Appendix B: Model and Supporting Evidence

In this appendix, we first present evidence in support of our modelling assumptions in Section IV. We then present formal proofs for the propositions discussed in Section IV and present similar analyses to those that appear in Section V.C, but with a different measure of information updating.

I. Motivating the Assumptions

In this subsection, we present evidence to support our modelling assumptions that students perceive the benefits of earning higher grades in categories and that they do not discriminate between grades that are equal to a letter grade of C or below.

Categorical Thinking

We first provide evidence that students think about the benefits of earning higher grades in discrete categories. Using survey data from the fifth year of experiment, panel (a) of Figure A.6 shows that nearly 40 percent of all students expect to earn an economics grade that is an exact multiple of ten—a far larger fraction than for expected grades at any other (integer) distance from the closest multiple of ten. At UofT, grade multiples of ten always indicate a change of letter grades, suggesting that students are bunching their grade expectations around clear letter grade cutoffs. Panel (b) of Figure A.6 presents direct evidence that students approach their studies by thinking about grade categories, showing the distribution of student test preparation strategies. Only 30 percent of students report studying until they completely understand the material, while the remaining 70 percent report studying only until they feel confident that they will earn a specific percentage grade that is a multiple of ten. Students' tendencies to think about their performance in grade categories is perhaps not surprising, given that most institutions (UofT included) produce transcripts that report letter grade performance (or GPA categories) for each course—measures that do not vary continuously with students' underlying percentage grades and only change when these grades cross specified thresholds.

Grouping All Grades Up To And Below a C Into One Category

Panel (b) of Figure A.6 shows that only about 9 percent of students approach preparing for a test by studying enough to earn only a C or below; 5 percent of students study just enough to earn a C (60 percent average grade) and 4 percent study just enough to pass. Panel (c) shows that only 1.5 percent of students expect to earn a grade of C or below across all their courses at the start of the semester, while Panel (d) shows the full distribution of expected percentage grades in economics, revealing that a very small mass of students expect to earn a C or less (60 percent or below). In summary, it appears that very few students expect to earn a grade below a B, and even among those who do, most do not expect to earn less than a C.

II. Proofs

In this section, we present formal proofs of the propositions made in Section IV in the main text. To begin, recall that the optimal study choice of student i in period t is given by equation (7) in the main text:

$$s_{it}^{*} = \begin{cases} s_{it}^{A} \text{ if } \theta_{it}^{A} - \theta_{it}^{B} \geq c\left(s_{it}^{A}\right) - c\left(s_{it}^{B}\right) \\ s_{it}^{B} \text{ if } \theta_{it}^{A} - \theta_{it}^{B} < c\left(s_{it}^{A}\right) - c\left(s_{it}^{B}\right) \end{cases},$$

where $s_{it}^{j} = \frac{y^{j} - \hat{\alpha}_{it}}{\hat{\beta}_{it}}$ for $j = A$ and B. We first establish that the RHS of (7) is decreasing in both $\hat{\alpha}_{it}$ and $\hat{\beta}_{it}$.

Lemma 1: Define $k\left(\hat{\alpha}_{it}, \hat{\beta}_{it}\right) = c\left(s_{it}^{A}\right) - c\left(s_{it}^{B}\right)$. $k\left(\hat{\alpha}_{it}, \hat{\beta}_{it}\right)$ is decreasing in both $\hat{\alpha}_{it}$ and $\hat{\beta}_{it}$.

Proof. Taking the partial derivative of $k\left(\hat{\alpha}_{it}, \hat{\beta}_{it}\right)$ with respect to each object and noting that $c(\cdot)$ is strictly increasing and convex gives the desired result.

We now present a proof for Proposition 1 in the main text, establishing how the behavior of students who are originally aiming for an A changes as they learn new information.

