Online Appendix for the paper "Business Cycle during Structural Change: Arthur Lewis' Theory from a Neoclassical Perspective"

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E Online Appendix

E.1 Details of Samples used in Figure 3

For panels a in Figure 3 (aggregate employment and GDP), the sample comprises 66 countries.

Countrycode	Start	End	Countrycode	Start	End
ALB	1996	2014	ITA	1977	2015
AUS	1970	2015	JPN	1970	2015
AUT	1983	2015	KAZ	2001	2015
AZE	1992	2012	KOR	1970	2015
BEL	1983	2015	LTU	1998	2015
ALB	1994	2014	ITA	1977	2015
AUS	1970	2015	JAM	1992	2015
AUT	1983	2015	JPN	1970	2015
AZE	1990	2015	KOR	1970	2015
BEL	1983	2015	LTU	1998	2015
BGR	2000	2015	LUX	1983	2015
BHS	1989	2011	LVA	1996	2015
BRA	1981	2014	MDA	1999	2015
BRB	1981	2015	MEX	1995	2015
CAN	1970	2012	MLT	2000	2015
CHE	1991	2015	MMR	1978	1994
CHL	1975	2015	MYS	1980	2015
CHN	1985	2012	NLD	1987	2015
CRI	1980	2013	NOR	1972	2015
CUB	1995	2014	NZL	1986	2015
CYP	1999	2015	PAK	1973	2008
CZE	1993	2015	PAN	1982	2015
DEU	1983	2015	PHL	1971	2015
DNK	1972	2015	POL	1999	2015
DOM	1996	2015	PRI	1970	2011
EGY	1989	2015	PRT	1974	2015
ESP	1970	2015	PRY	1997	2015
EST	1995	2015	ROU	1997	2015
FIN	1970	2015	RUS	1997	2015
FRA	1970	2015	SLV	1994	2015
GBR	1983	2015	SVK	1994	2015
GRC	1983	2015	SVN	1995	2015
HND	1990	2015	SWE	1970	2015
HUN	1992	2015	THA	1971	2015
IDN	1985	2015	TTO	1977	2010
IRL	1983	2015	USA	1970	2015
ISL	1991	2015	VEN	1975	2013
ISR	1970	2015	ZAF	2000	2015

For panel b in Figure 3 (agricultural versus nonagricultural employment), the sample comprises 66 countries. The sample time periods for each country are the following,

Countrycode	Start	End	Countrycode	Start	End
ALB	1994	2014	ITA	1977	2015
AUS	1970	2015	JAM	1992	2015
AUT	1983	2015	JPN	1970	2015
AZE	1983	2015	KOR	1970	2015
BEL	1983	2015	LTU	1998	2015
BGR	2000	2015	LUX	1983	2015
BHS	1989	2011	LVA	1996	2015
BRA	1981	2014	MDA	1999	2015
BRB	1981	2015	MEX	1995	2015
CAN	1970	2012	MLT	2000	2015
CHE	1991	2015	MMR	1978	1994
CHL	1975	2015	MYS	1980	2015
CHN	1985	2012	NLD	1987	2015
CRI	1980	2013	NOR	1972	2015
CUB	1995	2014	NZL	1986	2015
CYP	1999	2015	PAK	1973	2008
CZE	1993	2015	PAN	1982	2015
DEU	1983	2015	PHL	1971	2015
DNK	1972	2015	POL	1999	2015
DOM	1996	2015	PRI	1970	2011
EGY	1989	2015	PRT	1974	2015
ESP	1970	2015	PRY	1997	2015
EST	1989	2015	ROU	1997	2015
FIN	1970	2015	RUS	1997	2015
FRA	1970	2015	SLV	1994	2015
GBR	1983	2015	SVK	1994	2015
GRC	1983	2015	SVN	1993	2015
HND	1990	2015	SWE	1970	2015
HUN	1992	2015	THA	1971	2015
IDN	1985	2015	TTO	1977	2010
IRL	1983	2015	USA	1970	2015
ISL	1991	2015	VEN	1975	2013
ISR	1970	2015	ZAF	2000	2015

For panel c in Figure 3 (productivity gap versus nonagricultural employment), the sample comprises 63 countries. The sample time periods for each country are the following

