A Global View of Creative Destruction

Online Appendix

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In Section A.1 we document changes in job reallocation in Canada at the 2-digit level following CUSFTA tariff changes. In Section A.2 of this Appendix we infer the innovation arrival rates from data on relative wages and relative employment, the trade share, the growth rate, and the TFPR gap between exporters and non-exporters. In section A.3 we characterize comparative statics in the exogenous innovation case. Section A.4 describes how we infer the innovation cost functions to rationalize the innovation arrival rates. As in the main text, we refer to the rest of the OECD as simply "the OECD" in this Appendix.

A.1 2-digit Canadian evidence

Figure 1 plots the change in the job destruction rate in two digit Canadian industries from 1973–1988 to 1988–2003 against the change in the tariff rate on Canadian imports in those same industries over the same period due to CUSFTA.[1] Job destruction rates increased more in industries where tariffs declined the most, though the relationship is not particularly tight.[2]

Figure 2 shows that a similar fact holds across two-digit Canadian industries. Job creation from exports increased more in sectors where U.S. tariffs declined the most, though again this relationship is not a tight one.[3]

A.2 Inferring innovation arrival rates

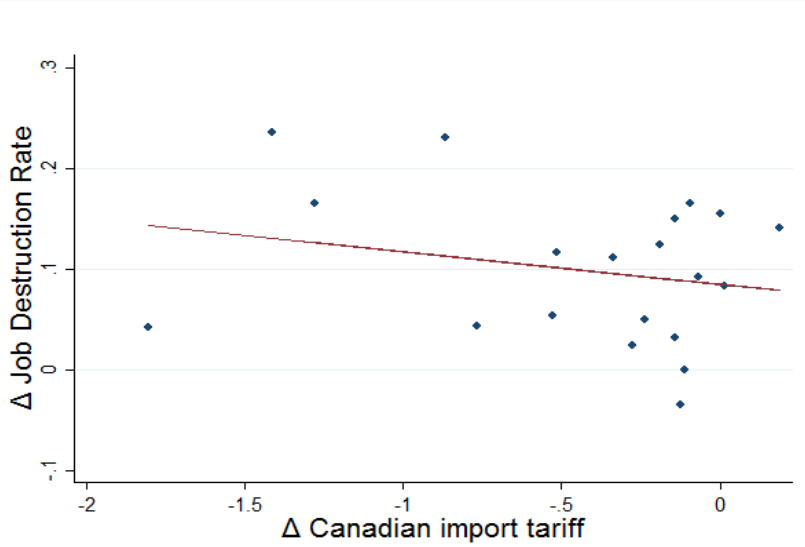
For generality, we present the CES case where the elasticity of substitution between varieties is $\sigma > 0$ first. In the main text we specialize to the Cobb-Douglas case with $\sigma = 1$, so we also show that here.

[1] We use the tariff cuts constructed by Trefler (2004), which give changes in bilateral tariffs between Canada and the U.S. following CUSFTA net of the changes in the respective most-favored-nation tariffs.

[2] The coefficient of the OLS regression in Figure 1 is $-.096$ with a standard error of $.031$.

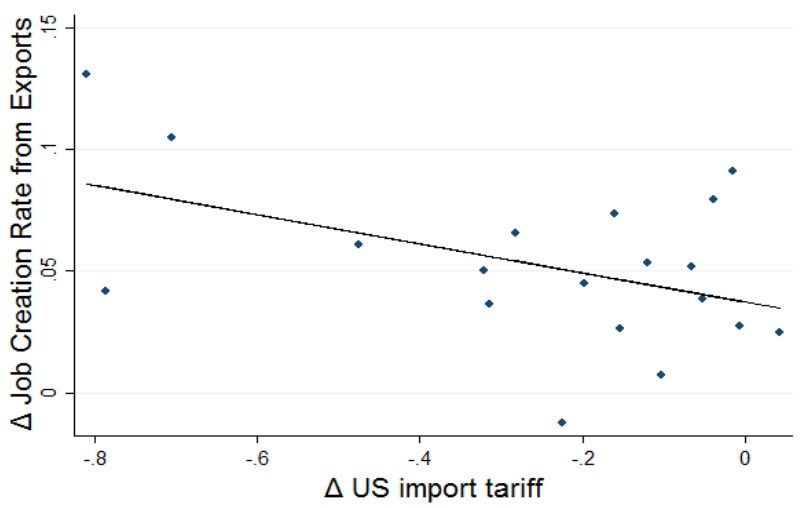
[3] The coefficient of the OLS regression in Figure 2 is $-.189$ with a standard error of $.096$.

Figure 1: Δ Job Destruction in Canada vs. Δ Canadian Tariffs



Note: Each observation is a 2-digit Canadian industry. Δ job destruction is the difference between the average job destruction rate (calculated over five years) in 1988 to 2003 and 1978 to 1988.

Figure 2: Δ Job Creation in Canada from Exports vs. Δ U.S. Tariffs



Note: Each observation is a 2-digit Canadian industry. Δ job creation from exports is the difference between the average job creation rate from exports from 1988 to 2003 and 1978 to 1988.

Output per variety

The $\sigma > 0$ case

With preferences given by $U = \left(\int_0^1 c_j^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}}$, we have the following:

- U.S. variety in the OECD market: $y_j = \left(\frac{P}{p_j}\right)^{\sigma} \times \frac{I}{P}$.
- U.S. variety in the OECD market: $y_j^f = \left(\frac{P^*}{p_j^f}\right)^{\sigma} \times \frac{I^*}{P^*}$.
- OECD variety in the OECD market: $y_j^* = \left(\frac{P^*}{p_j^*}\right)^{\sigma} \times \frac{I^*}{P^*}$.
- OECD variety in the U.S. market: $y_j^{*f} = \left(\frac{P}{p_j^{*f}}\right)^{\sigma} \times \frac{I}{P}$.

The $\sigma = 1$ case

With preferences given by $U = \prod_0^1 c_j^{dj}$, we have the following:

- U.S. variety in the U.S. market: $y_j = \frac{I}{p_j}$.
- U.S. variety in the OECD market: $y_j^f = \frac{I^*}{p_j^f}$.
- OECD variety in the OECD market: $y_j^* = \frac{I^*}{p_j^*}$.
- OECD variety in the U.S. market: $y_j^{*f} = \frac{I}{p_j^{*f}}$.

