A Regulatory Details: For Online Publication

This appendix presents the formulas for programmed withdrawal and the minimum pension guarantee in detail.

A.1 Programmed Withdrawal

The exposition in this subsection follows Pino (2005). PW payouts in each month of year t for an individual of age x and gender g are

$$PW_t(x,g) = \frac{Balance_t}{CNU_t(x,g) \cdot 12}$$

where balance is the beginning of year account balance in the PFA and CNU is the expected present discounted value of paying out a unit pension. To calculate the CNU, we need to define a few objects. A mortality table issued in year m defines a gender-specific death probability $q^m(x,g)$ for every age x and an adjustment factor $AF^m(x,g)$ - a value meant to correct for increasing longevity expectations for a fixed mortality table.

In year t, the appropriate value for $q^t(x,g)$ is

$$q^t(x,g) = q^m(x,g) \cdot *(1 - AF^m(x,g))^{t-m}$$

Regardless of gender, the tables assume that the probability of being alive at 20 equals 1 and that the probability of being alive at 110 equals 0. For intermediate values, define $l^t(x,g)$, the year

t probability of being alive at age x, as

$$l^t(x,g) = l^t(x-1,g)\cdot(1-q^t(x-1,g)) \text{ for } x \in (20,110]$$

Then $CNU_t(x,g)$ is

$$CNU_t(x,g) = \sum_{j=x}^{110} \frac{l^t(j,g)}{l^t(x,g)\cdot(1+r_{RP})^{j-x}} - \frac{11}{24} \text{ for } x \in (20,110]$$

where $r_{RP} = 0.8\cdot r_A + 0.2\cdot\bar{r}$, r_A is the previous year's implicit interest rate for annuities and $\bar{r}$ is the 10 year average return for PW balances. Finally, note that CNU calculations vary for individuals with dependents. We do not report those adjustments, as we work with a no-dependents sample. See Pino (2005) for details. Readers wishing to obtain CNU values will benefit from also reading Vega (2014) and the accompanying Stata module.

A.2 Minimum Pension Guarantee

There are two minimum pension regimes in Chile during our sample period: pre and post 2008. In the first period, any individual with at least 20 years of contributions into the pension system who receives a pension below a minimum guaranteed amount receives a top-up from the government. Since annuity offers cannot fall below this amount, during this period the minimum guaranteed amount is only relevant for valuing programmed withdrawal contracts and for calculating annuity payouts after a default. We value both contracts by taking the UF denominated value of the pension guarantee at the time of retirement and holding it fixed throughout the lifetime of the contract.

Starting in 2008, this guarantee is replaced by an expanded top-up that is available to individuals whose pension falls below a maximum amount. To be precise, the new regime sets a new floor, called the "Pensión Básica Solidaria" or PBS, and a maximum, called the "Pensión Máxima con Aporte Solidario", or PMAS. Annuity offers after this reform cannot fall below the PBS, and individuals funding offers above the PMAS receive no subsidy. For individuals who fund an offer ("Pensión Base", or PB) in between the PBS and the PMAS, the government top up ("Complemento Solidario", or CS) is

$$CS = PBS \cdot (1 - \frac{PB}{PMAS})$$

This amount is added to any annuity offer accepted, regardless of contract type, provided the retiree is 65 or older, has lived in Chile for 20 years after the age of 20, has lived in Chile for 4 of the last 5 years, and is in the 60% percentile or lower in a needs-based poverty index ("Puntaje de Focalización Previsional").

For PW offers, a corrected version of the CS is added to the payout schedule. The correction is meant to ensure that the expected present discounted value of the subsidy is equal under PW and an annuity. See (2018) for details.

B Grid Selection: For Online Publication

As mentioned in the main text, we incorporate a grid selection step into the estimation procedure. Through this step we are able to start with a grid that plausibly spans the support of the distribution of types, but that is infeasible to take to the data, and reduce its dimensionality without greatly affecting the outcomes that the model can cover or the predictions that will later be made in the counterfactuals of interest.

We start with an initial grid that has 17 grid points per dimension of the type space, which corresponds to 83,251 total points. For bequest motive β, the grid is logarithmic and spans from 0 to 7.88e03. For intuition, an individual with risk aversion coefficient equal to 3 who knew they will be dead tomorrow would consume all their wealth today if their bequest motive were 0, while they would consume 5% of their wealth today if $\beta = 7.88e03$. Table 6 presents this mapping for every point in the grid of bequest motives. For risk aversion, the grid spans from 0 to 10, while for outside wealth the grid spans from 0.2 to 20 thousand UFs.[38] Finally, the mortality shifter grid spans from -15 to 15. Recall that a retiree aged x with a mortality shifter value of y solves the optimal consumption savings problem using the mortality probabilities of an $x+y$ year old. Table 7 presents the grid points for each dimension.

We then group these points into 16 bins according to their choices under four counterfactual scenarios: a choice between allocating the full pension balance to an actuarially fair annuity or to programmed withdrawal, and a choice between allocating the remaining pension balance to an actuarially fair annuity or to lumpsum withdrawal when 0%, 50% and 90% of pension balances are placed in an actuarially fair annuity. We do this separately by gender. For women, we value these counterfactuals for a 60 year old with median pension balance, while for men we value them for a 65 year old with median pension balance.

[38] In December 12, 2017, the dollar equivalent range was between 8,170 and 817,000

	Bequest Motive	Percentage Consumed
1	0	100.00%
2	8.99E-07	99.09%
3	6.07E-05	96.38%
4	7.58E-04	91.99%
5	4.85E-03	86.09%
6	2.20E-02	78.90%
7	8.21E-02	70.68%
8	2.72E-01	61.79%
9	8.52E-01	52.50%
10	2.60E+00	43.25%
11	8.05E+00	34.33%
12	2.61E+01	26.10%
13	9.06E+01	18.92%
14	3.44E+02	13.01%
15	1.37E+03	8.62%
16	4.63E+03	5.91%
17	7.89E+03	5.00%

Table 6: Map from bequest motive to fraction of wealth consumed before certain death

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
1	0	0	0.2	-15
2	8.99E-07	0.09	0.39	-14
3	6.07E-05	0.38	0.95	-13
4	7.58E-04	0.84	1.87	-12
5	4.85E-03	1.46	3.1	-10
6	2.20E-02	2.22	4.59	-8
7	8.21E-02	3.08	6.31	-5
8	2.72E-01	4.02	8.17	-2
9	8.52E-01	5	10.1	0
10	2.60E+00	5.98	12	2
11	8.05E+00	6.91	13.9	5
12	2.61E+01	7.78	15.6	8
13	9.06E+01	8.54	17.1	10
14	3.44E+02	9.16	18.3	12
15	1.37E+03	9.62	19.2	13
16	4.63E+03	9.9	19.8	14
17	7.89E+03	10	20	15

Table 7: Gridpoints by dimension of types for initial grid

We take a random 5% subsample of individuals and solve the optimal consumption-savings problem for every annuity offer and programmed withdrawal offer these individuals receive, and for every one of these 83,251 points. To reduce the dimensionality of this grid, as it is computationally infeasible to estimate demand on the whole sample with this grid size, we group together points that predict the same choice for at least 99% of offer sets in this subsample and that predict the same choice in each of the 16 counterfactual bins. This yields 2089 groups for men and 5812 for women.

We select an element of each bin by random sampling, and estimate equation in our subsample. This estimation procedure yields that 106 points for women and 36 points for men have estimated mass over 10^{-5}, and that these points have a cumulative mass of 99.99%. This is the grid we take to our full dataset. The full list of points for each gender is available upon request.

C Additional Tables and Figures: For Online Publication

	(1)
	Time to Death
Choose annuity	-0.164**
	(0.0601)
Insurance co. agent	0.195**
	(0.0646)
Insurance broker	0.160*
	(0.0682)
Financial advisor	0.0841
	(0.103)
Direct thru insurance co.	0.133
	(0.189)
Wealth/age controls	✓
Observations	45091

Figure 16: Correlation between death hazard and choice to annuitize, Gompertz baseline hazard

Number of Previous Policies	Percentage of Acceptances
0	96.80%
1	2.90%
2	0.20%
3 or more	0.10%

Figure 17: Evidence that majority of retirees have never previously interacted with annuity company (Castro et al. (2018))

	(1)	(2)	(3)	(4)
	Log(0-0 Offer Amount)	Log(0-0 Offer Amount)	0-0 Offer Amount	0-0 Offer Amount
1[Request a Guarantee Period]	-0.00504		0.0724	
	(0.00149)		(0.0361)	
1[Request a Deferral Period]	-0.00490		-0.0630	
	(0.000681)		(0.0173)	
1[Request Both]	-0.000122	-0.0101	-0.188	-0.178
	(0.00152)	(0.000655)	(0.0367)	(0.0168)
1[Only Request a Guarantee Period]		-0.00490		-0.0630
		(0.000681)		(0.0173)
1[Only Request a Deferral Period]		-0.00504		0.0724
		(0.00149)		(0.0361)
Pension Balance Spline	X	X	X	X
Age, Gender, Month-Year FE	X	X	X	X
Observations	430921	430921	430921	430921

Notes: This table reports results of a linear regression of an individual's received offers for a 0 guarantee, 0 deferral period contract ("0-0 contract") on dummies for whether the individual also requested offers for contracts with guarantee periods or deferral periods. Offer amounts are in UFs – in December 12, 2017, one UF was worth 40.85 USD. 1[Request a Guarantee Period] is a dummy variable that is equal to 1 if the individual requests at least one contract type that includes a guarantee period. 1[Request a Deferral Period] and 1[Request Both] are defined analogously. 1[Only Request a Guarantee Period] is a dummy variable that is equal to 1 if the individual requests at least one contract type that includes a guarantee period and does not request any contracts with a deferral period. 1[Only Request a Deferral Period] is defined analogously. All regressions include a spline of pension balances and the full interaction of age, gender and retirement month-year fixed effects. Robust standard errors in parentheses.

Figure 18: Testing for Information Revelation in the Request Stage

Panel A: CDF Summary				
Mass Cutoff	1.00E-01	1.00E-02	1.00E-03	1.00E-04
Number of Points with Mass Greater than Cutoff	1	24	43	45
Total Mass for these Points	27.93%	89.01%	99.85%	100.00%

Panel B: Top 10 Mass Points						
	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass	95% CI
1	621	0.000	10.100	15	27.93%	(26.12%, 29.74%)
2	7.89E+03	1.250	17.525	-9	5.43%	(4.42%, 6.45%)
3	7.89E+03	1.875	17.525	15	4.74%	(3.19%, 6.29%)
4	0.414	0.000	0.200	15	4.74%	(3.47%, 6.01%)
5	44.6	0.625	18.762	-5	4.48%	(2.82%, 6.15%)
6	7.89E+03	4.375	0.200	-1	4.29%	(2.86%, 5.71%)
7	0.000289	1.250	20.000	1	3.88%	(3.24%, 4.52%)
8	137	2.500	0.200	-15	3.46%	(0.81%, 6.11%)
9	7.89E+03	5.625	0.200	-5	3.26%	(1.47%, 5.04%)
10	17.5	1.250	0.200	-3	3.06%	(1.93%, 4.19%)

Notes: Panel A reports the number of points whose estimated mass is above different values and the total mass for those points. Panel B reports the ten points with the highest estimated masses, as well as their estimated weights and 95% confidence regions. These confidence intervals are calculated using clustered standard errors at the individual level.