Countrycode	Start	End	Countrycode	Start	End
ALB	1994	2014	ITA	1990	2015
AUS	1990	2015	JAM	1993	2015
AUT	1983	2015	JPN	1970	2015
AZE	1990	2015	KOR	1970	2015
BEL	1995	2015	LTU	1998	2015
BGR	2000	2015	LUX	1995	2015
BHS	1989	2011	LVA	1996	2015
BRA	1981	2014	MDA	1999	2015
BRB	1990	2014	MEX	1995	2015
CHE	1991	2015	MLT	2000	2015
CHL	1975	2015	MYS	1980	2015
CHN	1985	2012	NLD	1987	2015
CRI	1980	2013	NOR	1972	2015
CUB	1995	2014	NZL	1986	2014
CYP	1999	2015	PAK	1973	2008
CZE	1993	2015	PAN	1982	2015
DEU	1991	2015	PHL	1971	2015
DNK	1972	2015	POL	1999	2015
DOM	1996	2015	PRI	1971	2011
EGY	1989	2015	PRT	1995	2015
ESP	1995	2015	PRY	1997	2015
EST	1995	2015	ROU	1997	2015
FIN	1975	2015	RUS	1997	2015
FRA	1970	2015	SLV	1994	2015
GBR	1990	2015	SVK	1995	2015
GRC	1995	2015	SVN	1995	2015
HND	1990	2015	SWE	1980	2015
HUN	1995	2015	THA	1971	2015
IDN	1987	2015	TTO	1984	2010
IRL	1995	2015	USA	1970	2015
ISL	1997	2015	VEN	1975	2013
			ZAF	2000	2015

For panel d in Figure 3 (relative consumption to output volatility versus nonagricultural employment), the sample comprises 64 countries. The sample time periods for each country are the following,

Countrycode	Start	End	Countrycode	Start	End
BGR	2000	2015	LUX	1983	2015
BHS	1989	2011	LVA	1996	2015
BRA	1981	2014	MDA	1999	2015
CAN	1970	2012	MEX	1995	2015
CHE	1991	2015	MLT	2000	2015
CHL	1975	2015	MYS	1980	2015
CHN	1990	2012	NLD	1987	2015
CRI	1980	2013	NOR	1972	2015
CUB	1995	2014	NZL	1986	2015
CYP	1999	2015	PAK	1986	2008
CZE	1993	2015	PAN	1982	2015
DEU	1983	2015	PHL	1971	2015
DNK	1972	2015	POL	1999	2015
DOM	1996	2015	PRI	1972	2011
EGY	1989	2015	PRT	1974	2015
ESP	1970	2015	PRY	1997	2015
EST	1995	2015	ROU	1997	2015
FIN	1970	2015	RUS	1997	2015
FRA	1970	2015	SLV	1994	2015
GBR	1983	2015	SVK	1994	2015
GRC	1983	2015	SVN	1995	2015
HND	1990	2015	SWE	1970	2015
HUN	1992	2015	THA	1971	2015
IDN	1985	2015	UKR	2001	2015
IRL	1983	2015	USA	1970	2015
ISL	1991	2015	VEN	1975	2013
ISR	1995	2015	ZAF	2000	2015

E.2 Discrete Time Model

In this section we provide a complete description of the discrete time model with endogenous labor supply estimated in Section 4. Our baseline discrete time model adds the following model features to the continuous-time model: (1) endogenous labor supply (2) land as a factor of production in modern-agriculture sector (3) TFP shocks (4) capital stocks in each sector are predetermined.

Time t is discrete, indexed by $0, 1, 2, \ldots$. Given the initial capital stock in each sector, i.e., $\bar{K}_0 \kappa_0$ and $\bar{K}_0 (1 - \kappa_0)$, and initial TFP levels, $Z_0^i, i = AM, M, S$, the representative household maximizes expected utility:

$$\max E_0 \sum_{t=0}^{\infty} \mu^t \left(\theta \log c_t + (1-\theta) \log (1 - h_t)\right)$$

subject to the budget constraint

$$N_t c_t + K_{t+1} = W_t N_t + (R_t - \delta) K_t + Tr_t$$

where $K_t = K_t^M + K_t^{AM}, N_t = N_t^M + N_t^{AM} + N_t^S$. W_t denotes the after-tax equilibrium wage. $Tr_t = \tau W_t^M h_t N_t^M$ denotes the lump-sum transfer from the government to the representative household. Note that μ denotes the discount factor.

The production side is identical to the model in the text, except that the production of modern agriculture has been modified to include land

$$Y_t^{AM} = \left(K_t^{AM}\right)^{1-\beta-\beta_T} \left(Z_t^{AM} H_t^{AM}\right)^{\beta}$$

where the land income share is denoted by $\beta_T \geq 0$. We assume $\beta + \beta_T < 1$.

