Technical Appendix to: Quantifying the Benefits of Labor Mobility in a Currency Union*

Christopher L. House
University of Michigan and NBER

Christian Proebsting
EPFL | École Polytechnique Fédérale de Lausanne

Linda L. Tesar
University of Michigan and NBER

December 2, 2018

Contents

*House: chouse@umich.edu; Proebsting: Christian.Probsting@epfl.ch; Tesar: ltesar@umich.edu.

1 Calibration

1.1 Bilateral Trade Preference Weights

We calibrate the trade weights $\bar{\omega}_i^j$ using data on bilateral trade across U.S. states. Information on bilateral trade in goods across U.S. states comes from the freight analysis framework, which calculates trade in goods based on the commodity flow survey and other sources. Importantly, this database also contains within-state trade. We first calculate state j's share in total goods absorbed by state i to

$$\tilde{\bar{\omega}}_i^j = \frac{y_i^j}{\sum_j y_i^j},$$

where y_i^j is trade from state j to state i as observed in the data. We adjust this parameter in a second step because our data only provides an imperfect measure of y_i^j. As a matter of fact, we observe large net export positions, which we believe to be the product of either a small time frame (data is only available at five-year intervals starting in 1997) or a lack of data on trade in services.

We decide to log-linearize our model around a steady state with zero net export positions for every state. We choose our bilateral trade matrix $\bar{\omega}$ to satisfy this condition and to look as "similar" as possibile to the trade matrix implied by the data, $\tilde{\bar{\omega}}$. In particular, we minimize

$$\min_{\bar{\omega}_i^j} \sum_j \sum_i \frac{1}{2} \frac{\left(\tilde{\bar{\omega}}_j^i - \bar{\omega}_j^i\right)^2}{k + \tilde{\bar{\omega}}_j^i}$$

subject to

$$\begin{aligned} \sum_i \bar{\omega}_i^j \frac{\mathbb{N}_i Y_i}{\mathbb{N}_j Y_j} &= 1 + \frac{NX_j}{P_j Y_j} \qquad \forall j \\ \sum_j \bar{\omega}_i^j &= 1 \qquad \forall i \\ \bar{\omega}_i^j &\geq 0 \\ 1 &\geq \bar{\omega}_i^j, \end{aligned}$$

with $k > 0$.[1] Our loss function specifies our idea of "similarity" between the two matrices. The first constraint describes the relationship between the trade preference weight and net exports. The second to fourth constraints are purely technical constraints on the parameters. In practice we set $k = 0.1$ and $\frac{NX_j}{P_j Y_j} = 0$. This is a simple problem to solve. Let λ_j and λ_i denote the Lagrange multiplier on the first two constraints. We solve for these parameters using the two constraints and

[1] Notice that we require $k > 0$ because elements in $\tilde{\bar{\omega}}_j^i$ might be equal to 0.

setting the preference weights to

$$\bar{\omega}_j^i = min\left(1, max\left[0, \tilde{\bar{\omega}}_j^i - (k + \tilde{\bar{\omega}}_j^i)\left(\lambda_i + \lambda_j \frac{\mathbb{N}_i Y_i}{\mathbb{N}_j Y_j}\right)\right]\right).$$

2 Equilibrium Conditions

In the Technical Appendix, we allow for an intensive margin of labor supply. In particular, the labor supplied by a member of household i living in country j is denoted by $l_{j,t}^i$. Total labor supply in country j is then:

$$l_{j,t}\mathbb{N}_{j,t} = \sum_i n_{j,t}^i l_{j,t}^i \mathbb{N}^i \tag{2.1}$$

where $l_{j,t}$ is labor supply per capita in country j. Similarly, total labor supplied by household i is

$$l_t^i = \sum_j n_{j,t}^i l_{j,t}^i.$$

The utility function over consumption and labor is described by

$$u(c_{j,t}^i, l_{j,t}^i) = \upsilon_i \left(c_{j,t}^i - \kappa_j \frac{\left(l_{j,t}^i\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right)^{1-\frac{1}{\sigma}},$$

where σ is the intertemporal elasticity of substitution, υ_i is a household-specific utility weight, κ_j is a disutility weight on labor and η is the Frisch elasticity of labor supply.

2.1 Household's Budget Constraint

The household's budget constraint is given by[2]

$$\mathbb{N}^i \left(\sum_j S_{i,t}^j P_{j,t} n_{j,t}^i c_{j,t}^i \right) + \mathbb{N}_{i,t} P_{i,t} X_{i,t} + \mathbb{N}^i \frac{B_t^i}{(1+i_t)S_{i,t}}$$

$$= \mathbb{N}^i \left(\sum_j S_{i,t}^j n_{j,t}^i \left(W_{j,t}^h l_{j,t}^i + Tr_j^i \right) \right) + \mathbb{N}_{i,t-1} K_{i,t-1} \left(R_{i,t}^k u_{i,t} - P_{i,t} a(u_{i,t}) \right) + \mathbb{N}_{i,t} \left(\Pi_{i,t} - T_{i,t} \right) + \mathbb{N}^i \frac{B_{t-1}^i}{S_{i,t}},$$

[2] We ignore the quadratic penalty term on foreign bond holdings and the moving cost because they do not affect the log-linearized equations and are zero in steady state.

Replacing $T_{i,t}$ by the government's budget constraint

$$P_{i,t}\left(b_i \mathbb{N}_{i,t} U_{i,t} + \mathbb{N}_{i,t} G_i\right) + \sum_j \mathbb{N}^j n^j_{i,t} Tr^j_i = \mathbb{N}_{i,t} T_{i,t}.$$

gives

$$\mathbb{N}^i \left(\sum_j S^j_{i,t} P_{j,t} n^i_{j,t} c^i_{j,t} \right) + \mathbb{N}_{i,t} P_{i,t} (X_{i,t} + G_{i,t} + b_i U_{i,t}) + \mathbb{N}^i \left(\frac{B^i_t}{(1+i_t) S_{i,t}} - \frac{B^i_{t-1}}{S_{i,t}} \right)$$
$$= \mathbb{N}^i \left(\sum_j S^j_{i,t} n^i_{j,t} \left(W^h_{j,t} l^i_{j,t} + Tr^i_j \right) \right) + \mathbb{N}_{i,t-1} K_{i,t-1} \left(R^k_{i,t} u_{i,t} - P_{i,t} a(u_{i,t}) \right) + \mathbb{N}_{i,t} \Pi_{i,t} - \sum_j \mathbb{N}^j n^j_{i,t} Tr^j_i.$$

Replacing $\mathbb{N}_{i,t} P_{i,t} (X_{i,t} + G_{i,t}) + \mathbb{N}_{i,t-1} K_{i,t-1} P_{i,t} a(u_{i,t})$ by the market clearing for the final good

$$\mathbb{N}_{i,t} Y_{i,t} = \mathbb{N}_{i,t} C_{i,t} + \mathbb{N}_{i,t} X_{i,t} + \mathbb{N}_{i,t} G_{i,t} + a(u_{i,t}) \mathbb{N}_{i,t-1} K_{i,t-1} + \varsigma \mathbb{N}_{i,t} V_{i,t},$$

gives

$$\mathbb{N}^i \left(\sum_j S^j_{i,t} P_{j,t} n^i_{j,t} c^i_{j,t} \right) + \mathbb{N}_{i,t} P_{i,t} (Y_{i,t} - C_{i,t} - \varsigma V_{i,t} + b_i U_{i,t}) + \mathbb{N}^i \left(\frac{B^i_t}{(1+i_t) S_{i,t}} - \frac{B^i_{t-1}}{S_{i,t}} \right)$$
$$= \mathbb{N}^i \left(\sum_j S^j_{i,t} n^i_{j,t} \left(W^h_{j,t} l^i_{j,t} + Tr^i_j \right) \right) + R^k_{i,t} u_{i,t} \mathbb{N}_{i,t-1} K_{i,t-1} + \mathbb{N}_{i,t} \Pi_{i,t} - \sum_j \mathbb{N}^j n^j_{i,t} Tr^j_i.$$

The profit term $\mathbb{N}_{i,t} \Pi_{i,t}$ consists of profits by monopolistically competitve producers of varieties, $\mathbb{N}_{i,t} \Pi^f_{i,t}$, and profits by labor market firms (employment agencies and HR firms), $\mathbb{N}_{i,t} \Pi^l_{i,t}$. Profits by variety producers are given by

$$\mathbb{N}_{i,t} \Pi^f_{i,t} = p_{i,t} \mathbb{N}_{i,t} Q_{i,t} - W^f_{i,t} \mathbb{N}_{i,t} L_{i,t} - R^k_{i,t} u_{i,t} \mathbb{N}_{i,t-1} K_{i,t-1}.$$

Employment agencies pay $W^h_{i,t} l_{i,t}$ to households, receive $W_{i,t} L_{i,t}$ from HR firms and $P_{i,t} b_i U_{i,t}$ from the government. HR firms pay $W_{i,t} L_{i,t}$ to employment agencies, pay a vacancy cost $P_{i,t} \varsigma V_{i,t}$ and receive $W^f_i L_i$ from producing firms. Total profits of employment agencies and HR firms are therefore

$$\mathbb{N}_{i,t} \Pi^l_{i,t} = W^f_{i,t} \mathbb{N}_{i,t} L_{i,t} - \varsigma P_{i,t} \mathbb{N}_{i,t} V_{i,t} + b_i P_{i,t} \mathbb{N}_{i,t} U_{i,t} - \frac{W^h_{i,t} L_{i,t}}{1 - ur_{i,t}}.$$

It follows that overall profits, $\mathbb{N}_{i,t}\Pi_{i,t} = \mathbb{N}_{i,t}\Pi^f_{i,t} + \mathbb{N}_{i,t}\Pi^l_{i,t}$, are

$$\mathbb{N}_{i,t}\Pi_{i,t} = p_{i,t}\mathbb{N}_{i,t}Q_{i,t} - R^k_{i,t}u_{i,t}\mathbb{N}_{i,t-1}K_{i,t-1} - \frac{W^h_{i,t}L_{i,t}}{1-ur_{i,t}} - \varsigma P_{i,t}\mathbb{N}_{i,t}V_{i,t} + b_i P_{i,t}\mathbb{N}_{i,t}U_{i,t}.$$

Inserting this term into the budget constraint gives

$$\begin{aligned}
&\mathbb{N}^i\left(\sum_j S^j_{i,t}P_{j,t}n^i_{j,t}c^i_{j,t}\right) + \mathbb{N}_{i,t}P_{i,t}(Y_{i,t} - C_{i,t}) + \mathbb{N}^i\left(\frac{B^i_t}{(1+i_t)S_{i,t}} - \frac{B^i_{t-1}}{S_{i,t}}\right)\\
&= \mathbb{N}^i\left(\sum_j S^j_{i,t}n^i_{j,t}\left(W^h_{j,t}l^i_{j,t} + Tr^i_j\right)\right) + \mathbb{N}_{i,t}\left(p_{i,t}Q_{i,t} - W^h_{i,t}l_{i,t}\right) - \sum_j \mathbb{N}^j n^j_{i,t}Tr^j_i.
\end{aligned}$$

