that only the fraction $2\int_{\underline{p}}^{\bar{p}} g_1(p)\mathrm{d}p$ of cheap adjustment opportunities will trigger an actual price change, the distribution of (non-zero) price changes $p \in [-\bar{p}, -\underline{p}] \cup [\underline{p}, \bar{p}]$ is symmetric and is given by (we only report the formulas for $x > 0$) . Thus the distribution of (positive) price changes is

$$\text{Price changes} \sim \begin{cases} \text{density for a price change of size } p \in [\underline{p}, \bar{p}) & : \ \frac{\lambda}{N_a} g_1(p) \\ \text{mass point at } \bar{p} & : \ \frac{1}{2} - \ \frac{\lambda}{N_a} \int_{\underline{p}}^{\bar{p}} g_1(p)\mathrm{d}p \end{cases}$$

The $j-th$ moment of price changes for j even is

$$\mathbb{E}\left(\Delta p^j\right) = \frac{\lambda}{N_a} 2 \int_{\underline{p}}^{\bar{p}} x^j \, g_1(p) \, \mathrm{d}p + \left(1 - \frac{\lambda \, 2 \int_{\underline{p}}^{\bar{p}} g_1(p)\mathrm{d}p}{N_a}\right) \bar{p}^j$$

Using that $Var\left(\Delta p\right) N\left(\Delta p\right) = \sigma^2$, the kurtosis of price changes is given by:

$$Kur\left(\Delta p\right) = \frac{\mathbb{E}\left(\Delta p^4\right)}{\left(\sigma^2 / N\left(\Delta p\right)\right)^2} \,. \tag{82}$$

Area under impulse response. To find an expression for $\mathcal{M}'(0)$ we first define the contribution to the area under impulse response of a firm that starts with price gap p. Letting $m(p)$ the integral of the (minus) expected price gap until the first time the firms adjusts its price, and starting the economy with a distribution of price gaps with density f we have

$$\mathcal{M}(\delta) \;\; = \;\; \int_{-\bar{p}}^{\bar{p}} m(p-\delta) \; g(p) \, dp \tag{83}$$

and differentiating it:

$$\mathcal{M}'(0) \;\; = \;\; -\int_{-\bar{p}}^{\bar{p}} m'(p) \; g(p) \, dp \tag{84}$$

To obtain the solution for m we consider two functions in each segments which solves:

$$0 \;\; = \;\; -p + \frac{\sigma^2}{2} m_0''(p) \text{ for } 0 \le p \le \underline{p} \tag{85}$$

$$\lambda \, m_1(p) \;\; = \;\; -p + \frac{\sigma^2}{2} m_1''(p) \text{ for } \underline{p} \le p \le \bar{p} \tag{86}$$

The boundary conditions are that these functions meet in a continuously differentiable manner in the lower boundary, i.e. $m_0(\underline{p}) = m_1(\underline{p})$, $m_0'(\underline{p}) = m_1'(\underline{p})$, and that a price change occurs at the upper boundary, i.e. $m_1(\bar{p}) = 0$. The solution, with three constant of integration is:

$$m_0(p) \;\; = \;\; A_1 \, p + \frac{p^3}{3 \, \sigma^2} \tag{87}$$

$$m_1(p) \;\; = \;\; -\frac{p}{\lambda} + A_2 \, e^{p\varphi} + A_3 \, e^{-p\varphi} \tag{88}$$

Thus, given $\underline{p}, \bar{p}$ boils down to solving three linear equations in three unknowns, as detailed in Appendix S.6.

Hence, given any pair $(\underline{p}, \bar{p})$ we can find the solution for the density g, the solution to the function m and compute:

$$\mathcal{M}'(0) \;\; = \;\; -2 \int_0^{\bar{p}} m'(p) \; g(p) \, dp$$

Likewise, given any pair $(\underline{p}, \bar{p})$, we can find the solution for g, $N(\Delta p)$ and compute $Kur(\Delta p)$ as in equation (82). In Appendix S.7 we collect the solutions as function of the thresholds $(\underline{p}, \bar{p})$ and constants $(A_1, A_2, A_3, J, C_3, C_4)$. From this one can easily compute both expressions and check the equality in

$$\mathcal{M}'(0) = \frac{Kur(\Delta p)}{6\, N(\Delta p)} \ .$$

S.1 Solution of ode for value function in inaction

$$\begin{aligned} v_0(p) &= \frac{B\, p^2}{r} + \frac{B\, \sigma^2}{r^2} + K_0 \left(e^{p\sqrt{\frac{2r}{\sigma^2}}} + e^{-p\sqrt{\frac{2r}{\sigma^2}}} \right) \\ v_1(p) &= \frac{B\, p^2 + \lambda\,(v_0(0) + b\psi)}{\lambda + r} + \frac{B\, \sigma^2}{(\lambda + r)^2} + K_1 \left(e^{p\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-p\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} \right) \end{aligned}$$