Proposition 1: *Suppose student i is originally studying enough to expect to earn a letter grade of A. Hold fixed the difference between the perceived benefit of earning an A and the benefit of earning a B, $\theta_{it}^{A} - \theta_{it}^{B}$. If student i receives a positive update about her academic ability (α_i) or return to studying (β_i), she continues aiming for an A but with less study effort. If she receives a small negative update, she continues aiming for an A but with more study effort; if she receives an intermediate negative update, she lowers her expected grade to a B but decreases or does not change study effort; if she receives a large negative update, she lowers her expected grade to a B and increases study effort.*

Proof. Suppose student i is studying enough to expect to earn an A at time 0 such that $\theta_{it}^{A} - \theta_{it}^{B} \geq k\left(\hat{\alpha}_{i0}, \hat{\beta}_{i0}\right)$ in equation (7).

Case 1: Suppose student i receives a positive information shock, such that $\hat{\alpha}_{i1} > \hat{\alpha}_{i0}$ or $\hat{\beta}_{i1} > \hat{\beta}_{i0}$. Because $k\left(\hat{\alpha}_{i0}, \hat{\beta}_{i0}\right)$ is decreasing in both objects, the RHS of (7) falls, ensuring the inequality remains satisfied. The student responds by continuing to study enough to expect an *A* but reduces study time, such that $s_{i1}^{*} = s_{i1}^{A} < s_{i0}^{*} = s_{i0}^{A}$.

Case 2: Suppose student i receives a negative information shock, such that $\hat{\alpha}_{i1} < \hat{\alpha}_{i0}$ or $\hat{\beta}_{i1} < \hat{\beta}_{i0}$. Because $k\left(\hat{\alpha}_{i0}, \hat{\beta}_{i0}\right)$ is decreasing in both objects, the RHS of (7) increases. For the remainder of the proof, we consider a decrease in $\hat{\alpha}_i$, such that $\hat{\alpha}_{i1} < \hat{\alpha}_{i0}$, assuming that $\hat{\beta}_{i1} = \hat{\beta}_{i0}$. (Following analogous steps would establish the results when $\hat{\beta}_{i1} < \hat{\beta}_{i0}$ and $\hat{\alpha}_{i1} = \hat{\alpha}_{i0}$.) Let $\hat{\alpha}_i^*(\hat{\beta}_{it}, \theta_{it}^A, \theta_{it}^B)$ denote the value of $\hat{\alpha}_i$ that, for a given return to studying and benefits to grades, ensures that equation (7) is satisfied as an equality – that is, student i is indifferent between studying enough to earn an A or B (in which case we assume she aims for an *A*). Let the student i's new belief over her academic ability be $\hat{\alpha}_{i1} = \hat{\alpha}_{i0} - \Delta\hat{\alpha}_i$ for some $\Delta\hat{\alpha}_i > 0$.

Case 2(i): Suppose the change in $\hat{\alpha}_i$ is relatively small, such that $\hat{\alpha}_{i1} = \hat{\alpha}_{i0} - \Delta\hat{\alpha}_i > \hat{\alpha}_i^*(\hat{\beta}_{it}, \theta_{it}^A, \theta_{it}^B)$. Then it is still that case that $\theta_{it}^A - \theta_{it}^B > k(\hat{\alpha}_{i1}, \hat{\beta}_{i1})$. Student i continues aiming for an *A* but increases study effort, such that $s_{i1}^* = s_{i1}^A > s_{i0}^* = s_{i0}^A$.

Case 2(ii): Suppose the change in $\hat{\alpha}_i$ is such that $\hat{\alpha}_{i1} = \hat{\alpha}_{i0} - \Delta\hat{\alpha}_i < \hat{\alpha}_i^*(\hat{\beta}_{it}, \theta_{it}^A, \theta_{it}^B)$ but that the downward revision $\Delta\hat{\alpha}_i$ is not too big, such that $\Delta\hat{\alpha}_i \leq y^A - y^B$. In this case, because $\theta_{it}^A - \theta_{it}^B < k(\hat{\alpha}_{i1}, \hat{\beta}_{i1})$, student i switches to aiming for a *B* but either reduces or does not change study time. The change in study time is given by $s_{i1}^* - s_{i0}^* = \frac{y^B - y^A - (\hat{\alpha}_{i1} - \hat{\alpha}_{i0})}{\hat{\beta}_{i0}}$, which is negative when $\Delta\hat{\alpha}_i = \hat{\alpha}_{i0} - \hat{\alpha}_{i1} < y^A - y^B$ and zero when $\Delta\hat{\alpha}_i = \hat{\alpha}_{i0} - \hat{\alpha}_{i1} = y^A - y^B$.

