

Online Appendix for
Trade and Domestic Production Networks

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A Theoretical Results

A.1 Firm-level cost changes under general foreign shocks

Here we derive expressions analogous to Proposition 1, but considering more general foreign shocks. We consider import price changes that are heterogeneous across firms, $\{\hat{p}_{Fj}\}$, and in addition consider firm-level changes in export demand, $\{\hat{\beta}_{jF}\}$. To compute the changes in equilibrium variables given these shocks, we solve the system of equations by following the steps below.

1. Guess \hat{w} . Compute $\{\hat{c}_i\}$ from

$$\hat{c}_j = \left((1 - s_{Fj}^{Total}) \hat{w}^{1-\rho} + t_{Fj} \right)^{\frac{1}{1-\rho}},$$

where t_{Fj} is the obtained by solving the following system:

$$t_{Fj} = s_{Fj} \hat{p}_{Fj}^{1-\rho} + \sum_{i \in Z_j^D} s_{ij} t_{Fi}.$$

This is in contrast with the definition of s_{Fj}^{Total} , where the analogous system is $s_{Fj}^{Total} = s_{Fj}^{Total} + \sum_{i \in Z_j^D} s_{ij} s_{Fi}^{Total}$.

2. Compute the following hat variables.

$$\begin{aligned} \hat{x}_{iF} &= \hat{\beta}_{iF}^{1-\sigma} \hat{c}_i^{1-\sigma} \quad (\text{if } I_{iF} = 1) \\ \hat{P}^{1-\sigma} &= \sum_i s_{iH} \hat{c}_i^{1-\sigma} \\ \hat{s}_{Li} &= \hat{w}^{1-\rho} \hat{c}_i^{\rho-1} \\ \hat{s}_{ij} &= \hat{c}_i^{1-\rho} \hat{c}_j^{\rho-1} \quad (\text{if } i \in Z_j), \end{aligned}$$

where \hat{x}_{iF} is defined for exporting firms.

3. Solve for $\{\hat{x}_i\}$ from

$$\hat{x}_i = \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{x_{iH}}{x_i} \hat{c}_i^{1-\sigma} \hat{P}^{\sigma-1} \left(\frac{wL}{E} \hat{w} + \sum_k \frac{\mu_k - 1}{\mu_k} \frac{x_k}{E} \hat{x}_k - \frac{TB}{E} \right) + \sum_{j \in W_i} \frac{\hat{s}_{ij} x_{ij}}{x_i} \hat{x}_j.$$

4. Update the guess of \hat{w} with

$$\hat{w} = \sum_i \frac{s_{Li} x_i}{\mu_i w L} \hat{s}_{Li} \hat{x}_i,$$

and iterate from Step 1 until \hat{w} converges.

A.2 Proposition 1 and 2 under continuum of firms

Here we show that analogous results to Propositions 1 and 2 can be obtained under the assumption of a continuum of firms. To represent a network of a continuum of firms, we use a notation similar to Lim (2018). Denote $m(\chi', \chi)$ the probability a type- χ firm sourcing from a type- χ' firm. The key difference between the continuous network representation from the model described in the main text is that the continuous network is characterized by a probability measure that any two types of firms are matched, while the discrete network features a finite number of possible linkages between firms taking values of either 0 (not connected) or 1 (connected). χ is a collection of firm characteristics which are all continuous and have bounded support: $\{\beta_H(\chi), \beta_F(\chi), \alpha_L(\chi), \alpha_F(\chi)\}$. $g(\chi', \chi)$ measures the efficiency of a match between a firm with type- χ' and a firm with type- χ . In addition, $v(\chi)$ is the probability density function of type- χ firms. We assume that the functions $m(\cdot, \cdot)$, $g(\cdot, \cdot)$, and $v(\chi)$ are all continuous and have bounded supports.

We first list the key variables analogous to ones derived in the main text. The consumer preference is:

$$U = \left(\int (\beta_H(\chi) q(\chi, H))^{\frac{\sigma-1}{\sigma}} dV(\chi) \right)^{\frac{\sigma}{\sigma-1}}.$$

The aggregate final consumer demand for a type- χ firm is:

$$q(\chi, H) = \beta_H(\chi)^{\sigma-1} \frac{p(\chi, H)^{-\sigma}}{P^{1-\sigma}} E.$$

The price index is represented by:

$$P^{1-\sigma} = \int \beta_H(\chi)^{\sigma-1} p(\chi, H)^{1-\sigma} dV(\chi).$$

Foreign sales takes the following form:

$$q(\chi, F) = \beta_F(\chi)^{\sigma-1} \frac{p(\chi, F)^{-\sigma}}{P_F^{1-\sigma}} E_F.$$

The unit cost function for a type- χ firm:

$$c(\chi) = \frac{1}{\phi} \left(\alpha_L(\chi)^{\rho-1} w^{1-\rho} + m(F, \chi) \alpha_F(\chi)^{\rho-1} p_F^{1-\rho} + \int m(\chi', \chi) g(\chi', \chi)^{\rho-1} p(\chi', \chi)^{1-\rho} dV(\chi') \right)^{\frac{1}{1-\rho}}.$$

The sourcing capability of firm with type χ :

$$\Theta(\chi) = \alpha_L(\chi)^{\rho-1} w^{1-\rho} + m(F, \chi) \alpha_F(\chi)^{\rho-1} p_F^{1-\rho} + \int m(\chi', \chi) g(\chi', \chi)^{\rho-1} p(\chi', \chi)^{1-\rho} dV(\chi').$$

The share of variable cost of type- χ firm spent on type- χ' firm:

$$s(\chi', \chi) = \frac{m(\chi', \chi) g(\chi', \chi)^{\rho-1} p(\chi', \chi)^{1-\rho}}{\Theta(\chi)}.$$

Cost share spent on labor is:

$$s(L, \chi) = \frac{\alpha_L(\chi)^{\rho-1} w^{1-\rho}}{\Theta(\chi)}.$$

Cost share spent on foreign inputs is:

$$s(F, \chi) = \frac{m(F, \chi) \alpha_F(\chi)^{\rho-1} p_F^{1-\rho}}{\Theta(\chi)}.$$

The total foreign share becomes:

$$s^{Total}(F, \chi) = s(F, \chi) + \int s^{Total}(F, \chi') s(\chi', \chi) dV(\chi').$$

Now, Proposition 1 holds for a continuum of firms: Given fixed linkages between firms, $m(\chi', \chi)$, the change in firm j 's unit cost, $\hat{c}_j |^{\hat{p}_F}$ given a uniform change in foreign prices, \hat{p}_F , is:

$$\hat{c}(\chi) |^{\hat{p}_F} = \left((1 - s^{Total}(F, \chi)) \hat{w}^{1-\rho} + s^{Total}(F, \chi) \hat{p}_F^{1-\rho} \right)^{1/(1-\rho)}.$$

For a small percentage point change in the foreign price, $\frac{dp_F}{p_F}$, the first-order approximation to firm j 's unit cost is

$$\frac{dc(\chi)}{c(\chi)} = (1 - s^{Total}(F, \chi)) \frac{dw}{w} + s^{Total}(F, \chi) \frac{dp_F}{p_F}.$$

The proof largely follows the one for Proposition 1.

$$\begin{aligned}
(\hat{c}(\chi) |^{\hat{p}_{F.}})^{1-\rho} &= s(L, \chi) \hat{w}^{1-\rho} + s(F, \chi) \hat{p}_{F.}^{1-\rho} + \int s(\chi', \chi) \hat{p}(\chi', \chi)^{1-\rho} dV(\chi') \\
&= \left(s(L, \chi) + \int s(\chi', \chi) s(L, \chi') dV(\chi') + \right. \\
&\quad \left. \int s(\chi', \chi) \int s(\chi'', \chi') s(L, \chi'') dV(\chi') dV(\chi'') + \dots \right) \hat{w}^{1-\rho} \\
&\quad + \left(s(F, \chi) + \int s(\chi', \chi) s(F, \chi') dV(\chi') + \right. \\
&\quad \left. \int s(\chi', \chi) \int s(\chi'', \chi') s(F, \chi'') dV(\chi') dV(\chi'') + \dots \right) \hat{p}_{F.}^{1-\rho} \\
&= (1 - s^{Total}(F, \chi)) \hat{w}^{1-\rho} + s^{Total}(F, \chi) \hat{p}_{F.}^{1-\rho}.
\end{aligned}$$

Proposition 2 also holds for a continuum of firms. Given fixed linkages between firms, the change in the real wage, $\frac{\hat{w}}{\hat{P}}$, due to an uniform change in foreign price, $\hat{p}_{F.}$, is:

$$\begin{aligned}
\frac{\hat{w}}{\hat{P}} |^{\hat{p}_{F.}} &= \hat{w} \left(\int s(\chi, H) (\hat{c}(\chi) |^{\hat{p}_{F.}})^{1-\sigma} dV(\chi) \right)^{\frac{1}{\sigma-1}} \\
&= \left(\int s(\chi, H) \left((1 - s^{Total}(F, \chi)) + s^{Total}(F, \chi) \frac{\hat{p}_{F.}^{1-\rho}}{\hat{w}^{1-\rho}} \right)^{\frac{1-\sigma}{1-\rho}} dV(\chi) \right)^{\frac{1}{\sigma-1}}.
\end{aligned}$$

For a small change in the foreign price, $\frac{dp_{F.}}{p_{F.}}$, the first-order approximation to the change in the real wage is:

$$\begin{aligned}
\frac{dw}{w} - \frac{dP}{P} &= \frac{dw}{w} - \int s(\chi, H) \frac{dc(\chi)}{c(\chi)} \\
&= \left(\frac{dw}{w} - \frac{dp_{F.}}{p_{F.}} \right) \int s(\chi, H) s^{Total}(F, \chi) dV(\chi).
\end{aligned}$$

A.3 System of hat equations allowing for changes in wages

Here we describe the system of equations to solve for the changes in equilibrium variables in general equilibrium given $\hat{p}_{F.}$. We also outline the firm-level cost changes and change in real wage when foreign price goes to infinity and the economy goes into autarky. We do this for three different models: the baseline model that utilizes the observed firm-to-firm linkages, the roundabout production economy, and the sectoral roundabout production economy.

Throughout the three models, the labor market clearing condition and the household's

budget constraint are the same. The labor market clearing condition is

$$wL = \sum_i \frac{1}{\mu_i} s_{Li} x_i,$$

where μ_i is the average firm-level markups that are constant, $\mu_i = \frac{x_i}{c_i q_i}$. Household's budget constraint is

$$\begin{aligned} E &= wL + \sum_i \pi_i - TB \\ &= wL + \sum_i \frac{\mu_i - 1}{\mu_i} x_i - TB \end{aligned}$$

where TB is the exogenous trade balance, $TB = \sum_i x_{iF} - \sum_i x_{Fi}$.