S.2 Solution for value function

Note that smooth pasting $v_1'(\bar{p}) = 0$ gives

$$0 = \frac{2B\bar{p}}{\lambda + r} + K_1 \sqrt{\frac{2(\lambda + r)}{\sigma^2}} \left(e^{\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} - e^{-\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} \right)$$

or K_1 as function of $\bar{p}$

$$K_1 = \frac{2B\bar{p}}{\lambda + r} \left[\sqrt{\frac{2(\lambda + r)}{\sigma^2}} \left(e^{-\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} - e^{\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} \right) \right]^{-1} \tag{89}$$

Using $v_0(0) = \frac{B\sigma^2}{r^2} + 2K_0$ and value matching $v_0(0) + \psi = v_1(\bar{p})$ gives

$$\frac{r}{\lambda + r} v_0(0) + \psi = \frac{\lambda b \psi + B\bar{p}^2}{\lambda + r} + \frac{B\sigma^2}{(\lambda + r)^2} + K_1 \left(e^{\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} \right)$$

or K_0 as function of $\bar{p}$

$$2rK_0 = B\bar{p}^2 - (\lambda(1-b) + r)\psi - \frac{\lambda B \sigma^2}{r(\lambda + r)} + (\lambda + r) K_1 \left(e^{\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} \right) \tag{90}$$

Value matching at $\underline{p}$ gives

$$\frac{B\underline{p}^2}{r} + \frac{B\sigma^2}{r^2} + K_0 \left(e^{\underline{p}\sqrt{\frac{2r}{\sigma^2}}} + e^{-\underline{p}\sqrt{\frac{2r}{\sigma^2}}} \right) = \frac{B\underline{p}^2 + \lambda\,(v_0(0) + b\psi)}{\lambda + r} + \frac{B\sigma^2}{(\lambda + r)^2} + K_1 \left(e^{\underline{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-\underline{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} \right)$$

or an equation implicitly defining $\underline{p}$ in terms of $\bar{p}$

$$\frac{B\underline{p}^2 \lambda}{r(r + \lambda)} + \frac{B\sigma^2 \lambda}{(\lambda + r)^2 r} + K_0 \left(e^{\underline{p}\sqrt{\frac{2r}{\sigma^2}}} + e^{-\underline{p}\sqrt{\frac{2r}{\sigma^2}}} - \frac{2\lambda}{\lambda + r} \right) = \frac{\lambda b \psi}{\lambda + r} + K_1 \left(e^{\underline{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-\underline{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} \right)$$

Given these 3 equations implicitly defining $K_0, K_1, \underline{p}$ as function of $\bar{p}$, the smooth pasting at $\underline{p}$ gives one equation in one unknown to solve for $\bar{p}$, namely

$$\left(\frac{2B}{r} - \frac{2B}{r+\lambda}\right)\underline{p} + \sqrt{\frac{2r}{\sigma^2}}K_0\left(e^{\underline{p}\sqrt{\frac{2r}{\sigma^2}}} - e^{-\underline{p}\sqrt{\frac{2r}{\sigma^2}}}\right) = \sqrt{\frac{2(\lambda+r)}{\sigma^2}}K_1\left(e^{\underline{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} - e^{-\underline{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}}\right)$$

S.3 Value function approximation

Recall

$$\begin{aligned} v_0(p) &= \frac{Bp^2}{r} + \frac{B\sigma^2}{r^2} + K_0\left(e^{p\sqrt{\frac{2r}{\sigma^2}}} + e^{-p\sqrt{\frac{2r}{\sigma^2}}}\right) \\ v_1(p) &= \frac{Bp^2 + \lambda\left(v_0(0) + b\psi\right)}{\lambda + r} + \frac{B\sigma^2}{(\lambda+r)^2} + K_1\left(e^{p\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-p\sqrt{\frac{2(\lambda+r)}{\sigma^2}}}\right) \end{aligned}$$