As explained in Section 3.2, we can exploit the equivalence between the competitive equilibrium and the distorted social planner problem, and write the Lagrangian as:

$$L = E_0 \sum_{t=0}^{\infty} \mu^t \left\{ \begin{array}{c} \theta \log c_t + (1-\theta) \log (1-h_t) + \\ \xi_t \left[\begin{array}{c} Y_t + (1-\delta) K_t - c_t N_t - K_{t+1} \\ -\tau W_t^M H_t^M + Tr_t \end{array} \right] \end{array} \right\}$$

where we use the notation $\chi_t, \kappa_t, \nu_t^M, \nu_t^{AM}, \nu_t^S, \upsilon_t$ introduced in the main text, but we modify the notations of η_t and introduce $\tilde{\eta}_t$

$$\begin{aligned} \eta_t &\equiv \left[\gamma \left(\frac{Y_t^G}{Y_t^M}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ \tilde{\eta}_t &\equiv \left[\gamma + (1-\gamma) \left(\frac{Y_t^M}{Y_t^G}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

Therefore, by definition $Y_t = Y_t^M \eta_t$ and $Y_t = Y_t^G \tilde{\eta}_t$. Recall that $H_t^i \equiv h_t N_t^i$.

The FOC with respect to ν_t^{AM} and ν_t^S are as in the continuous time problem.

$$\begin{aligned}\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}\upsilon_t\beta\frac{1}{\nu_t^A} &= (1-\tau)(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\alpha\frac{1}{\nu_t^M}\\ \gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}(1-\upsilon_t)\frac{1}{\nu_t^S} &= (1-\tau)(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\alpha\frac{1}{\nu_t^M}\end{aligned}$$

Therefore, we have

$$\begin{aligned}\frac{\nu_t^{AM}}{\nu_t^M} &= \frac{\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}\upsilon_t\beta}{(1-\tau)(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\alpha}\\ \frac{\nu_t^S}{\nu_t^M} &= \frac{\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}(1-\upsilon_t)}{(1-\tau)(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\alpha}\end{aligned}$$

sum up together to have the expenditure ratio agr./non-agr. as

$$\frac{\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}}{(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}} = \frac{1-\nu_t^M}{\nu_t^M}\frac{(1-\tau)\alpha}{(1-\upsilon_t)+\upsilon_t\beta}$$

and express ν_t^M as

$$\nu_t^M = \left(1+\frac{\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}}{(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}}\frac{(1-\upsilon_t)+\upsilon_t\beta}{(1-\tau)\alpha}\right)^{-1}$$

Those with respect to c_t and h_t yield, respectively:

$$\theta\frac{1}{c_t} = \xi_t N_t, \tag{50}$$

$$\frac{1-\theta}{1-h_t} = \xi_t Y_t^{\frac{1}{\varepsilon}}\left[\begin{array}{c}\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}\left(\upsilon_t\frac{\beta}{h_t}+(1-\upsilon_t)\frac{1}{h_t}\right)\\ +(1-\tau)(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\frac{\alpha}{h_t}\end{array}\right]. \tag{51}$$

Substituting the FOCs with respect to ν_t^{AM} and ν_t^S into (51) yields

$$\frac{1-\theta}{1-h_t} = \xi_t Y_t^{\frac{1}{\varepsilon}}(1-\tau)\alpha(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\frac{1}{h_t\nu_t^M}. \tag{52}$$

Combining (52) with (50) yields

$$\frac{1-\theta}{\theta}\frac{c_t}{1-h_t} = (1-\tau)Y_t^{\frac{1}{\varepsilon}}\alpha(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\frac{1}{\nu_t^M h_t N_t}.$$

The FOC w.r.t. κ_{t+1} and K_{t+1} yield, respectively (after combining the two equations and rearranging terms):

$$\xi_t = E_t\left[\mu\xi_{t+1}\left(Y_{t+1}^{\frac{1}{\varepsilon}}\gamma\left(Y_{t+1}^G\right)^{-\frac{1}{\varepsilon}}\varsigma\left(\frac{\upsilon_{t+1}}{\varsigma}\right)^{-\frac{1}{\omega-1}}(1-\beta-\beta_T)\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}}+1-\delta\right)\right] \quad (53)$$

$$\xi_t = \mu E_t\left[\xi_{t+1}\left(Y_{t+1}^{\frac{1}{\varepsilon}}\frac{1}{K_{t+1}}\frac{1}{\kappa_{t+1}}(1-\gamma)\left(Y_{t+1}^M\right)^{1-\frac{1}{\varepsilon}}(1-\alpha)+1-\delta\right)\right]. \quad (54)$$