Table 8: Descriptive Statistics for Estimated Type Distribution - First Quartile Females

Panel A: CDF Summary				
Mass Cutoff	1.00E-01	1.00E-02	1.00E-03	1.00E-04
Number of Points with Mass Greater than Cutoff	1	30	59	60
Total Mass for these Points	28.06%	85.66%	99.95%	100.00%

Panel B: Top 10 Mass Points						
	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass	95% CI
1	621	0.000	10.100	15	28.06%	(26.70%, 29.41%)
2	7.89E+03	1.250	17.525	-9	4.31%	(0.38%, 8.25%)
3	7.89E+03	1.875	17.525	15	3.20%	(1.80%, 4.61%)
4	0.414	0.000	0.200	15	3.03%	(2.22%, 3.84%)
5	44.6	0.625	18.762	-5	3.03%	(1.89%, 4.17%)
6	7.89E+03	4.375	0.200	-1	2.88%	(1.68%, 4.08%)
7	0.000289	1.250	20.000	1	2.86%	(2.47%, 3.25%)
8	137	2.500	0.200	-15	2.78%	(0.83%, 4.74%)
9	7.89E+03	5.625	0.200	-5	2.37%	(0.98%, 3.75%)
10	17.5	1.250	0.200	-3	2.35%	(1.29%, 3.40%)

Notes: Panel A reports the number of points whose estimated mass is above different values and the total mass for those points. Panel B reports the ten points with the highest estimated masses, as well as their estimated weights and 95% confidence regions. These confidence intervals are calculated using clustered standard errors at the individual level.

Table 9: Descriptive Statistics for Estimated Type Distribution - Second Quartile Females

Panel A: CDF Summary						
Mass Cutoff			1.00E-01	1.00E-02	1.00E-03	1.00E-04
Number of Points with Mass Greater than Cutoff			1	27	48	51
Total Mass for these Points			35.19%	90.03%	99.85%	100.00%
Panel B: Top 10 Mass Points						
	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass	95% CI
1	621	0.000	10.100	15	35.19%	(33.51%, 36.86%)
2	7.89E+03	1.250	17.525	-9	4.83%	(2.70%, 6.96%)
3	7.89E+03	1.875	17.525	15	3.72%	(1.70%, 5.74%)
4	0.414	0.000	0.200	15	3.27%	(2.15%, 4.40%)
5	44.6	0.625	18.762	-5	3.19%	(1.50%, 4.87%)
6	7.89E+03	4.375	0.200	-1	3.12%	(1.45%, 4.80%)
7	0.000289	1.250	20.000	1	2.73%	(2.11%, 3.34%)
8	137	2.500	0.200	-15	2.52%	(1.33%, 3.71%)
9	7.89E+03	5.625	0.200	-5	2.43%	(0.36%, 4.50%)
10	17.5	1.250	0.200	-3	2.38%	(1.10%, 3.66%)

Notes: Panel A reports the number of points whose estimated mass is above different values and the total mass for those points. Panel B reports the ten points with the highest estimated masses, as well as their estimated weights and 95% confidence regions. These confidence intervals are calculated using clustered standard errors at the individual level.

Table 10: Descriptive Statistics for Estimated Type Distribution - Fourth Quartile Females

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
Bequest Motive	1.00	0.62	0.02	-0.06
Risk Aversion	0.62	1.00	-0.60	-0.23
Outside Wealth	0.02	-0.60	1.00	0.15
Health Shifter	-0.06	-0.23	0.15	1.00

Table 11: Correlations Across Dimensions of Unobserved Type - First Quartile Females

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
Bequest Motive	1.00	0.73	-0.23	-0.08
Risk Aversion	0.73	1.00	-0.61	-0.39
Outside Wealth	-0.23	-0.61	1.00	0.21
Health Shifter	-0.08	-0.39	0.21	1.00

Table 12: Correlations Across Dimensions of Unobserved Type - Second Quartile Females

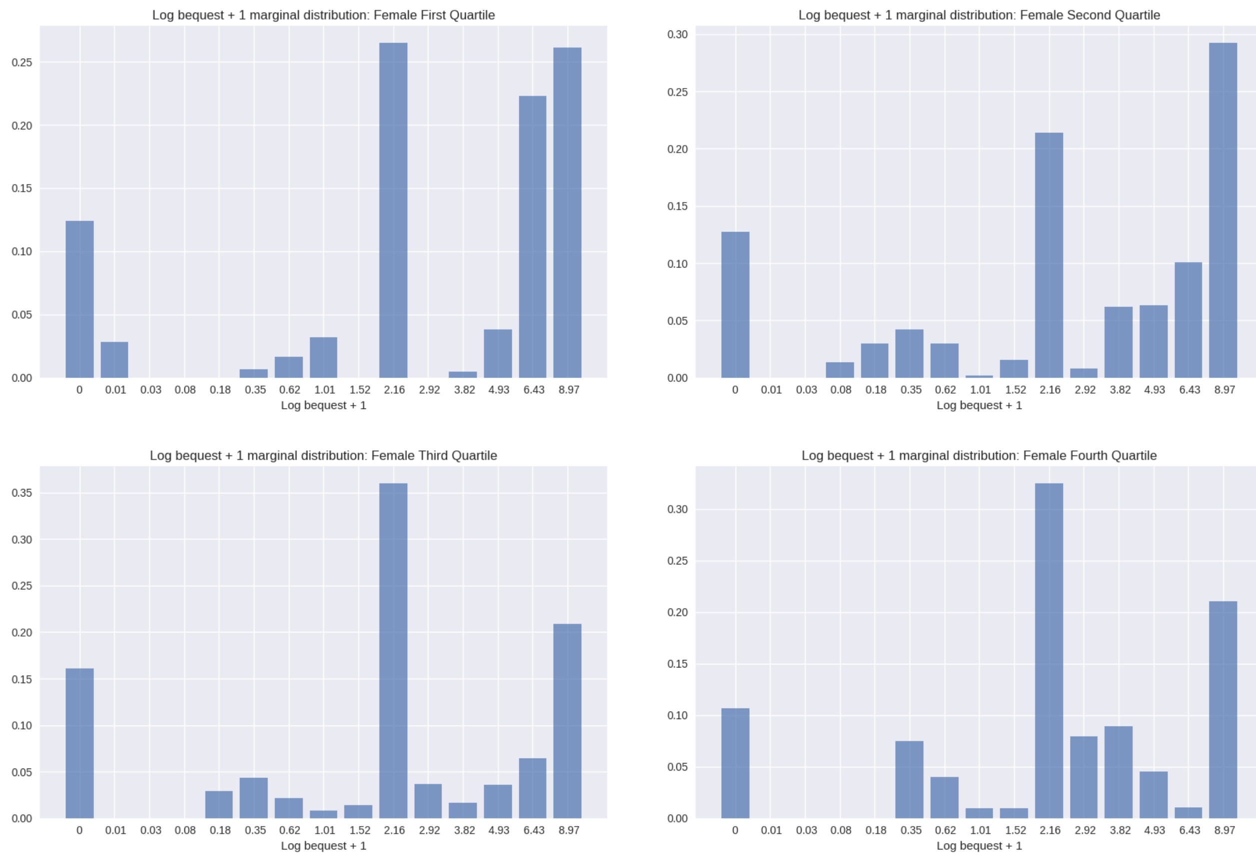


Figure 19: Marginal Distribution of Bequest Motive - Females

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
Bequest Motive	1.00	0.66	-0.12	-0.22
Risk Aversion	0.66	1.00	-0.49	-0.45
Outside Wealth	-0.12	-0.49	1.00	0.23
Health Shifter	-0.22	-0.45	0.23	1.00

Table 13: Correlations Across Dimensions of Unobserved Type - Fourth Quartile Females

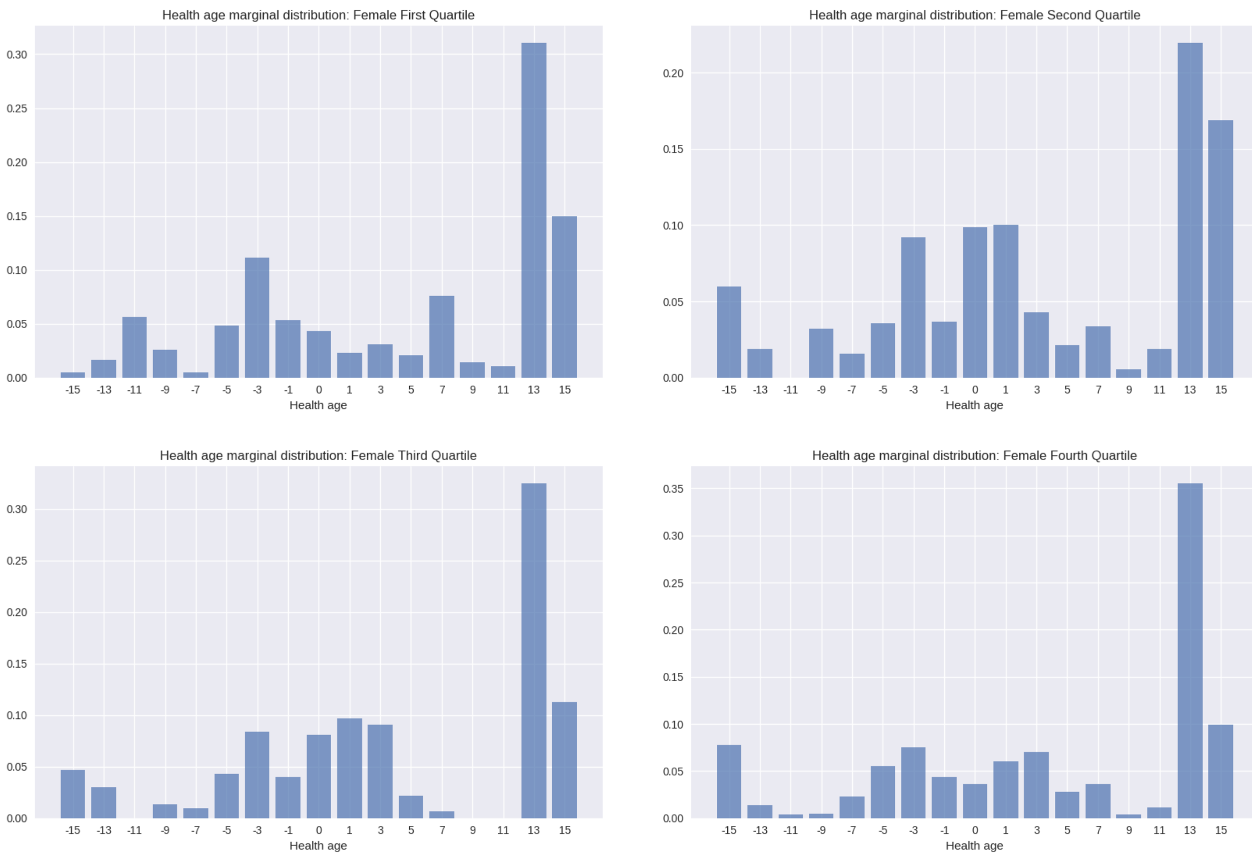


Figure 20: Marginal Distribution of Health Shifter - Females

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
Bequest Motive	1.00	0.45	0.45	-0.17
Risk Aversion	0.45	1.00	-0.05	-0.61
Outside Wealth	0.45	-0.05	1.00	-0.05
Health Shifter	-0.17	-0.61	-0.05	1.00