Re-arranging terms yields

$$\begin{aligned}
\mathbb{N}^i\left(\frac{B^i_t}{(1+i_t)S_{i,t}} - \frac{B^i_{t-1}}{S_{i,t}}\right) &= \mathbb{N}_{i,t}p_{i,t}Q_{i,t} - \mathbb{N}_{i,t}P_{i,t}Y_{i,t}\\
&+ \mathbb{N}^i\left(\sum_j S^j_{i,t}n^i_{j,t}W^h_{j,t}l^i_{j,t}\right) - \mathbb{N}_{i,t}W^h_{i,t}l_{i,t} - \left[\mathbb{N}^i\left(\sum_j S^j_{i,t}P_{j,t}n^i_{j,t}c^i_{j,t}\right) - \mathbb{N}_{i,t}P_{i,t}C_{i,t}\right]\\
&+ \mathbb{N}^i\sum_j S^j_{i,t}n^i_{j,t}Tr^i_j - \sum_j \mathbb{N}^j n^j_{i,t}Tr^j_i.
\end{aligned}$$

The definition of consumption and the labor force are

$$\begin{aligned}
C_{i,t}\mathbb{N}_{i,t} &= \sum_j n^j_{i,t}c^j_{i,t}\mathbb{N}^j\\
l_{i,t}\mathbb{N}_{i,t} &= \sum_j n^j_{i,t}l^j_{i,t}\mathbb{N}^j.
\end{aligned}$$

Substituting this in and re-arranging yields:

$$\begin{aligned}
\mathbb{N}^i\left(\frac{B^i_t}{(1+i_t)S_{i,t}} - \frac{B^i_{t-1}}{S_{i,t}}\right) &= \mathbb{N}_{i,t}p_{i,t}Q_{i,t} - \mathbb{N}_{i,t}P_{i,t}Y_{i,t}\\
&+ \sum_{j\neq i}\left(S^j_{i,t}n^i_{j,t}\mathbb{N}^iW^h_{j,t}l^i_{j,t} - n^j_{i,t}\mathbb{N}^jW^h_{i,t}l^j_{i,t}\right) - \left(\sum_j S^j_{i,t}n^i_{j,t}\mathbb{N}^iP_{j,t}c^i_{j,t} - n^j_{i,t}\mathbb{N}^jP_{i,t}C_{i,t}\right)\\
&+ \sum_{j\neq i}\left(S^j_{i,t}n^i_{j,t}\mathbb{N}^iTr^i_j - n^j_{i,t}\mathbb{N}^jTr^j_i\right).
\end{aligned}$$

The current account (LHS) equals net exports (1st row), net primary income from abroad (2nd row) and current transfers (3rd row). Net exports is the value of total production less the value of total domestic absorption. Net primary income is labor income earned abroad less labor income earned at home by foreigners, minus consumption expenditure abroad less consumption expenditure at home by foreigners. Current transfers consist of total government transfers received abroad less government transfers paid out at home to foreigners.

In steady state, this equation becomes

$$\mathbb{N}^i(\beta-1)\frac{B^i}{S_i} = \mathbb{N}_i(p_iQ_i - P_iY_i) + \sum_{j\neq i}\left(S_i^j n_j^i \mathbb{N}^i Tr_j^i - n_i^j \mathbb{N}^j Tr_i^j\right)$$
$$+\sum_{j\neq i}\left(S_j^i n_j^i \mathbb{N}^i W_j^h l_j^i - n_i^j \mathbb{N}^j W_i^h l_i^j\right) - \left(\sum_j S_j^i n_j^i \mathbb{N}^i P_j c_j^i - n_i^j \mathbb{N}^j P_i C_i\right)$$

Re-arranging yields:

$$\mathbb{N}^i(\beta-1)\frac{B^i}{S_i} = \mathbb{N}_i(p_iQ_i - P_iY_i) + \mathbb{N}^i\sum_{j\neq i} S_j^i n_j^i\left(Tr_j^i - W_j^h l_j^i + P_j c_j^i\right) - \sum_{j\neq i}\mathbb{N}^j n_i^j\left(Tr_i^j - W_i^h l_i^j - P_i c_i^j\right).$$

Since governments set transfers in steady state such that

$$Tr_i^j = P_i c_i^j - W_i^h l_i^j,$$

we are left with

$$\mathbb{N}^i(\beta-1)\frac{B^i}{S_i} = \mathbb{N}_i(p_iQ_i - P_iY_i).$$

That is, in steady state net primary income and current transfers cancel each other out and the net foreign asset position is proportional to net exports. We start from a steady state with zero net exports, i.e. the net foreign asset position is zero as well.

Log-linearizing the household's budget constraint around a steady state with a zero net foreign asset position ($B^i = 0$ for all i) yields

$$\mathbb{N}^i\left(\frac{1}{\beta}\Delta B_{t-1}^i - \Delta B_t^i\right) = \mathbb{N}_i\left(Y_i\widetilde{Y}_{i,t} - Q_i\left(\widetilde{\frac{p_{i,t}}{P_{i,t}}} + \tilde{Q}_{i,t}\right)\right)$$
$$-\left[\sum_{j\neq i} n_j^i \mathbb{N}^i w_j^h l_j^i\left(\widetilde{w}_{j,t}^h + \tilde{l}_{j,t}^i\right) - n_i^j \mathbb{N}^j w_i^h l_i^j\left(\widetilde{w}_{i,t}^h + \tilde{l}_{i,t}^j\right)\right]$$
$$+\left[\sum_{j\neq i} n_j^i \mathbb{N}^i c_j^i \tilde{c}_{j,t}^i - n_i^j \mathbb{N}^j c_i^j \tilde{c}_{i,t}^j\right].$$

2.2 Steady State

To solve for the steady state, we proceed in three steps:

1. We first solve for real prices (rental price of capital, r_i^k, real price of the intermediate good,

$\frac{p_i}{P_i}$) and shares in GDP (share of consumption expenditure in GDP, $\frac{C_i}{Q_i}$, share of investment in GDP, $\frac{X_i}{Q_i}$, share of net exports in GDP, $\frac{NX_i}{Q_i}$). This requires data on relative country size in terms of countries' domestic absorption, $\frac{\mathbb{N}_j Y_j}{\mathbb{N}_i Y_i}$, but it does not require separate information on population, $\mathbb{N}_i$ or domestic absorption per capita, Y_i. In a standard international DSGE model, only total country size matters, but not GDP *per capita*. Importantly, at this stage we cannot and do not need to solve for the real wage w_i^f. The model only pins down wage payments as a share of GDP, i.e. $\frac{w_i^f L_i}{Q_i}$, but not the real wage w_i^f.

2. We next solve for the steady-state values related to migration (labor income of nationals, $w_j^h l_j^i$, and consumption of nationals, c_j^i, for all locations j). Here, we require information on population measured in terms of persons, $\mathbb{N}_i$, and the share of the labor force in total population.

3. We then solve for the steady-state values related to the search and matching block. Here, we require data on unemploment rates, ur_i, to solve for employment and the real wage w_i^f. Given this real wage and unemployment benefits (as percent of GDP), b_i, we can solve for all remaining variables pertaining to the search and matching block.

2.2.1 DSGE block

We solve the model in a neighborhood of a non-stochastic steady state with zero inflation. Because inflation is zero, the Euler equations associated with the noncontingent nominal bonds imply that the nominal interest rate is $1 + i_i = \frac{1}{\beta}$ for all i. Next, we use the capital Euler equation

$$\frac{\mu_i}{P_i} = \beta \left[u_i r_i^k + \frac{\mu_i}{P_i} (1 - \delta) - a(u_i) \right].$$

Note that the households' first-order condition for investment implies that $\mu_i = P_i$ because $\Lambda = \Lambda' = 0$ in steady state. Inserting this back into the entrepreneurs' first-order condition for capital and noting that $a(u_i) = 0$ and $u_i = 1$ gives

$$r_i^k = \frac{1}{\beta} - 1 + \delta. \tag{2.2}$$

Tthis equation determines the real rental price of capital r_i^k in each country.

With zero inflation, the steady state price of intermediates is a constant markup over the nominal marginal cost,

$$p_i = \frac{\psi_q}{\psi_q - 1} MC_i.$$

This can be seen from the reset equation and the law of motion for the nominal price of the intermediate good.

Next, cost minimization of the first-stage producers implies

$$R_i^k = MC_i \alpha Z_i \left[\frac{K_i}{L_i}\right]^{\alpha-1}$$

$$r_i^k = \frac{\psi_q - 1}{\psi_q}\frac{p_i}{P_i}\alpha Z_i \left[\frac{K_i}{L_i}\right]^{\alpha-1}$$

$$\frac{p_i}{P_i} = r_i^k \frac{\psi_q}{\psi_q - 1}\frac{1}{\alpha Z_i}\left[\frac{K_i}{L_i}\right]^{1-\alpha}$$

We adjust the technology levels Z_i so that all intermediate goods prices equal the price of the respective final good: $p_i = P_i$. Notice that the real wage paid by the producing firms is given by

$$w_i^f = \frac{\psi_q - 1}{\psi_q}\frac{p_i}{P_i}(1-\alpha)\frac{Q_i}{L_i}.$$

The price index formula for the final good states

$$P_i = \left(\sum_{j=1}^{\mathcal{N}} \bar{\omega}_i^j \left[\frac{S_j}{S_i}p_j\right]^{1-\psi_y}\right)^{\frac{1}{1-\psi_y}}$$

$$P_i S_i = \left(\sum_{j=1}^{\mathcal{N}} \bar{\omega}_i^j \left[P_j S_j \frac{p_j}{P_j}\right]^{1-\psi_y}\right)^{\frac{1}{1-\psi_y}}$$

One can easily verify that $P_i S_i = 1$ solves this equation, that is the real exchange rate $s_i = P_i S_i$ is unity.

We directly calibrate some steady-state variables to match their empirical counterparts. Those are the shares of government purchases, G_i, the relative country sizes, $\frac{\mathbb{N}_i Y_i}{\mathbb{N}_i Y_i}$ and the bilateral import shares $\frac{y_n^i}{Y_i}$. We now derive the shares of the remaining variables, NX_i, C_i and X_i, and later show that these non-targeted shares implied by our model match their empirical counterparts quite closely.