Threshold varieties

Suppose the variety index j is decreasing in A_j/A_j^*. Then varieties $j \in [0, x]$ are traded and produced at home, $j \in [x, x^*]$ are non-traded and $j \in [x^*, 1]$ are traded and produced abroad. The cutoff varieties x and x^* are defined by equating U.S. and OECD marginal costs:

$$\frac{w\tau^*}{A_x} = \frac{w^*}{A_x^*} \quad \text{and} \quad \frac{w}{A_{x^*}} = \frac{w^*\tau}{A_{x^*}^*},$$

Here $\omega = w/w^*$ is the relative wage, and we can rewrite the above as:

$$\omega\tau^* = \frac{A_x}{A_x^*} \quad \text{and} \quad \frac{\omega}{\tau} = \frac{A_{x^*}}{A_{x^*}^*}.$$

Price per variety

Denote the technology of the second most competitive firm in each market as A_j' and $A_j'^*$. Correspondingly denoting the marginal cost of the leading firm in variety j's closest competitor as m_j', we have:

- U.S. firm selling in the U.S. market: $m_j' = \min\left\{\frac{w}{A_j'}, \frac{w^*\tau}{A_j^*}\right\}$.
- U.S. firm selling in the OECD market: $m_j'^f = \min\left\{\frac{w\tau^*}{A_j'}, \frac{w^*}{A_j^*}\right\}$.
- OECD firm selling in the OECD market: $m_j'^* = \min\left\{\frac{w\tau^*}{A_j}, \frac{w^*}{A_j'^*}\right\}$.
- OECD firm selling in the U.S. market: $m_j'^{*f} = \min\left\{\frac{w}{A_j}, \frac{w^*\tau}{A_j'^*}\right\}$.

Dividing by the marginal cost of the leading firm in variety j, we obtain the following Bertrand competition markups denoted by ϵ_j:

- U.S. firm selling in the U.S. market: $\epsilon_j = \min\left\{\frac{A_j}{A_j'}, \frac{A_j\tau}{A_j^*\omega}\right\}$.
- U.S. firm selling in the OECD market: $\epsilon_j^f = \min\left\{\frac{A_j}{A_j'}, \frac{A_j}{A_j^*\tau^*\omega}\right\}$.
- OECD firm selling in the OECD market: $\epsilon_j^* = \min\left\{\frac{A_j^*}{A_j'^*}, \frac{A_j^*\tau^*\omega}{A_j}\right\}$.
- OECD firm selling in the U.S. market: $\epsilon_j^{*f} = \min\left\{\frac{A_j^*}{A_j'^*}, \frac{A_j^*\omega}{A_j\tau}\right\}$.

By Bertrand competition, the leading firm sets its price to the minimum between the marginal cost of its closest competitor and the monopolistic competition price. Letting $\rho = \frac{\sigma-1}{\sigma}$, we have the following prices where the term multiplying the marginal cost is the gross markup denoted by μ_j:

The $\sigma > 0$ case

- U.S. variety in the U.S. market: $p_j = \min\{\epsilon_j, 1/\rho\} \times \frac{w}{A_j}$.
- U.S. variety in the OECD market: $p_j^f = \min\left\{\epsilon_j^f, 1/\rho\right\} \times \frac{w\tau^*}{A_j}$.
- OECD variety in the OECD market: $p_j^* = \min\left\{\epsilon_j^*, 1/\rho\right\} \times \frac{w^*}{A_j^*}$.
- OECD variety in the U.S. market: $p_j^{*f} = \min\left\{\epsilon_j^{*f}, 1/\rho\right\} \times \frac{w^*\tau}{A_j^*}$.

The $\sigma = 1$ case

- U.S. variety in the U.S. market: $p_j = \epsilon_j \times \frac{w}{A_j}$.
- U.S. variety in the OECD market: $p_j^f = \epsilon_j^f \times \frac{w\tau^*}{A_j}$.
- OECD variety in the OECD market: $p_j^* = \epsilon_j^* \times \frac{w^*}{A_j^*}$.
- OECD variety in the U.S. market: $p_j^{*f} = \epsilon_j^{*f} \times \frac{w^*\tau}{A_j^*}$.

Consumption price index

The U.S. and OECD consumption price indices are given by:

The $\sigma > 0$ case

- U.S. consumption price index: $P = \left[\int_0^{x^*}(p_j)^{1-\sigma}dj + \int_{x^*}^1(p_j^{*f})^{1-\sigma}dj\right]^{\frac{1}{1-\sigma}}$.
- OECD consumption price index: $P^* = \left[\int_0^{x}(p_j^{f})^{1-\sigma}dj + \int_{x}^1(p_j^{*})^{1-\sigma}dj\right]^{\frac{1}{1-\sigma}}$.

The $\sigma = 1$ case

- U.S. consumption price index: $P = \prod_0^{x^*}(p_j)^{dj} \times \prod_{x^*}^1(p_j^{*f})^{dj}$.
- OECD consumption price index: $P^* = \prod_0^{x}(p_j^{f})^{dj} \times \prod_{x}^1(p_j^{*})^{dj}$.

Real consumption wage

Substituting in prices and rearranging, we get the following expressions for the U.S. and OECD real consumption wage:

The $\sigma > 0$ case

- U.S. real consumption wage: $W \equiv \frac{w}{P} = \left[\int_0^{x^*} \left(\frac{A_j}{\mu_j}\right)^{\sigma-1} dj + \int_{x^*}^1 \left(\frac{A_j^* \omega}{\mu_j^{*f} \tau}\right)^{\sigma-1} dj\right]^{\frac{1}{\sigma-1}}$.

- OECD real consumption wage: $W^* \equiv \frac{w^*}{P^*} = \left[\int_0^{x} \left(\frac{A_j}{\mu_j^{f} \omega \tau^*}\right)^{\sigma-1} dj + \int_{x}^1 \left(\frac{A_j^*}{\mu_j^{*}}\right)^{\sigma-1} dj\right]^{\frac{1}{\sigma-1}}$.