Table 14: Correlations Across Dimensions of Unobserved Type - First Quartile Males

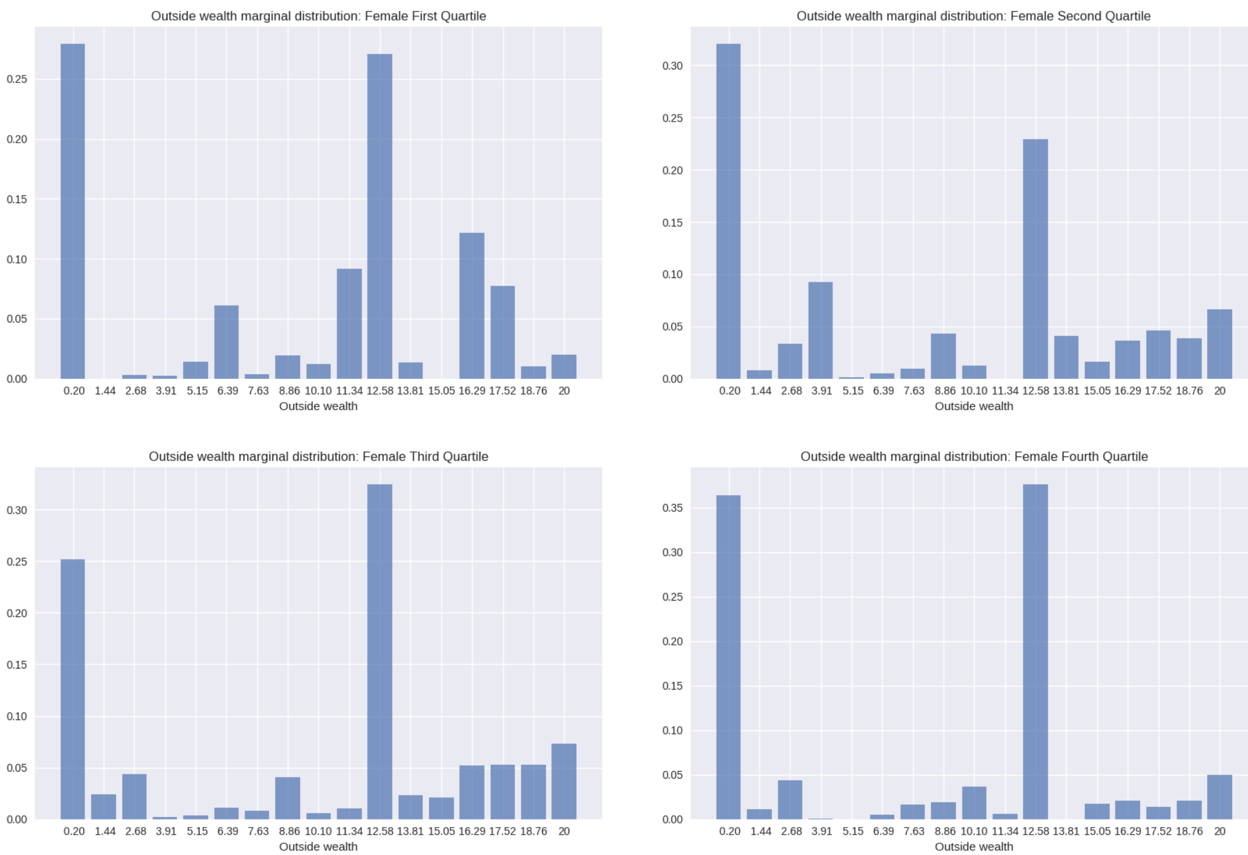


Figure 21: Marginal Distribution of Outside Wealth - Females

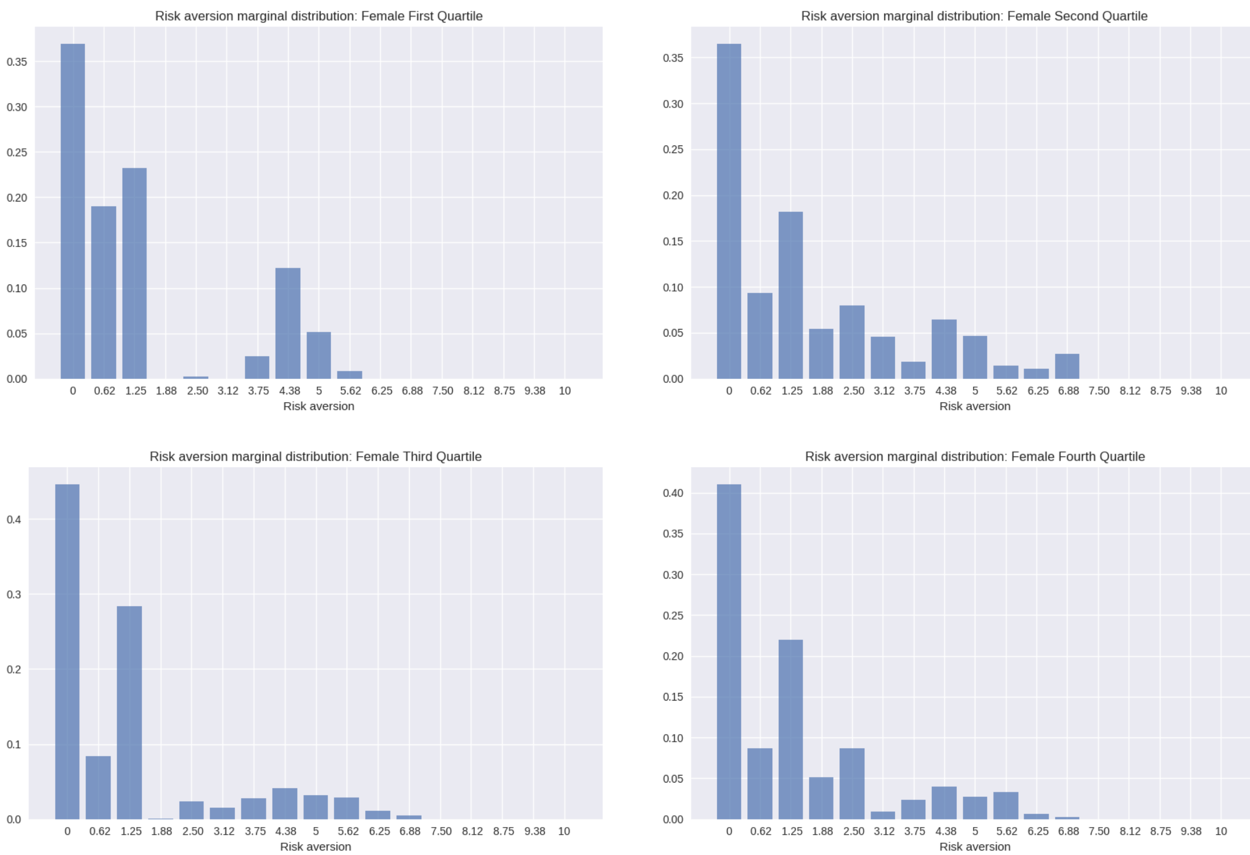
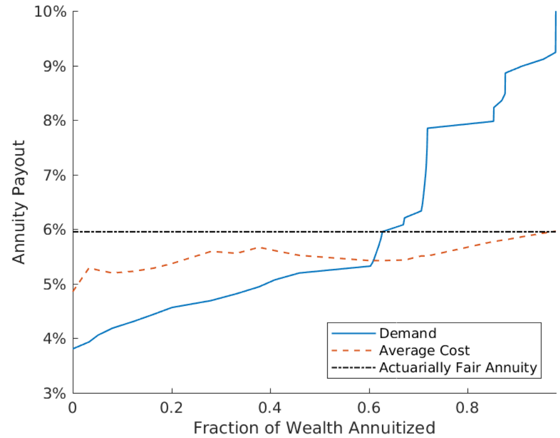
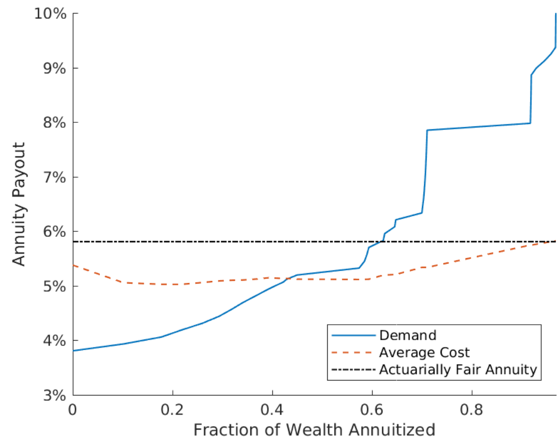


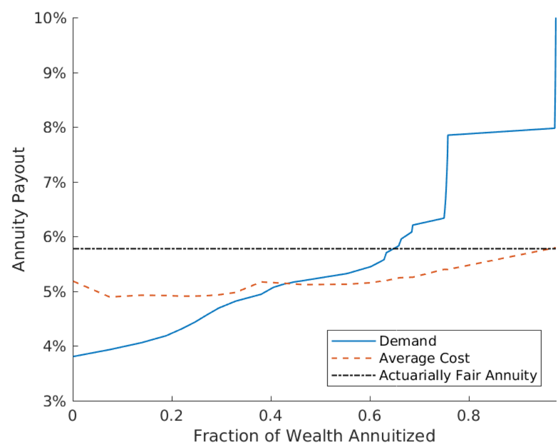
Figure 22: Marginal Distribution of Risk Aversion - Females