A.3.1 Baseline

Given a shock of \hat{p}_F , we solve the system of equations by following the steps below. In the special case of autarky, $\hat{p}_F \rightarrow \infty$, the firm-level cost changes are given by $\hat{c}_j |^{\hat{p}_F \rightarrow \infty} = (1 - s_{Fj}^{Total})^{1/(1-\rho)}$, and the change in real wage is $\frac{\hat{w}}{\hat{P}} |^{\hat{p}_F \rightarrow \infty} = \left(\sum_i s_{iH} (1 - s_{Fi}^{Total})^{\frac{1-\sigma}{1-\rho}} \right)^{\frac{1}{\sigma-1}}$. Using these firm-level cost changes one can proceed with the Steps 2 and 3 below by substituting \hat{w} with 1.

1. Guess \hat{w} . Compute $\{\hat{c}_i\}$ from

$$\hat{c}_j^{1-\rho} = (1 - s_{Fj}^{Total}) \hat{w}^{1-\rho} + s_{Fj}^{Total} \hat{p}_F^{1-\rho}.$$

2. Compute the following hat variables.

$$\begin{aligned} \hat{x}_{iF} &= \hat{c}_i^{1-\sigma} \quad (\text{if } I_{iF} = 1) \\ \hat{P}^{1-\sigma} &= \sum_i s_{iH} \hat{c}_i^{1-\sigma} \\ \hat{s}_{Li} &= \hat{w}^{1-\rho} \hat{c}_i^{\rho-1} \\ \hat{s}_{ij} &= \hat{c}_i^{1-\rho} \hat{c}_j^{\rho-1} \quad (\text{if } i \in Z_j), \end{aligned}$$

where \hat{x}_{iF} is defined for exporting firms.

3. Solve for $\{\hat{x}_i\}$ from

$$\hat{x}_i = \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{x_{iH}}{x_i} \hat{c}_i^{1-\sigma} \hat{P}^{\sigma-1} \left(\frac{wL}{E} \hat{w} + \sum_k \frac{\mu_k - 1}{\mu_k} \frac{x_k}{E} \hat{x}_k - \frac{TB}{E} \right) + \sum_{j \in W_i} \frac{\hat{s}_{ij} x_{ij}}{x_i} \hat{x}_j.$$

4. Update the guess of \hat{w} with

$$\hat{w} = \sum_i \frac{s_{Li}x_i}{\mu_i w L} \hat{s}_{Li} \hat{x}_i,$$

and iterate from Step 1 until \hat{w} converges.

A.3.2 Roundabout

For general shocks of \hat{p}_F , we solve the system of equations by following the steps below. In the special case of autarky, the firm-level cost changes are given by solving the system

$$(\hat{c}_j |^{p_F \rightarrow \infty})^{1-\rho} = s_{Dj} \left(\sum_i s_{iD} (\hat{c}_i |^{p_F \rightarrow \infty})^{1-\sigma} \right)^{\frac{1-\rho}{1-\sigma}} + s_{Lj},$$

and the change in real wage is $\frac{\hat{w}}{\bar{p}} |^{\hat{p}_F \rightarrow \infty} = \left(\sum_i s_{iD} (\hat{c}_i |^{p_F \rightarrow \infty})^{1-\sigma} \right)^{\frac{-1}{1-\sigma}}$. Using these firm-level cost changes one can proceed with the Steps 2 and 3 below by substituting \hat{w} with 1.

1. Guess \hat{w} . Compute $\{\hat{c}_i\}$ from

$$\hat{c}_j^{1-\rho} = s_{Dj} \hat{P}^{1-\rho} + s_{Lj} \hat{w}^{1-\rho} + s_{Fj} \hat{p}_F^{1-\rho}$$

where $\hat{P}^{1-\sigma} = \sum_i s_{iD} \hat{c}_i^{1-\sigma}$.

2. Compute the following hat variables.

$$\hat{x}_{iF} = \hat{c}_i^{1-\sigma} \quad (\text{if } I_{iF} = 1)$$

$$\hat{s}_{Li} = \hat{w}^{1-\rho} \hat{c}_i^{\rho-1}$$

$$\hat{s}_{Di} = \hat{P}^{1-\rho} \hat{c}_i^{\rho-1}$$

$$\hat{s}_{iD} = \hat{c}_i^{1-\sigma} \hat{P}^{\sigma-1}.$$

3. Solve for $\{\hat{x}_i\}$ from

$$\begin{aligned} \hat{x}_i &= \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{x_{iD}}{x_i} \hat{x}_{iD} \\ &= \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{x_{iD}}{x_i} \frac{s_{iD} \hat{E}}{x_{iD}} \hat{s}_{iD} \hat{E} + \frac{x_{iD}}{x_i} \frac{s_{iD}}{x_{iD}} \hat{s}_{iD} \left(\sum_j s_{Dj} \frac{1}{\mu_j} x_j \hat{s}_{Dj} \hat{x}_j \right) \\ \hat{E} &= \frac{wL}{E} \hat{w} + \sum_i \frac{\mu_i - 1}{\mu_i} \frac{x_i}{E} \hat{x}_i - \frac{TB}{E}. \end{aligned}$$

Combined, the above two equation can be expressed as

$$\hat{x}_i = \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{s_{iD}}{x_i} \hat{s}_{iD} (wL\hat{w} - TB) + \frac{s_{iD}}{x_i} \hat{s}_{iD} \left(\sum_j \frac{\mu_j - 1}{\mu_j} x_j \hat{x}_j + \sum_j s_{Dj} \frac{1}{\mu_j} x_j \hat{s}_{Dj} \hat{x}_j \right).$$

4. Update the guess of \hat{w} with

$$\hat{w} = \sum_i \frac{s_{Li} x_i}{\mu_i w L} \hat{s}_{Li} \hat{x}_i,$$

and iterate from Step 1 until \hat{w} converges.

A.3.3 Sectoral roundabout

Consider firm i in sector v , firm j in sector u . Let the price index of sector v be P_v . We calculate the share of sector u goods in household consumption as $\gamma_{vH} = \frac{\sum_{i \in v} x_{iH}}{\sum_k x_{kH}}$ and the share of sector v goods in the domestic intermediate input bundle for sector u as $\gamma_{vu} = \frac{\sum_{i \in v} \sum_{j \in u} x_{ij}}{\sum_k \sum_{j \in u} x_{kj}}$.

For general shocks of \hat{p}_F , we solve the system of equations by following the steps below. In the special case of autarky, the firm-level cost changes are given by solving the system

$$\begin{aligned} (\hat{c}_j |^{p_{F \rightarrow \infty}})^{1-\rho} &= s_{Dj} \left(\prod_v \left(\hat{P}_v |^{p_{F \rightarrow \infty}} \right)^{\gamma_{vu(j)}} \right)^{1-\rho} + s_{Lj} \\ \left(\hat{P}_{v(i)} |^{p_{F \rightarrow \infty}} \right)^{1-\sigma} &= \sum_{i \in v} s_{iv(i)} (\hat{c}_i |^{p_{F \rightarrow \infty}})^{1-\sigma}, \end{aligned}$$

and the change in real wage is $\frac{\hat{w}}{\hat{P}} |^{\hat{p}_{F \rightarrow \infty}} = \prod_v \left(\hat{P}_v |^{p_{F \rightarrow \infty}} \right)^{-\gamma_{vH}}$. Using these firm-level cost changes one can proceed with the Steps 2 and 3 below by substituting \hat{w} with 1.

1. Guess \hat{w} . Compute $\{\hat{c}_i\}$ from

$$\hat{c}_j^{1-\rho} = s_{Dj} \left(\prod_v \hat{P}_v^{\gamma_{vu(j)}} \right)^{1-\rho} + s_{Lj} \hat{w}^{1-\rho} + s_{Fj} \hat{p}_F^{1-\rho},$$

where $\hat{P}_{v(i)}^{1-\sigma} = \sum_{i \in v} s_{iv(i)} \hat{c}_i^{1-\sigma}$. $s_{iv(i)}$ is computed as firm i 's share of domestic sales among other firms in the same sector.

2. Compute the following hat variables.

$$\begin{aligned}
\hat{P} &= \prod_v \hat{P}_v^{\gamma_{vH}} \\
\hat{s}_{Li} &= \hat{w}^{1-\rho} \hat{c}_i^{\rho-1} \\
\hat{s}_{iv(i)} &= \hat{c}_i^{1-\sigma} \hat{P}_{v(i)}^{\sigma-1} \\
\hat{x}_{iF} &= \hat{c}_i^{1-\sigma} \quad (\text{if } I_{iF} = 1) \\
\hat{s}_{Dj} &= \left(\prod_v \hat{P}_v^{\gamma_{vu(j)}} \right)^{1-\rho} \hat{c}_j^{\rho-1}.
\end{aligned}$$

3. Solve for $\{\hat{x}_i\}$ from

$$\begin{aligned}
\hat{x}_i &= \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{x_{iD}}{x_i} \hat{x}_{iD} \\
&= \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{s_{iv(i)} \gamma_{v(i)H} E}{x_i} \hat{s}_{iv(i)} \hat{E} + \frac{1}{x_i} s_{iv(i)} \hat{s}_{iv(i)} \left(\sum_u \sum_{j \in u} s_{Dj} \gamma_{v(i)u(j)} \frac{x_j}{\mu_j} \hat{s}_{Dj} \hat{x}_j \right) \\
\hat{E} &= \frac{wL}{E} \hat{w} + \sum_i \frac{\mu_i - 1}{\mu_i} \frac{x_i}{E} \hat{x}_i - \frac{TB}{E},
\end{aligned}$$

which can be summarized as

$$\begin{aligned}
\hat{x}_i &= \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{s_{iv(i)} \hat{s}_{iv(i)} \gamma_{v(i)H}}{x_i} \left(wL \hat{w} - TB + \sum_j \frac{\mu_j - 1}{\mu_j} x_j \hat{x}_j \right) \\
&\quad + \frac{s_{iv(i)} \hat{s}_{iv(i)}}{x_i} \left(\sum_u \sum_{j \in u} s_{Dj} \gamma_{v(i)u(j)} \frac{x_j}{\mu_j} \hat{s}_{Dj} \hat{x}_j \right).
\end{aligned}$$

4. Update the guess of \hat{w} with

$$\hat{w} = \sum_i \frac{s_{Li} x_i}{\mu_i w L} \hat{s}_{Li} \hat{x}_i,$$

and iterate from Step 1 until \hat{w} converges.

A.4 System of equations under small changes of p_F .

A.4.1 Baseline

Under the baseline case where we take the observed Belgian firm-to-firm trade data, the change in real wage given $\frac{dP}{P}$ is expressed as follows (Lemma 2):

$$\frac{dw}{w} - \frac{dP}{P} = \left(\frac{dw}{w} - \frac{dp_F}{p_F} \right) \sum_j s_{jH} s_{Fj}^{Total}.$$

The term $\sum_j s_{jH} s_{Fj}^{Total}$ corresponds to the import content in domestic final demand (ICD), and analogously, import content in exports (ICE) is $\sum_j s_{jF} s_{Fj}^{Total}$ where $s_{jF} = \frac{x_{jF}}{\sum_j s_{jF}}$. We obtain the system of equations that determines $\frac{dw}{w}$ by log-linearizing the system described in Section A.3.1.