The $\sigma = 1$ case

- U.S. real consumption wage: $W \equiv \frac{w}{P} = \prod_0^{x^*} \left(\frac{A_j}{\mu_j}\right)^{dj} \times \prod_{x^*}^1 \left(\frac{A_j^* \omega}{\mu_j^{*f} \tau}\right)^{dj}$.

- OECD real consumption wage: $W^* \equiv \frac{w^*}{P^*} = \prod_0^{x} \left(\frac{A_j}{\mu_j^{f} \omega \tau^*}\right)^{dj} \times \prod_{x}^1 \left(\frac{A_j^*}{\mu_j^{*}}\right)^{dj}$.

Gross domestic product

U.S. and OECD GDP net of tariff revenue are given by:

- U.S. GDP net of tariff revenue: $I - T = \int_0^{x^*} p_j y_j dj + \frac{1}{\tau^*} \times \int_0^x p_j^f y_j^f dj$.

- OECD GDP net of tariff revenue: $I^* - T^* = \int_x^1 p_j^* y_j^* dj + \frac{1}{\tau} \times \int_{x^*}^1 p_j^{*f} y_j^{*f} dj$.

Hence, balanced trade implies:

$$\frac{1}{\tau^*} \times \int_0^x p_j^f y_j^f dj = \frac{1}{\tau} \times \int_{x^*}^1 p_j^{*f} y_j^{*f} dj.$$

Substituting in the expression for output per variety, we get:

The $\sigma > 0$ case

$$\frac{I^*}{\tau^*} \times \int_0^x \left(\frac{P^*}{p_j^f}\right)^{\sigma-1} dj = \frac{I}{\tau} \times \int_{x^*}^1 \left(\frac{P}{p_j^{*f}}\right)^{\sigma-1} dj.$$

The $\sigma = 1$ case

$$\frac{I^* x}{\tau^*} = \frac{I\left(1 - x^*\right)}{\tau}.$$

Producer price index

The U.S. and OECD producer price indices are given by:

The $\sigma > 0$ case

- U.S. producer price index: $P^P \equiv \left[\int_0^x \left(\frac{p_j^f y_j^f}{\tau^*(I-T)}\right)(p_j^f)^{1-\sigma} dj + \int_0^{x^*} \left(\frac{p_j y_j}{I-T}\right)(p_j)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}}$.
- OECD producer price index: $P^{*P} \equiv \left[\int_{x^*}^1 \left(\frac{p_j^{*f} y_j^{*f}}{\tau(I^*-T^*)}\right)(p_j^{*f})^{1-\sigma} dj + \int_x^1 \left(\frac{p_j^* y_j^*}{I^*-T^*}\right)(p_j^*)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}}$.

The $\sigma = 1$ case

- U.S. producer price index: $P^P \equiv \prod_0^x \left[(p_j^f)^{\frac{p_j^f y_j^f}{\tau^*(I-T)}}\right]^{dj} \times \prod_0^{x^*} \left[(p_j)^{\frac{p_j y_j}{I-T}}\right]^{dj}$.
- OECD producer price index: $P^{*P} \equiv \prod_{x^*}^1 \left[(p_j^{*f})^{\frac{p_j^{*f} y_j^{*f}}{\tau(I^*-T^*)}}\right]^{dj} \times \prod_x^1 \left[(p_j^*)^{\frac{p_j^* y_j^*}{I^*-T^*}}\right]^{dj}$.

Real product wage

Substituting in prices and rearranging, we get the following expressions for the U.S. and OECD real product wage:

The $\sigma > 0$ case

- U.S. real product wage: $\frac{w}{P^P} \equiv \left[\int_0^x \left(\frac{p_j^f y_j^f}{\tau^*(I-T)}\right)\left(\frac{A_j}{\mu_j^f \tau^*}\right)^{\sigma-1} dj + \int_0^{x^*} \left(\frac{p_j y_j}{I-T}\right)\left(\frac{A_j}{\mu_j}\right)^{\sigma-1} dj\right]^{\frac{1}{\sigma-1}}$.

- OECD real product wage: $\frac{w^*}{P^{*P}} \equiv \left[\int_{x^*}^1 \left(\frac{p_j^{*f} y_j^{*f}}{\tau(I^*-T^*)}\right)\left(\frac{A_j^*}{\mu_j^{*f} \tau}\right)^{\sigma-1} dj + \int_x^1 \left(\frac{p_j^* y_j^*}{I^*-T^*}\right)\left(\frac{A_j^*}{\mu_j^*}\right)^{\sigma-1} dj\right]^{\frac{1}{\sigma-1}}$.

The $\sigma = 1$ case

- U.S. real product wage: $\frac{w}{P^P} = \prod_0^x \left[\left(\frac{A_j}{\mu_j^f \tau^*}\right)^{\frac{p_j^f y_j^f}{\tau^*(I-T)}}\right]^{dj} \times \prod_0^{x^*} \left[\left(\frac{A_j}{\mu_j}\right)^{\frac{p_j y_j}{I-T}}\right]^{dj}$.

- OECD real product wage: $\frac{w^*}{P^{*P}} = \prod_{x^*}^1 \left[\left(\frac{A_j^*}{\mu_j^{*f} \tau}\right)^{\frac{p_j^{*f} y_j^{*f}}{\tau(I^*-T^*)}}\right]^{dj} \times \prod_x^1 \left[\left(\frac{A_j^*}{\mu_j^*}\right)^{\frac{p_j^* y_j^*}{I^*-T^*}}\right]^{dj}$.

Notice that the real product wage is the same as real GDP per worker in this economy with exogenous innovation.

Aggregate markup

The U.S. and OECD aggregate markups are defined as:

- U.S. aggregate markup: $\overline{\mu}^{-1} \equiv \int_0^{x^*} \frac{p_j y_j}{\mu_j (I-T)} dj + \frac{1}{\tau^*} \times \int_0^x \frac{p_j^f y_j^f}{\mu_j^f (I-T)} dj$.

- OECD aggregate markup: $\overline{\mu}^{*-1} \equiv \int_x^1 \frac{p_j^* y_j^*}{\mu_j^* (I^*-T^*)} dj + \frac{1}{\tau} \times \int_{x^*}^1 \frac{p_j^{*f} y_j^{*f}}{\mu_j^{*f} (I^*-T^*)} dj$.