Substitute $Y_t = \eta_t Y_t^M$ and $Y_t = Y_t^G\tilde{\eta}_t$ Substituting in this into (53) and (54) respectively to eliminate Y yields

$$\xi_t = \mu E_t\left[\xi_{t+1}\left(\gamma\tilde{\eta}_{t+1}^{\frac{1}{\varepsilon}}\varsigma\left(\frac{\upsilon_{t+1}}{\varsigma}\right)^{-\frac{1}{\omega-1}}(1-\beta-\beta_T)\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}}+1-\delta\right)\right] \quad (55)$$

$$\xi_t = \mu E_t\left[\xi_{t+1}\left(\eta_{t+1}^{\frac{1}{\varepsilon}}\frac{Y_{t+1}^M}{K_{t+1}^M}(1-\gamma)(1-\alpha)+1-\delta\right)\right]. \quad (56)$$

Equation (50), 55), and (56) yields the standard Euler equations for consumption:

$$1 = \frac{\mu}{1+n}E_t\left\{\frac{c_t}{c_{t+1}}\left[\begin{array}{c}\gamma\tilde{\eta}_{t+1}^{\frac{1}{\varepsilon}}\varsigma\left(\frac{\upsilon_{t+1}}{\varsigma}\right)^{-\frac{1}{\omega-1}}(1-\beta-\beta_T)\left((1-\kappa_{t+1})\chi_{t+1}\right)^{-\beta-\beta_T}\\ \times\left(Z_{t+1}^{AM}h_{t+1}\nu_{t+1}^{AM}\right)^{\beta}N_{t+1}^{-\beta_T}+1-\delta\end{array}\right]\right\}$$

$$1 = \frac{\mu}{1+n}E_t\left\{\frac{c_t}{c_{t+1}}\left[1+\eta_{t+1}^{\frac{1}{\varepsilon}}(1-\gamma)(1-\alpha)\left(\frac{Z_{t+1}^M h_{t+1}\nu_{t+1}^M}{\kappa_{t+1}\chi_{t+1}}\right)^{\alpha}-\delta\right]\right\}$$

We can simplify the intertemporal condition:

$$\frac{1-\theta}{\theta}\frac{c_t}{1-h_t} = (1-\tau)\eta_t^{\frac{1}{\varepsilon}}\alpha(1-\gamma)\left(\frac{Z_t^M h_t\nu_t^M}{\kappa_t\chi_t}\right)^{\alpha-1}$$

The resource constraint becomes

$$\chi_{t+1}(1+n) = \eta_t(\kappa_t)^{1-\alpha}\left(Z_t^M\nu_t^M h_t\right)^{\alpha}\chi_t^{1-\alpha}+(1-\delta)\chi_t-c_t$$

E.3 Algorithm to Solve the Rational Expectation Equilibrium

We can rewrite the model in discrete time recursively. Denote the state space as $\Theta_t \equiv \left(\hat{\chi}_t, \kappa_t, z_t^M, z_t^{AM}, z_t^S, t\right)$. The Bellman Equation (during the structural change transition) is given by

$$V(\Theta_t) = \max_{\hat{c}_t, h_t, \kappa_{t+1}, \hat{\chi}_{t+1}, \upsilon_t, \nu_t^M}\left\{u(\hat{c}_t, h_t)+\mu E_t V'(\Theta_{t+1})\right\}$$

where use the following notations to detrend the variables

$$\hat{\chi}_t = \frac{\chi_t}{\Lambda_t^M}, \hat{c}_t = \frac{c_t}{\Lambda_t^M}, z_t^i = \frac{Z_t^i}{\Lambda_t^i}, i = AM, M, S$$

and

$$\Lambda_t^i = Z_0^i \Pi_{k=1}^t \left(1+g_k^i\right), i = AM, M, S$$

We solve the Bellman Equation using its detrended first order conditions and budget constraint.