1. Guess $\frac{dw}{w}$. Compute $\left\{ \frac{dc_j}{c_j} \right\}$ from

$$\frac{dc_j}{c_j} = (1 - s_{Fj}^{Total}) \frac{dw}{w} + s_{Fj}^{Total} \frac{dp_F}{p_F}.$$

2. Compute the following variables.

$$\begin{aligned} \frac{dx_{iF}}{x_{iF}} &= (1 - \sigma) \frac{dc_i}{c_i} \\ \frac{dP}{P} &= \sum_j s_{jH} \frac{dc_j}{c_j} \\ \frac{ds_{Li}}{s_{Li}} &= (1 - \rho) \frac{dw}{w} + (\rho - 1) \frac{dc_i}{c_i} \\ \frac{ds_{ij}}{s_{ij}} &= (1 - \rho) \frac{dc_i}{c_i} + (\rho - 1) \frac{dc_j}{c_j}. \end{aligned}$$

3. Solve for $\left\{ \frac{dx_i}{x_i} \right\}$ from

$$\begin{aligned} \frac{dx_i}{x_i} &= \frac{x_{iF}}{x_i} \frac{dx_{iF}}{x_{iF}} + \frac{x_{iH}}{x_i} (1 - \sigma) \left(\frac{dc_i}{c_i} - \frac{dP}{P} \right) + \frac{x_{iH}}{x_i} \frac{wL}{E} \frac{dw}{w} + \frac{x_{iH}}{x_i} \sum_k \frac{\mu_k - 1}{\mu_j} \frac{x_k}{E} \frac{dx_k}{x_k} \\ &\quad + \sum_{j \in W_i} \frac{x_{ij}}{x_i} \left(\frac{ds_{ij}}{s_{ij}} + \frac{dx_j}{x_j} \right). \end{aligned}$$

4. Update the guess of $\frac{dw}{w}$ with

$$\frac{dw}{w} = \sum_i \frac{s_{Li}x_i}{\mu_i w L} \left(\frac{ds_{Li}}{s_{Li}} + \frac{dx_i}{x_i} \right),$$

and iterate from Step 1 until $\frac{dw}{w}$ converges.

A.4.2 Roundabout

Under the roundabout production model, the production functions are

$$\begin{aligned} c_j^{1-\rho} &= \phi_j^{\rho-1} (\alpha_{Dj}^{\rho-1} P^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{Lj}^{\rho-1} w^{1-\rho}) \\ P^{1-\sigma} &= \sum_j \beta_{jH}^{\sigma-1} \mu^{1-\sigma} c_j^{1-\sigma}. \end{aligned}$$

By log-linearizing the system by considering small changes, one obtains:

$$\begin{aligned} \frac{dc_j}{c_j} &= s_{Dj} \frac{dP}{P} + s_{Fj} \frac{dp_{Fj}}{p_{Fj}} + s_{Lj} \frac{dw}{w} \\ \frac{dP}{P} &= \sum_j s_{jD} \frac{dc_j}{c_j}. \end{aligned}$$

Rearranging yields the change in real wage given $\frac{dw}{w}$:

$$\frac{dw}{w} - \frac{dP}{P} = \left(\frac{dw}{w} - \frac{dp_{Fj}}{p_{Fj}} \right) \frac{\sum_j s_{jD} s_{Fj}}{1 - \sum_j s_{jD} s_{Dj}}.$$

The import content in domestic final demand (ICD) in this model is $\frac{\sum_j s_{jD} s_{Fj}}{1 - \sum_j s_{jD} s_{Dj}}$, and the import content in exports (ICE) is $\sum_j s_{jF} (s_{Fj} + s_{Dj} \text{ICD})$. We obtain the system of equations that determines $\frac{dw}{w}$ by log-linearizing the system described in Section A.3.2.

1. Guess $\frac{dw}{w}$. Compute $\left\{ \frac{dc_j}{c_j} \right\}$ from

$$\frac{dc_j}{c_j} = s_{Dj} \frac{dP}{P} + s_{Lj} \frac{dw}{w} + s_{Fj} \frac{dp_{Fj}}{p_{Fj}}.$$

where $\frac{dP}{P} = \sum_j s_{jD} \frac{dc_j}{c_j}$.

2. Compute the following variables.

$$\begin{aligned}\frac{dx_{iF}}{x_{iF}} &= (1 - \sigma) \frac{dc_i}{c_i} \text{ (if } I_{iF} = 1) \\ \frac{ds_{Li}}{s_{Li}} &= (1 - \rho) \frac{dw}{w} + (\rho - 1) \frac{dc_i}{c_i} \\ \frac{ds_{Di}}{s_{Di}} &= (1 - \rho) \frac{dP}{P} + (\rho - 1) \frac{dc_i}{c_i} \\ \frac{ds_{iD}}{s_{iD}} &= (\sigma - 1) \frac{dP}{P} + (1 - \sigma) \frac{dc_i}{c_i}.\end{aligned}$$

3. Solve for $\left\{\frac{dx_i}{x_i}\right\}$ from

$$\begin{aligned}\frac{dx_i}{x_i} &= \frac{x_{iF}}{x_i} \frac{dx_{iF}}{x_{iF}} + \frac{s_{iD}}{x_i} wL \frac{dw}{w} + \frac{s_{iD}}{x_i} \left(E + \sum_j s_{Dj} \frac{x_j}{\mu_j} \right) \frac{ds_{iD}}{s_{iD}} \\ &+ \frac{s_{iD}}{x_i} \left(\sum_j \frac{\mu_j - 1}{\mu_j} x_j \frac{dx_j}{x_j} + \sum_j s_{Dj} \frac{x_j}{\mu_j} \frac{dx_j}{x_j} \right) \\ &+ \frac{s_{iD}}{x_i} \left(\sum_j s_{Dj} \frac{x_j}{\mu_j} \frac{ds_{Dj}}{s_{Dj}} \right).\end{aligned}$$

4. Update the guess of $\frac{dw}{w}$ with

$$\frac{dw}{w} = \sum_i \frac{s_{Li} x_i}{\mu_i w L} \left(\frac{ds_{Li}}{s_{Li}} + \frac{dx_i}{x_i} \right),$$

and iterate from Step 1 until $\frac{dw}{w}$ converges

A.4.3 Sectoral roundabout

Log-linearizing the system of equations for firm-level cost changes yields:

$$\begin{aligned}\frac{dc_j}{c_j} &= s_{Dj} \sum_v \gamma_{vu(j)} \frac{dP_v}{P_v} + s_{Fj} \frac{dp_F}{p_F} + s_{Lj} \frac{dw}{w} \\ \frac{dP_v}{P_v} &= \sum_{i \in v} s_{w(i)} \frac{dc_i}{c_i} \\ \frac{dP}{P} &= \sum_v \gamma_{vH} \frac{dP_v}{P_v}.\end{aligned}$$

Rearranging and solving the system of equations below yields the change in sectoral goods' prices, $\frac{dP_v}{P_v}$, and thus the change in real wage given $\frac{dp_F}{p_F}$:

$$\begin{aligned}\frac{dP_v}{P_v} &= \sum_{i \in v} s_{iv(i)} \left(s_{Di} \sum_u \gamma_{uv(i)} \frac{dP_u}{P_u} + s_{Fi} \frac{dP_F}{P_F} + s_{Li} \frac{dw}{w} \right) \\ \frac{dw}{w} - \frac{dP}{P} &= \frac{dw}{w} - \sum_v \gamma_{vH} \frac{dP_v}{P_v}.\end{aligned}$$

The import content in domestic goods at the sector level, IC_v , and the import content in domestic final demand, ICD , can be obtained by solving the following:

$$\begin{aligned}IC_v &= \sum_{i \in v} s_{Fi} s_{iv(i)} + \sum_u \gamma_{uv} \sum_{i \in v} s_{Di} s_{iv(i)} IC_u \\ ICD &= \sum_v \gamma_{vH} IC_v.\end{aligned}$$

The import content in exports is computed by:

$$ICE = \sum_i s_{iF} \left(s_{Fi} + s_{Di} \sum_u \gamma_{uv(i)} IC_u \right).$$

We obtain the system of equations that determines $\frac{dw}{w}$ by log-linearizing the system described in Section A.3.3.

1. Guess $\frac{dw}{w}$. Compute $\left\{ \frac{dc_j}{c_j} \right\}$ from

$$\begin{aligned}\frac{dc_j}{c_j} &= s_{Dj} \sum_v \gamma_{vu(j)} \frac{dP_v}{P_v} + s_{Fj} \frac{dp_F}{p_F} + s_{Lj} \frac{dw}{w} \\ \frac{dP_v}{P_v} &= \sum_{i \in v} s_{iv(i)} \frac{dc_i}{c_i}.\end{aligned}$$

2. Compute the following variables.

$$\begin{aligned}\frac{dP}{P} &= \sum_v \gamma_{vH} \frac{dP_v}{P_v} \\ \frac{ds_{Li}}{s_{Li}} &= (1 - \rho) \left(\frac{dw}{w} - \frac{dc_i}{c_i} \right) \\ \frac{ds_{iv(i)}}{s_{iv(i)}} &= (1 - \sigma) \left(\frac{dc_i}{c_i} - \frac{dP_{v(i)}}{P_{v(i)}} \right) \\ \frac{dx_{iF}}{x_{iF}} &= (1 - \sigma) \frac{dc_i}{c_i} \quad (\text{if } I_{iF} = 1) \\ \frac{ds_{Dj}}{s_{Dj}} &= (1 - \rho) \left(\sum_v \gamma_{vu(j)} \frac{dP_v}{P_v} - \frac{dc_j}{c_j} \right).\end{aligned}$$

3. Solve for $\left\{ \frac{dx_i}{x_i} \right\}$ from

$$\begin{aligned}\frac{dx_i}{x_i} &= \frac{x_{iF}}{x_i} \frac{dx_{iF}}{x_{iF}} + \frac{s_{iv(i)}}{x_i} \left(\sum_u \sum_{j \in u} s_{Dj} \gamma_{v(i)u(j)} \frac{x_j}{\mu_j} + \gamma_{v(i)HE} \right) \frac{ds_{iv(i)}}{s_{iv(i)}} \\ &\quad + \frac{s_{iv(i)} \gamma_{v(i)H}}{x_i} \left(wL \frac{dw}{w} + \sum_j \frac{\mu_j - 1}{\mu_j} x_j \frac{dx_j}{x_j} \right) \\ &\quad + \frac{s_{iv(i)}}{x_i} \left(\sum_u \sum_{j \in u} s_{Dj} \gamma_{v(i)u(j)} \frac{x_j}{\mu_j} \left(\frac{ds_{Dj}}{s_{Dj}} + \frac{dx_j}{x_j} \right) \right).\end{aligned}$$

4. Update the guess of $\frac{dw}{w}$ with

$$\frac{dw}{w} = \sum_i \frac{s_{Li} x_i}{\mu_i w L} \left(\frac{ds_{Li}}{s_{Li}} + \frac{dx_i}{x_i} \right),$$

and iterate from Step 1 until $\frac{dw}{w}$ converges.