Labor market clearing

The wage expenditure per variety is given by:

- U.S. variety in the U.S. market: $wl_j = \frac{p_j y_j}{\mu_j}$.

- U.S. variety in the OECD market: $wl_j = \frac{p_j^f y_j^f}{\tau^* \mu_j^f}$.

- OECD variety in the OECD market: $w^* l_j^* = \frac{p_j^* y_j^*}{\mu_j^*}$.
- OECD variety in the U.S. market: $w^* l_j^* = \frac{p_j^{*f} y_j^{*f}}{\tau \mu_j^{*f}}$.

Aggregating across varieties, we get:

- U.S. aggregate wage expenditure: $wL = \int_0^{x^*} \frac{p_j y_j}{\mu_j} dj + \frac{1}{\tau^*} \times \int_0^{x} \frac{p_j^f y_j^f}{\mu_j^f} dj$.
- OECD aggregate wage expenditure: $w^* L^* = \int_x^1 \frac{p_j^* y_j^*}{\mu_j^*} dj + \frac{1}{\tau} \times \int_{x^*}^1 \frac{p_j^{*f} y_j^{*f}}{\mu_j^{*f}} dj$.

One can see that GDP net of tariff revenue is the product of the aggregate markup and the aggregate wage expenditure:

$$I - T = \overline{\mu} w L \quad \text{and} \quad I^* - T^* = \overline{\mu}^* w^* L^*.$$

Correspondingly, GDP is given by:

$$I = \frac{\overline{\mu} w L}{1 - \left(\frac{\tau - 1}{\tau}\right)(1 - x^*)} \quad \text{and} \quad I^* = \frac{\overline{\mu}^* w^* L^*}{1 - \left(\frac{\tau^* - 1}{\tau^*}\right) x}.$$

Solution steps

1. Guess the value of $\omega \equiv {}^{w}/_{w^*}$.
2. The guess for ω will pin down the set of products that are exported, non-traded, and imported (from the U.S. perspective) given the A_j, A_j', A_j^* and $A_j^{*\prime}$ levels.
3. Calculate markups for each variety in each market.
4. Calculate the real wage in the home country using the markups, relative wages, and realized distribution of quality.
5. Calculate the prices of each variety and the exact OECD consumer price index (the U.S. aggregate consumer price index is normalized to one).

6. Use data on U.S. and OECD export shares as initial guesses for export shares. Use them to calculate the $\overline{\mu}$ and $\overline{\mu}^*$ implied by the distribution of prices and qualities. Given data on L and L^*, the initial guess for ω, and the implied real wage at home w, we then calculate $I - T$, $I^* - T^*$, C and C^*.

7. Calculate $I - T$ and $I^* - T^*$ by adding up sales of each variety.

8. Calculate $\overline{\mu}$ and $\overline{\mu}^*$ implied by the distribution of revenues and by GDP net of tariff revenues.

9. Iterate over ω until the following conditions hold:

   (a) Trade is balanced.

   (b) The initial guesses for $\overline{\mu}$ and $\overline{\mu}^*$ in step 4 are equal to $\overline{\mu}$ and $\overline{\mu}^*$ calculated in step 8.

   (c) $I - T$ and $I^* - T^*$ implied by the initial guesses from step 6 are equal to $I - T$ and $I^* - T^*$ calculated in step 7.

A.3 Growth rates with exogenous innovation

The expected growth rate g of the domestic consumption wage is given by:[4]

$$
\begin{aligned}
g = {} & (\lambda + \widetilde{\eta}) \left[\frac{x^*}{\theta - 1} + (1 - x^*) \min \left\{ 1, \left(\frac{\tau}{\omega} \right)^{\theta} \right\} \left(\frac{\theta}{\theta - 1} \cdot \max \left\{ \frac{\omega}{\tau}, 1 \right\} - 1 \right) \right] \\
& + \left(\widetilde{\lambda}^* + \widetilde{\eta}^* \right) \left[\frac{1 - x^*}{\theta - 1} + (x^* - x) \min \left\{ 1, \left(\frac{\omega A_j^*}{\tau A_j} \right)^{\theta} \right\} \left(\frac{\theta}{\theta - 1} \cdot \max \left\{ \frac{\tau A_j}{\omega A_j^*}, 1 \right\} - 1 \right) \right] \\
& + \left(\widetilde{\lambda}^* + \widetilde{\eta}^* \right) \left[x \cdot min \left\{ 1, \left(\frac{\omega}{\tau} \right)^{\theta} \right\} \left(\frac{\theta}{\theta - 1} \cdot \max \left\{ \frac{\tau}{\omega}, 1 \right\} - 1 \right) \right]
\end{aligned}
$$

The first line is contribution of innovation by domestic firms; the second and third lines is the contribution of foreign firms. Table 1 shows the expected growth

[4] Here we abstract from the reflecting barrier at the low end of the quality distribution.

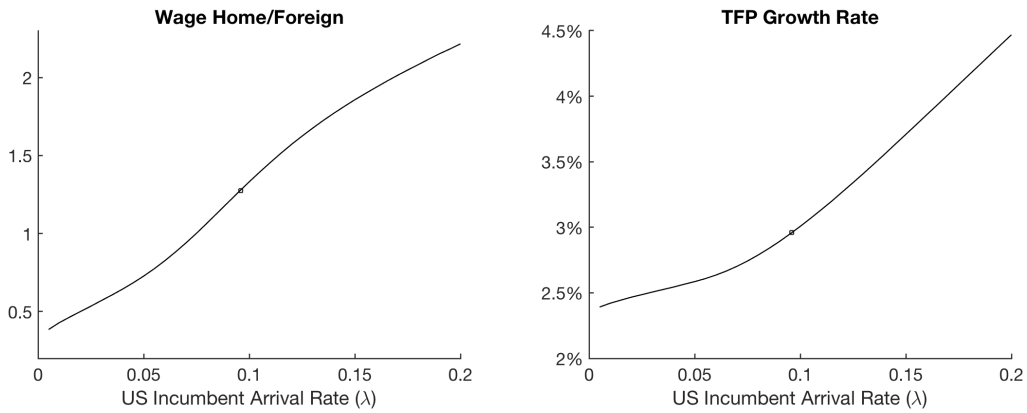
Table 1: Growth Rate under Autarky and Free Trade