(a) First Quartile

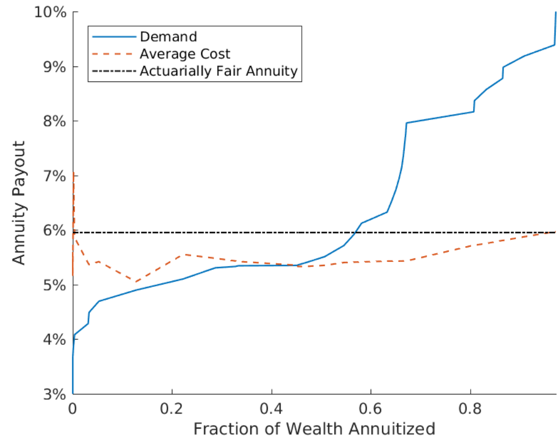


(b) Second Quartile

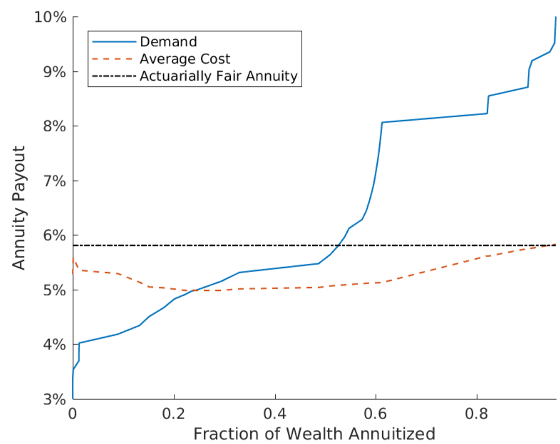


(c) Third Quartile

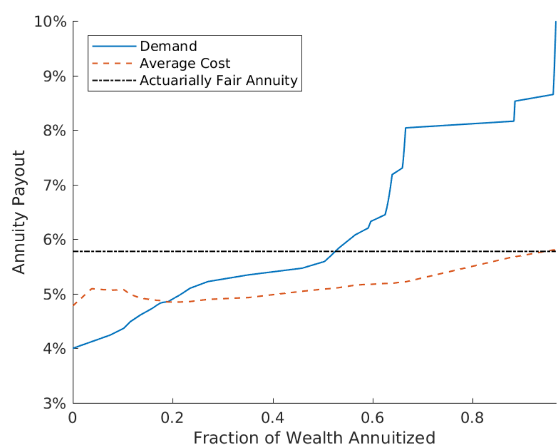
Figure 23: Simulated Equilibria under Chilean System - Females



(a) First Quartile

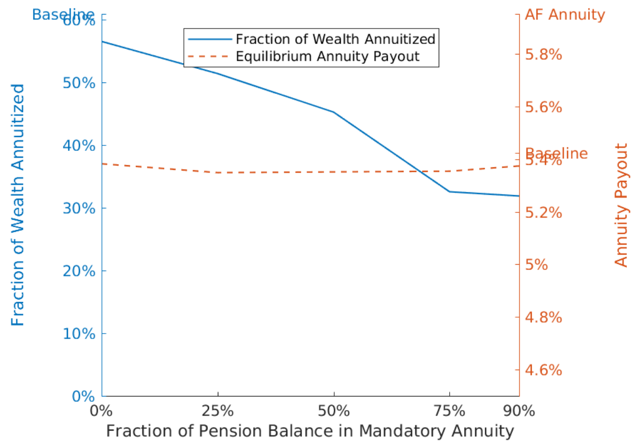


(b) Second Quartile

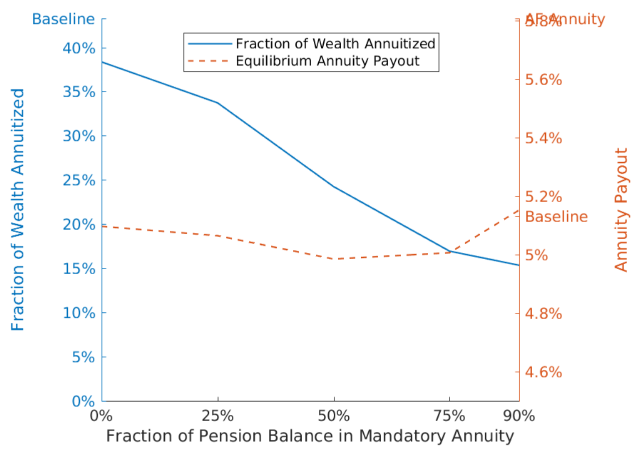


(c) Third Quartile

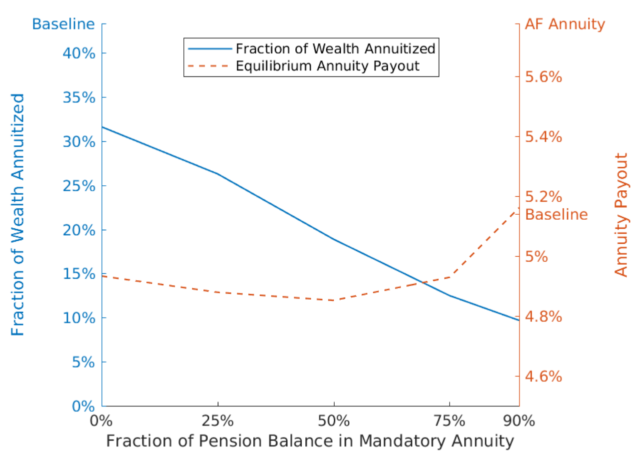
Figure 24: Simulated Equilibrium with 50% Mandatory Annuitization and Lump-Sum Withdrawal - Females



(a) First Quartile



(b) Second Quartile



(c) Third Quartile

Figure 25: Simulated equilibria for different levels of wealth in the mandatory annuity - Females

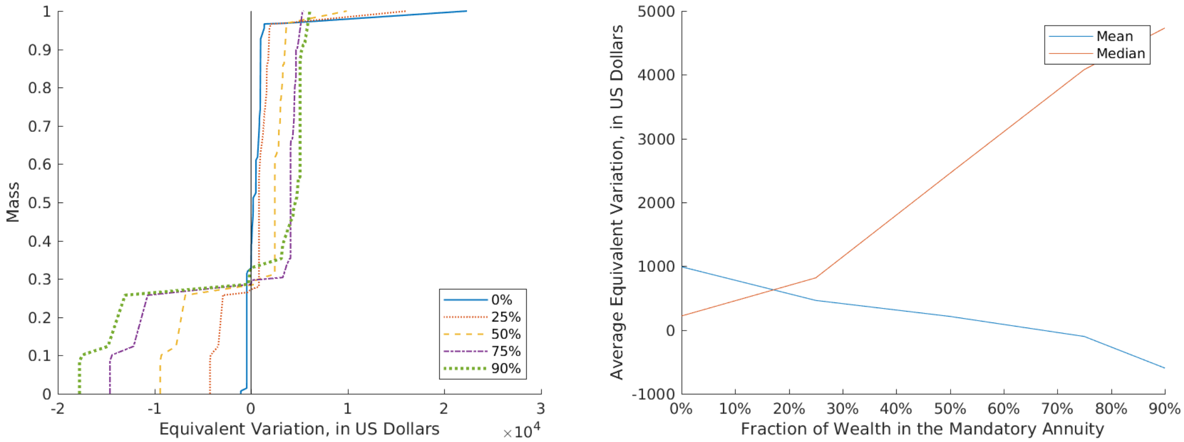


Figure 26: CDF of equivalent variation (left) and mean equivalent variation (right), for first quartile females, across different amounts in the mandatory annuity.

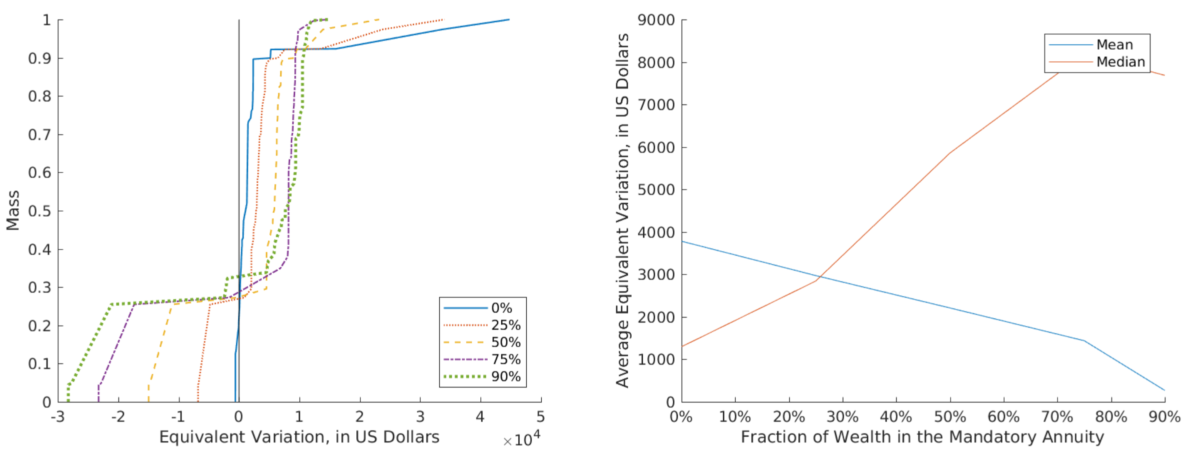
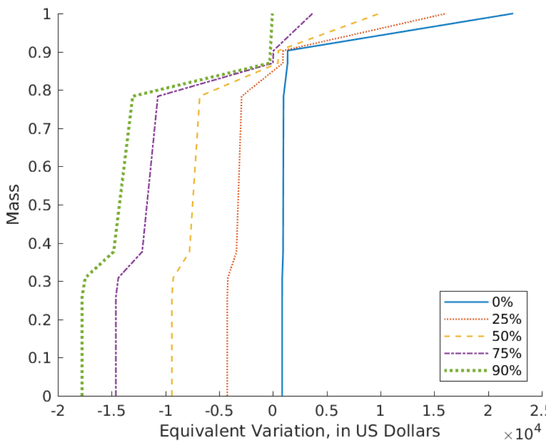
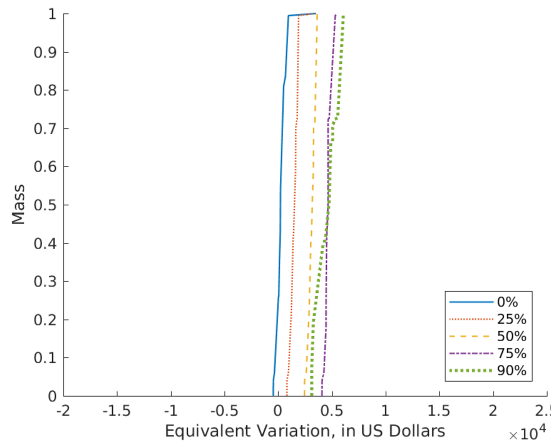


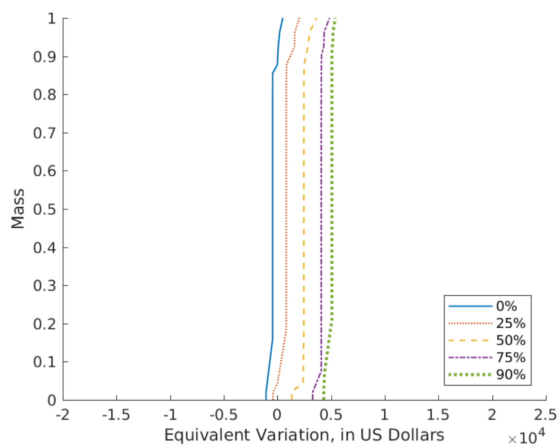
Figure 27: CDF of equivalent variation (left) and mean equivalent variation (right), for second quartile females, across different amounts in the mandatory annuity.