A.5 Real income changes

Here we present the expression for the change in real income. Across the three models, the change in real income can be expressed in terms of the change variables that are computed from the system of equations in Online Appendix A.3:

$$\frac{\hat{E}}{\hat{P}} = \left(\frac{wL}{E} \hat{w} + \sum_i \frac{\mu_i - 1}{\mu_i} \frac{x_i}{E} \hat{x}_i - \frac{TB}{E} \right) \hat{P}^{-1}. \quad (1)$$

We then present the expression for the first-order approximated change in real income. Across the three models, the log-change in real income can be expressed in terms of the log-change variables that are computed from the system of equations in Online Appendix A.4:

$$\frac{dE}{E} - \frac{dP}{P} = \frac{wL}{E} \frac{dw}{w} + \sum_k \frac{\mu_k - 1}{\mu_k} \frac{x_k}{E} \frac{dx_k}{x_k} - \frac{dP}{P}. \quad (2)$$

B Numerical example

Here we demonstrate that economies with identical sets of aggregate exports, aggregate imports, aggregate gross production and GDP, but with different firm-to-firm network structures, can potentially generate different import content in domestic final demand. We consider two economies with both consisting of three firms. Table 1 lays out the details of the two economies. In the two economies, firm-level imports, exports, gross production, domestic sales, labor costs, and value added are the same. The only difference between the two economies are how firms allocate their domestic sales to sales to households or to sales to other firms. In Table 1, the first seven rows are identical across the two economies, but the entries for Firm-to-firm sales and sales to households differ.

Table 1: Two economies

	Economy 1			Economy 2		
	Firm 1	Firm 2	Firm 3	Firm 1	Firm 2	Firm 3
Imports	100	0	0	100	0	0
Exports	0	0	100	0	0	100
Gross production	200	200	200	200	200	200
Domestic Sales	200	200	100	200	200	100
Labor cost	50	100	50	50	100	50
Domestic purchases	0	50	100	0	50	100
Profits	50	50	50	50	50	50
Firm-to-firm sales	$x_{12} = 50$ $x_{13} = 50$	$x_{23} = 50$		$x_{23} = 100$	$x_{32} = 50$	
Sales to households	100	150	100	200	100	50

Now let us compute the direct and total shares of foreign inputs, as well as the firms' shares in household consumption. Table 2 summarizes the firms' shares. Firms' direct shares of foreign inputs are the same across the two economies, as firm-level imports and total inputs are the same. But because the firm-to-firm sales structure is not the same in the two economies, the total shares of foreign inputs and firms' shares in household consumption are different.

Table 2: Direct and total shares of foreign inputs, and shares in household consumption

	Economy 1			Economy 2		
	Firm 1	Firm 2	Firm 3	Firm 1	Firm 2	Firm 3
s_{Fi}	2/3	0	0	2/3	0	0
s_{Fi}^{Total}	2/3	2/9	2/27	2/3	0	0
s_{iH}	2/7	2/7	3/7	4/7	1/7	2/7

Next, we compute the import content in domestic final demand in the two economies, $\sum_i s_{iH} s_{Fi}^{Total}$. In economy 1, the import content in domestic final demand is 6/21, while in economy 2 it is 8/21. Because there is no leakage of imports in economy 2 to Foreign, the import content in domestic final demand is about 10 percentage points higher in economy 2.

We can also easily calculate what that implies for the real wage change of going to autarky. Under $\rho = 2$ and $\sigma = 4$,

$$\frac{\hat{w}}{\hat{P}} \Big|_{\hat{p}_F \rightarrow \infty} = \left(\sum_i s_{iH} (1 - s_{Fj}^{Total})^{\frac{1-\sigma}{1-\rho}} \right)^{\frac{1}{\sigma-1}} = \begin{cases} 0.79 & \text{if economy 1} \\ 0.77 & \text{if economy 2} \end{cases} .$$

Hence, as expected, the real wage changes are larger in economy 2.

C Data Appendix

C.1 Definition of variables

We describe how we compute each variable that we use in the analyses in the paper. Firms' variable inputs consist of their labor costs reported in the annual accounts, their imports reported in the international trade dataset, and the goods purchased from other Belgian firms that are reported in the B2B dataset. Note that we do not include goods purchased from firms that do not meet the sample selection criteria. Firms' sales consist of their sales to other Belgian firms that meet the sample selection criteria, their exports reported in the international trade dataset, and their sales to domestic final demand. A firm's sales to domestic final demand is computed as the residual of the firm's total turnover reported in the annual accounts, after subtracting B2B sales and exports. This procedure counts firms' sales to other firms that do not meet the sample selection criteria as part of sales to domestic final demand.

C.2 Reporting thresholds of the international trade dataset

There are different reporting thresholds for the international trade dataset, depending on if the trade occurred with an extra-EU country or within the EU. The dataset covers all extra-EU exports and imports by firms with values higher than 1,000 Euro or with weights bigger than 1,000kg. Nevertheless, we also observe values less than 1,000 Euro as more firms use electronic reporting procedures. For intra-EU trade prior to 2006, the dataset covers all exports and imports by firms whose combined imports from intra-EU countries that are more than 250,000 Euro a year. For intra-EU trade from 2006 onward, the thresholds for exports and imports changed to 1,000,000 Euro and 400,000 Euro, respectively. Import reporting thresholds became 700,000 Euro per year in 2010. While these reporting threshold for intra-EU trade imply we miss some trade transaction, they are set to capture at least 93% of aggregate Belgian trade in the micro-data, hence our data still contains the overwhelming majority of the value of Belgian trade.

C.3 Mapping CN codes into NACE codes

Our international trade dataset records products in Combined Nomenclature (CN) codes, up to 8 digits. On the other hand, all other datasets that we use record the enterprise's primary sector in NACE Rev.2 code. To concord the two classifications, we convert the CN 8 digit codes into NACE Rev.2 codes. As the first 6 digits of CN codes are identical to the contemporary Harmonized System (HS) codes, we first convert those HS 6-digit codes to Classification of Products by Activity (CPA) codes. We then convert CPA codes to NACE

codes, using the fact that CPA 2008 codes are identical to NACE Rev.2 codes up to 4 digits. This conversion allows us to convert more than 98% of all international trade recorded in our dataset, in terms of values (in 2012).

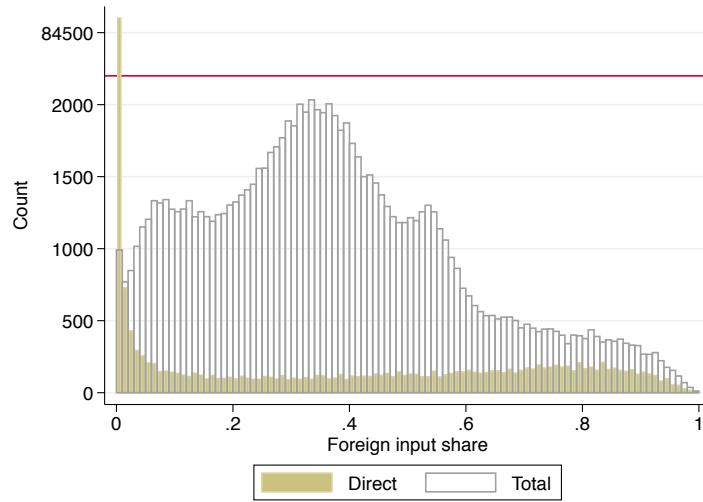
D Descriptive statistics

D.1 Taking into account capital usage

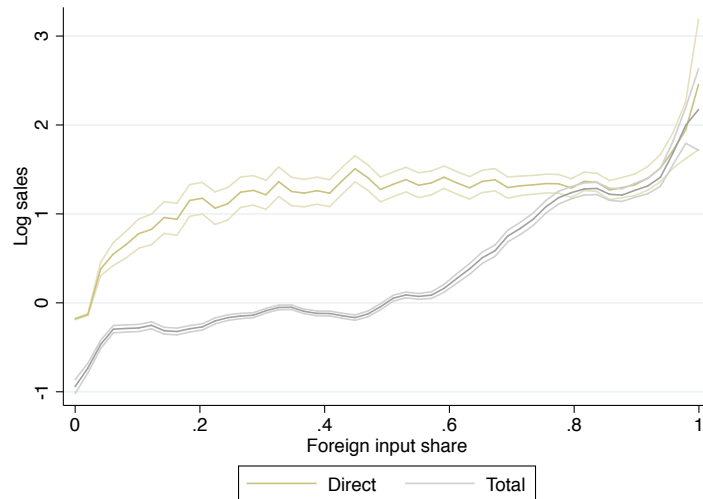
In Figure 1 we show figures analogous to Figures 1a and 2a, but taking into account firms' capital usage. Following Dhyne, Petrin, Smeets, and Warzynski (2017), we set the yearly depreciation rate as 8% and set the interest rate as the long-term interest rate in Belgium. We compute the capital rental costs using fixed tangible assets reported in the annual accounts.