	Domestic	Foreign
Autarky	$(\lambda+\widetilde{\eta})\left(\frac{1}{\theta-1}\right)$	$(\lambda^*+\widetilde{\eta}^*)\left(\frac{1}{\theta-1}\right)$
Free Trade	$\underbrace{(\lambda+\widetilde{\eta})\left[\frac{x^*}{\theta-1}+(1-x^*)\min\left\{1,\omega^{-\theta}\right\}\left(\frac{\theta}{\theta-1}\times\max\{\omega,1\}-1\right)\right]}_{\text{domestic innovation}}$	
	$+\underbrace{\left(\widetilde{\lambda}^*+\widetilde{\eta}^*\right)\left[\frac{1-x^*}{\theta-1}+x^*\min\left\{1,\omega^{\theta}\right\}\left(\frac{\theta}{\theta-1}\times\max\left\{\frac{1}{\omega},1\right\}-1\right)\right]}_{\text{foreign innovation}}$	

rate of the real consumption wage for the polar cases of complete autarky and free trade. The first row in Table 1 says that under autarky, a country's growth rate only depends on its innovation rate (and the innovation step size). A country with a higher innovation rate grows faster than a country with a lower innovation rate. The second row in Table 1 shows the growth rate with free trade. The first term is the contribution of domestic firms. Domestic firms attempt to innovate over *all* products, including imported ones, so $\lambda+\widetilde{\eta}$ is the rate at which a domestically produced variety is replaced by another domestic firm and $(\lambda+\widetilde{\eta})\min\left\{1,\omega^{-\theta}\right\}$ is the probability an imported product is taken over by a domestic firm. The second term captures the impact of innovation by *foreign* firms on domestic consumers. The rate at which an imported variety is replaced by another foreign firm is $\lambda^*+\widetilde{\eta}^*$ and the probability a variety produced by a domestic firm is replaced by a foreign firm is $(\lambda^*+\widetilde{\eta}^*)\min\left\{1,\omega^{\theta}\right\}$.

Given innovation rates in the two countries, the relative wage ω has opposite effects on the contribution of domestic vs. foreign innovation. A higher ω lowers the contribution of innovation by domestic firms and raises the contribution of foreign firms. In a steady state, differences in innovation rates show up as differences in the relative wage, but the real consumption wage grows at the same

Figure 3: Effect of Home Innovation on Growth and Relative Income



Note: The left panel shows the steady-state wage at home relative to the foreign country as a function of the innovation rate of domestic incumbent firms, holding fixed the other variables. The right panel shows the effect on the steady state growth rate of aggregate TFP in *both* countries.

rate in the two countries.[5] Figure 3 shows the relative wage (in the left panel) and the growth rate in *both* countries (in the panel on the right) as a function of the innovation rate of domestic incumbent firms λ.[6] A faster innovation rate by domestic firms increases the domestic wage relative to the foreign wage, but increases the growth rate of the real wage equally in the two countries.

A.4 Endogenous arrival rates

After we back out the innovation arrival rates, we endogenize the arrival rates. Productivity draws are relative to the productivity of the seller (incumbent producer) of each variety. For an imported variety, this is the productivity of the foreign producer. For an exporter and a non-traded variety, this is the productivity of the domestic producer.

[5]This result is reminiscent of Acemoglu and Ventura (2002).

[6]The values of the other parameters used in the simulation shown in Figure 3 are $\eta = .0295$, $\lambda^* = .115$, $\eta^* = .034$, $\theta = 10.94$, and $\tau = 1.491$. These are the parameter values in our baseline simulation (Table 9).

Innovation process

Productivity of domestic firms for variety j follows a power law with shape parameter θ and scale parameter ζ_j. Productivity of foreign firms for variety j also follows a power law with the same shape parameter but with scale parameter ζ_j^*. Since innovations build on the existing productivity level of varieties that are consumed domestically, ζ_j and ζ_j^* are given by:

$$\zeta_j = \begin{cases} A_j & \text{if } j \in [0, x^*], \\ A_j^* & \text{if } j \in [x^*, 1] \end{cases} \quad \text{and} \quad \zeta_j^* = \begin{cases} A_j & \text{if } j \in [0, x], \\ A_j^* & \text{if } j \in [x, 1]. \end{cases}$$

Hence, the ***conditional*** probabilities of replacing the incumbent for variety j are given in Table 2. From the perspective of a U.S. firm, we denote the conditional probability of gaining an exported variety by β_x and that of gaining a non-traded variety by β_n such that:

$$\beta_x \equiv x + (x^* - x) \min\left\{\left(\frac{A_j}{A_j^* \omega \tau^*}\right)^\theta, 1\right\} + (1 - x^*) \min\left\{\left(\frac{1}{\omega \tau^*}\right)^\theta, 1\right\},$$

$$\beta_n \equiv (x^* - x)\left(1 - \min\left\{\left(\frac{A_j}{A_j^* \omega \tau^*}\right)^\theta, 1\right\}\right) + (1 - x^*)\left(\min\left\{\left(\frac{\tau}{\omega}\right)^\theta, 1\right\} - \min\left\{\left(\frac{1}{\omega \tau^*}\right)^\theta, 1\right\}\right).$$

Unconditional probabilities of innovation

The ***unconditional*** probabilities of innovation are given by:

- U.S. incumbent: λ.
- U.S. entrant: $\widetilde{\eta} \equiv (1 - \lambda)\eta$.
- OECD incumbent: $\widetilde{\lambda}^* \equiv (1 - \lambda)(1 - \eta)\lambda^*$.
- OECD entrant: $\widetilde{\eta}^* \equiv (1 - \lambda)(1 - \eta)\,(1 - \lambda^*)\,\eta^*$.

tp]

Table 2: Conditional probability of replacing the incumbent

Market	Product Type	U.S. Firm	OECD Firm
U.S.	$j \in [0, x]$	1	$\min\left\{\left(\frac{\omega}{\tau}\right)^{\theta}, 1\right\}$
	$j \in [x, x^*]$	1	$\min\left\{\left(\frac{A_j^*\omega}{A_j\tau}\right)^{\theta}, 1\right\}$
	$j \in [x^*, 1]$	$\min\left\{\left(\frac{\tau}{\omega}\right)^{\theta}, 1\right\}$	1
OECD	$j \in [0, x]$	1	$\min\left\{(\omega\tau^*)^{\theta}, 1\right\}$
	$j \in [x, x^*]$	$\min\left\{\left(\frac{A_j}{A_j^*\omega\tau^*}\right)^{\theta}, 1\right\}$	1
	$j \in [x^*, 1]$	$\min\left\{\left(\frac{1}{\omega\tau^*}\right)^{\theta}, 1\right\}$	1

Unconditional probabilities of creative destruction

The ***unconditional*** probabilities of creative destruction are given in Table 3.