To derive the share of net exports, we first use the demand equation for intermediate goods,

$$y_i^j = Y_i \bar{\omega}_i^j \left[\frac{S_j}{S_i}\frac{p_j}{P_i}\right]^{-\psi_y}$$

$$= Y_i \bar{\omega}_i^j \left[\frac{s_j}{s_i}\frac{p_j}{P_j}\right]^{-\psi_y}.$$

It follows that $\bar{\omega}_i^j$ is country i's import share of country j's good, measured in terms of the privately-produced good Y_i:

$$\bar{\omega}_i^j = \frac{y_i^j}{Y_i}.$$

The implied net export share can then be expressed in terms of country sizes and the import preference parameters. Inserting the market clearing condition for Q_i into the definition of net exports, $NX_i = p_i Q_i - P_i Y_i$, we have

$$\begin{aligned}
\frac{NX_i}{P_i Y_i} &= \left(\sum_{j=1}^{N} \frac{\mathbb{N}_j p_i y_j^i}{\mathbb{N}_i P_i Y_i} \right) - 1 \\
&= \left(\sum_{j=1}^{N} \frac{\mathbb{N}_j Y_j}{\mathbb{N}_i Y_i} \bar{\omega}_j^i \right) - 1. \qquad (2.3)
\end{aligned}$$

Notice that this also gives us

$$\frac{p_i Q_i}{P_i Y_i} = 1 + \frac{NX_i}{P_i Y_i}. \qquad (2.4)$$

To derive the share of investment, we insert the marginal product of capital equation, $p_i Q_i = \frac{\psi_q}{\psi_q - 1} \frac{R_i}{\alpha} K_i$, into the definition of net exports, $NX_i = p_i Q_i - P_i Y_i$:

$$\begin{aligned}
\frac{\psi_q}{\psi_q - 1} \frac{R_i}{\alpha \delta} X_i &= P_i Y_i + NX_i \\
\frac{X_i}{Y_i} &= \frac{\alpha \delta}{\frac{\psi_q}{\psi_q - 1} r_i^k} \left(1 + \frac{NX_i}{P_i Y_i} \right), \qquad (2.5)
\end{aligned}$$

where $X_i = \delta K_i$.

Finally, the consumption share is the residual of the market clearing condition $Y_i = C_i + X_i + G_i + \varsigma V_i$:

$$\frac{C_i}{Y_i} = 1 - \frac{X_i}{Y_i} - \frac{G_i}{Y_i} - \varsigma \frac{V_i}{Y_i}. \qquad (2.6)$$

To summarize, we solve for the steady state values as follows:

1. Calibrate the government expenditure shares $\frac{G_i}{Y_i}$ to the counterpart in the data.
2. Solve for the real rental price r_i^k using equation (2.2).
3. Calibrate the import preference parameters $\bar{\omega}_i^j$ using data on country j's share of country i's imports, and calibrate the relative size of countries in terms of their domestic absorption, $\frac{\mathbb{N}_j Y_j}{\mathbb{N}_i Y_i}$.
4. Solve for the net export share $\frac{NX_i}{P_i Y_i}$ using equation (2.3), the GDP share using equation (2.4), the investment share $\frac{X_i}{Y_i}$ using equation (2.5) and the consumption share $\frac{C_i}{Y_i}$ using equation (2.6)

2.2.2 Migration

For the DSGE block, we only require data on a country's total domestic absorption, $\mathbb{N}_i Y_i$, as a measure of an economy's size. Given $\mathbb{N}_i Y_i$, the values for population, $\mathbb{N}_i$, and domestic absorption per capita, Y_i, are irrelevant. This is no longer true if we allow for migration. For the migration block, we require data on countries' population, $\mathbb{N}_i$. Given values for $\mathbb{N}_i$, we immediately have domestic absorption per capita, Y_i from

$$Y_i = \frac{\mathbb{N}_i Y_i}{\mathbb{N}_i}. \tag{2.7}$$

This allows us to write all shares previously expressed in terms of domestic absorption, (2.6), (2.5), (2.3) and (2.4) in absolute values. For instance, Q_i, is then calculated from equation (2.4) as

$$Q_i = \frac{Q_i}{Y_i} Y_i.$$

Next, we solve for steady-state shares of migrants, n_i^j, population size of nationals, $\mathbb{N}^j$, the bilateral labor supply matrix, l_i^j, and the bilateral consumption matrix, c_i^j.

The steady-state shares of migrants, n_i^j, are determined by the equation governing the location choice:

$$u(c_j^i, l_j^i) - u(c_i^i, l_i^i) + A_j^i - \frac{1}{\gamma}\left(\ln(n_j^i) + 1\right) = \left(c_j^i - w_j^h l_j^i - tr_j^i\right) u_{1,j}^i - \left(c_i^i - w_i^h l_i^i - tr_i^i\right) u_{1,i}^i.$$

Notice that the terms related to migration costs are zero in steady state. Given our assumption on how transfers tr_j^i are set in steady state, we have

$$u(c_j^i, l_j^i) - u(c_i^i, l_i^i) + A_j^i = \frac{1}{\gamma}\left(\ln(n_j^i) + 1\right).$$

Since steady-state values for c_j^i and l_j^i are independent of A_j^i, migration shares n_j^i are pinned down by A_j^i. In practice, we adjust A_j^i to match bilateral stocks of migrants observed in the data.

Given the matrix of migration shares n_j^i, we solve for the population size of nationals (household size), $\mathbb{N}^j$, from the linear equation system

$$\mathbb{N}_i = \sum_j n_i^j \mathbb{N}^j. \tag{2.8}$$

Consumption and labor supply. Given GHH preferences, labor supply in steady state satisfies

$$\kappa_j \left(l_j^i\right)^{\frac{1}{\eta}} = w_j^h$$
$$l_j^i = \left(\frac{w_j^h}{\kappa_j}\right)^{\eta}.$$

which implies that labor supply only depends on a household member's current location, i.e. $l_j^i = l_j^j = l_j$. We can therefore write

$$l_j = \left(\frac{w_j^h}{\kappa_j}\right)^{\eta}.$$

Given values for the household wage, w_j^h, and the disutility weight κ_j, we could solve for l_j. Here, we use a slightly different strategy and calibrate l_j directly to the data. We then adjust κ_j accordingly.[3]

To solve for the consumption matrix, we make use of the Backus-Smith risk-sharing condition:

$$c_j^i - \kappa_j \frac{(l_j)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} = c_i^i - \kappa_i \frac{(l_i)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}$$
$$c_j^i - \frac{w_j^h l_j}{1+\frac{1}{\eta}} = c_i^i - \frac{w_i^h l_i}{1+\frac{1}{\eta}}$$

Then, c_j^j can be solved from the linear equation system

$$C_i \mathbb{N}_i = \sum_j n_i^j c_i^j \mathbb{N}^j$$
$$C_i \mathbb{N}_i = \sum_j n_i^j \left(c_j^j - \frac{w_j^h l_j - w_i^h l_i}{1+\frac{1}{\eta}}\right) \mathbb{N}^j. \tag{2.9}$$

Notice that we have solved for C_i in equation (2.6) together with equation (2.7), $\mathbb{N}_i$ is country i's population, given by the data, n_i^j are migration shares given by the data, $\mathbb{N}^j$ is the size of household j, given by (2.8), labor supply is calibrated to the data and the household wage is solved below (see equation (2.13)).[4]

[3] We choose this strategy because there is little guidance on the actual value of κ_j. If the disutility on labor κ_j was the same across countries, our calibration would imply large differences in the labor force, l_j, because countries substantially differ in their real wage rates.

[4] With separable preferences, the risk-sharing condition

$$\left(c_j^i\right)^{-\frac{1}{\sigma}} = \left(c_i^i\right)^{-\frac{1}{\sigma}},$$

ensures that $c_i^i = c_j^i = c^i$, independent of κ_j. We can solve for c^j using

$$C_i \mathbb{N}_i = \sum_j n_i^j c^j \mathbb{N}^j.$$

To summarize, we solve for the steady state values as follows:

1. Calibrate population sizes $\mathbb{N}_i$ to rewrite quantities in per capita terms $(Y_i, Q_i, ...)$
2. Calibrate migrant shares n_i^j and obtain the household size from (2.8)
3. Calibrate the size of the labor force (per capita), l_i, and use $l_i^j = l_i$
4. Solve for the consumption matrix c_j^i from (2.9)

2.2.3 Search block

In this section, we solve for the real wage paid by the HR firm to the employment agency, w_i, the wage received by the household, w_i^h, and the values of a filled vacancy and of being employed, $\mathcal{J}_i$ and $\mathcal{E}_i$. We solve for these values as functions of the real wage paid by the producing firm, w_i^f and the parameters of the model. The real wage paid by firms equals the marginal product of labor:

$$w_i^f = (1-\alpha)\frac{\psi_q - 1}{\psi_q}\frac{Q_i}{L_i}. \tag{2.10}$$

Employment, L_i, is calculated as

$$L_i = l_i(1 - ur_i), \tag{2.11}$$

where l_i is the labor force and $ur_i = \frac{U_i}{l_i}$ is the unemployment rate. We calibrate both the labor force and the unemployment rate to the data.

From labor supply in steady state, we have

$$w_i^h l_i^j = \kappa_i \left(l_i^i\right)^{1+\frac{1}{\eta}} \left(c^j\right)^{\frac{1}{\sigma}}$$

and

$$w_i^h l_i^i = \kappa_i \left(l_i^i\right)^{1+\frac{1}{\eta}} \left(c^i\right)^{\frac{1}{\sigma}}.$$

Combining, we obtain

$$l_i^j = l_i^i \left(\frac{c^j}{c_i}\right)^{\frac{\eta}{\sigma}}.$$

Then, we can solve for l_i^i using

$$\begin{aligned} l_i \mathbb{N}_{i,t} &= \sum_j n_i^j l_i^j \mathbb{N}^j \\ l_i \mathbb{N}_{i,t} &= l_i^i \sum_j n_i^j \left(\frac{c^j}{c_i}\right)^{\frac{\eta}{\sigma}} \mathbb{N}^j \\ l_i^i &= l_i \mathbb{N}_{i,t} \left(\sum_j n_i^j \left(\frac{c^j}{c_i}\right)^{\frac{\eta}{\sigma}} \mathbb{N}^j\right)^{-1}. \end{aligned}$$

The matching function, $f_i = \frac{M_i}{H_i}$, combined with the law of motion for employed workers, $L_i = (1-d)L_i + M_i$ and the equation describing the number of job hunters, $H_i = U_i + dL_i$ gives

$$f_i = \frac{dL_i}{U_i + dL_i} = \frac{d}{\frac{ur_i}{1-ur_i} + d}, \tag{2.12}$$

where we used that $\frac{U_i}{L_i} = ur_i \frac{l_i}{L_i} = \frac{ur_i}{1-ur_i}$. The equation describing the value of being employed is given by

$$w_i^h = w_i - [1-(1-d)\beta]\,\mathcal{E}_i \tag{2.13}$$

where we can replace w_i^h by the free entry condition for employment agencies

$$\mathcal{E}_i = \frac{1-f_i}{f_i}\left(w_i^h - b_i\right),$$

to get

$$\left[1-(1-d)\beta + \frac{f_i}{1-f_i}\right]\mathcal{E}_i = w_i - b_i.$$

To solve for w_i as a function of parameters, we combine the wage bargaining equation,

$$\varrho \mathcal{J}_i = (1-\varrho)\left(\mathcal{E}_i - b_i + w_i^h\right),$$

with the free entry condition for employment agencies and the equation describing the value of a filled vacancy

$$\mathcal{J}_i = \frac{w_i^f - w_i}{1-(1-d)\beta} \tag{2.14}$$

to obtain

$$\mathcal{E}_i = (1-f_i)\frac{\varrho}{1-\varrho}\frac{w_i^f - w_i}{1-(1-d)\beta}. \tag{2.15}$$

Using this expression to replace $\mathcal{E}_i$, we obtain

$$\begin{aligned}
\frac{1-(1-f_i)(1-d)\beta}{1-(1-d)\beta}\frac{\varrho}{1-\varrho}(w_i^f - w_i) &= w_i - b_i \\
\Xi_i w_i^f &= (1+\Xi)w_i - b_i \\
w_i &= \frac{\Xi_i}{1+\Xi_i}w_i^f + \frac{1}{1+\Xi_i}b_i,
\end{aligned}$$

where $\Xi_i = \frac{1-(1-f_i)(1-d)\beta}{1-(1-d)\beta}\frac{\varrho}{1-\varrho}$. We calibrate b_i using data on replacement values, $rv_i = \frac{b_i}{w_i}$. Given

rv_i, the wage w_i is given by

$$w_i = \frac{\Xi_i}{1 + \Xi_i - rv_i} w_i^f. \tag{2.16}$$

We can then directly solve for the value of a filled vacancy, $\mathcal{J}_i$, the value of being employed, $\mathcal{E}_i$ and the wage rate paid to the household, w_i^h.