Figure 1: Figures 1a and 2a with capital usage

(a) Direct and total foreign input share



(b) Sales and foreign input shares

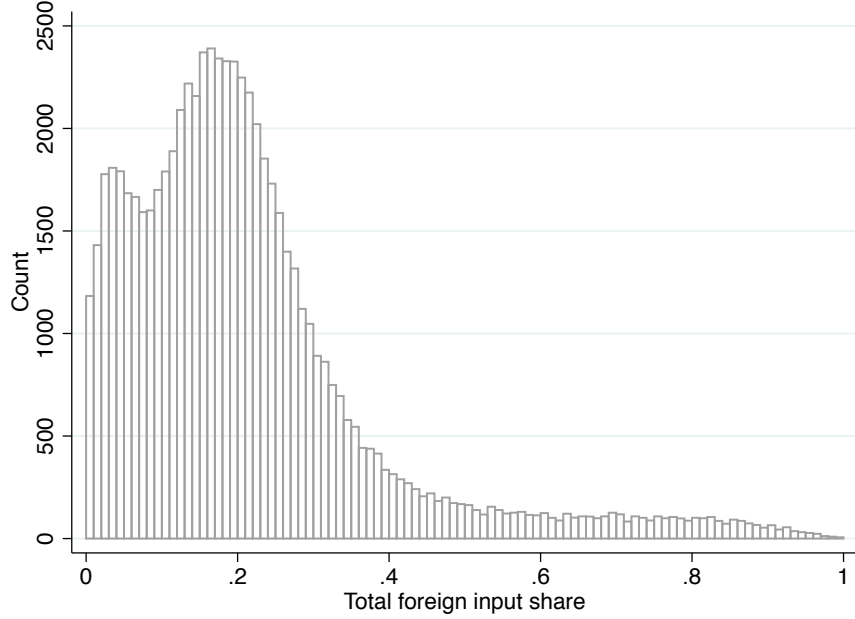


D.2 Total exposures to foreign trade when excluding wholesale and retail sectors

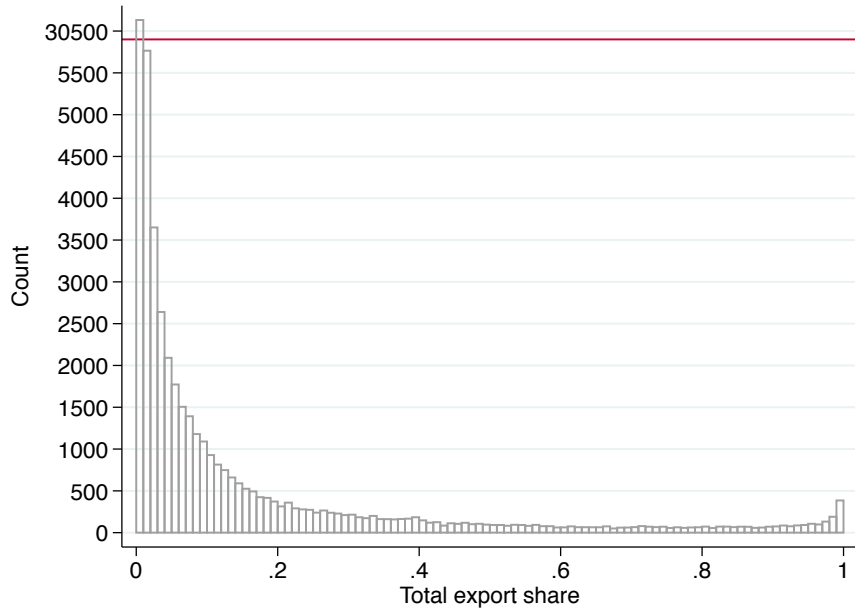
Figure 2a plots the distribution of total foreign input shares for firms outside the wholesale and retail sectors, where the total foreign input shares are computed by setting direct foreign input shares for firms in wholesale and retail sectors as 0. 15% of firms outside the wholesale and retail sectors were importers. Analogously, Figure 2b plots the distribution of total export shares for firms outside the wholesale and retail sectors, where the total export shares are computed by setting direct export shares for firms in wholesale and retail sectors as 0. 10% of firms outside the wholesale and retail sectors were exporters.

Figure 2: Histograms of total exposures to foreign trade, when excluding direct imports and exports by wholesale and retail sectors

(a) Total foreign input share



(b) Total export share



Notes: The histograms plot the total foreign input shares and total export shares for firms not in the wholesale or retail sectors. Total foreign input share of firm i , s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i 's direct foreign input share, and s_{ji} is j 's share among i 's inputs. s_{Fi} for firms in wholesale and retail sectors are set to be 0. Total export share firm i , r_{iF}^{Total} is calculated by solving $r_{iF}^{Total} = r_{iF} + \sum_{j \in W_i} r_{ij} r_{jF}^{Total}$ where r_{iF} is i 's share of exports in its revenue, and r_{ij} is share of i 's revenue that arises from sales to firm j . r_{iF} for firms in wholesale and retail sectors are set to be 0.

D.3 Import content in domestic final demand and import content in exports

As explained in Online Appendix A.4.1, we define the import content in domestic final demand (ICD) as $\sum_j s_{jH} s_{Fj}^{Total}$ as , and analogously, import content in exports (ICE) as $\sum_j s_{jF} s_{Fj}^{Total}$ where $s_{jF} = \frac{x_{jF}}{\sum_j s_{jF}}$. Table 3 reports the two numbers across broad sectors.

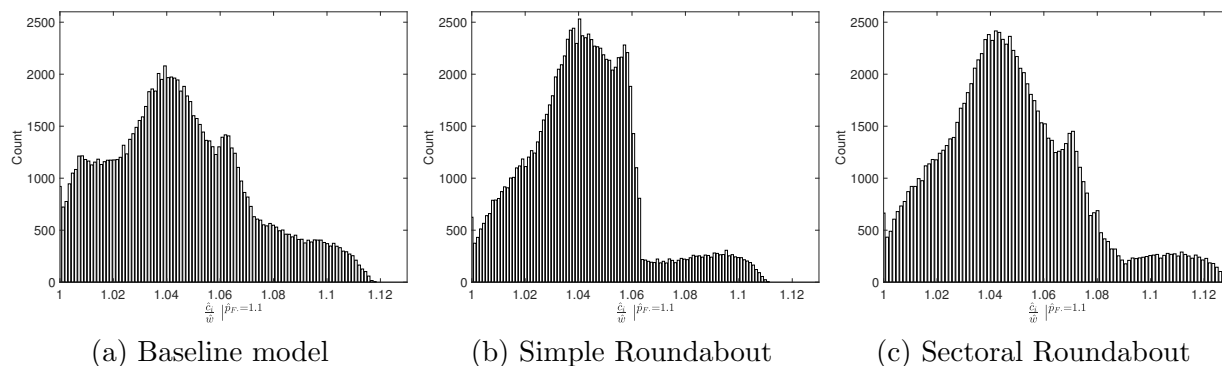
Table 3: ICD and ICE across sectors

Sector	ICD	ICE
Agriculture and Mining	0.068	0.033
Manufacturing	0.158	0.522
Utility and Construction	0.059	0.052
Wholesale and Retail	0.237	0.155
Service	0.062	0.038
Total	0.584	0.800

D.4 Histogram of $\frac{\hat{c}_i}{\hat{w}} \mid \hat{p}_F=1.1$

Figure 3 plots histograms of firm-level cost changes relative to the change in nominal wage, $\frac{\hat{c}_i}{\hat{w}}$, under 10% increase in the foreign price.

Figure 3: Histograms of firm-level cost changes relative to change in nominal wage

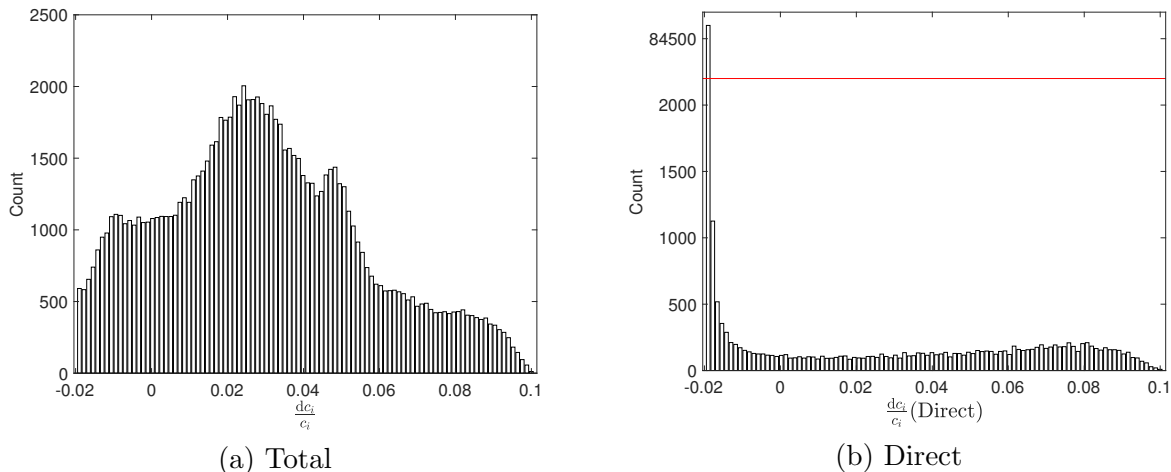


D.5 Histogram of $\frac{dc_i}{c_i}$

Figure 4 plots the first-order approximated cost increases at the firm-level, upon a 10% increase in the foreign input price. Figure 4a shows the firm-level cost changes in equation

(12) from Proposition 1, and Figure 4b plots $\frac{dc_i}{c_i}$ in which total foreign input share, s_{Fj}^{Total} , is substituted to direct foreign input share, s_{Fj} .

Figure 4: Histograms of firm-level cost changes (First-order approximation)



D.6 Real wage changes across sectors

Here we investigate how final good prices (weighted by household expenditure on these goods) change across broad sectors in the economy. Table 4 reports the increase in the price index, $\frac{dP}{P}$, upon a 10% increase in the foreign price. We take the additive decomposition coming from the first-order approximation of the price index changes in equation (20).

Table 4: Log-changes in price index across sectors

Sector	$\frac{dP}{P}$
Agriculture and Mining	0.007
Manufacturing	0.014
Utility and Construction	0.005
Wholesale and Retail	0.022
Service	0.004
Total	0.051

D.7 Changes in real income

Analogous to Table 3, in Table 5 we report the change in real income across the three models both upon 10% increase in the foreign price and upon autarky. In addition, analogous

to Table 16, in Table 6, we decompose the change in real income. Rearranging equation (2), the log-change in real income can be decomposed into four additive terms: (1) the term that arises from the change in nominal wage, (2) the term arising from the changes in firms' profits from their domestic sales, (3) the term arising from the changes in firms' profits from their exports, and (4) the term coming from the change in the price index.

$$\frac{dE}{E} - \frac{dP}{P} = \underbrace{\frac{wL}{E} \frac{dw}{w}}_{=0.28} \underbrace{(1)} + \underbrace{\sum_k \frac{\mu_k - 1}{\mu_k} \frac{x_{kD}}{E} \frac{dx_{kD}}{x_{kD}}}_{(2)} + \underbrace{\sum_k \frac{\mu_k - 1}{\mu_k} \frac{x_{kF}}{E} \frac{dx_{kF}}{x_{kF}}}_{(3)} - \underbrace{\frac{dP}{P}}_{(4)}. \quad (3)$$

Table 5: Changes in real income

(a) Changes in real income upon 10% increase in foreign price			
	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{E}}{\hat{P}} \hat{p}_F = 1.1$	0.883	0.878	0.887
(b) Changes in real income upon autarky			
	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{E}}{\hat{P}} \hat{p}_F \rightarrow \infty$	0.374	0.390	0.291

Table 6: Log-changes in real income upon 10% increase in foreign price

	Baseline	Simple Roundabout	Sectoral Roundabout
(1)	-0.018	-0.012	-0.030
(2)	-0.040	-0.042	-0.036
(3)	-0.043	-0.044	-0.040
(4)	0.051	0.056	0.049
$\frac{dE}{E} - \frac{dP}{P}$	-0.139	-0.145	-0.133

Notes: The fourth row is computed as $0.28 \times (1) + (2) + (3) - (4)$ in equation (3), where 0.28 is the laborcost to income ratio in the data. Note that labor is the only primary input our model, and all other primary inputs such as capital enter this calculation as part of profits.