Now, from the perspective of a U.S. firm, we denote the probability of losing an exported variety in both markets by δ_x, that of losing an exported variety in the foreign market only by δ'_x and that of losing a non-traded variety by δ_n such that:

$$\delta_x \equiv \lambda + \widetilde{\eta} + \left(\widetilde{\eta}^* + \widetilde{\lambda}^*\right) \min\left\{\left(\frac{\omega}{\tau}\right)^{\theta}, 1\right\},$$
$$\delta'_x \equiv \left(\widetilde{\eta}^* + \widetilde{\lambda}^*\right)\left[\min\left\{(\omega\tau^*)^{\theta}, 1\right\} - \min\left\{\left(\frac{\omega}{\tau}\right)^{\theta}, 1\right\}\right],$$
$$\delta_n \equiv \lambda + \widetilde{\eta} + \left(\widetilde{\lambda}^* + \widetilde{\eta}^*\right) \min\left\{\left(\frac{A_j^*\omega}{A_j\tau}\right)^{\theta}, 1\right\}.$$

Return and cost of innovation

Table 3: Probability of creative destruction

Market	Product Type	U.S. Firm	OECD Firm
U.S.	$j \in [0, x]$	$\lambda + \widetilde{\eta}$	$\left(\widetilde{\lambda}^* + \widetilde{\eta}^*\right) \min\left\{\left(\frac{\omega}{\tau}\right)^\theta, 1\right\}$
	$j \in [x, x^*]$	$\lambda + \widetilde{\eta}$	$\left(\widetilde{\lambda}^* + \widetilde{\eta}^*\right) \min\left\{\left(\frac{A_j^* \omega}{A_j \tau}\right)^\theta, 1\right\}$
	$j \in [x^*, 1]$	$(\lambda + \widetilde{\eta}) \min\left\{\left(\frac{\tau}{\omega}\right)^\theta, 1\right\}$	$\widetilde{\lambda}^* + \widetilde{\eta}^*$
OECD	$j \in [0, x]$	$\lambda + \widetilde{\eta}$	$\left(\widetilde{\lambda}^* + \widetilde{\eta}^*\right) \min\left\{(\omega\tau^*)^\theta, 1\right\}$
	$j \in [x, x^*]$	$(\lambda + \widetilde{\eta}) \min\left\{\left(\frac{A_j}{A_j^* \omega \tau^*}\right)^\theta, 1\right\}$	$\widetilde{\lambda}^* + \widetilde{\eta}^*$
	$j \in [x^*, 1]$	$(\lambda + \widetilde{\eta}) \min\left\{\left(\frac{1}{\omega\tau^*}\right)^\theta, 1\right\}$	$\widetilde{\lambda}^* + \widetilde{\eta}^*$

Domestic Incumbent firms

The production function of innovation per variety owned by a U.S. incumbent firm is given by:

$$\lambda = \left(\frac{R_i}{\gamma \chi_i \overline{A}^{(1-\phi)/\gamma}}\right)^\gamma,$$

where R_i denotes units of labor used for R&D ***per variety*** owned by the incumbent firm and $\overline{A}$ is aggregate TFP (in terms of consumed varieties). We have semi-endogenous growth *a la* Jones when $\phi < 1$ and endogenous growth when $\phi = 1$. Holding R&D labor constant, innovation rates are declining in aggregate TFP when $\phi < 1$. After normalizing the nominal domestic wage to one, the cost of innovation per variety as a function of innovation intensity is:

$$C(\lambda) = R_i = \gamma \chi_i \left(\overline{A}^{1-\phi} \lambda\right)^{1/\gamma}.$$

The marginal cost of innovation per variety is given by:

$$C'(\lambda) = \chi_i \left(\overline{A}^{1-\phi}\lambda^{1-\gamma}\right)^{1/\gamma}.$$

The expected return for a given innovation intensity is:

$$R(\lambda) = \lambda\left(\beta_x V_x + \beta_n V_n\right),$$

where V_x is the expected value of an exported variety, V_n is the expected value of a non-traded variety, $\lambda\beta_x$ is the probability of gaining an exported variety and $\lambda\beta_n$ is the probability of gaining a non-traded variety. Markups vary across varieties so the value of a variety differs across products. Innovation is undirected so effort cannot be targeted toward varieties with the highest expected markup. The return to a marginal increase in innovation intensity is thus given by:

$$R'(\lambda) = \beta_x V_x + \beta_n V_n.$$

The privately optimal ***unconditional*** innovation rate is given by equating the marginal return and marginal cost of innovation effort:

$$\lambda = \left(\frac{\beta_x v_x + \beta_n v_n}{\chi_i}\right)^{\frac{\gamma}{1-\gamma}}, \tag{1}$$

where we have the following normalization:

$$v_x = \frac{V_x}{\overline{A}^{(1-\phi)/\gamma}} \quad \text{and} \quad v_n = \frac{V_n}{\overline{A}^{(1-\phi)/\gamma}}.$$

Here v_x and v_n are the expected value of the two types of varieties normalized by $\overline{A}^{(1-\phi)/\gamma}$. V_x and V_n should grow at a constant rate in steady state equal to the rate of population growth. Since a steady state will feature a fixed fraction of labor devoted to research, the innovation rate will be constant in steady state with the growth rate of population equal to $(1-\phi)/\gamma$ times the growth rate of aggregate

TFP. As long as population grows at a constant rate, v_x and v_n are constant in a steady state. In the absence of population growth, v_x and v_n decline as TFP rises and the innovation rate goes to zero.