Notice that if the bargaining power of the worker, ϱ goes to 1, we have $\Xi_i \to \infty$ and the steady state wage, w_i, equals the marginal product of labor, w_i^f. If, in addition, the steady-state unemployment rate goes to zero, $ur_i \to 0$, then, the probability to find a job goes to 1, $f_i \to 1$, the value of being employed goes to 0, $\mathcal{E}_i \to 0$, so that $w_i^h = w_i = w_i^f$.
To summarize, we solve for the steady state values as follows:

1. Solve for employment per capita, L_i, from (2.11), using data on unemployment rates, the real wage paid by producing firms, w_i^f, from (2.10) and the job finding rate, f_i, from (2.12).
2. Solve for the real wage, w_i, from (2.16), using data on replacement values, rv_i. Solve for real unemployment benefits, $b_i = \frac{b_i}{w_i^f} w_i^f$.
3. Solve for the value of a filled vacancy, $\mathcal{J}_i$, from (2.14), the value of being employed, $\mathcal{E}_i$, from (2.15), and the the real wage received by households, w_i^h, from (2.13)

2.3 Log-linearized Equilibrium Conditions

1. Marginal utility of consumption[5]
   With separable preferences:

$$\widetilde{u}^i_{1,j,t} = -\frac{1}{\sigma}\tilde{c}^i_{j,t}$$

   with GHH preferences:

$$-\sigma\left(u^i_{1,j}\right)^{-\sigma}\tilde{u}^i_{1,j,t} = c^i_j\tilde{c}^i_{j,t} - w^h_j l^i_j \tilde{l}^i_{j,t}$$

2. Marginal rate of substitution
   With separable preferences

$$\widetilde{mrs}^i_{j,t} = \frac{1}{\eta}\tilde{l}^i_{j,t} - \tilde{u}^i_{1,j,t}$$

   With GHH:

$$\widetilde{mrs}^i_{j,t} = \frac{1}{\eta}\tilde{l}^i_{j,t}$$

3. Risk sharing among household members (FOC $c^i_{j,t}$)

$$u^i_{1,i,t}s^j_{i,t} = u^i_{1,j,t}, \qquad \text{for} \quad j \neq i$$
$$\tilde{u}^i_{1,i,t} + \tilde{s}_{j,t} - \tilde{s}_{i,t} = \tilde{u}^i_{1,j,t} \qquad \text{for} \quad j \neq i$$

4. Labor supply

$$mrs^i_{j,t} = w^h_{j,t}$$
$$\widetilde{mrs}^i_{j,t} = \widetilde{w}^h_{j,t}$$

[5] **GHH preferences:**

$$u^i_{j,t} = \frac{1}{1-\frac{1}{\sigma}}\left(c^i_{j,t} - \kappa_j\frac{\left(l^i_{j,t}\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right)^{1-\frac{1}{\sigma}}$$

$$\left(u^i_{1,j,t}\right)^{-\sigma} = c^i_{j,t} - \kappa_j\frac{\left(l^i_{j,t}\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}$$

$$-\sigma\left(u^i_{1,j}\right)^{-\sigma}\tilde{u}^i_{1,j,t} = c^i_j\tilde{c}^i_{j,t} - l^i_j\kappa_j\left(l^i_j\right)^{\frac{1}{\eta}}\tilde{l}^i_{j,t},$$

where we can replace $\kappa_j\left(l^i_j\right)^{\frac{1}{\eta}} = w^h_j$.

5. Definition of consumption

$$C_{i,t}\mathbb{N}_{i,t} = \sum_j n_{i,t}^j c_{i,t}^j \mathbb{N}^j$$
$$\left(\tilde{C}_{i,t} + \tilde{\mathbb{N}}_{i,t}\right) = \sum_j \frac{\mathbb{N}^j}{\mathbb{N}_i}\frac{c_i^j}{C_i} n_i^j \left(\tilde{n}_{i,t}^j + \tilde{c}_{i,t}^j\right)$$

6. Definition of the labor force

$$l_{i,t}\mathbb{N}_{i,t} = \sum_j n_{i,t}^j l_{i,t}^j \mathbb{N}^j$$
$$\left(\tilde{l}_{i,t} + \tilde{\mathbb{N}}_{i,t}\right) = \sum_j \frac{\mathbb{N}^j}{\mathbb{N}_i}\frac{l_i^j}{l_i} n_i^j \left(\tilde{n}_{i,t}^j + \tilde{l}_{i,t}^j\right)$$

7. Capital Euler equation

$$\frac{\mu_{i,t}}{P_{i,t}} = \beta\mathbb{E}_t\left\{\frac{u_{1,i,t+1}^i}{u_{1,i,t}^i}\left[u_{i,t+1}r_{i,t+1}^k + \frac{\mu_{i,t+1}}{P_{i,t+1}}(1-\delta) - a\left(u_{i,t+1}\right)\right]\right\}$$
$$\beta r_i^k \tilde{r}_{i,t+1}^k = \beta\Delta i_{i,t} - \tilde{\pi}_{i,t+1} + \widetilde{\left(\frac{\mu_{i,t}}{P_{i,t}}\right)} - \beta(1-\delta)\widetilde{\left(\frac{\mu_{i,t+1}}{P_{i,t+1}}\right)}$$

8. Price of capital

$$1 = \frac{\mu_{i,t}}{P_{i,t}}\left(1 - \Lambda_{i,t} - \frac{\mathbb{N}_{i,t}X_{i,t}}{\mathbb{N}_{i,t-1}X_{i,t-1}}\Lambda_{i,t}'\right) + \beta\mathbb{E}_t\left\{\frac{u_{1,i,t+1}^i}{u_{1,i,t}^i}\frac{\mu_{i,t+1}}{P_{i,t+1}}\left(\frac{\mathbb{N}_{i,t+1}X_{i,t+1}}{\mathbb{N}_{i,t}X_{i,t}}\right)^2\Lambda_{i,t+1}'\right\},$$
$$\widetilde{\left(\frac{\mu_{i,t}}{P_{i,t}}\right)}\frac{1}{\Lambda''} = (1+\beta)\left(\tilde{\mathbb{N}}_{i,t} + \widetilde{X}_{i,t}\right) - \left(\tilde{\mathbb{N}}_{i,t-1} + \widetilde{X}_{i,t-1}\right) - \beta\left(\tilde{\mathbb{N}}_{i,t+1} + \widetilde{X}_{i,t+1}\right)$$

9. Law of motion for the capital stock

$$\mathbb{N}_{i,t}K_{i,t} = \mathbb{N}_{i,t-1}K_{i,t-1}(1-\delta) + \left[1 - \Lambda\left(\frac{\mathbb{N}_{i,t}X_{i,t}}{\mathbb{N}_{i,t-1}X_{i,t-1}}\right)\right]\mathbb{N}_{i,t}X_{i,t}$$
$$\tilde{K}_{i,t} = (1-\delta)\left(\tilde{\mathbb{N}}_{i,t-1} + \tilde{K}_{i,t-1} - \tilde{\mathbb{N}}_{i,t}\right) + \delta\tilde{X}_{i,t}$$

10. Optimal capital utilization

$$r_{i,t}^k = a'\left(u_{i,t}\right)$$
$$r_i^k\tilde{r}_{i,t}^k = a'' u_i \tilde{u}_{i,t}$$

11. Optimal factor employment

$$\frac{\alpha}{1-\alpha}\frac{W^f_{i,t}}{R^k_{i,t}} = \frac{u_{i,t}\mathbb{N}_{i,t-1}K_{i,t-1}}{\mathbb{N}_{i,t}L_{i,t}}$$
$$\tilde{r}^k_{i,t} - \widetilde{w}^f_{i,t} = \tilde{\mathbb{N}}_{i,t} + \tilde{L}_{i,t} - \tilde{u}_{i,t} - \tilde{K}_{i,t-1} - \tilde{\mathbb{N}}_{i,t-1}$$

12. Real marginal costs

$$MC_{i,t} = \frac{\left(W^f_{i,t}\right)^{1-\alpha}\left(R^k_{i,t}\right)^{\alpha}}{Z_{i,t}}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}$$
$$\widetilde{mc}_{i,t} = -\tilde{Z}_{i,t} + \alpha\tilde{r}^k_{i,t} + (1-\alpha)\widetilde{w}^f_{i,t}$$

13. FOC wrt $y^j_{i,t}$

$$y^j_{i,t} = Y_{i,t}\omega^j_{i,t}\left[\frac{S_{j,t}}{S_{i,t}}\frac{p_{j,t}}{P_{i,t}}\right]^{-\psi_y}$$
$$\widetilde{\left(\frac{p_{j,t}}{P_{j,t}}\right)} + \tilde{s}_{j,t} - \tilde{s}_{i,t} = \frac{1}{\psi_y}\left(\tilde{Y}_{i,t} + \left(\varepsilon^j_t - \sum_k \bar{\omega}^k_i\varepsilon^k_t\right) - \tilde{y}^j_{i,t}\right) \quad \forall j$$

14. Production of intermediate good

$$\mathbb{N}_{i,t}Q_{i,t} = Z_{i,t}\left(u_{i,t}\mathbb{N}_{i,t-1}K_{i,t-1}\right)^{\alpha}\left(\mathbb{N}_{i,t}L_{i,t}\right)^{1-\alpha}$$
$$\tilde{Q}_{i,t} = \tilde{Z}_{i,t} + \alpha\left(\tilde{u}_{i,t} + \tilde{\mathbb{N}}_{i,t-1} + \tilde{K}_{i,t-1} - \tilde{\mathbb{N}}_{i,t}\right) + (1-\alpha)\tilde{L}_{i,t}$$

15. Production of final good[6]

$$Y_{i,t} = \left(\sum_{j=1}^{\mathcal{N}}\left(\omega^j_{i,t}\right)^{\frac{1}{\psi_y}}\left(y^j_{i,t}\right)^{\frac{\psi_y-1}{\psi_y}}\right)^{\frac{\psi_y}{\psi_y-1}}$$
$$\tilde{Y}_{i,t} = \sum_{j=1}^{\mathcal{N}}\bar{\omega}^j_i\left(\tilde{y}^j_{i,t} + \frac{1}{\psi_y-1}\left(\varepsilon^j_t - \sum_k \bar{\omega}^k_i\varepsilon^k_t\right)\right)$$

[6] Our calibration of the shares $\bar{\omega}^j_i$ is $\bar{\omega}^j_i = \frac{y^j_n}{Y_n}$, so that

$$Y^{\frac{\psi_y-1}{\psi_y}}\tilde{Y}_{i,t} = \sum_{j=1}^{\mathcal{N}}\left[\left(\bar{\omega}^j_i\right)^{\frac{1}{\psi_y}}\left(y^j_n\right)^{\frac{\psi_y-1}{\psi_y}}\left(\tilde{y}^j_{i,t} + \frac{1}{\psi_y-1}\left(\varepsilon^j_t - \sum_k \bar{\omega}^k_i\varepsilon^k_t\right)\right)\right]$$

can be simplified.