Case 2(iii): Suppose the change in $\hat{\alpha}_i$ is such that $\hat{\alpha}_{i1} = \hat{\alpha}_{i0} - \Delta\hat{\alpha}_i < \hat{\alpha}_i^*(\hat{\beta}_{it}, \theta_{it}^A, \theta_{it}^B)$ and that the downward revision $\Delta\hat{\alpha}_i$ is large, such that $\Delta\hat{\alpha}_i > y^A - y^B$. In this case, because $\theta_{it}^A - \theta_{it}^B < k(\hat{\alpha}_{i1}, \hat{\beta}_{i1})$, student i switches to aiming for a *B* but increases study time. The change in study time is given $s_{i1}^* - s_{i0}^* = \frac{y^B - y^A - (\hat{\alpha}_{i1} - \hat{\alpha}_{i0})}{\hat{\beta}_{i0}}$, which is positive because the downward revision to beliefs about academic ability is sufficiently large, such that $\Delta\hat{\alpha}_i = \hat{\alpha}_{i0} - \hat{\alpha}_{i1} > y^A - y^B$.

We now present a proof for Proposition 2 in the main text, establishing how the behavior of students who are originally aiming for a B changes as they learn new information.

Proposition 2: *Suppose student i is originally studying enough to expect to earn a letter grade of B. Hold fixed the difference between the perceived benefit of earning an A and the benefit of earning a B, $\theta_{it}^A - \theta_{it}^B$. If student i receives a negative update about her academic ability (α_i) or return to studying (β_i), she continues aiming for a B but with more study effort. If she receives a small positive update, she continues aiming for a B but with less study effort; if she receives an intermediate positive update, she increases her expected grade to an A and increases or does not change study effort; if she receives a large positive update, she raises her expected grade to an A but decreases study effort.*

Proof. Suppose student i is studying enough only to expect to earn a B at time 0 such that $\theta_i^A - \theta_i^B < k(\hat{\alpha}_{i0}, \hat{\beta}_{i0})$ in equation (7).

Case 1: Suppose student i receives a negative information shock, such that $\hat{\alpha}_{i1} < \hat{\alpha}_{i0}$ or $\hat{\beta}_{i1} < \hat{\beta}_{i0}$. Because $k(\hat{\alpha}_{i0}, \hat{\beta}_{i0})$ is decreasing in both objects, the RHS of (7) increases, ensuring the inequality remains satisfied. The student responds by continuing to study enough to expect to earn only a *B* but increases study time, such that $s_{i1}^* = s_{i1}^B > s_{i0}^* = s_{i0}^B$.

Case 2: Suppose student i receives a positive information shock, such that $\hat{\alpha}_{i1} > \hat{\alpha}_{i0}$ or $\hat{\beta}_{i1} > \hat{\beta}_{i0}$. Because $k(\hat{\alpha}_{i0}, \hat{\beta}_{i0})$ is decreasing in both objects, the RHS of (7) decreases. For the remainder of the proof, we consider an increase in $\hat{\alpha}_i$, such that $\hat{\alpha}_{i1} > \hat{\alpha}_{i0}$, assuming that $\hat{\beta}_{i1} = \hat{\beta}_{i0}$. (Following analogous steps would establish the results when $\hat{\beta}_{i1} > \hat{\beta}_{i0}$ and $\hat{\alpha}_{i1} = \hat{\alpha}_{i0}$.) As above, let $\hat{\alpha}_i^*(\hat{\beta}_{it}, \theta_{it}^A, \theta_{it}^B)$ denote the value of $\hat{\alpha}_i$ that, for a given return to studying and benefits to grades, ensures that equation (7) is satisfied as an equality and let the student i's new belief over their academic ability be $\hat{\alpha}_{i1} = \hat{\alpha}_{i0} + \Delta\hat{\alpha}_i$ for some $\Delta\hat{\alpha}_i > 0$.