In each period, we have 6 unknown policy function to solve

$$\hat{c}_t\left(\Theta_t\right), h_t\left(\Theta_t\right), \kappa_{t+1}\left(\Theta_t\right), \hat{\chi}_{t+1}\left(\Theta_t\right), \upsilon_t\left(\Theta_t\right), \nu_t^M\left(\Theta_t\right).$$

We can reduce the number of unknown policy function further by expressing $\hat{c}_t$ and $\hat{\chi}_{t+1}$ in terms of other state variables and decision variables.

$$\begin{aligned}
\hat{c}_t &= (1-\tau)\,\eta_t^{\frac{1}{\varepsilon}}\,(1-\gamma)\,\alpha\left(\kappa_t\hat{\chi}_t\right)^{1-\alpha}\left(z_t^M h_t \nu_t^M\right)^{\alpha-1}(1-h_t)\,\frac{\theta}{1-\theta} \\
\hat{\chi}_{t+1} &= \frac{\eta_t\left(\kappa_t\right)^{1-\alpha}\left(z_t^M\nu_t^M h_t\right)^{\alpha}\left(\hat{\chi}_t\right)^{1-\alpha}+(1-\delta)\,\hat{\chi}_t-\hat{c}_t}{(1+n)\left(1+g_{t+1}^M\right)}
\end{aligned}$$

where

$$\begin{aligned}
\eta_t &= \left[\gamma\left(\frac{Y_t^G}{Y_t^M}\right)^{\frac{\varepsilon-1}{\varepsilon}}+(1-\gamma)\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
\frac{Y_t^G}{Y_t^M} &= \frac{\left(z_t^{AM}\Lambda_t^{AM}h_t\frac{\nu_t^{AM}}{\nu_t^M}\nu_t^M N_t\right)^{\beta}\left(\left(1-\kappa_t\right)\hat{\chi}_t\Lambda_t^M N_t\right)^{1-\beta-\beta_T}}{\left(z_t^M\Lambda_t^{AM}h_t\nu_t^M N_t\right)^{\alpha}\left(\kappa_t\hat{\chi}_t\Lambda_t^M N_t\right)^{1-\alpha}}\left(\frac{\upsilon_t}{\varsigma}\right)^{-\frac{\omega}{\omega-1}} \\
\frac{\nu_t^{AM}}{\nu_t^M} &= \frac{1-\nu_t^M}{\nu_t^M}\,\frac{(1-\tau)\,\alpha}{(1-\upsilon_t)+\upsilon_t\beta}\,\frac{\upsilon_t\beta}{(1-\tau)\,\alpha}
\end{aligned}$$

Therefore, we are left with four unknowns $h_t\left(\Theta_t\right), \kappa_{t+1}\left(\Theta_t\right), \nu_t^M\left(\Theta_t\right), \upsilon_t\left(\Theta_t\right)$. We need to solve four nonlinear equations as well.

The first two are the detrended the Euler Equations

$$\hat{c}_t^{-1} = \frac{\mu}{(1+n)\left(1+g_{t+1}^M\right)}E_t\left\{\hat{c}_{t+1}^{-1}\times\left[\eta_{t+1}^{\frac{1}{\varepsilon}}\left(\frac{z_{t+1}^M\nu_{t+1}^M h_{t+1}}{\kappa_{t+1}\hat{\chi}_{t+1}}\right)^{\alpha}(1-\gamma)(1-\alpha)+(1-\delta)\right]\right\} \tag{57}$$

$$\hat{c}_t^{-1} = \frac{\mu}{(1+n)\left(1+g_{t+1}^M\right)}E_t\left[\hat{c}_{t+1}^{-1}\left(\gamma\tilde{\eta}_{t+1}^{\frac{1}{\varepsilon}}\varsigma\left(\frac{\upsilon_{t+1}}{\varsigma}\right)^{-\frac{1}{\omega-1}}\left(1-\beta-\beta_T\right)\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}}+1-\delta\right)\right] \tag{58}$$

where

$$\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}} = \left(\left(1-\kappa_{t+1}\right)\hat{\chi}_{t+1}\Lambda_{t+1}^M\right)^{-\beta-\beta_T}\left(z_{t+1}^{AM}\Lambda_{t+1}^{AM}\lambda_{t+1}^{AM}h_{t+1}\right)^{\alpha_A}N_{t+1}^{-\beta_T}$$

and

$$\frac{Y_{t+1}^G}{Y_{t+1}^M} = \frac{\left(z_{t+1}^{AM}\Lambda_{t+1}^{AM}h_{t+1}\nu_{t+1}^{AM}N_{t+1}\right)^{\beta}\left(\left(1-\kappa_{t+1}\right)\hat{\chi}_{t+1}\Lambda_{t+1}^M N_{t+1}\right)^{1-\beta-\beta_T}}{\left(z_{t+1}^M\Lambda_{t+1}^{AM}h_{t+1}\nu_{t+1}^M N_{t+1}\right)^{\alpha}\left(\kappa_{t+1}\hat{\chi}_{t+1}\Lambda_{t+1}^M N_{t+1}\right)^{1-\alpha}}\left(\frac{\upsilon_{t+1}}{\varsigma}\right)^{-\frac{\omega}{\omega-1}}$$

