

# THE DESIGN OF TRADE AGREEMENTS: ONLINE APPENDIX

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## Abstract

This online Appendix extends to a many-good setting the main features of reciprocity emphasized in section 3.1 of our Handbook chapter.

## Online Appendix: Reciprocity with Many Goods

In this Online Appendix we discuss the extension of the main features of reciprocity emphasized in section 3.1 of our Handbook chapter to a setting with more than two goods. We have already described in the text how Bagwell and Staiger (2015) show that these features are preserved in a partial equilibrium setting where the non-numeraire sector is a monopolistically competitive industry with many varieties. Bagwell and Staiger (2001a) demonstrate how these features extend to a 3-good partial equilibrium setting where each of the two non-numeraire goods is a competitive homogeneous-good industry. And Bagwell and Staiger (1999, note 16) describe how the terms-of-trade fixing property of reciprocity extends to an  $N$ -good version of the two-country competitive general equilibrium trade model featured above.<sup>1</sup> Here we work within an  $N$ -good general equilibrium trade model and derive the three key properties of reciprocity emphasized in section 3.1: (i) if a common terms of exchange of market access is to be applied for both countries, then it must be one for one, the same terms of exchange embodied in GATT's reciprocity principle; (ii) beginning from their Nash tariffs, both countries can gain from reciprocal liberalization provided they do not go too far; and (iii) beginning from their efficient politically optimal tariffs, neither country could gain from renegotiation subject to reciprocity.

To accommodate  $N$  goods, we choose good 1 as the numeraire and now let  $p_i^{w0}$  denote the world price of good  $i$  relative to the world price of the numeraire good 1 under an initial set of trade policies, and we let  $\mathbf{p}^{w0'}$  denote the  $(1 \times N)$  vector of this set of initial world prices (that is,  $\mathbf{p}^{w0'}$  is composed of the set of  $N - 1$  initial relative world prices, plus the first element of  $\mathbf{p}^{w0'}$  which is equal to 1). We then define  $\mathbf{E}^0$  as the  $(N \times 1)$  vector of home country export volumes, where the  $j^{th}$  element of  $\mathbf{E}^0$  equals 0 if the home country does not export good  $j$  under the initial set of trade policies and equals the home country export volume of good  $j$  otherwise; and similarly we define

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<sup>1</sup>Bagwell and Staiger, 2002, Appendix B.3 considers extensions of the properties of reciprocity in a many-good many-country setting when MFN is also imposed.

$\mathbf{M}^0$  as the  $(N \times 1)$  vector of home country import volumes, where the  $j^{th}$  element of  $\mathbf{M}^0$  equals 0 if the home country does not import good  $j$  under the initial set of trade policies and equals the home country import volume of good  $j$  otherwise. The analogous vectors for the foreign country are denoted by  $\mathbf{E}^{*0}$  and  $\mathbf{M}^{*0}$ . And finally, we denote with a superscript “1” these vectors under an alternative set of trade policies.

With this new minimal notation for the  $N$ -good model now defined, let us as in section 3.1 begin with the home country and consider first a general version of reciprocity for the  $N$ -good setting defined as any change in tariffs that leads to a change in home-country export and import volumes satisfying

$$\mathbf{p}^{w0'}[\mathbf{E}^1 - \mathbf{E}^0] = \gamma \mathbf{p}^{w0'}[\mathbf{M}^1 - \mathbf{M}^0] \quad (1)$$

where as before  $\gamma$  is a parameter specifying the terms of exchange of market access. We next observe that market clearing implies

$$[\mathbf{E}^1 - \mathbf{E}^0] = [\mathbf{M}^{*1} - \mathbf{M}^{*0}] \text{ and } [\mathbf{E}^{*1} - \mathbf{E}^{*0}] = [\mathbf{M}^1 - \mathbf{M}^0],$$

whence (1) then implies  $\mathbf{p}^{w0'}[\mathbf{E}^1 - \mathbf{E}^0] = \gamma \mathbf{p}^{w0'}[\mathbf{E}^{*1} - \mathbf{E}^{*0}]$  and therefore  $\mathbf{p}^{w0'}[\mathbf{M}^{*1} - \mathbf{M}^{*0}] = \gamma \mathbf{p}^{w0'}[\mathbf{E}^{*1} - \mathbf{E}^{*0}]$  or

$$\mathbf{p}^{w0'}[\mathbf{E}^{*1} - \mathbf{E}^{*0}] = \frac{1}{\gamma} \mathbf{p}^{w0'}[\mathbf{M}^{*1} - \mathbf{M}^{*0}], \quad (2)$$

which describes the foreign-country terms of exchange of market access that must accompany (1) according to the market clearing requirements. From (1) and (2), it follows that for a *common* terms-of-exchange across countries we must have  $\gamma = 1$ , and thus a one-for-one exchange of import volumes for export volumes. Hence, in this  $N$ -good setting it remains the case that the adding-up constraint imposed by market clearing makes it inevitable that, if governments wish to adopt a common terms of exchange for all countries, they must adopt the one-for-one terms of exchange that characterizes GATT’s reciprocity principle.