We approximate the value functions $v_0(p), v_1(p)$ using a fourth order expansion around $p = 0$. We get

$$\begin{aligned} v_0(p) &= \frac{B\sigma^2}{r^2} + 2K_0 + \left(\frac{B}{r} + K_0\varphi_0^2\right)\ p^2 + \frac{K_0}{12}\varphi_0^4\ p^4 \\ v_1(p) &= \frac{\lambda(v_0(0) + b\psi)}{\lambda + r} + \frac{B\sigma^2}{(\lambda+r)^2} + 2K_1 + \left(\frac{B}{\lambda+r} + K_1\varphi_1^2\right)\ p^2 + \frac{K_1}{12}\varphi_1^4\ p^4 \\ &\text{where } \varphi_0 \equiv \sqrt{\frac{2r}{\sigma^2}} \quad \text{and} \quad \varphi_1 \equiv \sqrt{\frac{2(\lambda+r)}{\sigma^2}} \end{aligned}$$

The smooth pasting at $\underline{p}$, namely $v_0'(\underline{p}) - v_1'(\underline{p}) = 0$, gives

$$p\left[\left(\frac{B}{r} + K_0\varphi_0^2\right) - \left(\frac{B}{\lambda+r} + K_1\varphi_1^2\right) + \left(K_0\varphi_0^4 - K_1\varphi_1^4\right)\frac{p^2}{6}\right] \quad = \quad 0$$

which gives

$$\underline{p} \quad = \quad \pm\sqrt{\frac{\left(\frac{B}{\lambda+r} + K_1\varphi_1^2\right) - \left(\frac{B}{r} + K_0\varphi_0^2\right)}{\left(K_0\varphi_0^4 - K_1\varphi_1^4\right)/6}}$$

Similarly smooth pasting at $\bar{p}$ gives

$$\bar{p} \quad = \quad \pm\sqrt{\frac{\left(\frac{B}{\lambda+r} + K_1\varphi_1^2\right)}{-K_1\varphi_1^4/6}}$$

S.4 Boundary conditions for T_i

We have the following three linear equations for T_i:

$$
\begin{aligned}
-\frac{1}{\lambda} &= K e^{\varphi \bar{p}} + L e^{-\varphi \bar{p}} \\
-\frac{2\,\underline{p}}{\sigma^2} &= \varphi \left(K e^{\varphi \underline{p}} - L e^{-\varphi \underline{p}} \right) \\
J &= \frac{\left(\underline{p}\right)^2}{\sigma^2} + \frac{1}{\lambda} + K e^{\varphi \underline{p}} + L e^{-\varphi \underline{p}}
\end{aligned}
$$

S.5 Density function

The 4 unknowns of the density function, using $g_1(\bar{p}) = 0$ and $g_0'(\underline{p}) = g_1'(\underline{p})$, give

$$
C_3 = -C_4 e^{-2\varphi \bar{p}} \quad \text{and} \quad C_2 = -C_4 \varphi \left(e^{-2\varphi \bar{p} + \varphi \underline{p}} + e^{-\varphi \underline{p}} \right)
$$

Next, using $g_0(\underline{p}) = g_1(\underline{p})$ gives

$$
C_1 = -C_2 \underline{p} - C_4 \left(e^{-2\varphi \bar{p} + \varphi \underline{p}} - e^{-\varphi \underline{p}} \right) = C_4 \left[e^{-2\varphi \bar{p} + \varphi \underline{p}} \left(\varphi \underline{p} - 1 \right) + e^{-\varphi \underline{p}} \left(\varphi \underline{p} + 1 \right) \right]
$$

Finally we solve for C_4 by imposing $1/2 = \int_0^{\underline{p}} g_0(p)\, \mathrm{d}p + \int_{\underline{p}}^{\bar{p}} g_1(p)\, \mathrm{d}p$ i.e.