(a) No Annuitization at Baseline

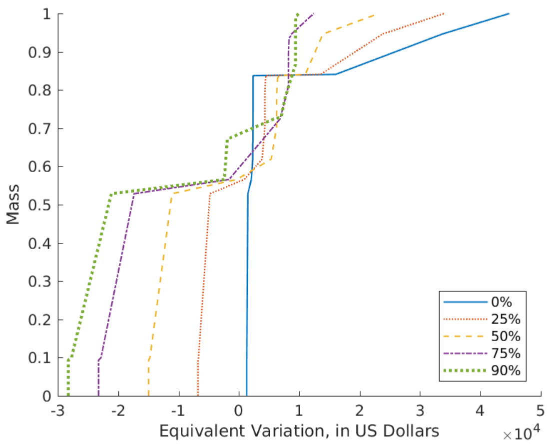


(b) Partial Annuitization at Baseline

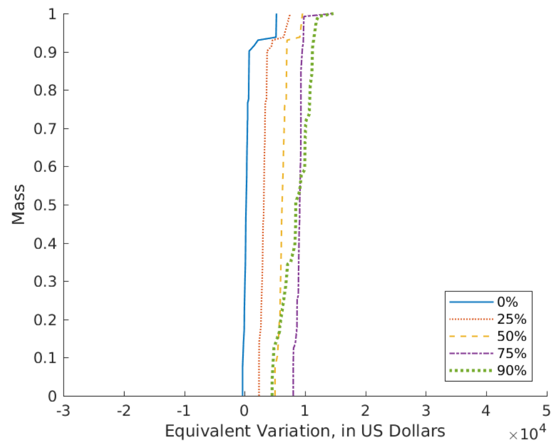


(c) Full Annuitization at Baseline

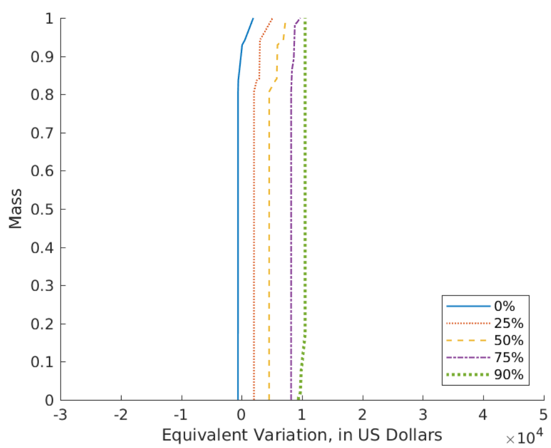
Figure 28: CDF of equivalent variation for first quartile females, across different amounts in the mandatory annuity, divided by annuitization at baseline.



(a) No Annuitization at Baseline

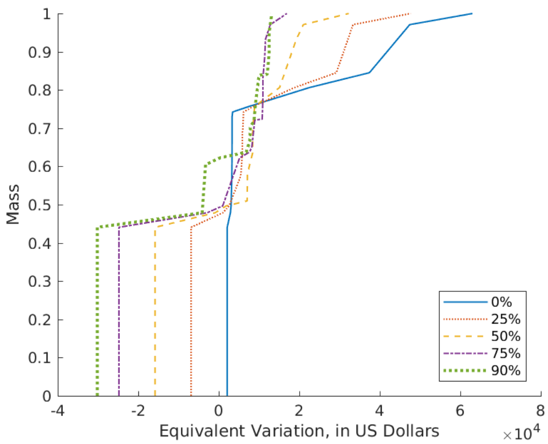


(b) Partial Annuitization at Baseline

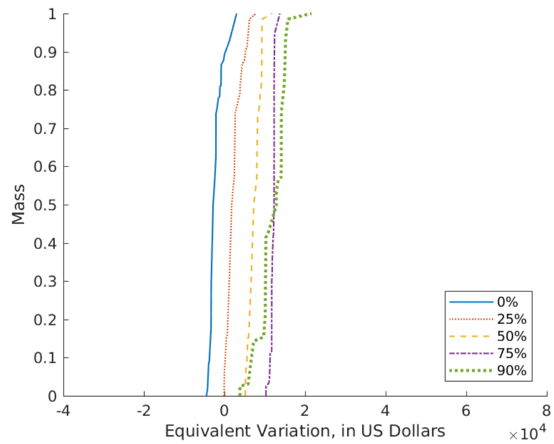


(c) Full Annuitization at Baseline

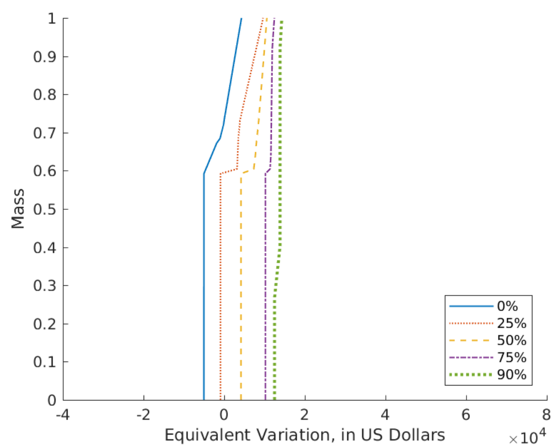
Figure 29: CDF of equivalent variation for second quartile females, across different amounts in the mandatory annuity, divided by annuitization at baseline.



(a) No Annuitization at Baseline



(b) Partial Annuitization at Baseline



(c) Full Annuitization at Baseline

Figure 30: CDF of equivalent variation for third quartile females, across different amounts in the mandatory annuity, divided by annuitization at baseline.

D Model: For Online Publication

This appendix section presents the detailed explanation of how the values of annuity and programmed withdrawal offers are calculated. It is divided into four subsections. The first derives the Euler equations for the annuity problem; the second derives the Euler equations for the PW problem; the third presents the computational details of how to solve the annuity problem; and the fourth does the same for the PW problem.

D.1 Derivations for the Annuity Problem

Consider the problem presented in Equation 3. For expositional clarity, we ignore the no borrowing constraint and derive a solution in an unconstrained setting, and then bring the constraint back in. It is well known that the problems of the previous form can be re-written recursively. In any arbitrary period t, the value of the remaining consumption problem given the current death state d_t, bankruptcy state b_t and liquid assets m_t is $V_t(d_t,q_t,m_t)$, and the Bellman equations are:

$$V_t(d_t,q_t,m_t) = \max_{c_t(d_t,q_t)} \frac{c_t(d_t,q_t)^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_t(d_t,q_t)' \begin{bmatrix} E_t\left[V_{t+1}(0,0,m_{t+1})\right] \\ E_t\left[V_{t+1}(0,1,m_{t+1})\right] \\ E_t\left[V_{t+1}(1,0,m_{t+1})\right] \\ E_t\left[V_{t+1}(1,1,m_{t+1})\right] \end{bmatrix}$$

where $\Gamma_t(0,0) = \begin{bmatrix} (1-\mu_{t+1})(1-\psi_{t+1}) \\ (1-\mu_{t+1})\psi_{t+1} \\ \mu_{t+1}(1-\psi_{t+1}) \\ \mu_{t+1}\psi_{t+1} \end{bmatrix}$, $\Gamma_t(0,1) = \begin{bmatrix} 0 \\ (1-\mu_{t+1}) \\ 0 \\ \mu_{t+1} \end{bmatrix}$, $\Gamma_t(1,0) = \begin{bmatrix} 0 \\ 0 \\ (1-\psi_{t+1}) \\ \psi_{t+1} \end{bmatrix}$, and $\Gamma_t(1,1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, and each equation is subject to the appropriate dynamic budget constraints and transition rules. We can simplify the previous equation by noting that there is no optimization after death, so for the absorbing state $(d_t = 1, q_t = 1)$ we have that:

$$V_t(1,1,m_t) = \beta \frac{[m_t + PDV_t^z(1,1,D,G)]^{1-\gamma}}{1-\gamma}$$
$$E_t\left[V_{t+1}(1,1,m_{t+1})\right] = \beta \frac{\left[m_{t+1} + PDV_{t+1}^z(1,1,D,G)\right]^{1-\gamma}}{1-\gamma}$$

where $PDV_t^z(1,1,D,G) = \sum_{\tau=t+1}^{G+D} R^{t-\tau} \cdot z_\tau(1,1,D,G)$ is the PDV in period t of the payment stream of the guarantee period from $t+1$ to $G+D$.

The expressions are similar in the "dead but not bankrupt" case $(d_t = 1, q_t = 0)$, but take into account that for guaranteed annuities there is uncertainty in the value of future payments:

$$V_t(1,0,m_t) = \beta \frac{[m_t + E[PDV_t^z(1,0,D,G)]]^{1-\gamma}}{1-\gamma}$$
$$E_t\left[V_{t+1}(1,0,m_{t+1})\right] = \beta \frac{\left[m_{t+1} + E[PDV_{t+1}^z(1,0,D,G)]\right]^{1-\gamma}}{1-\gamma}$$

where $E[PDV_t^z(1,0,D,G)]$ is the expected present value in t of the payment stream of the guarantee period from $t+1$ to $G+D$:

$$E[PDV_t^z(1,0,D,G)] = \sum_{\tau=t+1}^{G+D} R^{t-\tau} \cdot ((1-\Psi_\tau) \cdot z_\tau(1,0,D,G) + \Psi_\tau \cdot z_\tau(1,1,D,G))$$
$$\Psi_\tau = \sum_{\kappa=t+1}^{\tau} \left(\prod_{\tilde{\kappa}=t+1}^{\kappa-1} (1-\psi_{\tilde{\kappa}}) \right) \psi_\kappa$$

and Ψ_τ is the probability that the firm is bankrupt in $\tau > t$, conditional on not being bankrupt in t. As for the remaining states (when the individual is alive), the FOCs from (D.1) are:

$$c_t(0,q_t)^{-\gamma} = \delta \cdot R \cdot \Gamma_t(0,q_t)' \begin{bmatrix} E_t\left[V'_{t+1}(0,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(0,1,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,1,m_{t+1})\right] \end{bmatrix}$$