We now turn to the remaining tasks of this Appendix, and show that, beginning from their Nash tariffs, both countries can gain from reciprocal liberalization provided they do not go too far, and that beginning from their efficient politically optimal tariffs, neither country could gain from renegotiation subject to reciprocity. As before, for these purposes we adopt the perfectly competitive version of the two-country general equilibrium trade model described in Section 2, extended here to the  $N$ -good case. As in our discussion just above, there are  $N - 1$  relative prices, and we suppose without loss of generality that each country imposes trade taxes on the same  $N - 1$  goods, goods 2 through  $N$ . Let  $\tau_i > 0$  denote a home-country import tariff or export tax, with  $\tau_i < 0$  a home-country import or export subsidy. Similarly, let  $\tau_i^* > 0$  denote a foreign-country import tariff or export tax, with  $\tau_i^* < 0$  a foreign-country import or export subsidy. Then, for  $i = 2, \dots, N$ , we have world prices given by  $p_i^w(\tau_2, \dots, \tau_N, \tau_2^*, \dots, \tau_N^*)$ , home local prices given by  $p_i(\tau_i, p_i^w(\tau_2, \dots, \tau_N, \tau_2^*, \dots, \tau_N^*))$ , and foreign local prices given by  $p_i^*(\tau_i^*, p_i^w(\tau_2, \dots, \tau_N, \tau_2^*, \dots, \tau_N^*))$ .

We assume away the Lerner paradox for all  $i$ , so that  $\frac{\partial p_i^w}{\partial \tau_i} < 0$  for  $i$  a home import good,  $\frac{\partial p_i^w}{\partial \tau_i} > 0$  for  $i$  a home export good,  $\frac{\partial p_i^w}{\partial \tau_i^*} < 0$  for  $i$  a foreign import good, and  $\frac{\partial p_i^w}{\partial \tau_i^*} > 0$  for  $i$  a foreign export good. And we also assume away the Metzler paradox for any  $i$ ; hence,  $\frac{dp_i}{d\tau_i} > 0$  for  $i$  a Home import good,  $\frac{dp_i}{d\tau_i} < 0$  for  $i$  a Home export good,  $\frac{dp_i^*}{d\tau_i^*} > 0$  for  $i$  a Foreign import good, and  $\frac{dp_i^*}{d\tau_i^*} < 0$  for  $i$  a Foreign export good.

We also assume that “direct tariff effects dominate indirect tariff effects” in the following sense.

First, we assume that,

$$\text{sign}\left(\sum_{k=2}^N \frac{\partial p_i^w}{\partial \tau_k}\right) = \text{sign}\left(\frac{\partial p_i^w}{\partial \tau_i}\right); \text{sign}\left(\sum_{k=2}^N \frac{\partial p_i^w}{\partial \tau_k^*}\right) = \text{sign}\left(\frac{\partial p_i^w}{\partial \tau_i^*}\right) \text{ for } i = 2, \dots, N, \quad (3)$$

so that the impact of an increase in a country's good- $i$  trade tax on the world price of good  $i$  dominates the indirect impact on that world price of an increase in all of that country's other (good  $j \neq i$ ) tariffs combined. Second, we assume that

$$\text{sign}\left(\sum_{k=2}^N \frac{dp_i}{d\tau_k}\right) = \text{sign}\left(\frac{dp_i}{d\tau_i}\right) \text{ for } i = 2, \dots, N, \quad (4)$$

$$\text{sign}\left(\sum_{k=2}^N \frac{dp_i^*}{d\tau_k^*}\right) = \text{sign}\left(\frac{dp_i^*}{d\tau_i^*}\right) \text{ for } i = 2, \dots, N. \quad (5)$$

so that the impact of an increase in a country's good- $i$  trade tax on its local price of good  $i$  dominates the indirect impact on that local price of an increase in all of that country's other (good  $j \neq i$ ) tariffs combined. These assumptions are stronger than necessary, but serve to make the basic argument transparent. In effect, (3) will be used in combination with (6) below and the absence of the Lerner paradox to ensure that costs are shifted abroad when a country increases its trade taxes, while (4) and (5) will be used together with the absence of the Metzler paradox to ensure that the reciprocal trade tax changes characterized below involve reductions (no increases) in every trade tax.