The other two are the equations about υ_t and υ_t^M.

$$\upsilon_t = \frac{\varsigma \left(Y_t^{AM}\right)^{\frac{\omega-1}{\omega}}}{\varsigma \left(Y_t^{AM}\right)^{\frac{\omega-1}{\omega}} + (1-\varsigma)\left(Y_t^{S}\right)^{\frac{\omega-1}{\omega}}} \tag{59}$$

$$\nu_t^M = \left(1 + \frac{\gamma \left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}}{(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}} \frac{(1-\upsilon_t)+\upsilon_t\beta}{(1-\tau)\alpha}\right)^{-1} \tag{60}$$

where

$$\begin{aligned}
Y_t^{AM} &= (1-\kappa_t)^{1-\beta-\beta_T}\left(\hat{\chi}_t\Lambda_t^M\right)^{1-\beta-\beta_T}\left(\frac{\nu_t^{AM}}{\nu_t^M}\right)^{\beta}\left(z_t^{AM}\Lambda_t^{AM}\nu_t^M\right)^{\beta} h_t^{\beta} N_t^{-\beta_T} \\
Y_t^S &= z_t^S \nu_t^M \frac{\nu_t^S}{\nu_t^M} h_t \Lambda_t^S N_t \\
\frac{\nu_t^{AM}}{\nu_t^M} &= \frac{1-\nu_t^M}{\nu_t^M}\frac{(1-\tau)\alpha}{(1-\upsilon_t)+\upsilon_t\beta}\frac{\upsilon_t\beta}{(1-\tau)\alpha} \\
\frac{\nu_t^S}{\nu_t^M} &= \frac{1-\nu_t^M}{\nu_t^M}\frac{(1-\tau)\alpha}{(1-\upsilon_t)+\upsilon_t\beta}\frac{(1-\upsilon_t)}{(1-\tau)\alpha}
\end{aligned}$$

Summary 1 *We solve the Bellman Equation backwards from the last period, which corresponds to the long-run approximate balanced growth path with (approximately) only one sector, that is, the non-agr. sector.*

1. *We choose to number of transition period to be a large number ($T = 250$). In practice, we can increases the number until the beginning transition period we are interested in are no longer affected by the choice of T. We can also check whether the economy will converge to the long-run ABGP within the period T.*

2. *We discretize the state space. In the deterministic case, we choose the state space for $\hat{\chi}$ as $[0.5\hat{\chi}_0, 1.5\hat{\chi}^*]$ and choose the state space for κ as $[0.5, 1]$. We discretize both $\hat{\chi}$ and κ using 250 equally spaced grid points; In the stochastic case, we choose the state space for $\hat{\chi}$ as $[0.9\hat{\chi}_t^*, 1.1\hat{\chi}_t^*]$ and choose state space for κ as $[\kappa_t^* - 0.025, \kappa_t^* + 0.025]$. We discretize both $\hat{\chi}$ and κ using 75 equally spaced grid points, where $\hat{\chi}_t^*$ and κ_t^* are the realized path in the deterministic model. We further discretize the joint process for the three types of shocks using 27 grid points using Tauchen's method (Tauchen 1986).*

3. *We solve the transitional path backwards.*

   (a) *In the last period, the economy is almost identical to a one-sector RBC model. Therefore,* *we set*

$$\begin{aligned}\kappa_{T+1}(\Theta_T) &= 1, \upsilon_T(\Theta_T) = 1 \\ \nu_T^M(\Theta_T) &= 1, \nu_T^{AM}(\Theta_T) = 0, \nu_T^S(\Theta_T) = 0\end{aligned}$$

We solve the Bellman Equation using value function iteration, with linear interpolation between grid points. We can solve for the rest of the policy functions

$$c_T(\Theta_T), h_T(\Theta_T), \hat{\chi}_{T+1}(\Theta_T)$$

(b) *From period $t = T-1$ to 1, we solve the nonlinear system of Equations (57), (58), (59), and (60) for the policy functions $\kappa_{t+1}(\Theta_t), \upsilon_t(\Theta_t), \nu_t^M(\Theta_t), h_t(\Theta_t)$. In each period, we first express $\hat{c}_t(\Theta_t), \hat{\chi}_{t+1}(\Theta_t)$ in terms of other state variables and decision variables*