16. Market clearing for intermediate goods[7]

$$\mathbb{N}_{i,t} Q_{i,t} = \sum_{j=1}^{\mathcal{N}} \mathbb{N}_{j,t} y_{j,t}^{i}$$
$$\frac{Q_i}{Y_i}\left(\tilde{\mathbb{N}}_{i,t} + \tilde{Q}_{i,t}\right) = \sum_{j=1}^{\mathcal{N}} \frac{\mathbb{N}_j Y_j}{\mathbb{N}_i Y_i} \bar{\omega}_i^j \left(\tilde{\mathbb{N}}_{j,t} + \tilde{y}_{j,t}^i\right)$$

17. Market clearing for final goods[8]

$$Y_{i,t} = C_{i,t} + X_{i,t} + G_{i,t} + a\left(u_{i,t}\right) \frac{\mathbb{N}_{i,t-1} K_{i,t-1}}{\mathbb{N}_{i,t}} + \varsigma V_{i,t}$$
$$\tilde{Y}_{i,t} = \frac{C_i}{Y_i}\tilde{C}_{i,t} + \frac{X_i}{Y_i}\tilde{X}_{i,t} + \frac{G_i}{Y_i}\tilde{G}_{i,t} + r_i^k(1-\tau_i^K)\frac{K_i}{Y_i}\tilde{u}_{i,t} + \frac{\varsigma V_i}{Y_i}\left(\tilde{f}_{i,t} - \tilde{g}_{i,t} + \tilde{H}_{i,t}\right)$$

18. Domestic Euler equation

$$\frac{u_{1,i,t}^i}{P_{i,t}} = (1+i_{i,t}) \sum_{s^{t+1}} \pi(s^{t+1}|s^t)\beta \frac{u_{1,i,t+1}^i}{P_{i,t+1}}$$
$$\beta \Delta i_{i,t} - \tilde{\pi}_{i,t+1} = \tilde{u}_{1,i,t}^i - \tilde{u}_{1,i,t+1}^i$$

19. Phillips curve

$$\theta_p\left(\tilde{\pi}_{i,t} + \widetilde{ToT}_{i,t}\right) = (1-\theta_p)(1-\theta_p\beta)\left[\widetilde{mc}_{i,t} - \widetilde{\left(\frac{p_{i,t}}{P_{i,t}}\right)}\right] + \theta_p\beta\left(\tilde{\pi}_{i,t+1} + \widetilde{ToT}_{i,t+1}\right)$$

20. Definition of Terms of Trade

$$\widetilde{ToT}_{i,t} = \widetilde{\left(\frac{p_{i,t}}{P_{i,t}}\right)} - \widetilde{\left(\frac{p_{i,t-1}}{P_{i,t-1}}\right)}$$

21. Monetary Policy

[7] Note that
$$Q_i\left(\tilde{\mathbb{N}}_{i,t} + \tilde{Q}_{i,t}\right) = \sum_{j=1}^{\mathcal{N}}\left(\frac{\mathbb{N}_j}{\mathbb{N}_i} y_j^i \left(\tilde{\mathbb{N}}_{j,t} + \tilde{y}_{j,t}^i\right)\right)$$

[8] Note that
$$a(u_i) = u_i r_i^k + 1 - \frac{F_i}{\beta} - \delta$$
and is zero if $u_i = 1$. Also: $V_{i,t} = \frac{f_{i,t} H_{i,t}}{g_{i,t}}$ and $a'(u_i) = r_i^k(1-\tau_i^K)$

- Floating exchange rate:

$$\Delta i_{i,t} = \phi_i \Delta i_{i,t-1} + (1-\phi_i)\left(\phi_Q \widetilde{Q}_{i,t} + \phi_\pi \tilde{\pi}_{i,t}\right)$$

- Fixed exchange rate:
  - Leader n:

$$\Delta i_{i,t} = \phi_i \Delta i_{i,t-1} + (1-\phi_i)\sum_{j\in CU} weight_j \left(\phi_Q \widetilde{Q}_{j,t} + \phi_\pi \tilde{\pi}_{j,t}\right),$$

    where $weight_j$ is the share of Q_j in the gdp of the currency union.
  - Follower j:

$$\widetilde{\Delta S}_{j,t} = \widetilde{\Delta S}_{i,t}$$

22. Definition of change in nominal exchange rate

$$\widetilde{\Delta S}_{i,t} = (\tilde{s}_{i,t} - \tilde{s}_{i,t-1}) - \tilde{\pi}_{i,t}$$

23. International Euler equation

- Complete markets

$$\tilde{u}_{1,i,t} = \tilde{s}_{i,t}$$

- Incomplete markets (Uncovered interest rate parity)

$$\begin{aligned}
0 &= \tilde{s}_{1,t} \\
\beta\Delta i_{i,t} - \tilde{\pi}_{i,t+1} + \tilde{s}_{i,t+1} - \tilde{s}_{i,t} &= \beta\Delta i_{1,t} - \tilde{\pi}_{1,t+1} + \tilde{s}_{1,t+1} - \tilde{s}_{1,t} + \iota\frac{B_1^*}{Y_1}\tilde{B}_{1,t}^*. \qquad \text{for} \quad n>1
\end{aligned}$$

24. Budget constraint (for incomplete market case)

$$\begin{aligned}
\frac{\Delta B_t^1}{Y_{1,t}} &= 0 \\
\mathbb{N}^i\left(\frac{1}{\beta}\Delta B_{t-1}^i - \Delta B_t^i\right) &= \mathbb{N}_i\left(Y_i\widetilde{Y}_{i,t} - Q_i\left(\widetilde{\frac{p_{i,t}}{P_{i,t}}} + \tilde{Q}_{i,t}\right)\right) \\
&- \left[\sum_{j\neq i} n_j^i \mathbb{N}^i w_j^h l_j^i\left(\widetilde{w}_{j,t}^h + \tilde{l}_{j,t}^i\right) - n_i^j \mathbb{N}^j w_i^h l_i^j\left(\widetilde{w}_{i,t}^h + \tilde{l}_{i,t}^j\right)\right] \\
&+ \left[\sum_{j\neq i} n_j^i \mathbb{N}^i c_j^i \tilde{c}_{j,t}^i - n_i^j \mathbb{N}^j c_i^j \tilde{c}_{i,t}^j\right] \qquad \text{for} \quad i>1
\end{aligned}$$

25. Location choice[9]

$$\frac{1}{\gamma}\left(\ln(n^i_{j,t})+1\right) = u(c^i_{j,t},l^i_{j,t}) - u(c^i_{i,t},l^i_{i,t}) + A^i_j - \left[\left(c^i_{j,t} - w^h_{j,t}l^i_{j,t} + tr^i_j\right)u^i_{1,j,t} - \left(c^i_{i,t} - w^h_{i,t}l^i_{i,t} + tr^i_i\right)u^i_{1,i,t}\right]$$
$$- \left(\frac{n^i_{j,t}}{n^i_{j,t-1}}\left(\Phi^i_{j,t}\right)' + \Phi^i_{j,t}\right)u^i_{1,j,t} + \beta\left(\frac{n^j_{i,t+1}}{n^j_{i,t}}\right)^2\left(\Phi^i_{j,t+1}\right)' u^i_{1,j,t+1} \qquad \text{for} \quad i \neq j$$
$$\frac{1}{\gamma u^i_{1,i}}\tilde{n}^i_{j,t} = w^h_j l^i_j \tilde{w}^h_{j,t} - w^h_i l^i_i \tilde{w}^h_{i,t} - \Phi''\left[(1+\beta)\tilde{n}^i_{j,t} - \tilde{n}^i_{j,t-1} - \beta\tilde{n}^i_{j,t+1}\right] \qquad \text{for} \quad i \neq j$$

$$\sum_j n^i_{j,t} = 1$$
$$\sum_j n^i_j \tilde{n}^i_{j,t} = 0$$

26. Definition of population

$$\mathbb{N}_{i,t} = \sum_j n^j_{i,t}\mathbb{N}^j$$
$$\tilde{\mathbb{N}}_{i,t} = \sum_j \frac{\mathbb{N}^j}{\mathbb{N}_i} n^j_i \tilde{n}^j_{i,t}$$

27. Number of job hunters

$$\mathbb{N}_{i,t}H_{i,t} = \mathbb{N}_{i,t-1}U_{i,t-1} + d\mathbb{N}_{i,t-1}L_{i,t-1} + \mathbb{N}_{i,t}l_{i,t} - \mathbb{N}_{i,t-1}l_{i,t-1}$$
$$\left[(1-d)ur_i + d\right]\tilde{H}_{i,t} = ur_i\tilde{U}_{i,t-1} + d\left(1-ur_i\right)\tilde{L}_{i,t-1} + \left(\tilde{l}_{i,t} - \tilde{l}_{i,t-1}\right) + (1-d)(1-ur_i)\left(\tilde{\mathbb{N}}_{i,t} - \tilde{\mathbb{N}}_{i,t-1}\right)$$

[9] Log-linearizing yields

$$\frac{1}{\gamma}\tilde{n}^i_{j,t} = \left(u^i_{1,j} - u^i_{1,j}\right)c^i_j\tilde{c}^i_{j,t} + \left(u^i_{2,j} - w^h_j u^i_{1,j}\right)l^i_j\tilde{l}^i_{j,t} - \left(c^i_j - w^h_j l^i_j - tr^i_j\right)u^i_{1,j}\tilde{u}^i_{1,j,t} + w^h_j l^i_j u^i_{1,j}\tilde{w}^h_{j,t}$$
$$- \left[\left(u^i_{1,i} - u^i_{1,i}\right)c^i_i\tilde{c}^i_{i,t} + \left(u^i_{2,i} - w^h_i u^i_{1,i}\right)l^i_i\tilde{l}^i_{i,t} - \left(c^i_i - w^h_i l^i_i - tr^i_i\right)u^i_{1,i}\tilde{u}^i_{1,i,t} + w^h_i l^i_i u^i_{1,i}\tilde{w}^h_{i,t}\right]$$
$$- u^i_{1,j}\Phi''\left[(1+\beta)\tilde{n}^i_{j,t} - \tilde{n}^i_{j,t-1} - \beta\tilde{n}^i_{j,t+1}\right]$$

The terms related to $\tilde{c}^i_{j,t}$ and $\tilde{c}^i_{i,t}$ drop out. Similarly, the terms related to $\tilde{l}^i_{j,t}$ and $\tilde{l}^i_{j,t}$ drop out because in steady state, $w^h_j = -\frac{u^i_{2,j}}{u^i_{1,j}}$ for all j. Given our assumption that governments set transfers so as to balance out labor income and consumption expenditures in steady state, the terms related to $\tilde{u}^i_{1,j,t}$ and $\tilde{u}^i_{1,i,t}$ drop out. Finally, notice that the Backus-Smith condition among household members in steady state is simply $u^i_{1,i} = u^i_{1,j}$ (since all real exchange rates are set to 1 in steady state).