$$
\frac{1}{2} = C_1 \underline{p} + \frac{1}{2} C_2 \underline{p}^2 + \frac{1}{\varphi} \left[C_3 \left(e^{\varphi \bar{p}} - e^{\varphi \underline{p}} \right) - C_4 \left(e^{-\varphi \bar{p}} - e^{-\varphi \underline{p}} \right) \right]
$$

or, substituting the expressions,

$$
\begin{aligned}
\frac{1}{2C_4} = {} & \left[e^{-2\varphi \bar{p} + \varphi \underline{p}} \left(\varphi \underline{p} - 1 \right) + e^{-\varphi \underline{p}} \left(\varphi \underline{p} + 1 \right) \right] \underline{p} - \frac{1}{2} \varphi \left(e^{-2\varphi \bar{p} + \varphi \underline{p}} + e^{-\varphi \underline{p}} \right) \underline{p}^2 \\
& - \frac{1}{\varphi} \left[e^{-2\varphi \bar{p}} \left(e^{\varphi \bar{p}} - e^{\varphi \underline{p}} \right) + e^{-\varphi \bar{p}} - e^{-\varphi \underline{p}} \right]
\end{aligned}
$$

S.6 Equation for the solution of m

The boundary conditions are: $m_1(\bar{p}) = 0$, $m_1(\underline{p}) = m_0(\underline{p})$ and $m_1'(\underline{p}) = m_0'(\underline{p})$. They give a linear system of equations on A_1, A_2, A_3 :

$$
0 = -\frac{\bar{p}}{\lambda} + A_2\, e^{\bar{p}\varphi} + A_3\, e^{-\bar{p}\varphi} \tag{91}
$$

$$
A_1 + \frac{(\underline{p})^2}{\sigma^2} = -\frac{1}{\lambda} + \varphi\, A_2\, e^{\underline{p}\varphi} - \varphi\, A_3\, e^{-\underline{p}\varphi} \tag{92}
$$

$$
A_1\, \underline{p} + \frac{(\underline{p})^3}{3\,\sigma^2} = -\frac{\underline{p}}{\lambda} + A_2\, e^{\underline{p}\varphi} + A_3\, e^{-\underline{p}\varphi} \tag{93}
$$

S.7 Algebraic details for main proposition

For the area under the IRF of output we get:

$$\begin{aligned}\mathcal{M}'(0) = & - \;\; 2\int_0^{\underline{p}} \left[A_1 + A_3 \frac{p^2}{\sigma^2}\right] [C_1 + C_2 p]\, \mathrm{d}p \\ & - \;\; 2\int_{\underline{p}}^{\bar{p}} \left[-\frac{1}{\lambda} + \varphi\, A_2\, e^{p\varphi} + \varphi\, A_3\, e^{-p\varphi}\right] \left[C_3 e^{\varphi p} + C_4 e^{-\varphi p}\right] \mathrm{d}p\end{aligned}$$

For the kurtosis of steady state price changes we get:

$$\begin{aligned}\frac{Kur\,(\Delta p)}{6\,N\,(\Delta p)} &= N\,(\Delta p)\,\frac{\mathbb{E}\,(\Delta p^4)}{6\,\sigma^4} \\ &= \frac{\lambda\,J\,2\,\int_{\underline{p}}^{\bar{p}} p^4\,\left[C_3 e^{\varphi p} + C_4 e^{-\varphi p}\right] \mathrm{d}p + \left(1 - \frac{\lambda\,2\,\int_{\underline{p}}^{\bar{p}}\left[C_3 e^{\varphi p} + C_4 e^{-\varphi p}\right]\mathrm{d}p}{N_a}\right)\bar{p}^4}{6\,J\,\sigma^4} \\ &= \frac{\lambda\,2}{6\,\sigma^4}\int_{\underline{p}}^{\bar{p}} p^4\,\left[C_3 e^{\varphi p} + C_4 e^{-\varphi p}\right] \mathrm{d}p + \frac{1}{6\,J\,\sigma^4} - \frac{\lambda\,2}{6\,\sigma^4}\int_{\underline{p}}^{\bar{p}} \bar{p}^4\,\left[C_3 e^{\varphi p} + C_4 e^{-\varphi p}\right] \mathrm{d}p \\ &= \frac{\lambda\,2}{6\,\sigma^4}\int_{\underline{p}}^{\bar{p}} \left(p^4 - \bar{p}^4\right)\,\left[C_3 e^{\varphi p} + C_4 e^{-\varphi p}\right] \mathrm{d}p + \frac{1}{6\,J\,\sigma^4}\end{aligned}$$