Domestic Entrant firms

The production function of innovation per variety owned by a U.S. entrant firm is given by:

$$\eta = \left(\frac{R_e}{\gamma \chi_e \overline{A}^{(1-\phi)/\gamma}} \right)^{\gamma} (1-\lambda)^{-1},$$

where R_e denotes units of labor used for R&D ***per variety*** owned by the entrant firm. The cost of innovation per variety (in units of domestic labor) as a function of innovation intensity is:

$$C(\eta) = R_e = \gamma \chi_e \left(\overline{A}^{1-\phi} \eta (1-\lambda) \right)^{1/\gamma}.$$

The marginal cost of innovation per variety is given by:

$$C'(\eta) = \chi_e \left(\overline{A}^{1-\phi} \eta^{1-\gamma} (1-\lambda) \right)^{1/\gamma}.$$

The expected return for a given innovation intensity is:

$$R(\eta) = \eta (1-\lambda) (\beta_x V_x + \beta_n V_n),$$

The return to a marginal increase in innovation intensity is thus given by:

$$R'(\eta) = (1-\lambda) (\beta_x V_x + \beta_n V_n).$$

The privately optimal ***unconditional*** innovation rate is given by equating the marginal return and marginal cost of innovation effort:

$$\widetilde{\eta} = \left(\frac{\beta_x v_x + \beta_n v_n}{\chi_e} \right)^{\frac{\gamma}{1-\gamma}} . \tag{2}$$

Foreign incumbent firms

The production function of innovation per variety owned by a foreign incumbent firm is given by:

$$\lambda^* = \left(\frac{R_i^*}{\gamma \chi_i^* \overline{A}^{*\,(1-\phi)/\gamma}} \right)^{\gamma} \left[(1-\widetilde{\eta})\,(1-\lambda) \right]^{-1} ,$$

The marginal cost of innovation per variety is given by:

$$C'(\lambda^*) = \omega^{-1} \chi_i^* \left(\overline{A}^{*1-\phi} \lambda^{*1-\gamma} \,(1-\widetilde{\eta})\,(1-\lambda) \right)^{1/\gamma} .$$

where ω is the relative wage (remember we normalize the domestic nominal wage to one). The marginal return to innovation intensity is:

$$R'(\lambda^*) = \left[(1-\widetilde{\eta})\,(1-\lambda) \right] \left(\beta_x^* V_x^* + \beta_n^* V_n^* \right) ,$$

The privately optimal ***unconditional*** innovation rate is then:

$$\widetilde{\lambda}^* = \left(\frac{\beta_x^* v_x^* + \beta_n^* v_n^*}{\chi_i^* / \omega} \right)^{\frac{\gamma}{1-\gamma}} , \tag{3}$$

where v_x^* and v_n^* are now defined as:

$$v_x^* = \frac{V_x^*}{\overline{A}^{*\,(1-\phi)/\gamma}} \quad \text{and} \quad v_n^* = \frac{V_n^*}{\overline{A}^{*\,(1-\phi)/\gamma}} .$$

4.1 Foreign entrant firms

The production function of innovation per variety owned by a foreign entrant firm is given by:

$$\eta^* = \left(\frac{R_e^*}{\gamma \chi_e^* \overline{A}^{*\,(1-\phi)/\gamma}} \right)^{\gamma} \left[(1 - \widetilde{\lambda}^*)\,(1 - \widetilde{\eta})\,(1 - \lambda) \right]^{-1},$$

The privately optimal ***unconditional*** innovation rate is then:

$$\widetilde{\eta}^* = \left(\frac{\beta_x^* v_x^* + \beta_n^* v_n^*}{\chi_e^* / \omega} \right)^{\frac{\gamma}{1-\gamma}}, \tag{4}$$

Value of a variety

Exported variety

The value of an exported variety j at time t is defined by:

$$\begin{aligned}(1 + r_t)\, V_{x,t}\,(j) = {} & (1 + r_t) \left[\Pi_{x,t}\,(j) - \gamma \chi_i \left(\overline{A}_t^{1-\phi} \lambda_t \right)^{1/\gamma} \right] \\ & + \lambda_t \left(\beta_{x,t} V_{x,t+1} + \beta_{n,t} V_{n,t+1} \right) \\ & + (1 - \delta_{x,t})\, V_{x,t+1}\,(j) \\ & + \delta'_{x,t} \left[V_{n,t+1}\,(j) - V_{x,t+1}\,(j) \right],\end{aligned} \tag{5}$$

where r_t is the interest rate between time t and $t + 1$, and innovation rates at time t affect arrival rates at time $t + 1$. The terms on the right hand side are:

1. The flow of profits (in real consumption terms) from variety j at time t.

2. The expected gain of grabbing a new variety at time $t+1$. The probabilities are a function of innovation rates at time t, the value of the new variety is at time $t + 1$ and it is not indexed by j since innovations are undirected.

3. The expected value of variety j at time $t+1$. Here, the probability of losing variety j in both markets is a function of innovation rates at time t.

4. The expected loss of losing variety j in the foreign market only at time $t+1$. Here, the probability of losing variety j in the foreign market only is a function of innovation rates at time t.