We know that:

$$E_t\left[V'_{t+1}(1,0,m_{t+1})\right] = \beta \cdot [m_{t+1} + \sum_{\tau=t+1}^{G+D} R^{t-\tau} \cdot ((1-\Psi_\tau) \cdot z_\tau(1,0,D,G) + \Psi_\tau \cdot z_\tau(1,1,D,G))]^{-\gamma}$$
$$E_t\left[V'_{t+1}(1,1,m_{t+1})\right] = \beta \cdot \left[m_{t+1} + \sum_{\tau=t+1}^{G+D} R^{t-\tau} \cdot z_\tau(1,1,D,G)\right]^{-\gamma}$$

Also, from the Envelope Theorem:

$$V'_t(0,q_t,m_t) = \delta \cdot R \cdot \Gamma_t(0,q_t)' \begin{bmatrix} E_t\left[V'_{t+1}(0,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(0,1,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,1,m_{t+1})\right] \end{bmatrix}$$

Combining (D.1) and (D.1), and rolling the equation forward by one year:

$$\begin{aligned} c_t(0,q_t)^{-\gamma} &= V_t'(0,q_t,m_t) \\ c_{t+1}(0,q_{t+1})^{-\gamma} &= V_{t+1}'(0,q_{t+1},a_t \cdot R + z_{t+1}(0,q_{t+1},D,G)) \end{aligned}$$

Substituting back into (D.1) yields the Euler equation:

$$c_t(0,q_t)^{-\gamma} = \delta \cdot R \cdot \Gamma_t(0,q_t)' \begin{bmatrix} E_t\left[c_{t+1}(0,0)^{-\gamma}\right] \\ E_t\left[c_{t+1}(0,1)^{-\gamma}\right] \\ E_t\left[V_{t+1}'(1,0,m_{t+1})\right] \\ E_t\left[V_{t+1}'(1,1,m_{t+1})\right] \end{bmatrix}$$

Following Carroll (2012), note that in equation (D.1) neither m_t nor c_t has any direct effect on V_{t+1}'. Instead, it is their difference, a_t, which enters into the function. This motivates the use of the Endogenous Gridpoint Method to approximate the optimal policy and value functions, as is derived in subsection D.3. Before moving to computation, however, the next section presents the analogous derivation for the PW problem.

D.2 Derivations for the PW Problem

Consider for now the problem free of borrowing constraint. As before, utility is CRRA, and in each state is given by:

$$\begin{aligned} u(c_t,d_t=0) &= \frac{c_t^{1-\gamma}}{1-\gamma} \\ u(d_t=1) &= \beta \cdot \frac{(m_t+PW_t)^{1-\gamma}}{1-\gamma} \end{aligned}$$

As in the annuity case, to obtain the value of taking a PW offer we re-write the problem in recursive form. The Bellman equation for the PDV of expected utility under the optimal state-contingent consumption path, for any period t, given the death state, PW account balance, and asset balance, denoted by $V_t(d_t, PW_t, m_t)$, is:

$$V_t(d_t = 0, PW_t, m_t) = \max_{c_t} \frac{c_t^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_t' \begin{bmatrix} E_t\left[V_{t+1}(0, m_{t+1}, PW_t)\right] \\ E_t\left[V_{t+1}(1, m_{t+1}, PW_t)\right] \end{bmatrix}$$

where $\Gamma_t = \begin{bmatrix} 1-\mu_{t+1} \\ \mu_{t+1} \end{bmatrix}$ and, as before, the problem is constrained by dynamic budget constraints and transition rules. Since there is no optimization after death, and inheritors receive the full PW balance, for the absorbing state $d_t = 1$ we have that:

$$V_t(1, m_t, PW_t) = \beta \frac{\left[m_t + PW_t\right]^{1-\gamma}}{1-\gamma}$$

Therefore we can write the expected continuation value for the death state as:

$$E_t[V_{t+1}(1, m_{t+1}, PW_t)] = \frac{\beta}{1-\gamma} \int \left[m_{t+1} + (PW_t - z_t(PW_t)) \cdot R^{PW}\right]^{1-\gamma} dF(R^{PW})$$

For the state where the individual is alive, the expected continuation value is:

$$E_t[V_{t+1}(0, m_{t+1}, PW_t)] = \int V_{t+1}(0, (PW_t - z_t(PW_t)) \cdot R^{PW}, m_{t+1}) dF(R^{PW})$$

With these definitions, the FOCs from (D.2) are:

$$c_t^{-\gamma} = \delta \cdot R \cdot \Gamma_t' \begin{bmatrix} E_t\left[V_{t+1}'(0, m_{t+1}, PW_t)\right] \\ E_t\left[V_{t+1}'(1, m_{t+1}, PW_t)\right] \end{bmatrix}$$

We know that:

$$E_t\left[V'_{t+1}(1,m_{t+1},PW_t)\right] = \beta \cdot R \int \left[m_{t+1} + (PW_t - z_t(PW_t)) \cdot R^{PW}\right]^{-\gamma} dF(R^{PW})$$

Also, from the Envelope Theorem:

$$V'_t(0,m_t) = \delta \cdot R \cdot \Gamma'_t \begin{bmatrix} E_t\left[V'_{t+1}(0,m_{t+1},PW_t)\right] \\ E_t\left[V'_{t+1}(1,m_{t+1},PW_t)\right] \end{bmatrix}$$

Combining (D.2) and (D.2), and rolling the equation forward by one year:

$$c_t^{-\gamma} = V'_t(0,m_t)$$
$$c_{t+1}^{-\gamma} = V'_{t+1}(0,m_{t+1},PW_{t+1})$$

Substituting back into (D.2) yields the Euler equation:

$$c_t^{-\gamma} = \delta \cdot R \cdot \Gamma'_t \begin{bmatrix} E_t\left[c_{t+1}^{-\gamma}\right] \\ E_t\left[V'_{t+1}(1,m_{t+1},PW_t)\right] \end{bmatrix}$$

D.3 Computation of the Solution to the Annuity Problem

Having derived the conditions that govern the optimal consumption policy and the value functions for both problems, this subsection presents the details of the numerical procedure used to solve these conditions. Since the problem is solved recursively, we will begin with the solution for period T and work our way backwards. In period T, $\mu_T = 1$ and $T > G+D$, so $m_T = a_{T-1} \cdot R$ and regardless of the bankruptcy state q_T:

$$V_T(0, q_T, m_T) = \beta \cdot \frac{m_T^{1-\gamma}}{1-\gamma}$$

Then in the next-to-last period:

$$V_{T-1}(0, q_{T-1}, m_{T-1}) = \max_{c_{T-1}} \frac{c_{T-1}^{1-\gamma}}{1-\gamma} + \delta \cdot \beta \cdot \frac{((m_{T-1} - c_{T-1}) \cdot R)^{1-\gamma}}{1-\gamma}$$

Which generates the optimal policy:

$$c_{T-1}^{-\gamma} = \delta \cdot \beta \cdot R^{1-\gamma} \cdot (m_{T-1} - c_{T-1})^{-\gamma}$$
$$c_{T-1}(0, q_{T-1}, m_{T-1}) = \frac{R}{((\delta \cdot \beta \cdot R)^{\frac{1}{\gamma}} + R)} \cdot m_{T-1}$$

And implies that the value function in $T-1$ is:

$$V_{T-1}(0, q_{T-1}, m_{T-1}) = \left(\frac{1 + (\delta \cdot \beta \cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma}\right) \left(\frac{R \cdot m_{T-1}}{(\delta \cdot \beta \cdot R)^{1/\gamma} + R}\right)^{1-\gamma}$$

Note that conditional on m_{T-1}, there is no dependence on q_{T-1}. That is, q_{T-1} will shift m_{T-1}, as $m_{T-1} = a_{T-2} \cdot R + z_{T-1}(0, q_{T-1}, D, G)$, but conditional on m_{T-1} it becomes irrelevant. Therefore, given a grid of m_{T-1} one could easily solve for $V_{T-1}(m_{T-1})$, and the value of m_{T-1}'s for other values would be found by interpolation/extrapolation. Note as well that as long as the bequest motive is positive the no-borrowing constraint can be omitted from this stage without loss as the the unconstrained solution always satisfies $c_{T-1} < m_{T-1}$.

Having solved for all the relevant quantities in $T-1$ and T, let us consider the unconstrained problem in $T-2$. From the Euler condition in (D.1) and the optimal policy in (D.3):

$$c_{T-2}(0,q_t)^{-\gamma} = \delta \cdot R \cdot \Gamma_{T-2}(0,q_t)' \begin{bmatrix} E_t\left[c_{T-1}(0,0)^{-\gamma}\right] \\ E_t\left[c_{T-1}(0,1)^{-\gamma}\right] \\ E_t\left[V'_{T-1}(1,0,m_{T-1})\right] \\ E_t\left[V'_{T-1}(1,1,m_{T-1})\right] \end{bmatrix}$$

$$= \delta \cdot R \cdot \Gamma_{T-2}(0,q_t)' \begin{bmatrix} \left(\frac{R}{((\delta\cdot\beta\cdot R)^{1/\gamma}+R)}\right)^{-\gamma}((m_{T-2}-c_{T-2}(0,q_{T-2}))\cdot R + z_{T-1}(0,0,D,G))^{-\gamma} \\ \left(\frac{R}{((\delta\cdot\beta\cdot R)^{1/\gamma}+R)}\right)^{-\gamma}((m_{T-2}-c_{T-2}(0,q_{T-2}))\cdot R + z_{T-1}(0,1,D,G))^{-\gamma} \\ \beta\cdot\left[(m_{T-2}-c_{T-2}(0,q_{T-2}))\cdot R + z_{T-1}(1,0,D,G) + E[PDV^z_{T-1}(1,0,D,G)]\right]^{-\gamma} \\ \\ \beta\cdot\left[(m_{T-2}-c_{T-2}(0,q_{T-2}))\cdot R + z_{T-1}(1,1,D,G) + E[PDV^z_{T-1}(1,1,D,G)]\right]^{-\gamma} \end{bmatrix}$$

Unfortunately, this is a non-linear system of equations. To find the value function in $T-2$, one could fix a grid of m_{T-2}, and for each point in the grid solve for optimal consumption and obtain the value function. Interpolation across m's would yield the value function for any m_{T-2}. Note also that the previous derivation is also valid for $0 < t < T-2$, so backward induction would allow us to unwind this problem and construct the value function in period 1. The problem in period 0 is slightly different, as the state is $(0,0)$ and wealth is $\omega + z_0(0,0,D,G) + FDA$ with certainty[39], but the same tools apply.