Case 2(i): Suppose the change in $\hat{\alpha}_i$ is relatively small, such that $\hat{\alpha}_{i1} = \hat{\alpha}_{i0} + \Delta\hat{\alpha}_i < \hat{\alpha}_i^*(\hat{\beta}_{it}, \theta_i^A, \theta_i^B)$. Then it is still that case that $\theta_{it}^A - \theta_{it}^B < k(\hat{\alpha}_{i1}, \hat{\beta}_{i1})$. Student i continues aiming for a *B* but decreases study effort, such that $s_{i1}^* = s_{i1}^B < s_{i0}^* = s_{i0}^B$.

Case 2(ii): Suppose the change in $\hat{\alpha}_i$ is such that $\hat{\alpha}_{i1} = \hat{\alpha}_{i0} + \Delta\hat{\alpha}_i > \hat{\alpha}_i^*(\hat{\beta}_{it}, \theta_{it}^A, \theta_{it}^B)$ but that the upward revision $\Delta\hat{\alpha}_i$ is not too big, such that $\Delta\hat{\alpha}_i \leq y^A - y^B$. In this case, because $\theta_{it}^A - \theta_{it}^B > k(\hat{\alpha}_{i1}, \hat{\beta}_{i1})$, student i switches to aiming for an *A* and either increases or does not change study time. The change in study time is given by $s_{i1}^* - s_{i0}^* = \frac{y^A - y^B - (\hat{\alpha}_{i1} - \hat{\alpha}_{i0})}{\hat{\beta}_{i0}}$, which is positve when $\Delta\hat{\alpha}_i = \hat{\alpha}_{i0} - \hat{\alpha}_{i1} < y^A - y^B$ and zero when $\Delta\hat{\alpha}_i = \hat{\alpha}_{i0} - \hat{\alpha}_{i1} = y^A - y^B$.

Case 2(iii): Suppose the change in $\hat{\alpha}_i$ is such that $\hat{\alpha}_{i1} = \hat{\alpha}_{i0} + \Delta\hat{\alpha}_i > \hat{\alpha}_i^*(\hat{\beta}_{it}, \theta_{it}^A, \theta_{it}^B)$ and that the upward revision $\Delta\hat{\alpha}_i$ is large, such that $\Delta\hat{\alpha}_i > y^A - y^B$. In this case, because $\theta_{it}^A - \theta_{it}^B > k(\hat{\alpha}_{i1}, \hat{\beta}_{i1})$, student i switches to aiming for a *A* but decreases study time. The change in study time is given $s_{i1}^* - s_{i0}^* = \frac{y^A - y^B - (\hat{\alpha}_{i1} - \hat{\alpha}_{i0})}{\hat{\beta}_{i0}}$, which is negative because the upward revision to beliefs about academic ability is sufficiently large, such that $\Delta\hat{\alpha}_i = \hat{\alpha}_{i0} - \hat{\alpha}_{i1} > y^A - y^B$.

Proposition 3: *Holding* $\hat{\alpha}_i$ and $\hat{\beta}_i$ *fixed, the maximum amount of time a student is willing to study for an A is increasing in the difference between the perceived benefit of earning an A and the perceived benefit of earning a B,* $\theta_{it}^A - \theta_{it}^B$.