Only the first and last two terms vary across varieties. Taking expectations and dividing by $\overline{A}_t^{(1-\phi)/\gamma}$ and $1+r_t$, we get:

$$\begin{aligned} v_{x,t} = \pi_{x,t} - \gamma\chi_i\lambda_t^{1/\gamma} + (1+r_t)^{-1}\left(\frac{\overline{A}_{t+1}}{\overline{A}_t}\right)^{(1-\phi)/\gamma} \\ \times\left[\left(\lambda_t\beta_{x,t} + 1 - \delta_{x,t} - \delta'_{x,t}\right)v_{x,t+1} + \left(\lambda_t\beta_{n,t} + \delta'_{x,t}\right)v_{n,t+1}\right]. \end{aligned} \tag{6}$$

The equivalent expression for a traded variety owned by the foreign firm is:

$$\begin{aligned} v^*_{x,t} = \pi^*_{x,t} - \gamma\omega_t^{-1}\chi_i^*\widetilde{\lambda}_t^{*1/\gamma} + (1+r_t)^{-1}\left(\frac{\overline{A}^*_{t+1}}{\overline{A}^*_t}\right)^{(1-\phi)/\gamma} \\ \times\left[\left(\widetilde{\lambda}^*_t\beta^*_{x,t} + 1 - \delta^*_{x,t} - \delta^{*\prime}_{x,t}\right)v^*_{x,t+1} + \left(\widetilde{\lambda}^*_t\beta^*_{n,t} + \delta^{*\prime}_{x,t}\right)v^*_{n,t+1}\right]. \end{aligned} \tag{7}$$

Non-traded variety

The value of a non-traded variety j at time t is defined by:

$$\begin{aligned} (1+r_t)\,V_{n,t}(j) = (1+r_t)\left[\Pi_{n,t}(j) - \gamma\chi_i\left(\overline{A}_t^{1-\phi}\lambda_t\right)^{1/\gamma}\right] \\ + \lambda_t\left(\beta_{x,t}V_{x,t+1} + \beta_{n,t}V_{n,t+1}\right) \\ + (1-\delta_{n,t})\,V_{n,t+1}(j). \end{aligned} \tag{8}$$

The terms on the right hand side are:

1. The flow of profits (in real consumption terms) from variety j at time t.

2. The expected gain of grabbing a new variety at time $t+1$. The probabilities are a function of innovation rates at time t, the value of the new variety is at time $t+1$ and it is not indexed by j since innovations are undirected.

3. The expected value of variety j at time $t+1$. Here, the probability of losing variety j is a function of innovation rates at time t.

Only the first and last terms vary across varieties. Taking expectations and dividing by $\overline{A}_t^{(1-\phi)/\gamma}$ and $1+r_t$, we get:

$$v_{n,t} = \pi_{n,t} - \gamma\chi_i\lambda_t^{1/\gamma} + (1+r_t)^{-1}\left(\frac{\overline{A}_{t+1}}{\overline{A}_t}\right)^{(1-\phi)/\gamma} \times \left[\left(\lambda_t\beta_{n,t} + 1 - \delta_{n,t}\right)v_{n,t+1} + \lambda_t\beta_{x,t}v_{x,t+1}\right]. \tag{9}$$

The equivalent expression for a non-traded variety owned by the foreign firm is:

$$v^*_{n,t} = \pi^*_{n,t} - \gamma\omega_t^{-1}\chi_i^*\widetilde{\lambda}_t^{*1/\gamma} + (1+r_t)^{-1}\left(\frac{\overline{A}^*_{t+1}}{\overline{A}^*_t}\right)^{(1-\phi)/\gamma} \times \left[\left(\widetilde{\lambda}^*_t\beta^*_{n,t} + 1 - \delta^*_{n,t}\right)v^*_{n,t+1} + \lambda^*_t\beta^*_{x,t}v^*_{x,t+1}\right]. \tag{10}$$

Steady state

In steady state, the values for exported and non-traded varieties owned by domestic firms are given by:

$$v_x = \frac{(1+r)\left(\pi_x - \gamma\chi_i\lambda^{1/\gamma}\right) + (1+g)^{(1-\phi)/\gamma}\left(\lambda\beta_n + \delta'_x\right)v_n}{1+r-(1+g)^{(1-\phi)/\gamma}\left(\lambda\beta_x + 1 - \delta_x - \delta'_x\right)}, \tag{11}$$

$$v_n = \frac{(1+r)\left(\pi_n - \gamma\chi_i\lambda^{1/\gamma}\right) + (1+g)^{(1-\phi)/\gamma}\lambda\beta_x v_x}{1+r-(1+g)^{(1-\phi)/\gamma}\left(\lambda\beta_n + 1 - \delta_n\right)}, \tag{12}$$

where g is the growth rate of TFP (the growth rate of $\overline{A}$). The steady state values of varieties owned by foreign firms are given by:

$$v^*_x = \frac{(1+r)\left(\pi^*_x - \gamma\omega^{-1}\chi^*_i\widetilde{\lambda}^{*1/\gamma}\right) + (1+g)^{(1-\phi)/\gamma}\left(\widetilde{\lambda}^*\beta^*_n + \delta^{*\prime}_x\right)v^*_n}{1+r-(1+g)^{(1-\phi)/\gamma}\left(\widetilde{\lambda}^*\beta^*_x + 1 - \delta^*_x - \delta^{*\prime}_x\right)}, \tag{13}$$

$$v^*_n = \frac{(1+r)\left(\pi^*_n - \gamma\omega^{-1}\chi^*_i\widetilde{\lambda}^{*1/\gamma}\right) + (1+g)^{(1-\phi)/\gamma}\widetilde{\lambda}^*\beta^*_x v^*_x}{1+r-(1+g)^{(1-\phi)/\gamma}\left(\widetilde{\lambda}^*\beta^*_n + 1 - \delta^*_n\right)}. \tag{14}$$

We use (11) and (12) to calculate the steady state values of v_x and v_n. To calculate transitional dynamics, we first calculate the final steady state values of v_x and v_n. After we have the final steady state values of v_x and v_n, we simulate the time path of realized profits, interest rates, R&D spending, and arrival rates and find a fixed point of the time path of v_x and v_n using equations (6) and (9).

Estimation

1. Infer the trade cost, the relative wage ω, the Pareto shape parameter θ, and innovation rates from the data as we currently do.

2. Use the simulated data from step 1 to calculate $\beta_x, \beta_n, \delta_x, \delta'_x, \delta_n$ and the corresponding arrival and destruction rates for the foreign country.

3. Calculate expected profits π_x and π_n and the corresponding expected profits for the foreign country.

4. Set γ and ϕ to match TFP growth, employment growth, and the share of labor in R&D. With these values and the results from steps 1 through 3, iterate over χ_i, χ_i^*, χ_e and χ_e^*, which pins down v_n, v_x, v_n^* and v_x^*, such that the four arbitrage equations that pin down the value of a variety and the four equations for the innovation rates (equations (4), (2) and their foreign counterparts) hold.

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