28. Labor market clearing

$$U_{i,t} = l_{i,t} - L_{i,t}$$
$$ur_i\tilde{U}_{i,t} = \tilde{l}_{i,t} - (1 - ur_i)\tilde{L}_{i,t}$$

29. Law of motion for employed workers

$$\mathbb{N}_{i,t}L_{i,t} = \mathbb{N}_{i,t-1}L_{i,t-1}(1-d) + \mathbb{N}_{i,t}M_{i,t}$$
$$\tilde{L}_{i,t} = (1-d)\tilde{L}_{i,t-1} + d\tilde{M}_{i,t} - (1-d)\left(\tilde{\mathbb{N}}_{i,t} - \tilde{\mathbb{N}}_{i,t-1}\right)$$

30. Matching function

$$M_{i,t} = H_{i,t}\bar{m}\lambda_{i,t}^{1-\zeta}$$
$$\tilde{M}_{i,t} = \tilde{H}_{i,t} + (1-\zeta)\tilde{\lambda}_{i,t}$$

31. Job finding rate

$$f_{i,t} = \bar{m}\lambda_{i,t}^{1-\zeta}$$
$$\tilde{f}_{i,t} = (1-\zeta)\tilde{\lambda}_{i,t}$$

32. Job filling rate

$$g_{i,t} = \bar{m}\lambda_{i,t}^{-\zeta}$$
$$\tilde{g}_{i,t} = -\zeta\tilde{\lambda}_{i,t}$$

33. Value of a posted vacancy

$$\mathcal{V}_{i,t} = -\varsigma + g_{i,t}\mathcal{J}_{i,t} + (1-g_{i,t})\beta\mathbb{E}_t\mathcal{V}_{i,t+1}$$
$$\mathcal{V}_i\tilde{\mathcal{V}}_{i,t} = g_i(\mathcal{J}_i - \beta\mathcal{V}_i)\tilde{g}_{i,t} + g_i\mathcal{J}_i\tilde{\mathcal{J}}_{i,t} + (1-g_i)\beta\mathcal{V}_i\tilde{\mathcal{V}}_{i,t+1}$$
$$\Delta\mathcal{V}_{i,t} = g_i\mathcal{J}_i\tilde{g}_{i,t} + g_i\mathcal{J}_i\tilde{\mathcal{J}}_{i,t} + (1-g_i)\beta\Delta\mathcal{V}_{i,t+1}$$

34. Value of a filled vacancy

$$\mathcal{J}_{i,t} = w_{i,t}^f - w_{i,t} + (1-d)\beta\mathcal{J}_{i,t+1} + d\beta\mathbb{E}_t\mathcal{V}_{i,t+1}$$
$$\mathcal{J}_i\tilde{\mathcal{J}}_{i,t} = w_i^f\widetilde{w}_{i,t}^f - w_i\widetilde{w}_{i,t} + (1-d)\beta\mathcal{J}_i\tilde{\mathcal{J}}_{i,t+1} + d\beta\Delta\mathcal{V}_{i,t+1}$$

35. Vacancy adjustment cost

$$\mathcal{V}_{i,t} = \Upsilon_{i,t} + \frac{V_{i,t}}{V_{i,t-1}}\Upsilon'_{i,t} - \beta\frac{u^i_{1,i,t+1}}{u^i_{1,i,t}}\left(\frac{V_{i,t+1}}{V_{i,t}}\right)^2\Upsilon'_{i,t+1}$$
$$\frac{1}{\Upsilon''}\Delta\mathcal{V}_{i,t} = (1+\beta)\left(\tilde{\lambda}_{i,t} + \tilde{H}_{i,t}\right) - \left(\tilde{\lambda}_{i,t-1} + \tilde{H}_{i,t-1}\right) - \beta\left(\tilde{\lambda}_{i,t+1} + \tilde{H}_{i,t+1}\right)$$

36. Value of being employed

$$\mathcal{E}_{i,t} = w_{i,t} - w^h_{i,t} + (1-d)\,\beta\mathcal{E}_{i,t+1}$$
$$\mathcal{E}_i\tilde{\mathcal{E}}_{i,t} = w_i\widetilde{w}_{i,t} - w^h_i\widetilde{w}^h_{i,t} + (1-d)\,\beta\mathcal{E}_i\tilde{\mathcal{E}}_{i,t+1}$$

37. Free entry of employment agencies

$$f_{i,t}\mathcal{E}_{i,t} = (1-f_{i,t})\left(w^h_{i,t} - b_i\right)$$
$$f_i\mathcal{E}_i\tilde{\mathcal{E}}_{i,t} = (1-f_i)w^h_i\widetilde{w}^h_{i,t} - (w^h_i - b_i + \mathcal{E}_i)\tilde{f}_{i,t}$$

38. Wage determination

$$\theta^w w_{i,t} = \theta^w w_{i,t-1} + (1-\theta^w)\left[\varrho\mathcal{J}_{i,t} - (1-\varrho)\left(\mathcal{E}_{i,t} - \mathcal{U}_{i,t}\right)\right]$$
$$\theta^w w_i\widetilde{w}_{i,t} = \theta^w w_i\widetilde{w}_{i,t-1} + (1-\theta^w)\left[\varrho\mathcal{J}_i\tilde{\mathcal{J}}_{i,t} - (1-\varrho)\left(\mathcal{E}_i\tilde{\mathcal{E}}_{i,t} + w^h_i\widetilde{w}^h_{i,t}\right)\right]$$

39. Percentage point change in the unemployment rate

$$\Delta ur_{i,t} = ur_i\left(\tilde{U}_{i,t} - \tilde{l}_{i,t}\right).$$

2.4 Combined Log-Linearized Equilibrium Conditions

We combine a few equations to reduce the number of equations and variables in the system.

2.4.1 DSGE block

We remove the following equations

- Marginal utility of consumption (1) ($c^i_{j,t}$)
- Marginal rate of substitution (2) ($l^i_{j,t}$)
- Risk sharing among household members (3) ($u^i_{j,t}$, $j \neq i$)
- Labor supply (4) ($mrs^i_{j,t}$)

- Optimal capital utilization (10) ($u_{i,t}$)
- FOC wrt $y_{i,t}^{j}$ (13) ($y_{j,t}^{i}$)
- Definition of Terms of Trade (20) ($ToT_{i,t}$)
- Definition of change in nominal exchange rate (22) ($\Delta S_{i,t}$)

Definition of consumption (5) Combining the Risk sharing among household members (3)

$$\tilde{u}_{1,j,t}^{i} = \tilde{u}_{1,i,t}^{i} + \tilde{s}_{j,t} - \tilde{s}_{i,t}$$

with the Marginal utility of consumption (1)

$$\begin{aligned} \widetilde{u}_{1,j,t}^{i} &= -\frac{1}{\sigma}\tilde{c}_{j,t}^{i} \qquad \text{separable} \\ -\sigma\left(u_{1,j}^{i}\right)^{-\sigma}\tilde{u}_{1,j,t}^{i} &= c_{j}^{i}\tilde{c}_{j,t}^{i} - l_{j}^{i}w_{j}^{h}\tilde{l}_{j,t}^{i} \qquad \text{GHH} \end{aligned}$$

and (for GHH preferences) with the Marginal rate of substitution (2)

$$\widetilde{mrs}_{j,t}^{i} = \frac{1}{\eta}\tilde{l}_{j,t}^{i}$$

plus the labor supply condition, (4) to replace $\widetilde{mrs}_{j,t}^{i} = \tilde{w}_{j,t}^{h}$ gives

$$\begin{aligned} \tilde{u}_{1,i,t}^{i} + \tilde{s}_{j,t} - \tilde{s}_{i,t} &= -\frac{1}{\sigma}\tilde{c}_{j,t}^{i} \qquad \text{separable} \\ -\sigma\left(u_{1,j}^{i}\right)^{-\sigma}\left(\tilde{u}_{1,i,t}^{i} + \tilde{s}_{j,t} - \tilde{s}_{i,t}\right) &= c_{j}^{i}\tilde{c}_{j,t}^{i} - \eta l_{j}^{i}w_{j}^{h}\tilde{w}_{j,t}^{h} \qquad \text{GHH} \end{aligned}$$

Inserting this into the Definition of consumption (5):

$$\left(\tilde{C}_{i,t} + \tilde{\mathbb{N}}_{i,t}\right) = \sum_{j}\frac{\mathbb{N}^{j}}{\mathbb{N}_{i}}\frac{c_{i}^{j}}{C_{i}}n_{i}^{j}\left(\tilde{n}_{i,t}^{j} + \tilde{c}_{i,t}^{j}\right)$$

gives

$$\begin{aligned} \left(\tilde{C}_{i,t} + \tilde{\mathbb{N}}_{i,t}\right) &= \sum_{j}\frac{\mathbb{N}^{j}}{\mathbb{N}_{i}}\frac{c_{i}^{j}}{C_{i}}n_{i}^{j}\left(\tilde{n}_{i,t}^{j} - \sigma\left(\tilde{u}_{1,j,t}^{j} + \tilde{s}_{i,t} - \tilde{s}_{j,t}\right)\right) \qquad \text{separable} \\ \left(\tilde{C}_{i,t} + \tilde{\mathbb{N}}_{i,t}\right) &= \sum_{j}\frac{\mathbb{N}^{j}}{\mathbb{N}_{i}}\frac{c_{i}^{j}}{C_{i}}n_{i}^{j}\left(\tilde{n}_{i,t}^{j} - \sigma\frac{\left(u_{1,j}^{i}\right)^{-\sigma}}{c_{i}^{j}}\left(\tilde{u}_{1,j,t}^{j} + \tilde{s}_{i,t} - \tilde{s}_{j,t}\right) + \eta l_{i}^{j}\frac{w_{i}^{h}}{c_{i}^{j}}\tilde{w}_{i,t}^{h}\right) \qquad \text{GHH} \end{aligned}$$