Proof. As above, let $\hat{\alpha}_i^*(\hat{\beta}_{it}, \theta_{it}^A, \theta_{it}^B)$ denote the value of $\hat{\alpha}_i$ that, for a given return to studying and benefits to grades, ensures that equation (7) is satisfied as an equality. Because the RHS of (7) $k(\hat{\alpha}_{it}, \hat{\beta}_{it})$ is decreasing in $\hat{\alpha}_{it}$, for a given return to studying $(\hat{\beta}_{it})$ and values of $\theta_{it}^A - \theta_{it}^B$, the maximum amount of study time that student i is willing to put forward to earn an A occurs at the level of $\hat{\alpha}_{it}$ when $\hat{\alpha}_{it} = \hat{\alpha}_{it}^*(\hat{\beta}_{it}, \theta_{it}^A, \theta_{it}^B)$ and is given by $s_{it}^A(\hat{\alpha}_{it}^*) = \frac{y^A - \hat{\alpha}_{it}^*}{\hat{\beta}_{it}}$. Because LHS of equation (7) is increasing in $\theta_{it}^A - \theta_{it}^B$, it follows that $\hat{\alpha}_{it}^*(\hat{\beta}_{it}, \theta_{it}^A, \theta_{it}^B)$ is decreasing in $\theta_{it}^A - \theta_{it}^B$, as higher values of $\theta_{it}^A - \theta_{it}^B$ allow equation (7) to hold as an equality for smaller values of $\hat{\alpha}_{it}$ (when the RHS, $k(\hat{\alpha}_{it}, \hat{\beta}_{it})$, is larger). Because the maximum amount one is willing to study for an A, $s_{it}^A(\hat{\alpha}_{it}^*)$, is decreasing in $\hat{\alpha}_{it}^*$, and $\hat{\alpha}_{it}^*$ is decreasing in $\theta_{it}^A - \theta_{it}^B$, it follows that $s_{it}^A(\hat{\alpha}_{it}^*)$ is increasing in $\theta_{it}^A - \theta_{it}^B$.

III. An Alternative Way to Measure Information Updating

As a robustness check of our main results for the relevance of information updating in Section V.C, we construct an alternative measure of information updating by combining students' initial expected weekly study time in economics with their two study gradients to determine the change in each student's implied expected economics grade ($\Delta\mathbb{E}(y_i|s_{i0})$) that arises from allowing only their study gradient to change while holding study hours fixed at the initial expectation (s_{i0}):

$$\Delta\mathbb{E}(y_i|s_{i0}) = (\hat{\alpha}_{i1} - \hat{\alpha}_{i0}) + (\hat{\beta}_{i1} - \hat{\beta}_{i0})s_{i0}. \tag{12}$$

When the value determined by equation (12) is positive, students received a positive overall information update, as the revisions to their gradients imply that they should expect to earn a higher grade for the same amount of study time. In contrast, a negative value implies that students received a negative update during the semester and should now expect to earn a lower grade for a given amount of study time.

We repeat the analyses from the main text done in Figure 6 and Table 12 with this alternative measure of information updating in Figure A.7 and Table A.7, respectively. In panel (a) of Figure A.7, we plot the difference between actual and expected weekly study time for economics against the measure of information updating. There is a clear negative relationship, indicating that students revise their study time down when the change in their study gradients implies that they should expect a higher grade in economics for the same amount of studying. Conversely, they study more hours than initially expected when their study gradients imply that they should expect a lower grade. Note, an asymmetric response again prevails, as students who learn they should expect a lower grade marginally increase their study hours while those who learn they should expect a higher grade substantially revise their study hours down.

Columns (1) and (2) in Table A.7 report the estimated slope coefficient corresponding to the linear fit in panel (a) of Figure A.7, with and without additional control variables, respectively. The point estimates are similar across both specifications and are economically significant, implying that when students should expect to earn a 10 percentage-point lower grade (for the same amount of study)—a one standard-deviation change in the information update measure—they study 0.8 hours (16 percent of a standard deviation) more per week for their economics course than originally expected.

Panel (b) of Figure A.7 shows a similar relationship when the dependent variable is the difference between actual and expected study time across all courses: a one standard deviation lower implied economics grade is associated with an increase in weekly study time across all courses of 1.3 hours per week. Columns (3) and (4) in Table A.7 show that the point estimates underlying these relationships remain qualitatively similar and statistically significant in specifications that include additional control variables.

In panel (c) of Figure A.7, we show the relationship between students' expected percentage grade revisions in economics and information updating. The relationship is again asymmetric, as students who should expect to earn a higher grade for the same amount of study time barely revise their grade expectations, while students who should expect to earn a lower grade revise their grade expectations down substantially. Overall, a clear positive relationship prevails between information updating and grade expectation revisions. Columns (5) and (6) in Table A.7 present the point estimates of the slope from the underlying linear fit, indicating that a one standard deviation increase in grade students should expect is associated with students expecting to earn grades that are approximately 6 percentage points higher than they originally believed. As in the main text, Panel (d) of Figure A.7 and the point estimates in columns (7) and (8) of Table A.7 show that students accurately revise their grade expectations upon learning new information.