One issue we've abstracted away from up to now is the no-borrowing constraint: $a_{T-1} \geq 0$. Incorporating this constraint implies that when m_{T-1} is sufficiently low, consumption will not be the solution to the aforementioned problem, but rather m_{T-1} itself. This creates a discontinuity in the optimal policy function. Since our approximations to the optimal policy and value functions are constructed by interpolation, it is crucial to incorporate the point where the discontinuity takes place into the grid of points to be evaluated. This ensures that the no-borrowing constraint is properly accounted for in the model. At the point where the no-borrowing constraint binds, $\hat{m}_{T-1}$, the

[39]Recall that FDA is the free disposal amount, another attribute of an annuity offer. In most cases, it is 0.

marginal value of consuming m_{T-1} must be equal to the marginal utility of saving 0.

We use the Endogenous Gridpoints Method (Carroll (2006)) to find the solution to the aforementioned problem. At a high level, the strategy is to solve the model for a grid of asset states, and then to interpolate across states to obtain the policy function and the value function. EGM allows us to solve the model efficiently, by re-writing the problem in a way that allows us to back out a solution using an inversion rather than root-finding. The details of the implementation for $T-2$ are presented below:

<u>Numerical Calculation of Policy Function in $T-2$:</u>

1. Select a grid of a_{T-2} with support $[0, \bar{a}_{T-2}]$, where:

$$\bar{a}_{T-2} = R^{T-2}\omega + \sum_{\tau=0}^{T-2} R^{T-2-\tau} z_{\tau}(0,0,D,G)$$

2. Calculate the relevant quantities for the unconstrained problem:

$$
\begin{aligned}
&m_{T-1}(d_{T-1},q_{T-1},D,G) = a_{T-2}\cdot R + z_{T-1}(d_{T-1},q_{T-1},D,G))\\
&\mathfrak{c}_{T-1}(0,q_{T-1}) = \left(\frac{R}{((\delta\cdot\beta\cdot R)^{1/\gamma}+R)}\cdot m_{T-1}(0,q_{T-1},D,G)\right)^{-\gamma}\\
&c_{T-2}(0,q_{T-2}) = \left[\delta\cdot R\cdot\Gamma_{T-2}(0,q_{T-2})'\begin{bmatrix}\mathfrak{c}_{T-1}(0,0)\\ \mathfrak{c}_{T-1}(0,1)\\ \beta\cdot[m_{T-1}(1,0,D,G)]^{-\gamma}\\ \beta\cdot[m_{T-1}(1,1,D,G)]]^{-\gamma}\end{bmatrix}\right]^{-\frac{1}{\gamma}}\\
&\mathfrak{c}_{T-2}(0,q_{T-2}) = c_{T-2}(0,q_{T-2})^{-\gamma}\\
&m_{T-2}(0,q_{T-2}) = c_{T-2}(0,q_{T-2}) + a_{T-2}\\
&V_{T-1}(0,q_{T-1}) = \left(\frac{1+(\delta\cdot\beta\cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma}\right)\left(\frac{R\cdot m_{T-1}(0,q_{T-1})}{(\delta\cdot\beta\cdot R)^{1/\gamma}+R}\right)^{1-\gamma}\\
&V_{T-1}(1,q_{T-1}) = \beta\left(\frac{m_{T-1}(1,q_{T-1})^{1-\gamma}}{1-\gamma}\right)\\
&V_{T-2}(0,q_{T-2}) = \frac{c_{T-2}(0,q_{T-2})^{1-\gamma}}{1-\gamma} + \delta\cdot\Gamma_{T-2}(0,q_{T-2})\begin{bmatrix}V_{T-1}(0,0)\\ V_{T-1}(0,1)\\ V_{T-1}(1,0)\\ V_{T-1}(1,1)\end{bmatrix}
\end{aligned}
$$

3. Denote $\hat{m}_{T-2}(0,q_{T-2})$ the solution to equation (2) when $a_{T-2,i}=0$. This is the lowest level

of wealth that is unconstrained. Define

$$
\begin{aligned}
&\hat{V}_{T-1}(0,q_{T-1}) = \left(\frac{1+(\delta\cdot\beta\cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma}\right)\left(\frac{R\cdot z_{T-1}(0,q_{T-1},D,G)}{(\delta\cdot\beta\cdot R)^{1/\gamma}+R}\right)^{1-\gamma} \\
&\hat{V}_{T-1}(1,q_{T-1}) = \beta\left(\frac{z_{T-1}(1,q_{T-1},D,G)^{1-\gamma}}{1-\gamma}\right) \\
&\grave{\mathfrak{c}}_{T-2,j}(0,q_{T-2}) = m_{T-2,j}^{-\gamma} \\
&\hat{V}_{T-2,j}(0,q_{T-2},m_{T-2}) = \frac{m_{T-2,j}^{1-\gamma}}{1-\gamma} + \delta\cdot\Gamma_{T-2}(0,q_{T-2})\begin{bmatrix}\hat{V}_{T-1}(0,0)\\ \hat{V}_{T-1}(0,1)\\ \hat{V}_{T-1}(1,0)\\ \hat{V}_{T-1}(1,1)\end{bmatrix}
\end{aligned}
$$

4. Use interpolation to obtain $\grave{\mathfrak{c}}_{T-2}(0,q_{T-2},m_{T-2})$, $\grave{\hat{\mathfrak{c}}}_{T-2,j}(0,q_{T-2})$, $\grave{V}_{T-2,}(0,q_{T-2},m_{T-2})$, and $\grave{\hat{V}}_{T-2,}(0,q_{T-2},m_{T-2})$ for the unconstrained problem.

5. Correct for the no-borrowing constraint by constructing a part exact, part interpolated policy and value function for this period [40]

$$
\grave{\mathfrak{c}}^*_{T-2}(0,q_{T-2},m_{T-2}) = \begin{cases} m_{T-2}^{-\gamma} & \text{if } m_{T-2} < \hat{m}_{T-2}(0,q_{T-2}) \\ \grave{\mathfrak{c}}_{T-2}(0,q_{T-2},m_{T-2}) & \text{otherwise} \end{cases}
$$

$$
\grave{V}^*_{T-2}(0,q_{T-2},m_{T-2}) = \begin{cases} \hat{V}_{T-2,}(0,q_{T-2},m_{T-2}) & \text{if } m_{T-2} < \hat{m}_{T-2}(0,q_{T-2}) \\ \grave{V}_{T-2,}(0,q_{T-2},m_{T-2}) & \text{otherwise} \end{cases}
$$

There are three issues worth discussing in this procedure: first, we assume that individuals cannot borrow against future annuity payments (the lower bound of a is 0). This is consistent with our knowledge of the Chilean banking system. Second, we set the upper bound of the support of assets as the PDV of initial wealth plus the PDV of the maximum sequence of previous annuity payments. This ensures that the grid of a's spans the optimal asset value in $T-2$, as in the model

[40] Note that the solution objects for the $T-2$ problem are exact when the constraint binds.

the agent cannot accumulate more wealth than this value. Third, we interpolate over $\mathfrak{c}(\cdot)$ instead of $c(\cdot)$. This is suggested by Carroll (2011), as the function that enters into the recursion in earlier periods is $\mathfrak{c}(\cdot)$, and not $c(\cdot)$. One could interpolate over $c(\cdot)$, and then raise the interpolated value to the power of $-\frac{1}{\gamma}$, but that is less accurate is simply interpolating over $\grave{\mathfrak{c}}$. With these objects, we can solve the problem for $T-3, T-4, ..., 0$ by recursion.

<u>Numerical Calculation of Policy Function in t:</u>

1. Select a grid of a_t with support $[0, \bar{a}_t]$:

$$\bar{a}_t = R^t \omega + \sum_{\tau=0}^{t} R^{t-\tau} z_\tau(0,0,D,G)$$

2. Calculate the relevant quantities for the unconstrained problem (suppressing the dependence

on D and G to simplify notation):

$$m_{t+1}(0,q_{t+1}) = a_t \cdot R + z_{t+1}(0,q_{t+1})$$

$$c_t(0,q_t) = \left[\delta \cdot R \cdot \Gamma_t(0,q_t)' \begin{bmatrix} \grave{\mathfrak{c}}^*_{t+1}(0,0,m_{t+1}(0,0)) \\ \grave{\mathfrak{c}}^*_{t+1}(0,1,m_{t+1}(0,0)) \\ \beta \cdot \left[m_{t+1}(1,0) + E[PDV^z_{t+1}(1,0,D,G)]\right]^{-\gamma} \\ \beta \cdot \left[m_{t+1}(1,1) + E[PDV^z_{t+1}(1,1,D,G)]\right]^{-\gamma} \end{bmatrix}\right]^{-\frac{1}{\gamma}}$$

$$\mathfrak{c}_t(0,q_{T-2}) = c_t(0,q_{T-2})^{-\gamma}$$

$$m_t(0,q_t) = c_t(0,q_t) + a_t$$

$$V_{t+1}(1,q_{t+1}) = \beta \left(\frac{m_{t+1}(1,q_{t+1})^{1-\gamma}}{1-\gamma}\right)$$

$$V_t(0,q_t) = \frac{c_t(0,q_t)^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_t(0,q_t) \begin{bmatrix} \grave{V}^*_{t+1}(0,0,m_{t+1}) \\ \grave{V}^*_{t+1}(0,1,m_{t+1}) \\ V_{t+1}(1,0) \\ V_{t+1}(1,1) \end{bmatrix}$$

3. Define $\hat{m}_t(0,q_t)$ as the level of wealth obtained at $a_t = 0$ and

$$\hat{V}_{t+1}(1,q_{t+1}) = \beta \left(\frac{E[PDV^z_{t+1}(1,q_{t+1},D,G)]^{1-\gamma}}{1-\gamma}\right)$$

$$\hat{V}_t(0,q_t) = \frac{\hat{m}_t(0,q_t)^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_{T-2}(0,q_{T-2}) \begin{bmatrix} \grave{V}^*_{t+1}(0,0,z_{t+1}(0,0)) \\ \grave{V}^*_{t+1}(0,1,z_{t+1}(0,1)) \\ \hat{V}_{t+1}(1,0) \\ \hat{V}_{t+1}(1,1) \end{bmatrix}$$

4. Use interpolation to obtain $\grave{\mathfrak{c}}_t(0,q_t,m_t)$, $\grave{\grave{\mathfrak{c}}}_{t,j}(0,q_t)$, $\grave{V}_{t,}(0,q_t,m_t)$, and $\grave{\grave{V}}_{t,}(0,q_t,m_t)$ for the unconstrained problem.

5. Correct for the no-borrowing constraint:

$$\grave{\mathfrak{c}}_t^*(0,q_t,m_t) = \begin{cases} m_t^{-\gamma} & \text{if } m_t < \hat{m}_t(0,q_{t+1}) \\ \grave{\mathfrak{c}}_t(0,q_t,m_t) & \text{otherwise} \end{cases}$$

$$\grave{V}_t^*(0,q_t,m_t) = \begin{cases} \hat{V}_t(0,q_t,m_t) & \text{if } m_t < \hat{m}_t(0,q_t) \\ \grave{V}_t(0,q_t,m_t) & \text{otherwise} \end{cases}$$

6. Repeat for $t-1$

Note that again, the constrained segment requires no additional interpolation and hence its implementation is both efficient and precise.We can recover the object of interest (the value of an annuity offer: $V(0,0,\omega_i,D,G)$) after the $t=0$ step in the previous recursion.