D.8 Sensitivity results under exogenous network

Table 7 reports the sensitivity results on \hat{c}_i and $\frac{\hat{w}}{\hat{P}}$ under different values of σ and ρ . Throughout we consider a 10% increase in the price of foreign inputs.

Table 8 reports how the change in real wage is affected when one considers an acyclic network structure. In the fourth and sixth columns, we make use of the acyclic network from

the algorithm explained in Online Appendix E, for the weighted case where we minimize the values of violating transactions.

Table 7: Results on \hat{c}_i and $\frac{\hat{w}}{\hat{P}}$ under different values of σ and ρ

(a) Median \hat{c}_i				
ρ	σ	Baseline	Simple Roundabout	Sectoral Roundabout
1.5	2	1.035	1.038	1.030
1.5	4	1.017	1.025	1.008
1.5	6	1.009	1.019	0.998
2	2	1.046	1.047	1.041
2	4	1.027	1.032	1.018
2	6	1.017	1.024	1.006

(b) 90th percentile \hat{c}_i				
ρ	σ	Baseline	Simple Roundabout	Sectoral Roundabout
1.5	2	1.069	1.058	1.059
1.5	4	1.061	1.047	1.045
1.5	6	1.057	1.042	1.038
2	2	1.075	1.064	1.065
2	4	1.065	1.052	1.051
2	6	1.060	1.046	1.043

(c) $\frac{\hat{w}}{\hat{P}}$				
ρ	σ	Baseline	Simple Roundabout	Sectoral Roundabout
1.5	2	0.945	0.944	0.939
1.5	4	0.931	0.934	0.922
1.5	6	0.927	0.932	0.915
2	2	0.955	0.953	0.949
2	4	0.940	0.941	0.931
2	6	0.934	0.937	0.923

Table 8: Change in real wage $\frac{\hat{w}}{\hat{P}}$ under acyclic network

ρ	σ	10% increase in p_F .		Autarky	
		$\frac{\hat{w}}{\hat{P}}$, Baseline	$\frac{\hat{w}}{\hat{P}}$, acyclic	$\frac{\hat{w}}{\hat{P}}$, Baseline	$\frac{\hat{w}}{\hat{P}}$, acyclic
1.5	2	0.945	0.942	0.251	0.223
1.5	4	0.931	0.927	0.437	0.408
1.5	6	0.927	0.922	0.541	0.519
2	2	0.955	0.952	0.391	0.388
2	4	0.940	0.936	0.558	0.530
2	6	0.934	0.929	0.634	0.609

E Ordering algorithm

In this section we describe the implementation of the ordering algorithm to solve the feedback arc set problem. We begin by defining some terms and notation.

E.1 Terms and notation

- *graph / network*, $G = (V, E)$ - A collection of a set of edges E and set of vertices V . Edges describe the relationship between vertices. Two basic classifications of graphs are based on whether the edges are *directed* or *undirected* and whether they are *weighted* or *unweighted*
- $n = |V|$, $m = |E|$
- *cycle* - A path within a graph where a vertex is reachable from itself
- $d^+(u)$ - For a vertex $u \in V$ in a directed graph, number of outgoing edges
- $d^-(u)$ - For a vertex $u \in V$ in a directed graph, number of incoming edges
- $w^+(u)$ - For a vertex $u \in V$ in a directed graph, cumulative sum of weights of outgoing edges
- $w^-(u)$ - For a vertex $u \in V$ in a directed graph, cumulative sum of weights of incoming edges
- *sink* - A vertex $u \in V$ in a directed graph with $d^+(u) = 0$
- *source* - A vertex $u \in V$ in a directed graph with $d^-(u) = 0$
- *feedback arc set* - A set of edges from a directed cyclic graph that when removed make the graph acyclic
- $s = s_{left}s_{right}$ - Given 2 finite sequences s_{left} and s_{right} with the indicated notation we symbolize the *concatenation* operation. For example, if $s_{left} = (A, B, C)$ and $s_{right} = (X, Y, Z)$, then $s = s_{left}s_{right} = (A, B, C, X, Y, Z)$
- $\lfloor x \rfloor$ is the greatest integer less than or equal to x

E.2 Overview

The Belgian B2B data describes a weighted directed graph $G = (V, E)$. Vertices are firms and edges are sales between firms. The goal of the ordering algorithm is to order firms in a way such that a given firm only sells to firms further along in the ordering and only buys from firms that precede it. The condition desired by this ordering is known in graph theory as a *topological ordering* (Black, 1999). A topological ordering exists if and only if a graph is directed and acyclic. The B2B data is cyclic. For the *unweighted* case our motivation is to find a feedback arc set of minimal cardinality, that is, what is the minimum number of transactions that we need to drop (i.e., the “violators”) from our network to satisfy our ordering condition? For the *weighted case*, we seek to find a feedback arc set such that the cumulative weight of the violating transactions is minimized. Finding a minimum feedback arc set is computationally difficult but approximation algorithms exist.

E.3 Unweighted case

The algorithm we use for the paper was first presented by Eades, Lin, and Smyth (1993). This algorithm was chosen because it has a linear run time complexity, $O(m+n)$, and because of its relative implementation simplicity. The algorithm uses a greedy heuristic through which it builds the proposed ordering $s = s_{left}s_{right}$.¹ Vertices are initialized into several buckets: sinks, sources, and δ buckets, where for a vertex $u \in V$, $\delta(u) = d^-(u) - d^+(u)$.² At each iteration, the algorithm removes all sinks from the network and prepends them to a sequence s_{right} , removes all sources and appends them to a sequence s_{left} , and then removes the vertex with the lowest δ score (the most “source”-like vertex) and appends it to s_{left} .³ Each removal requires updating the buckets to reflect the modified graph. The algorithm stops when the graph is empty. There will be $2n - 1$ buckets, which can be formalized as follows:⁴

¹According to Black (2005), a greedy algorithm is “an algorithm that always takes the best immediate, or local, solution while finding an answer. Greedy algorithms find the overall, or globally, optimal solution for some optimization problems, but may find less-than-optimal solutions for some instances of other problems.”

²We have flipped the sign here compared to Eades et al. (1993) to be consistent with the diagrams elsewhere in our paper.

³Eades et al. (1993) take the vertex with the maximum δ score.

⁴Eades et al. (1993) assume that the graph G is simple (no bidirectional edges), and hence their original algorithm only requires $2n - 3$ buckets.

$$V_{-n+1} = V_{sources} = \{u \in V \mid d^-(u) = 0; d^+(u) > 0\}$$

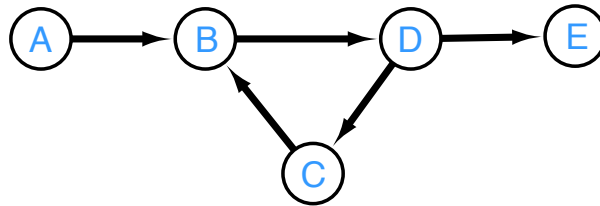
$$V_{n-1} = V_{sinks} = \{u \in V \mid d^-(u) > 0; d^+(u) = 0\}$$

$$V_d = \{u \in V \mid d = \delta(u); d^-(u) > 0; d^+(u) > 0\}$$

The bucket V_{-n+1} contains all the vertices that are only the sources of edges. The bucket V_{n-1} contains all the vertices that are only the sinks of edges (in other words, vertices that are only receiving edges). Each V_d bucket contains vertices with d net incoming edges (conditional on these vertices having both outgoing and incoming edges).

E.4 Example execution on unweighted network

Consider the following network:



Let's trace the execution of the algorithm described by Eades et al.

E.4.1 Initialization

Buckets:

A			D	C	B			E
<i>sources</i>	-3	-2	-1	0	1	2	3	<i>sinks</i>

Ordering : $s = s_{left} = s_{right} = ()$

E.4.2 First iteration:

Remove sinks

Updated buckets:

A				C, D	B			
<i>sources</i>	-3	-2	-1	0	1	2	3	<i>sinks</i>

Updated ordering : $s_{left} = ()$, $s_{right} = (E)$, $s = s_{left}s_{right} = (E)$

Remove sources

Updated buckets:

				C, D, B				
<i>sources</i>	-3	-2	-1	0	1	2	3	<i>sinks</i>

Updated ordering : $s_{left} = (A)$, $s_{right} = (E)$, $s = s_{left}s_{right} = (A, E)$

Remove vertex with lowest delta score

Updated buckets:

B								D
<i>sources</i>	-3	-2	-1	0	1	2	3	<i>sinks</i>

Updated ordering : $s_{left} = (A, C)$, $s_{right} = (E)$, $s = s_{left}s_{right} = (A, C, E)$

E.4.3 Second iteration

Remove sinks

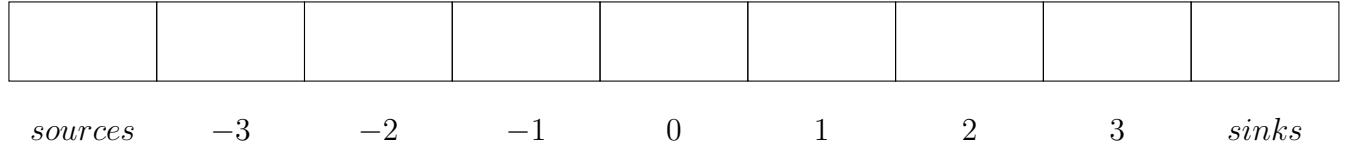
Updated buckets:

B								
<i>sources</i>	-3	-2	-1	0	1	2	3	<i>sinks</i>

Updated ordering : $s_{left} = (A, C)$, $s_{right} = (D, E)$, $s = s_{left}s_{right} = (A, C, D, E)$

Remove sources

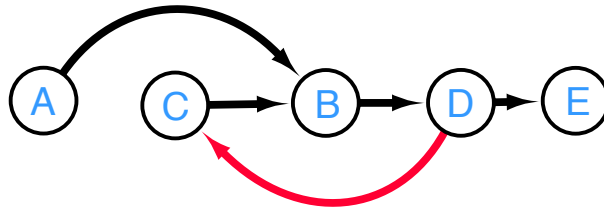
Updated buckets:



Updated ordering : $s_{left} = (A, C, B)$, $s_{right} = (D, E)$, $s = s_{left}s_{right} = (A, C, B, D, E)$

E.4.4 Final output

Ordering : $s = s_{left}s_{right} = (A, C, B, D, E)$, Violator edge set: $\{(D, C)\}$



E.5 Weighted case

Simpson, Srinivasan, and Thomo (2016) have proposed a modification to adapt the Eades et al. (1993) algorithm to solve the weighted problem. The required changes are:

1. In the initialization step, all edge weights need to be normalized to be between 0 and 1.
2. $\delta(u)$ is redefined as $\delta(u) = \lfloor w^-(u) - w^+(u) \rfloor$.

The key motivation behind these steps is to reformat the network so that the unweighted version of the algorithm could be used in an identical fashion as before, specifically without increasing the number of buckets. Without the floor in step 2, for any given network the number of buckets could become large.