Finally, we depict home and foreign government objectives respectively by the functions

$$W(p_2, \dots, p_N, p_2^w, \dots, p_N^w) \text{ and } W^*(p_2^*, \dots, p_N^*, p_2^w, \dots, p_N^w).$$

We impose the following structure on these objective functions:<sup>2</sup>

$$\begin{aligned} W_{p_i^w} &< 0 \text{ for } i \text{ a home import; } W_{p_i^w} > 0 \text{ for } i \text{ a home export; and} \\ W_{p_i^*} &< 0 \text{ for } i \text{ a foreign import; } W_{p_i^*} > 0 \text{ for } i \text{ a foreign export.} \end{aligned} \quad (6)$$

We begin by characterizing Nash trade taxes. The Nash first-order conditions are:

$$\sum_{i=2}^N W_{p_i} \frac{dp_i}{d\tau_k} + \sum_{i=2}^N W_{p_i^w} \frac{\partial p_i^w}{\partial \tau_k} = 0 \text{ for } k = 2, \dots, N, \quad (7)$$

$$\sum_{i=2}^N W_{p_i^*} \frac{dp_i^*}{d\tau_k^*} + \sum_{i=2}^N W_{p_i^w} \frac{\partial p_i^w}{\partial \tau_k^*} = 0 \text{ for } k = 2, \dots, N. \quad (8)$$

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<sup>2</sup>This approach to writing government objectives and imposing minimal structure on those objectives in a multi-good setting is analogous to that taken in Appendix B of Bagwell and Staiger (2002).

Summing each of the Nash conditions (7) and (8) over all  $N - 1$  tariffs yields:

$$\begin{aligned} \sum_{i=2}^N W_{p_i} \left[ \sum_{k=2}^N \frac{dp_i}{d\tau_k} \right] + \sum_{i=2}^N W_{p_i^w} \left[ \sum_{k=2}^N \frac{\partial p_i^w}{d\tau_k} \right] &= 0, \\ \sum_{i=2}^N W_{p_i^*} \left[ \sum_{k=2}^N \frac{dp_i^*}{d\tau_k^*} \right] + \sum_{i=2}^N W_{p_i^{*w}} \left[ \sum_{k=2}^N \frac{\partial p_i^{*w}}{d\tau_k^*} \right] &= 0. \end{aligned} \quad (9)$$

By the assumed absence of the Lerner paradox and conditions (3) and (6), it follows that

$$\sum_{i=2}^N W_{p_i^w} \left[ \sum_{k=2}^N \frac{\partial p_i^w}{d\tau_k} \right] > 0 \quad \text{and} \quad \sum_{i=2}^N W_{p_i^{*w}} \left[ \sum_{k=2}^N \frac{\partial p_i^{*w}}{d\tau_k^*} \right] > 0. \quad (10)$$

In words, and referring to (9), we then have by (10) that an increase in all of a country's trade taxes implies a positive "international cost shifting" component through the induced world price movements. And more specifically, using (10), the sum of the Nash conditions as displayed in (9) implies that at Nash trade taxes we must have:

$$\sum_{i=2}^N W_{p_i} \left[ \sum_{k=2}^N \frac{dp_i}{d\tau_k} \right] < 0 \quad \text{and} \quad \sum_{i=2}^N W_{p_i^*} \left[ \sum_{k=2}^N \frac{dp_i^*}{d\tau_k^*} \right] < 0. \quad (11)$$

We next express our formal definition of reciprocity for the  $N$ -good environment in terms of the notation we have introduced just above. From an initial set of tariffs,  $(\tau_2^0, \dots, \tau_N^0, \tau_2^{*0}, \dots, \tau_N^{*0})$ , suppose that a tariff negotiation results in a change to the new pair of tariffs,  $(\tau_2^1, \dots, \tau_N^1, \tau_2^{*1}, \dots, \tau_N^{*1})$ . Denoting the initial world and home local prices as  $\tilde{p}_i^{w0} \equiv \tilde{p}_i^w(\tau_2^0, \dots, \tau_N^0, \tau_2^{*0}, \dots, \tau_N^{*0})$  and  $p_i^0 \equiv p_i(\tau_i^0, \tilde{p}_i^{w0})$  for  $i = 2, \dots, N$ , and the new world and home local prices as  $\tilde{p}_i^{w1} \equiv \tilde{p}_i^w(\tau_2^1, \dots, \tau_N^1, \tau_2^{*1}, \dots, \tau_N^{*1})$  and  $p_i^1 \equiv p_i(\tau_i^1, \tilde{p}_i^{w1})$  for  $i = 2, \dots, N$ , and with  $p_1^0 = \tilde{p}_1^{w0} = \tilde{p}_1^{w1} = p_1^1 \equiv 1$  for the numeraire good 1, and finally letting  $\mathcal{M}$  and  $\mathcal{E}$  represent the set of home import goods and home export goods, respectively, we say that the tariff changes conform to *the principle of reciprocity* provided that

$$\sum_{i \in \mathcal{M}} \tilde{p}_i^{w0} [M_i(p^1, \tilde{p}^{w1}) - M_i(p^0, \tilde{p}^{w0})] = \sum_{i \in \mathcal{E}} \tilde{p}_i^{w0} [E_i(p^1, \tilde{p}^{w1}) - E_i(p^0, \tilde{p}^{w0})]. \quad (12)$$