Definition of the labor force (6) Combining the marginal rate of substitution (2)

$$\widetilde{mrs}^i_{j,t} = \frac{1}{\eta}\tilde{l}^i_{j,t} - \tilde{u}^i_{1,j,t} \qquad \text{separable}$$
$$\widetilde{mrs}^i_{j,t} = \frac{1}{\eta}\tilde{l}^i_{j,t} \qquad \text{GHH}$$

with the labor supply condition, (4) to replace $\widetilde{mrs}^i_{j,t} = \tilde{w}^h_{j,t}$ and (for separable preferences) the Risk sharing condition among household members (3) gives

$$\widetilde{w}^h_{j,t} = \frac{1}{\eta}\tilde{l}^i_{j,t} - \tilde{u}^i_{1,i,t} - \tilde{s}_{j,t} + \tilde{s}_{i,t} \qquad \text{separable}$$
$$\widetilde{w}^h_{j,t} = \frac{1}{\eta}\tilde{l}^i_{j,t} \qquad \text{GHH}$$

Inserting this into the Definition of the labor force (6)

$$\left(\tilde{l}_{i,t} + \tilde{\mathbb{N}}_{i,t}\right) = \sum_j \frac{\mathbb{N}^j}{\mathbb{N}_i}\frac{l^j_i}{l_i}n^j_i\left(\tilde{n}^j_{i,t} + \tilde{l}^j_{i,t}\right)$$

gives

$$\left(\tilde{l}_{i,t} + \tilde{\mathbb{N}}_{i,t}\right) = \sum_j \frac{\mathbb{N}^j}{\mathbb{N}_i}\frac{l^j_i}{l_i}n^j_i\left(\tilde{n}^j_{i,t} + \eta\left(\tilde{w}^h_{i,t} + \tilde{u}^j_{1,j,t} - \tilde{s}_{j,t} + \tilde{s}_{i,t}\right)\right) \qquad \text{separable}$$
$$\left(\tilde{l}_{i,t} + \tilde{\mathbb{N}}_{i,t}\right) = \sum_j \frac{\mathbb{N}^j}{\mathbb{N}_i}\frac{l^j_i}{l_i}n^j_i\left(\tilde{n}^j_{i,t} + \eta\tilde{w}^h_{i,t}\right) \qquad \text{GHH}$$

Optimal factor employment (11) Inserting the Optimal capital utilization (10)

$$\tilde{u}_{i,t} = \frac{r^k_i}{a''u_i}\tilde{r}^k_{i,t}$$

into the Optimal factor employment (11)

$$\tilde{r}^k_{i,t} - \widetilde{w}^f_{i,t} = \tilde{\mathbb{N}}_{i,t} + \tilde{L}_{i,t} - \tilde{u}_{i,t} - \tilde{K}_{i,t-1} - \tilde{\mathbb{N}}_{i,t-1}$$

gives

$$\tilde{r}^k_{i,t} - \widetilde{w}^f_{i,t} = \tilde{\mathbb{N}}_{i,t} + \tilde{L}_{i,t} - \frac{r^k_i}{a''u_i}\tilde{r}^k_{i,t} - \tilde{K}_{i,t-1} - \tilde{\mathbb{N}}_{i,t-1}.$$

Production of intermediate good (14) Inserting the Optimal capital utilization (10)

$$\tilde{u}_{i,t} = \frac{r^k_i}{a''u_i}\tilde{r}^k_{i,t}$$

into the Production of intermediate good (14)

$$\tilde{Q}_{i,t} = \tilde{Z}_{i,t} + \alpha\left(\tilde{u}_{i,t} + \tilde{\mathbb{N}}_{i,t-1} + \tilde{K}_{i,t-1} - \tilde{\mathbb{N}}_{i,t}\right) + (1-\alpha)\tilde{L}_{i,t}$$

gives

$$\tilde{Q}_{i,t} = \tilde{Z}_{i,t} + \alpha\left(\frac{r_i^k}{a'' u_i}\tilde{r}_{i,t}^k + \tilde{\mathbb{N}}_{i,t-1} + \tilde{K}_{i,t-1} - \tilde{\mathbb{N}}_{i,t}\right) + (1-\alpha)\tilde{L}_{i,t}.$$

Production of final good (15) Inserting the FOC wrt $y_{i,t}^j$ (13)

$$\tilde{y}_{i,t}^j = \tilde{Y}_{i,t} + \left(\varepsilon_t^j - \sum_k \bar{\omega}_i^k \varepsilon_t^k\right) - \psi_y\left(\widetilde{\left(\frac{p_{j,t}}{P_{j,t}}\right)} + \tilde{s}_{j,t} - \tilde{s}_{i,t}\right)$$

into the Production of final good (15)

$$\tilde{Y}_{i,t} = \sum_{j=1}^{\mathcal{N}} \bar{\omega}_i^j \left(\tilde{y}_{i,t}^j + \frac{1}{\psi_y - 1}\left(\varepsilon_t^j - \sum_k \bar{\omega}_i^k \varepsilon_t^k\right)\right)$$

gives

$$\psi_y \sum_{j=1}^{\mathcal{N}} \bar{\omega}_i^j \left(\widetilde{\left(\frac{p_{j,t}}{P_{j,t}}\right)} + \tilde{s}_{j,t} - \tilde{s}_{i,t}\right) = \sum_{j=1}^{\mathcal{N}} \bar{\omega}_i^j \left(1 + \frac{1}{\psi_y - 1}\right)\left(\varepsilon_t^j - \sum_k \bar{\omega}_i^k \varepsilon_t^k\right)$$

$$(\psi_y - 1)\sum_{j=1}^{\mathcal{N}} \bar{\omega}_i^j \left(\widetilde{\left(\frac{p_{j,t}}{P_{j,t}}\right)} + \tilde{s}_{j,t} - \tilde{s}_{i,t}\right) = \sum_{j=1}^{\mathcal{N}} \bar{\omega}_i^j \left(\varepsilon_t^j - \sum_k \bar{\omega}_i^k \varepsilon_t^k\right)$$

Market clearing for intermediate good (16) Inserting the FOC wrt $y_{i,t}^j$ (13)

$$\tilde{y}_{j,t}^i = \tilde{Y}_{j,t} + \left(\varepsilon_t^i - \sum_k \bar{\omega}_j^k \varepsilon_t^k\right) - \psi_y\left(\widetilde{\left(\frac{p_{i,t}}{P_{i,t}}\right)} + \tilde{s}_{i,t} - \tilde{s}_{j,t}\right)$$

into the Market clearing for intermediate good (16)

$$\frac{Q_i}{Y_i}\left(\tilde{\mathbb{N}}_{i,t} + \tilde{Q}_{i,t}\right) = \sum_{j=1}^{\mathcal{N}} \frac{\mathbb{N}_j Y_j}{\mathbb{N}_i Y_i}\bar{\omega}_i^j \left(\tilde{\mathbb{N}}_{j,t} + \tilde{y}_{j,t}^i\right)$$

gives

$$\frac{Q_i}{Y_i}\left(\tilde{\mathbb{N}}_{i,t} + \tilde{Q}_{i,t}\right) = \sum_{j=1}^{\mathcal{N}} \frac{\mathbb{N}_j Y_j}{\mathbb{N}_i Y_i}\bar{\omega}_i^j \left(\tilde{\mathbb{N}}_{j,t} + \tilde{Y}_{j,t} + \left(\varepsilon_t^i - \sum_k \bar{\omega}_j^k \varepsilon_t^k\right) - \psi_y\left(\widetilde{\left(\frac{p_{i,t}}{P_{i,t}}\right)} + \tilde{s}_{i,t} - \tilde{s}_{j,t}\right)\right)$$

Market clearing for final good (17) Inserting the relationship between $\tilde{f}_{i,t}$ and $\tilde{g}_{i,t}$,

$$\tilde{g}_{i,t} = -\frac{\zeta}{1-\zeta}\tilde{f}_{i,t},$$

and the Optimal capital utilization (10)

$$\tilde{u}_{i,t} = \frac{r_i^k}{a''u_i}\tilde{r}_{i,t}^k$$

into the Market clearing condition for final goods (17):

$$\tilde{Y}_{i,t} = \frac{C_i}{Y_i}\tilde{C}_{i,t} + \frac{X_i}{Y_i}\tilde{X}_{i,t} + \frac{G_i}{Y_i}\tilde{G}_{i,t} + r_i^k\frac{K_i}{Y_i}\tilde{u}_{i,t} + \frac{\varsigma V_i}{Y_i}\left(\tilde{f}_{i,t} - \tilde{g}_{i,t} + \tilde{H}_{i,t}\right)$$

gives

$$\tilde{Y}_{i,t} = \frac{C_i}{Y_i}\tilde{C}_{i,t} + \frac{X_i}{Y_i}\tilde{X}_{i,t} + \frac{G_i}{Y_i}\tilde{G}_{i,t} + \frac{K_i}{Y_i}\frac{\left(r_i^k\right)^2}{a''u_i}\tilde{r}_{i,t}^k + \frac{\varsigma V_i}{Y_i}\left(\frac{1}{1-\zeta}\tilde{f}_{i,t} + \tilde{H}_{i,t}\right).$$

Phillips curve (19) Inserting the Real marginal costs (12)

$$\widetilde{mc}_{i,t} = -\tilde{Z}_{i,t} + \alpha\tilde{r}_{i,t}^k + (1-\alpha)\widetilde{w}_{i,t}^f$$

and the Definition of Terms of Trade (20)

$$\widetilde{ToT}_{i,t} = \widetilde{\left(\frac{p_{i,t}}{P_{i,t}}\right)} - \widetilde{\left(\frac{p_{i,t-1}}{P_{i,t-1}}\right)}$$

into the Phillips curve (19)

$$\theta_p\left(\tilde{\pi}_{i,t} + \widetilde{ToT}_{i,t}\right) = (1-\theta_p)(1-\theta_p\beta)\left[\widetilde{mc}_{i,t} - \widetilde{\left(\frac{p_{i,t}}{P_{i,t}}\right)}\right] + \theta_p\beta\left(\tilde{\pi}_{i,t+1} + \widetilde{ToT}_{i,t+1}\right)$$

gives

$$\theta_p\left(\tilde{\pi}_{i,t} - \widetilde{\left(\frac{p_{i,t-1}}{P_{i,t-1}}\right)}\right) = (1-\theta_p)(1-\theta_p\beta)\left(-\tilde{Z}_{i,t} + \alpha\tilde{r}_{i,t}^k + (1-\alpha)\widetilde{w}_{i,t}^f\right) - (1-\theta_p^2\beta)\widetilde{\left(\frac{p_{i,t}}{P_{i,t}}\right)} + \theta_p\beta\left(\tilde{\pi}_{i,t+1} + \widetilde{\left(\frac{p_{i,t+1}}{P_{i,t+1}}\right)}\right).$$