F Algorithm to solve for the firm's sourcing strategy and export participation

A firm is solving the problem described in (25), where profits are defined in equation (24) and variable profits are defined in equation (14). For convenience, we re-state the problem of firm j here:

$$\max_{Z_j, I_{jF}} \pi_j(Z_j, I_{jF}) \quad \text{s.t.} \quad Z_j \subseteq \mathbf{Z}_j, I_{jF} \in \{0, 1\},$$

where

$$\begin{aligned} \pi_j(Z_j, I_{jF}) &= \frac{1}{\sigma} \beta_{jH}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j(Z_j)^{(\sigma-1)/(\rho-1)} \frac{E}{P^{1-\sigma}} \\ &\quad + I_{jF} \frac{1}{\sigma} \beta_{jF}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j(Z_j)^{(\sigma-1)/(\rho-1)} \tau^{1-\sigma} \frac{E_F}{P_F^{1-\sigma}} \\ &\quad - \sum_{k \in Z_j} f_{kj} w - I_{jF} f_{jF} w. \\ &= \pi_j^{var}(Z_j, I_{jF}) - \sum_{k \in Z_j} f_{kj} w - I_{jF} f_{jF} w \end{aligned}$$

In words, the firm is choosing its sourcing strategy, Z_j , and export participation, I_{jF} . We solve the firm's problem of choosing its sourcing strategy separately for $I_{jF} = 0$ and $I_{jF} = 1$. We then calculate the profits for these two cases and determine the firm is an exporter if and only if the profits under exporting are higher than under non-exporting.

Below we describe how we solve for the firm's optimal sourcing strategy for a given export participation choice.

F.1 Lower and upper bounds for the optimal sourcing strategy

We determine the lower and upper bound for the firm's sourcing strategy following the procedure in Jia (2008) and Antras, Fort, and Tintelnot (2017).

F.1.1 Lower bound

We start from a guess of no sourcing from any other domestic supplier and no importing, $S_l^{(0)}$. We then check supplier by supplier whether the benefit of adding a supplier (given the current guess of not purchasing from any supplier) is positive. At iteration t , starting from $S_l^{(t)}$, we calculate the marginal benefit of adding supplier $k \notin S_l^{(t)}$, $k \in \mathbf{Z}_j$:

$$\pi_j^{var}(S_l^{(t)} \cup k, I_{jF}) - \pi_j^{var}(S_l^{(t)}, I_{jF}) - f_{kj}w.$$

If the marginal benefit to include supplier k is positive, in the next iteration we include supplier k in the guess for the sourcing strategy of firm j . Note that given $\sigma > \rho$, one is the least likely to determine the benefit of a supplier is positive when the current guess is no supplier. Hence if it is possible to include a supplier in this iteration, in all the next iterations the marginal benefit from this supplier will be positive as well.

Starting from $S_l^{(t)}$, we consider firm-by-firm if trading with a firm not contained in $S_l^{(t)}$ brings positive marginal benefit (i.e., the additional variable profits under this sourcing strategy exceed the additional fixed cost) or not. Then, we add all those firms which bring positive benefit to form $S_l^{(t+1)}$.

The process ends when $S_l^{(t)} = S_l^{(t+1)}$ or all eligible suppliers are in $S_l^{(t)}$ already. When the process ends (i.e., $S_l^{(t)} = S_l^{(t+1)}$), we denote the lower bound of the sourcing strategy for firm j as $S_l^* = S_l^{(t)} = S_l^{(t+1)}$.

F.1.2 Upper bound

To determine the upper bound we start from a guess of purchasing from every supplier (incl. foreign), $S_u^{(0)}$. We then check supplier-by-supplier whether the marginal benefit from dropping the supplier is positive. At iteration t , starting from $S_u^{(t)}$, we calculate the marginal benefit of dropping supplier $k \in S_u^{(t)}$ as:

$$\pi_j^{var}(S_l^{(t)} \setminus k, I_{jF}) - \pi_j^{var}(S_l^{(t)}, I_{jF}) + f_{kj}w.$$

The remainder of the procedure is very similar to the iteration for the lower bound but starting from the opposite direction (i.e., we drop from the next iteration $S_u^{(t+1)}$ all those suppliers for which the marginal benefit of dropping is positive). The ending criteria is the same. We denote the upper bound for the sourcing strategy as S_u^* .

F.2 From lower and upper bounds to optimal sourcing strategy

Once we obtain S_u^* and S_l^* , we consider 3 alternative cases. Let $D = \{x \in S_u^* \mid x \notin S_l^*\}$ denote the set with the elements that are in the upper bound but not in the lower bound for the sourcing strategy.

F.2.1 $S_u^* = S_l^*$

If the upper and lower bounds for the sourcing strategy are the same, then we have obtained the optimal sourcing strategy for the firm (for a given exporting choice).

F.2.2 S_u^* is close to S_l^*

When the cardinality of set D is less than or equal to 15, we consider S_u^* to be close to S_l^* .

In that case we evaluate the profits at all possible combinations of sourcing strategies in between S_u^* and S_l^* , including S_u^* and S_l^* themselves. We choose the combination that yields the highest total profit as the optimal sourcing strategy for the firm.

F.2.3 S_u^* is far from S_l^*

When the cardinality of set D is larger than 15, then evaluating the profits at all combinations of feasible sourcing strategies in between the two bounds would be too computationally intensive. For that case, we have developed the following greedy algorithm to determine the firm's sourcing strategy:

Starting from S_l^* , we calculate the marginal benefit from adding separately each supplier in D to the sourcing strategy S_l^* . Note that by construction the marginal benefit from adding each of these suppliers individually to S_l^* is negative (otherwise the algorithm in Section F.1 would have already added those suppliers to the lower bound). We order the suppliers in D by their marginal benefit of being added to S_l^* . If the cardinality of D is K , we consider $K - 1$ alternative sourcing strategies. We first add the top 2 suppliers in D (those with the highest marginal benefit of being added evaluated at S_l^*) to S_l^* , then add the top 3 suppliers to S_l^* , and so forth. Hence, we calculate the profits for $K - 1$ alternative sourcing strategies.

In addition, we also follow a similar process starting from S_u^* and calculate the marginal benefit from dropping separately each supplier in D from the sourcing strategy S_u^* . Again, by construction, the benefit from dropping each of the suppliers individually is negative. We order the suppliers in D by their marginal benefit of being dropped from S_u^* . We then consider $K - 1$ alternative sourcing strategies, in which 2, 3, ..., K suppliers are removed from S_u^* (following the ranking of their marginal benefit of dropping individually at S_u^*).

Then, out of these $2K - 2$ sourcing strategies we choose the one with the highest total profit for the firm.

Note that, using the approach in Section F.2.2, the number of sourcing strategies we would need to calculate profits for would be 2^K (growing exponentially in K). The greedy algorithm developed here, requires evaluations of alternative sourcing strategies that grow

linearly in K , making it feasible even in the rare case that the difference between S_u^* and S_l^* is large.

We present statistics on the cardinality of the differences in the bounds in Table 9.

F.3 Statistics on the algorithm

Table 9: Cardinality of differences in the upper and lower bounds

Number of firm draws × parameter iterations	Bounds are perfectly overlapping	Percent of cases in which	
		Differences in bounds ≤ 15	Differences in bounds > 15
2,215,600,000	99.26	0.61	0.13

Notes: During the estimation we have to solve for each firm and parameter guess the firm's optimal sourcing strategy and exporting choice. This table presents aggregate statistics on the cardinality of the differences in the upper and lower bounds for the sourcing strategy summing over the outcomes for each firm, parameter guess, and exporting choice.

G Algorithm for network formation

Below we describe the algorithm to solve for the network formation in three contexts: In Section G.1, we describe the algorithm to solve for the network formation and equilibrium for a given set of parameters. In Section G.2, we describe the algorithm to estimate the parameters of the model. In Section G.3, we describe the algorithm to solve for network formation and equilibrium in a closed economy.

G.1 Network formation given parameters

Given a set of parameters, size of the labor force, price of foreign goods, and foreign demand, we follow the steps below to simulate the network formation.

1. Firms with productivities ϕ_i are randomly sorted, and indexed with $i = 1, 2, 3, \dots$. A firm's index determines the firm's set of eligible suppliers, \mathbf{Z}_i . The set of eligible suppliers is such that all feasible networks will be acyclic. Firms' draws of firm-pair-specific fixed cost of sourcing, fixed cost of importing and exporting, export demand, and benefits of importing, and firm-pair-specific cost shifters are also known at this point.
- 2a. All firms make a common guess of the wage level w .
- 2b. All firms make a common guess of aggregate demand term: $A = EP^{\sigma-1}$.
3. We assume that firms decide on their sourcing strategies in sequence of i . Firm 1 decides its sourcing strategy and determines c_1 , then firm 2 decides its sourcing strategy and determines c_2 , and so on. When firms make their sourcing decisions, we assume that all firms are able to use labor and foreign inputs, but firm i is only able to choose its suppliers from its eligible supplier set \mathbf{Z}_i . We determine which suppliers among \mathbf{Z}_i firm i sources from, using the algorithm described in Section F, and compute c_i . After the final firm $i = N$ decides its sourcing strategy, the whole vector \mathbf{c} and the supplier sets of all firms are determined. At this point we have also solved for the firm's optimal export participation choice and export sales.
4. Given the network, the guesses for A and w , we are able to compute the equilibrium variables.
 - (a) Sales to domestic final demand of firm i is computed as $X_{iH} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} A$ and to foreign final demand is computed as $X_{iF} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} \beta_{kF}^{\sigma-1} \frac{E_F}{P_F^{1-\sigma}}$.
 - (b) The cost of inputs used for firm i 's sales to domestic final demand is thus $C_{iH} = \frac{\sigma-1}{\sigma} X_{iH}$ and to foreign final demand is $C_{iF} = \frac{\sigma-1}{\sigma} X_{iF}$.

- (c) The total input costs of firms, C_i are calculated by solving the system of linear equations below:

$$C_i = C_{iH} + C_{iF} + \sum_j s_{ij} C_j,$$

$$\rightarrow \mathbf{C} = (\mathbf{I} - \mathbf{S})^{-1} (\mathbf{C}_H + \mathbf{C}_F)$$

where \mathbf{C} , \mathbf{C}_H , and \mathbf{C}_F are vectors of C_i , C_{iH} , and C_{iF} , respectively, and the i, j element of matrix S is s_{ij} .

- (d) The total sales of firm i is then $X_i = X_{iH} + X_{iF} + C_i - C_{iH}$.
- (e) Firm profits and total expenditure on fixed costs.
5. We solve for equilibrium variables of A and w in the following way: In the outer loop, we solve for wages such that the labor market clearing condition (26) is solved. In the inner loop, we iterate over steps 2b-4, such that a fixed point for the market demand level, A , is found.