E.4 Measuring sector-specific TFP levels: theory

We observe $\{Y, Y^M, K^M, N^M, Y^G, K^G, N^G\}$ but we do not have direct observations of allocations of labor and output across the two agricultural technologies (we also presume that we have already estimated all relevant parameters). We describe the procedure to estimate the three-shock process

To measure the sector-specific TFP levels we impose three equilibrium conditions: (i) marginal return to capital are equated across manufacturing and modern agriculture, and (ii) marginal return to labor is equated across the two agricultural technologies; and (iii) hours per worker h is equalized across sectors. In addition we assume that assume that h is constant over time so that employment is a sufficient statistic for measuring labor input.

Recall the following definitions of sectoral outputs;

$$\begin{aligned}Y^G &= \left[\varsigma\left(Y^{AM}\right)^{\frac{\omega-1}{\omega}} + (1-\varsigma)\left(Y^S\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \\ Y^M &= \left(K^M\right)^{1-\alpha_M} \times \left(Z^M H^M\right)^{\alpha_M} \\ Y^{AM} &= \left(K^{AM}\right)^{\beta_{AM}} \times \left(Z^{AM} H^{AM}\right)^{\alpha_{AM}} \\ Y^S &= Z^S H^S\end{aligned}$$

We express aggregate output using current prices,[30] $Y = P^G Y^G + P^M Y^M$.

[30] The advantage of focusing on aggregate output in terms of current prices instead of specifying the production function is that while a subsistence level in agricultural consumption will be subsumed in the relative prices, it will not affect the equations determining how TFP levels are measured.

The marginal products of sector-specific capital and labor are

$$
\begin{aligned}
\frac{\partial Y}{\partial K_M} &= P^M \frac{\partial Y_M}{\partial K_M} = P^M (1-\alpha_M) \left(\frac{K^M}{H^M}\right)^{-\alpha_M} \times \left(Z^M\right)^{\alpha_M} \\
\frac{\partial Y}{\partial K_G} &= P^G \frac{\partial Y_G}{\partial Y_{AM}} \frac{\partial Y_{AM}}{\partial K_G} = P^G \varsigma \left(\frac{Y^{AM}}{Y}\right)^{-\frac{1}{\omega}} \beta_{AM} \left(K^G\right)^{\beta_{AM}-1} \times \left(Z^{AM} H^{AM}\right)^{\alpha_{AM}} \\
\frac{\partial Y}{\partial N_S} &= P^G \frac{\partial Y_G}{\partial Y_S} \frac{\partial Y_S}{\partial N_S} = P^G (1-\varsigma) \left(\frac{Y^S}{Y^G}\right)^{-\frac{1}{\omega}} Z^S h \\
\frac{\partial Y}{\partial N_{AM}} &= P^G \frac{\partial Y_G}{\partial Y_{AM}} \frac{\partial Y_{AM}}{\partial N_{AM}} = P^G \varsigma \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} \alpha_{AM} \left(K^G\right)^{\beta_{AM}} \times \left(H^{AM}\right)^{\alpha_{AM}-1} \left(Z^{AM}\right)^{\alpha_{AM}} h
\end{aligned}
$$

We now proceed to measuring the TFP levels in five steps:

1. The marginal product of capital is the same in manufacturing and modern agriculture,

$$
\begin{aligned}
P^M (1-\alpha_M) \left(K^M\right)^{-\alpha_M} \times \left(Z^M H^M\right)^{\alpha_M} &= \varsigma \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} P^G \beta_{AM} \left(K^G\right)^{\beta_{AM}-1} \times \left(Z^{AM} H^{AM}\right)^{\alpha_{AM}} \\
&\Rightarrow
\end{aligned}
$$

$$
\left(\frac{Y^{AM}}{Y^G}\right)^{1-\frac{1}{\omega}} = \frac{K^G}{K^M} \frac{P^M Y^M}{P^G Y^G} \frac{1}{\varsigma} \frac{1-\alpha_M}{\beta_{AM}} \tag{61}
$$

   The variables on the right-hand side of Equation (61) are observable. This identifies the ratio Y^{AM}/Y^G.