As in section 3.1, using the balanced trade condition that must hold both at initial and new tariffs, it is straightforward to show that reciprocity implies

$$\sum_{i \in \mathcal{M}} [\tilde{p}_i^{w1} - \tilde{p}_i^{w0}] M_i(p^1, \tilde{p}^{w1}) = \sum_{i \in \mathcal{E}} [\tilde{p}_i^{w1} - \tilde{p}_i^{w0}] E_i(p^1, \tilde{p}^{w1}). \quad (13)$$

That is, in the  $N$ -good case, tariff changes that conform to reciprocity imply either that world prices are left unchanged as a result of the tariff changes, or if world prices are altered, that they are altered in a way that leaves net trade-tax revenue unchanged.

Now consider, beginning from Nash trade taxes, a small change in every home and foreign trade tax that (a) induces a change in  $p_i$  equal to  $-\sum_{k=2}^N \frac{dp_i}{d\tau_k}$  evaluated at Nash policies for  $i = 2, \dots, N$ , thereby replicating the local price changes for the home country induced by a small *unilateral reduction* in all of its trade taxes beginning from Nash, (b) induces a change in  $p_i^*$  equal

to  $-\sum_{k=2}^N \frac{dp_i^*}{d\tau_k^*}$  evaluated at Nash policies for  $i = 2, \dots, N$ , thereby replicating the local price changes for the foreign country induced by a small *unilateral reduction* in all of its trade taxes beginning from Nash, and (c) satisfies reciprocity as defined in (12). Achieving (a), (b) and (c) is feasible, because each local price change in each country can be targeted with the associated trade tax on that good, and the overall relative magnitudes of the home and foreign tariff changes can be adjusted to achieve reciprocity (just as in the 2-good case). And with the absence of the Metzler paradox and under assumptions (4) and (5) that direct tariff impacts outweigh indirect effects, each trade tax will be reduced under this maneuver. And finally, according to (13), by conforming to reciprocity these tariff changes either keep all world prices fixed, or alter world prices in a way that is welfare neutral for each country. In either case, with reciprocity ensuring that any world price movements are immaterial for each country's welfare, the change in home and foreign welfare from these reciprocal trade tax reductions beginning from Nash is then given by focusing only on the impact of the local price movements:

$$\sum_{i=2}^N W_{p_i} \left[ -\sum_{k=2}^N \frac{dp_i}{d\tau_k} \right] > 0 \quad \text{and} \quad \sum_{i=2}^N W_{p_i^*} \left[ -\sum_{k=2}^N \frac{dp_i^*}{d\tau_k^*} \right] > 0, \quad (14)$$

where the inequalities follow from (11). Given the absence of the Metzler paradox and with our assumptions (4) and (5) that direct tariff impacts outweigh indirect effects, (14) implies that both countries must gain. Hence, we have established that, beginning from their Nash tariffs, both countries can gain from at least a small amount of reciprocal liberalization.

The remaining task of this Appendix is to show that, beginning from their efficient politically optimal tariffs, neither country could gain from renegotiation subject to reciprocity. In this  $N$ -good setting, the political optimum is defined by the tariffs that conform to each government's politically optimal reaction curve and therefore satisfy

$$\sum_{i=2}^N W_{p_i} \frac{dp_i}{d\tau_k} = 0 \quad \text{for } k = 2, \dots, N; \quad \text{and} \quad \sum_{i=2}^N W_{p_i^*} \frac{dp_i^*}{d\tau_k^*} = 0 \quad \text{for } k = 2, \dots, N.$$

It is direct to show that the political optimum is efficient in the  $N$ -good setting. But arguing as above it is also now immediate that beginning from the political optimum, a small increase in any of the tariffs of one country can be met with reciprocal changes in the tariffs of its trading partner which together induce changes in the first country's local prices which are identical to those described by the first-order condition defining its politically optimal reaction curve tariffs. With this, the first-order conditions that define the first country's politically optimal reaction curve tariffs ensure that it could not gain from the local price movements implied by its tariff increases, while reciprocity neutralizes the welfare implications for the first country of any world price movements that are implied by its tariff increases as well. Therefore, in the  $N$ -good setting, beginning from the political optimum neither country can gain from renegotiation subject to reciprocity.