D.4 Computation of the Solution to the PW Problem

In period T, $\mu_T = 1$ and $PW_T = 0$, so $m_T = a_{T-1} \cdot R$ and:

$$V_T(0,m_T,PW_T) = \beta \cdot \frac{m_T^{1-\gamma}}{1-\gamma}$$

Then in the next-to-last period:

$$V_{T-1}(0,m_{T-1},PW_{T-1}) = \max_{c_{T-1}} \frac{c_{T-1}^{1-\gamma}}{1-\gamma} + \frac{\delta \cdot \beta}{1-\gamma}((m_{T-1}-c_{T-1}) \cdot R)^{1-\gamma}$$

The optimal policy and value functions in $T-1$ are then:

$$c_{T-1}(m_{T-1}) = \frac{R}{((\delta \cdot \beta \cdot R)^{\frac{1}{\gamma}} + R)} \cdot m_{T-1}$$
$$V_{T-1}(0, m_{T-1}) = \left(\frac{1 + (\delta \cdot \beta \cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma}\right)\left(\frac{R \cdot m_{T-1}}{(\delta \cdot \beta \cdot R)^{1/\gamma} + R}\right)^{1-\gamma}$$

Note that, conditional on m_{T-1}, there is no dependence on PW_{T-1}. This is because PW_{T-1} will shift m_{T-1}, as $m_{T-1} = a_{T-2} \cdot R + z_{T-1}(PW_{T-1}, a)$, but conditional on m_{T-1} it becomes irrelevant. Additionally, as in the annuity problem, as long as the bequest motive is not negative the unconstrained maximizer satisfies the no-borrowing constraint.

Having solved for all the relevant quantities in $T-1$ and T, we can proceed to solve the problem in $T-2$. There are a few additional objects that need to be introduced before proceeding. First, take K draws from the distribution of R^{PW}. Each draw will be denoted by k, and draws will be held fixed across time periods. Define $\bar{R}_K$ as the largest draw from the distribution of R^{PW}. Second, define the upper bound of the grid of PW, $P\bar{W}$, recursively:

$$P\bar{W}_1 = \bar{R}_K \cdot (PW_0 - z_t(PW_0))$$
$$P\bar{W}_t = \bar{R}_K \cdot (P\bar{W}_{t-1} - z_t(P\bar{W}_{t-1}))$$

Finally, define the upper bound of the grid of accumulated assets as:

$$\bar{a}_t = R^t \omega + \sum_{\tau=0}^{t} R^{t-\tau} z(P\bar{W}_\tau, 0, f)$$

<u>Numerical Calculation of Policy Function in $T-2$:</u>

1. Select a grid of $(a_{T-2,i}, PW_{T-2,i})$ with support $[0, \bar{a}_{T-2}] \times [0, P\bar{W}_{T-2}]$.

2. Calculate the relevant quantities for the unconstrained problem:

$$
\begin{aligned}
m_{T-1,k}(0) &= a_{T-2} \cdot R + z_{T-1}(R_k^{PW} \cdot (PW_{T-2} - z(PW_{T-2}), 0, a) \\
m_{T-1,k}(1) &= a_{T-2} \cdot R + R_k^{PW} \cdot (PW_{T-2} - z(PW_{T-2})) \\
E_{T-2}[\mathfrak{c}_{T-1}] &= \frac{1}{K} \sum_{k=1}^{K} \left[c_{T-1}(m_{T-1,k}(0))\right]^{-\gamma} \\
E_{T-2}[V'_{T-1}(1)] &= \frac{\beta}{K} \sum_{k=1}^{K} \left[m_{T-1,k}(1)\right]^{-\gamma} \\
c_{T-2} &= \left[\delta \cdot R \cdot \Gamma'_{T-2} \begin{bmatrix} E_{T-2}[\mathfrak{c}_{T-1}] \\ E_{T-2}[V'_{T-1}(1)] \end{bmatrix}\right]^{-\frac{1}{\gamma}} \\
m_{T-2} &= c_{T-2} + a_{T-2} \qquad (1) \\
\mathfrak{c}_{T-2} &= c_{T-2}^{-\gamma} \\
E_{T-2}[V_{T-1}(0)] &= \left(\frac{1 + (\delta \cdot \beta \cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma}\right) \cdot \left(\frac{R}{(\delta \cdot \beta \cdot R)^{1/\gamma} + R}\right)^{1-\gamma} \frac{1}{K} \sum_{k=1}^{K} \left[m_{T-1,k}(0)\right]^{1-\gamma} \\
E_{T-2}[V_{T-1}(1)] &= \frac{\beta}{1-\gamma} \cdot \frac{1}{K} \sum_{k=1}^{K} \left[m_{T-1,k}(1)\right]^{1-\gamma} \\
V_{T-2} &= \frac{c_{T-2}^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma'_{T-2} \begin{bmatrix} E_{T-2}[V_{T-1}(0)] \\ E_{T-2}[V_{T-1}(1)] \end{bmatrix}
\end{aligned}
$$

3. Denote $\hat{m}_{T-2}(PW_{T-2})$ the solution to (1) when $a_{T-2} = 0$ and the PW balance is PW_{T-2} define

$$
\hat{V}_{T-2}(m_{T-2}, PW_{T-2}) = \frac{m_{T-2}^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma'_{T-2} \begin{bmatrix} E_{T-2}[V_{T-1}(0)] \\ E_{T-2}[V_{T-1}(1)] \end{bmatrix}
$$

with the value of V_{T-1} determined by $a_{T-2} = 0$.

4. Use interpolation to obtain $\grave{\mathfrak{c}}_{T-2}(m_{T-2}, PW_{T-2})$ and $\grave{V}_{T-2}(0, m_{T-2}, PW_{T-2})$ for the unconstrained problem. Form the boundary interpolator $\hat{\grave{m}}_{T-2}(PW_{T-2})$ which determines the minimum level of unconstrained wealth for each value of the PW balance.

5. Correct for the no-borrowing constraint by constructing a part exact, part interpolated policy and value function for this period [41]

$$\grave{\mathfrak{c}}^*_{T-2}(m_{T-2}, PW_{T-2}) = \begin{cases} m_{T-2}^{-\gamma} & \text{if } m_{T-2} < \hat{\grave{m}}_{T-2}(PW_{T-2}) \\ \grave{\mathfrak{c}}_{T-2}(m_{T-2}, PW_{T-2}) & \text{otherwise} \end{cases}$$

$$\grave{V}^*_{T-2}(m_{T-2}, PW_{T-2}) = \begin{cases} \hat{V}_{T-2,}(m_{T-2}, PW_{T-2}) & \text{if } m_{T-2} < \hat{\grave{m}}_{T-2}(PW_{T-2}) \\ \grave{V}_{T-2,}(m_{T-2}, PW_{T-2}) & \text{otherwise} \end{cases}$$

Armed with these objects, we can solve the problem for $T-3, T-4, ..., 0$ by recursion.

<u>Numerical Calculation of Policy Function in t:</u>

1. Select a grid of (a_t, PW_t) with support $[0, \bar{a}_t] \times [0, P\bar{W}_t]$.

[41] Note that the solution objects for the $T-2$ problem are exact when the constraint binds.

2. Calculate the relevant quantities for the unconstrained problem:

$$
\begin{aligned}
&m_{t+1,k}(0) = a_t \cdot R + z_{t+1}(R_k^{PW} \cdot (PW_t - z_t(PW_t)), 0, a) \\
&m_{t+1,k}(1) = a_t \cdot R + R_k^{PW} \cdot (PW_t - z_t(PW_t)) \\
&E_t[\mathfrak{c}_{t+1}] = \frac{1}{K} \sum_{k=1}^{K} \grave{\mathfrak{c}}_{t+1}(m_{t+1,k}(0), PW_{t+1,k}) \\
&E_t[V'_{t+1}(1)] = \frac{\beta}{K} \sum_{k=1}^{K} \left[m_{t+1,k}(1)\right]^{-\gamma} \\
&c_t = \left[\delta \cdot R \cdot \Gamma'_t \begin{bmatrix} E_t[\mathfrak{c}_{t+1}] \\ E_t[V'_{t+1}(1)] \end{bmatrix}\right]^{-\frac{1}{\gamma}} \\
&m_t = c_t + a_{t,i} \\
&\mathfrak{c}_t = c_t^{-\gamma} \\
&E_t[V_{t+1}(0)] = \frac{1}{K} \sum_{k=1}^{K} \grave{V}(0, m_{t+1,k}(0), R_k^{PW} \cdot (PW_t - z(PW_t))) \\
&E_t[V_{t+1}(1)] = \frac{\beta}{1-\gamma} \cdot \frac{1}{K} \sum_{k=1}^{K} \left[m_{t+1,k}(1)\right]^{1-\gamma} \\
&V_t = \frac{c_t^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma'_t \begin{bmatrix} E_t[V_{t+1}(0)] \\ E_t[V_{t+1}(1)] \end{bmatrix}
\end{aligned}
$$

3. Denote $\hat{m}_t(PW_t)$ the solution when $a_t = 0$ and the PW balance is PW_t define

$$
\hat{V}_t(m_t, PW_t) = \frac{m_t^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma'_t \begin{bmatrix} E_t[V_{t+1}(0)] \\ E_t[V_{t+1}(1)] \end{bmatrix}
$$

with the value of V_{t+1} determined by $a_t = 0$.

4. Use interpolation to obtain $\grave{\mathfrak{c}}_t(m_t, PW_t)$ and $\grave{V}_t(0, m_t, PW_t)$ for the unconstrained problem. Form the boundary interpolator $\grave{\hat{m}}_t(PW_t)$ which determines the minimum level of uncon-

strained wealth for each value of the PW balance.

5. Correct for the no-borrowing constraint by constructing a part exact part interpolated policy and value function for this period:

$$\grave{\mathfrak{c}}_t^*(m_t, PW_t) = \begin{cases} m_t^{-\gamma} & \text{if } m_t < \grave{\hat{m}}_t(PW_t) \\ \grave{\mathfrak{c}}_t(m_t, PW_t) & \text{otherwise} \end{cases}$$

$$\grave{V}_t^*(m_t, PW_t) = \begin{cases} \hat{V}_{t,}(m_t, PW_t) & \text{if } m_t < \grave{\hat{m}}_t(PW_t) \\ \grave{V}_{t,}(m_t, PW_t) & \text{otherwise} \end{cases}$$

6. Repeat for $t-1$

We can recover the object of interest (the value of a PW offer: $V_0(0, \omega, PW_0)$) after the $t = 0$ step in the previous recursion.