2. Use the agricultural production function (3) to derive a relationship between the ratios Y^S/Y^G and Y^{AM}/Y^G,

$$
1 = \varsigma \left(\frac{Y^{AM}}{Y^G}\right)^{\frac{\omega-1}{\omega}} + (1-\varsigma) \left(\frac{Y^S}{Y^G}\right)^{\frac{\omega-1}{\omega}} \tag{62}
$$

   Given the imputed ratio Y^{AM}/Y^G, this equation identifies the ratio Y^S/Y^G.

3. The marginal product of labor is equated across the two agricultural sectors. This implies

$$
\begin{aligned}
P^G (1-\varsigma) \left(\frac{Y^S}{Y^G}\right)^{-\frac{1}{\omega}} Z^S h &= P^G \varsigma \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} \alpha_{AM} \left(K^G\right)^{\beta_{AM}} \times \left(H^{AM}\right)^{\alpha_{AM}-1} \left(Z^{AM}\right)^{\alpha_{AM}} h \\
&\Rightarrow
\end{aligned}
$$

$$
\frac{H^{AM}}{H^S} = \alpha_{AM} \frac{\varsigma}{1-\varsigma} \left(\frac{Y^{AM}}{Y^G} \frac{Y^G}{Y^S}\right)^{1-\frac{1}{\omega}} \tag{63}
$$

4. Use the accounting identity $N^G = N^{AM} + N^S$ to identify N^{AM} and N^S;

$$
\frac{H^{AM}}{H^S} = \frac{N^{AM}}{N^S} = \frac{N^{AM}}{N^G - N^{AM}} \Rightarrow
$$

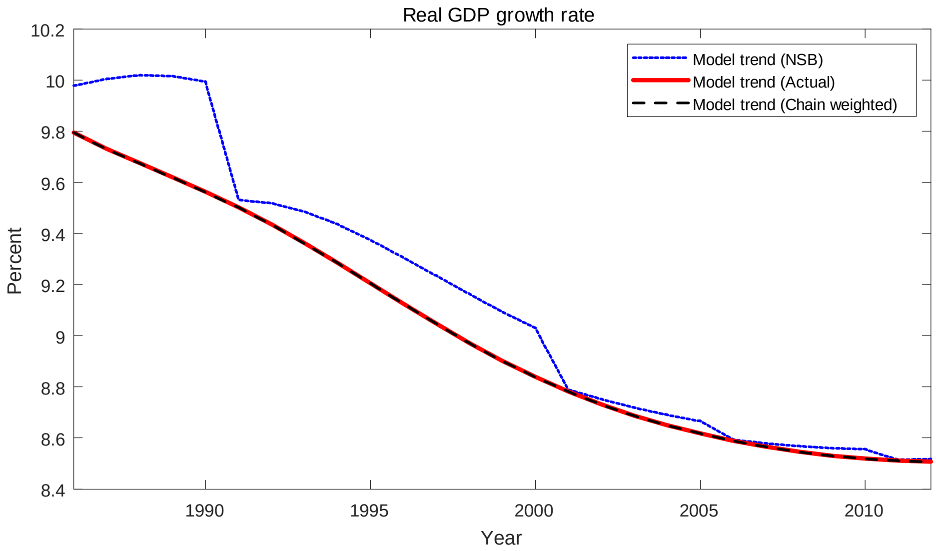

Figure 16: This figure compare different ways of calculating the real GDP growth rate with data.

$$N^{AM} = \frac{N^G}{1+\left(\frac{H^{AM}}{H^S}\right)^{-1}} \tag{64}$$

$$N^S = N^G - N^{AM} \tag{65}$$

5. Normalize hours per worker to $h = 1$ for all periods (so aggregate hours equals employment). Equations (61)-(65) then allows us to identify Z^M, Z^S, and Z^{AM};

$$\ln\left(Z^S\right) = \ln\left(Y^S\right) - \ln\left(N^S\right) \tag{66}$$

$$\ln\left(Z^{AM}\right) = \frac{1}{\alpha_{AM}}\ln\left(Y^{AM}\right) - \frac{\beta_{AM}}{\alpha_{AM}}\ln\left(K^{AM}\right) - \ln\left(N^{AM}\right) \tag{67}$$

and finally TFP in manufacturing;

$$\ln\left(Z^M\right) = \frac{1}{\alpha_M}\ln\left(Y^M\right) - \frac{1-\alpha_M}{\alpha_M}\ln\left(K^M\right) - \ln\left(N^M\right) \tag{68}$$

E.5 Additional Figures