G.2 Parameter estimation and network formation

One possible approach to estimating the parameters of the model is to simulate the model for each parameter guess according to the algorithm outlined in Section G.1, calculate the objective function in equation (29), and vary the parameter guesses to maximize the objective function. However, this requires for each parameter guess finding a fixed point in both the market demand, A , and a wage level, w . Below, we describe a more computationally attractive algorithm to estimate the model.

Throughout the estimation, we set the domestic wage, $w = 1$, as well as the domestic market demand, $A = 1$. We ensure labor market clearing condition and the fixed point in market demand in the following way:

1. Of the 8 parameters to estimate, the mean foreign demand parameter is implicitly pinned down to take the value that satisfies the trade balance condition.
2. Instead of treating the size of the labor force as data (note that the units are arbitrary), we choose its level by setting: $L = \frac{AP^{1-\sigma} - \sum_i \pi_i}{w}$.

Note that $A = w = 1$. Under this level of the size of the labor force, $L = \frac{AP^{1-\sigma} - \sum_i \pi_i}{w}$, the fixed point for the market demand, A is automatically satisfied. Also, since the trade balance holds, the domestic labor market clears as well.

Given a parameter guess for $\hat{\Phi}_{scale}^{\alpha_{dom}}$, $\hat{\Phi}_{scale}^{\alpha_F}$, $\hat{\Phi}_{disp}^{\alpha, \beta}$, $\hat{\Phi}_{scale}^{f_{dom}}$, $\hat{\Phi}_{scale}^{f_{imp}}$, $\hat{\Phi}_{scale}^{f_{exp}}$, and $\hat{\Phi}_{disp}^f$, we vary $\hat{\Phi}_{scale}^{\beta_F}$ and go through steps 3 and 4 in Section G.1 to calculate the aggregate trade

balance. Given the other parameters, the level of $\hat{\Phi}_{scale}^{\beta_F}$ is pinned down implicitly such that aggregate trade balance is equal to zero. Hence, instead of search for a fixed point in both A and w , we hold those fixed throughout the estimation and only need to find one fixed point in $\hat{\Phi}_{scale}^{\beta_F}$, that satisfies trade balance, for each guess of the other seven parameters.

G.3 Network formation in the closed economy

In the closed economy, we can normalize the domestic wage $w = 1$. We therefore only have to follow steps 2b-4 in Section G.1 to pin down the level of domestic market demand, A , in the closed economy.

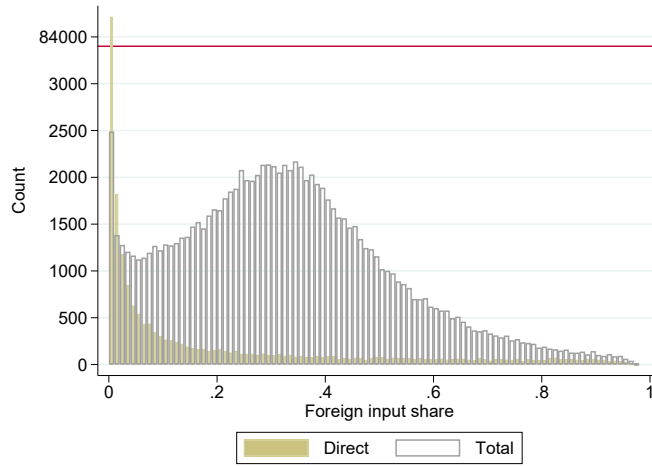
H Results from the estimated model

H.1 Direct and indirect linkages to foreign trade

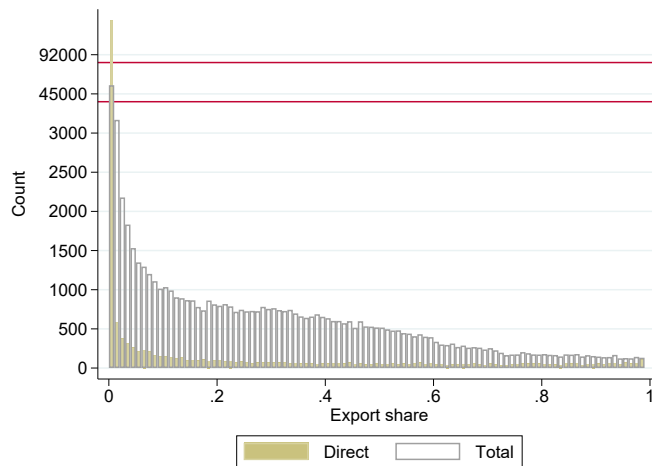
Figure 5 shows the histograms analogous to those in Figure 1, generated from the estimated model.

Figure 5: Histograms of direct and indirect linkages to foreign trade, from the estimated model

(a) Direct and total foreign input share



(b) Direct and total export share

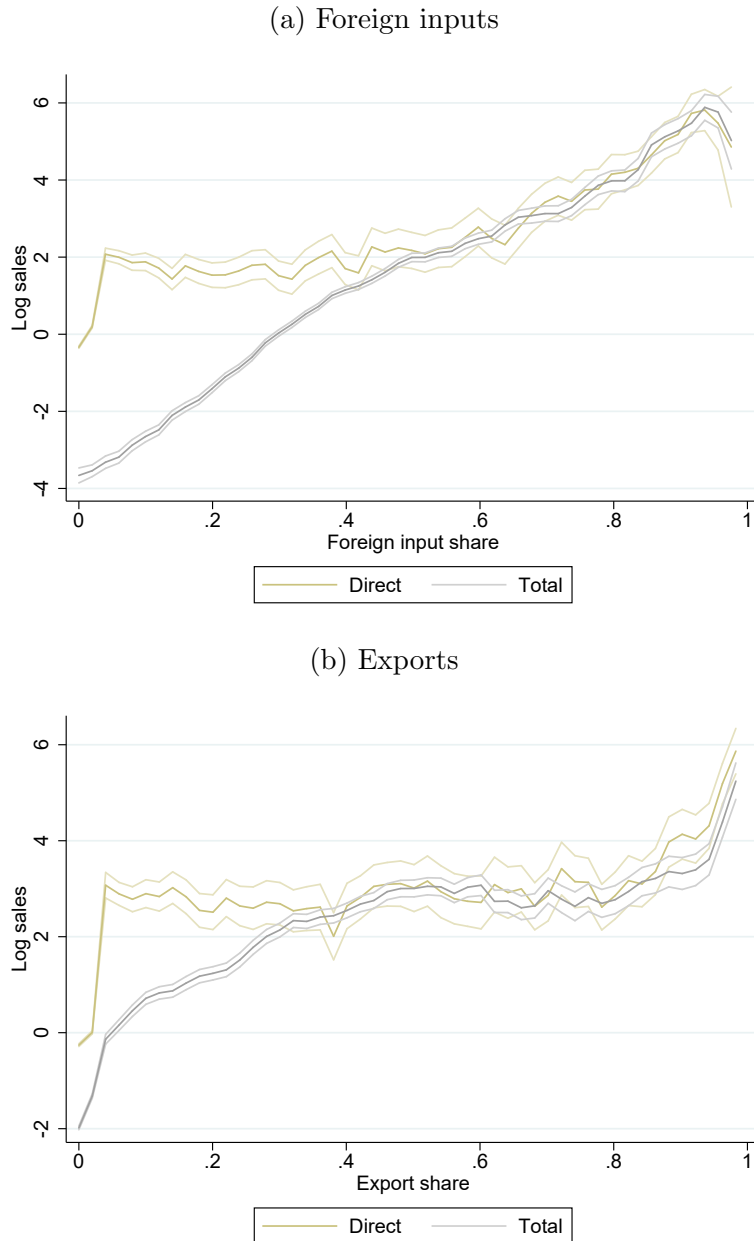


Notes: Total foreign input share of firm i , s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i 's direct foreign input share, and s_{ji} is j 's share among i 's inputs. Total export share firm i , r_{iF}^{Total} is calculated by solving $r_{iF}^{Total} = r_{iF} + \sum_{j \in W_i} r_{ij} r_{jF}^{Total}$ where r_{iF} is i 's share of exports in its revenue, and r_{ij} is share of i 's revenue that arises from sales to firm j . The figures are based on the estimated model. The horizontal lines represent scale breaks on the vertical axis.

H.2 Size premium of direct and indirect linkages to foreign trade

Figure 6 shows the plots analogous to those in Figure 2, generated from the estimated model.

Figure 6: Size premium of direct and indirect linkages to foreign trade, from the estimated model



Notes: The two figures display the smoothed values with 95% confidence intervals of kernel-weighted local polynomial regression estimates of the relationship between firms' sales and their levels of participation in foreign trade. We use the Epanechnikov kernel function with kernel bandwidth of 0.01, pilot bandwidth of 0.02, degree of polynomial smooth at 0, and smooth obtained at 50 points.

H.3 Statistics from the counterfactual exercise

Table 10 shows the results from linear and probit regressions where in its first four columns we regress the probability of having a lower unit cost upon a 10% increase in the foreign price, on firm-level variables. 21% of firms experienced a reduction in their costs. In its last four columns, we also show the results from regressing the probability of having a smaller number of domestic suppliers upon 10% increase in the foreign price, on firm-level variables. 4% of firms experienced a reduction in the number of domestic suppliers.

Table 10: Probabilities when cost increases by 10%

	Probability of having a cost reduction				Probability of having a reduction in the number of domestic suppliers			
	Linear		Probit		Linear		Probit	
	Est.	R^2	Est.	Pseudo R^2	Est.	R^2	Est.	Pseudo R^2
Import share	-0.40	0.02	-1.71	0.03	0.33	0.05	0.14	0.07
Total share of foreign inputs	-1.12	0.29	-1.32	0.40	0.28	0.08	0.22	0.20
Export share	-0.25	0.01	-0.40	0.01	0.45	0.09	0.16	0.11
Total share of exports	0.10	0.00	0.10	0.00	0.10	0.01	0.07	0.03
Labor share	0.86	0.31	0.63	0.29	-0.12	0.03	-0.23	0.12
Import status	-0.18	0.03	-0.24	0.04	0.07	0.02	0.05	0.05
Export status	-0.15	0.01	-0.19	0.01	0.16	0.06	0.08	0.11
Log import	-0.01	0.01	-0.01	0.03	0.02	0.05	0.02	0.08
Log export	-0.01	0.02	-0.01	0.04	0.03	0.09	0.03	0.10
Log sales to dom. fin. demand	-0.04	0.17	-0.04	0.18	0.01	0.04	0.01	0.12
Log total sales	-0.02	0.06	-0.02	0.05	0.01	0.02	0.01	0.07
Indegree	-0.01	0.11	-0.01	0.14	0.00	0.05	0.00	0.12

Notes: In the first four columns, the dependent variable equals to 1 if the firm's cost becomes lower after 10% increase in the foreign price, under endogenous networks. In the last four columns, the dependent variable equals to 1 if the number of domestic suppliers of the firm drops after 10% increase in the foreign price. All regressions are univariate. Estimates of Probit specification are scaled to match the interpretations of linear probability model.

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