Monetary policy (21) Inserting the Definition of change in nominal exchange rate (22)

$$\widetilde{\Delta S}_{i,t} = (\tilde{s}_{i,t} - \tilde{s}_{i,t-1}) - \tilde{\pi}_{i,t}$$

into the monetary policy rule for followers under fixed exchange rates (21)

$$\widetilde{\Delta S}_{j,t} = \widetilde{\Delta S}_{i,t}$$

gives

$$(\tilde{s}_{j,t} - \tilde{s}_{j,t-1}) - \tilde{\pi}_{j,t} = (\tilde{s}_{i,t} - \tilde{s}_{i,t-1}) - \tilde{\pi}_{i,t}$$

Budget constraint (24) Combining the marginal rate of substitution (2) with the labor supply condition, (4) to replace $\widetilde{mrs}^i_{j,t} = \tilde{w}^h_{j,t}$ and (for separable preferences) the Risk sharing condition among household members (3) gives

$$\begin{aligned}
\tilde{l}^i_{j,t} &= \eta \left(\widetilde{w}^h_{j,t} + \tilde{u}^i_{1,i,t} + \tilde{s}_{j,t} - \tilde{s}_{i,t} \right) \qquad \text{separable} \\
\tilde{l}^i_{j,t} &= \eta \widetilde{w}^h_{j,t} \qquad \text{GHH.}
\end{aligned}$$

Similarly, combining the Risk sharing among household members (3) with the Marginal utility of consumption (1) and (for GHH preferences) with the Marginal rate of substitution (2) plus the labor supply condition, (4) to replace $\widetilde{mrs}^i_{j,t} = \tilde{w}^h_{j,t}$ gives

$$\begin{aligned}
c^i_j \tilde{c}^i_{j,t} &= \sigma c^i_j \left(-\tilde{u}^i_{1,i,t} - \tilde{s}_{j,t} + \tilde{s}_{i,t} \right) \qquad \text{separable} \\
c^i_j \tilde{c}^i_{j,t} &= -\sigma \left(u^i_{1,j} \right)^{-\sigma} \left(\tilde{u}^i_{1,i,t} + \tilde{s}_{j,t} - \tilde{s}_{i,t} \right) + \eta l^i_j w^h_j \widetilde{w}^h_{j,t} \qquad \text{GHH.}
\end{aligned}$$

Inserting this into the Budget constraint (24) (for $i > 1$)

$$\begin{aligned}
\mathbb{N}^i \left(\frac{1}{\beta} \Delta B^i_{t-1} - \Delta B^i_t \right) = \mathbb{N}_i &\left(Y_i \widetilde{Y}_{i,t} - Q_i \left(\frac{\widetilde{p_{i,t}}}{P_{i,t}} + \tilde{Q}_{i,t} \right) \right) \\
&- \left[\sum_{j \neq i} n^i_j \mathbb{N}^i w^h_j l^i_j \left(\widetilde{w}^h_{j,t} + \tilde{l}^i_{j,t} \right) - n^j_i \mathbb{N}^j w^h_i l^j_i \left(\widetilde{w}^h_{i,t} + \tilde{l}^j_{i,t} \right) \right] \\
&+ \left[\sum_{j \neq i} n^i_j \mathbb{N}^i c^i_j \tilde{c}^i_{j,t} - n^j_i \mathbb{N}^j c^j_i \tilde{c}^j_{i,t} \right]
\end{aligned}$$

gives for separable preferences

$$
\begin{aligned}
\mathbb{N}^i\left(\frac{1}{\beta}\Delta B_{t-1}^i-\Delta B_t^i\right)=\mathbb{N}_i&\left(Y_i\widetilde{Y}_{i,t}-Q_i\left(\widetilde{\frac{p_{i,t}}{P_{i,t}}}+\tilde{Q}_{i,t}\right)\right)\\
&-\left[\sum_{j\neq i}n_j^i\mathbb{N}^iw_j^hl_j^i\left(\widetilde{w}_{j,t}^h(1+\eta)+\eta\left(\tilde{u}_{1,i,t}^i+\tilde{s}_{j,t}-\tilde{s}_{i,t}\right)\right)\right]\\
&+\left[\sum_{j\neq i}n_i^j\mathbb{N}^jw_i^hl_i^j\left(\widetilde{w}_{i,t}^h(1+\eta)+\eta\left(\tilde{u}_{1,j,t}^j+\tilde{s}_{i,t}-\tilde{s}_{j,t}\right)\right)\right]\\
&+\left[\sum_{j\neq i}n_j^i\mathbb{N}^i\sigma c_j^i\left(-\tilde{u}_{1,i,t}^i-\tilde{s}_{j,t}+\tilde{s}_{i,t}\right)-n_i^j\mathbb{N}^j\sigma c_i^j\left(-\tilde{u}_{1,j,t}^j-\tilde{s}_{i,t}+\tilde{s}_{j,t}\right)\right]
\end{aligned}
$$

and for GHH preferences

$$
\begin{aligned}
\mathbb{N}^i\left(\frac{1}{\beta}\Delta B_{t-1}^i-\Delta B_t^i\right)=\mathbb{N}_i&\left(Y_i\widetilde{Y}_{i,t}-Q_i\left(\widetilde{\frac{p_{i,t}}{P_{i,t}}}+\tilde{Q}_{i,t}\right)\right)\\
&-\left[\sum_{j\neq i}n_j^i\mathbb{N}^iw_j^hl_j^i(1+\eta)\widetilde{w}_{j,t}^h-n_i^j\mathbb{N}^jw_i^hl_i^j(1+\eta)\widetilde{w}_{i,t}^h\right]\\
&+\left[\sum_{j\neq i}n_j^i\mathbb{N}^i\left(\sigma\left(u_{1,j}^i\right)^{-\sigma}\left(-\tilde{u}_{1,i,t}^i-\tilde{s}_{j,t}+\tilde{s}_{i,t}\right)+\eta l_j^iw_j^h\widetilde{w}_{j,t}^h\right)\right]\\
&-\left[\sum_{j\neq i}n_i^j\mathbb{N}^j\left(\sigma\left(u_{1,i}^j\right)^{-\sigma}\left(-\tilde{u}_{1,j,t}^j-\tilde{s}_{i,t}+\tilde{s}_{j,t}\right)+\eta l_i^jw_i^h\widetilde{w}_{i,t}^h\right)\right].
\end{aligned}
$$

2.4.2 Search block

We remove the following equations

- Labor market clearing (28) ($U_{i,t}$)
- Matching function (30) ($M_{i,t}$)
- Job finding rate (31) ($\lambda_{i,t}$)
- Job filling rate (32) ($g_{i,t}$)

Number of job hunters (27) Inserting the Labor market clearing condition (28)

$$ur_i\tilde{U}_{i,t}=\tilde{l}_{i,t}-(1-ur_i)\tilde{L}_{i,t}$$

into the Number of job hunters (27):

$$[(1-d)ur_i + d]\,\tilde{H}_{i,t} = ur_i\tilde{U}_{i,t-1} + d\,(1-ur_i)\,\tilde{L}_{i,t-1} + \left(\tilde{l}_{i,t} - \tilde{l}_{i,t-1}\right) + (1-d)(1-ur_i)\,(\mathbb{N}_{i,t} - \mathbb{N}_{i,t-1})$$

gives

$$[(1-d)ur_i + d]\,\tilde{H}_{i,t} = (d-1)\,(1-ur_i)\,\tilde{L}_{i,t-1} + \tilde{l}_{i,t} + (1-d)(1-ur_i)\,(\mathbb{N}_{i,t} - \mathbb{N}_{i,t-1})\,.$$

Law of motion for employed workers (29) Inserting the Matching function (30)

$$\tilde{M}_{i,t} = \tilde{H}_{i,t} + (1-\zeta)\tilde{\lambda}_{i,t}$$

and the Job finding rate (31)

$$\tilde{f}_{i,t} = (1-\zeta)\tilde{\lambda}_{i,t}$$

into the Law of motion for employed workers (29)

$$\tilde{L}_{i,t} = (1-d)\tilde{L}_{i,t-1} + d\tilde{M}_{i,t} - (1-d)\left(\tilde{\mathbb{N}}_{i,t} - \tilde{\mathbb{N}}_{i,t-1}\right)$$

gives

$$\tilde{L}_{i,t} = (1-d)\tilde{L}_{i,t-1} + d\left(\tilde{H}_{i,t} + \tilde{f}_{i,t}\right) - (1-d)\left(\tilde{\mathbb{N}}_{i,t} - \tilde{\mathbb{N}}_{i,t-1}\right).$$

Value of a posted vacancy (33) Combining the Job filling rate (32)

$$\tilde{g}_{i,t} = -\zeta\tilde{\lambda}_{i,t}$$

with the Job finding rate (31)

$$\tilde{f}_{i,t} = (1-\zeta)\tilde{\lambda}_{i,t}$$

gives

$$\tilde{g}_{i,t} = -\frac{\zeta}{1-\zeta}\tilde{f}_{i,t}$$

Inserting this into the Value of a posted vacancy (33)

$$\Delta\mathcal{V}_{i,t} = g_i\mathcal{J}_i\tilde{g}_{i,t} + g_i\mathcal{J}_i\tilde{\mathcal{J}}_{i,t} + (1-g_i)\beta\Delta\mathcal{V}_{i,t+1}$$

yields

$$\Delta\mathcal{V}_{i,t} = -g_i\mathcal{J}_i\frac{\zeta}{1-\zeta}\tilde{f}_{i,t} + g_i\mathcal{J}_i\tilde{\mathcal{J}}_{i,t} + (1-g_i)\beta\Delta\mathcal{V}_{i,t+1}.$$

Vacancy adjustment cost (35) Inserting the Job finding rate (31)

$$\tilde{f}_{i,t} = (1-\zeta)\tilde{\lambda}_{i,t}$$

into the Vacancy adjustment cost (35)

$$\frac{1}{\Upsilon''}\Delta\mathcal{V}_t = (1+\beta)\left(\tilde{\lambda}_t + \tilde{H}_t\right) - \left(\tilde{\lambda}_{t-1} + \tilde{H}_{t-1}\right) - \beta\left(\tilde{\lambda}_{t+1} + \tilde{H}_{t+1}\right)$$

gives

$$\frac{1}{\Upsilon''}\Delta\mathcal{V}_t = (1+\beta)\left(\frac{1}{1-\zeta}\tilde{f}_t + \tilde{H}_t\right) - \left(\frac{1}{1-\zeta}\tilde{f}_{t-1} + \tilde{H}_{t-1}\right) - \beta\left(\frac{1}{1-\zeta}\tilde{f}_{t+1} + \tilde{H}_{t+1}\right)$$

Definition of unemployment rate (39) Inserting the Labor market clearing condition (28)

$$ur_i\tilde{U}_{i,t} = \tilde{l}_{i,t} - (1-ur_i)\tilde{L}_{i,t}$$

into the Percentage point change in the unemployment rate (39)

$$\Delta ur_{i,t} = ur_i\left(\tilde{U}_{i,t} - \tilde{l}_{i,t}\right)$$

gives

$$\Delta ur_{i,t} = (1-ur_i)\left(\tilde{l}_{i,t} - \tilde{L}_{i,t}\right)$$