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A Details on Theoretical Benchmarks

A.1 Simplest Consumption-Saving Model (Section 1.1)

A.1.1 Constant Asset Prices – Derivation of Saving Policy Function in (2)

Households maximize (1) subject to $\dot{a}_t = w + ra_t - c_t$. The corresponding HJB equation is

$$\rho v(a) = \max_c \; u(c) + v'(a)(w + ra - c) \tag{A18}$$

We solve this equation by using a guess-and-verify strategy: guess

$$v(a) = B\frac{(a + w/r)^{1-\gamma}}{1-\gamma}$$

which implies $v'(a) = B(a + w/r)^{-\gamma}$ and

$$c(a) = v'(a)^{-1/\gamma} = B^{-1/\gamma}(a + w/r) \tag{A19}$$

Substituting into (A18) and dividing by $(a + w/r)^{1-\gamma}$

$$\rho B\frac{1}{1-\gamma} = \frac{1}{1-\gamma}B^{-(1-\gamma)/\gamma} + \frac{B}{r} - BB^{-1/\gamma}$$

Dividing by B and collecting terms we have $B^{-1/\gamma} = \frac{\rho - r}{\gamma} + r$ and hence from (A19) we have

$$c(a) = \left(\frac{\rho - r}{\gamma} + r\right)\left(a + \frac{w}{r}\right) \tag{A20}$$

Since the saving policy function is given by $s(a) = w + ra - c$, this yields (2).

A.1.2 Changing Asset Prices – Derivation of Equation (6)

As mentioned in the text to derive (6), we assume that return innovations are Brownian, i.e. $\varepsilon dt = \sigma dW_t$ where W_t is a Wiener process. In this case (5) becomes

$$da_t = (w + \bar{r}a_t - c_t)dt + \sigma a_t dW_t, \quad \bar{r} := \theta + \mu \tag{A21}$$

Households maximize (1) subject to (A21). To ease notation we drop the bar from $\bar{r}$ and simply write $r = \theta + \mu$ for the remainder of this appendix. The corresponding HJB equation is

$$\rho v(a) = \max_c \; u(c) + v'(a)(w + ra - c) + \frac{\sigma^2 a^2}{2} v''(a). \tag{A22}$$

This problem can no longer be solved in closed form. However, we can use a perturbation method around small w. We first solve (A22) analytically when $w = 0$. We then perturb v that solves (A22) for $w > 0$ around the solution for the case $w = 0$, thus obtaining (6). Our argument makes use of a standard perturbation method as in Fleming (1971), Judd (1996), Anderson, Hansen and Sargent (2012) and Kasa and Lei (2018).

Step 1: Closed form with $w = 0$. For the first step, consider the value function for $w = 0$ which we denote by $v_0(a)$. It solves:

$$\rho v_0(a) = \max_c \; u(c) + v_0'(a)(ra - c) + \frac{\sigma^2 a^2}{2} v_0''(a) \tag{A23}$$

This is the HJB equation for a simplified version without portfolio choice of the problem analyzed by Merton (1969) and it is well-known to have a closed-form solution.

Lemma A1 *The value function and consumption policy function with $w = 0$ are*

$$v_0(a) = B_0^{-\gamma} \frac{a^{1-\gamma}}{1-\gamma}, \quad c_0(a) = B_0 a, \quad B_0 := \frac{\rho - r}{\gamma} + r + (1-\gamma)\frac{\sigma^2}{2} \tag{A24}$$

Proof of Lemma A1: the proof uses a guess-and-verify strategy. Start by guessing that $v_0(a) = B_0^{-\gamma} \frac{a^{1-\gamma}}{1-\gamma}$ for a constant B_0 to be determined. Then $v'(a) = (B_0 a)^{-\gamma}, c(a) = B_0 a$ and $v''(a) = -\gamma B_0^{-\gamma} a^{-\gamma-1}$. Substituting into (A23), we have

$$\rho B_0^{-\gamma} \frac{1}{1-\gamma} = B_0^{1-\gamma}/(1-\gamma) + B_0^{-\gamma} r - B_0^{1-\gamma} - \frac{\sigma^2}{2}\gamma B_0^{-\gamma}$$

Rearranging

$$\rho \frac{1}{1-\gamma} = \gamma B_0/(1-\gamma) + r - \frac{\sigma^2}{2}\gamma.$$

Rearranging again we obtain the expression for B_0 in (A24).■

Step 2: Perturbation around $w = 0$. As already mentioned, it is no longer possible to solve (A22) in closed form. However, we can look for approximate solutions of the form

$$v(a) = v_0(a) + wv_1(a) + O(w^2), \quad c(a) = c_0(a) + wc_1(a) + O(w^2) \tag{A25}$$

where v_0 and c_0 are the value and consumption policy functions from Lemma A1 and where v_1 and c_1 are to be determined.

Proposition A3 *The value and consumption policy functions solving* (A22) *satisfy*

$$\begin{aligned} v(a) &= \bar{c}^{-\gamma}\left(\frac{a^{1-\gamma}}{1-\gamma} + \frac{wa^{-\gamma}}{r-\gamma\sigma^2}\right) + O(w^2) \\ c(a) &= \bar{c}\left(a + \frac{w}{r-\gamma\sigma^2}\right) + O(w^2) \\ \bar{c} &:= \frac{\rho - r}{\gamma} + r + (1-\gamma)\frac{\sigma^2}{2} \end{aligned} \tag{A26}$$

Before proving the Proposition, we first note that it immediately implies the approximate saving policy function (6) in the main text. To see this simply substitute $c(a)$ in (A26) into (A21) to get

$$da_t \approx \left[w + \bar{r}a_t - \bar{c}\left(a + \frac{w}{r-\gamma\sigma^2}\right)\right]dt + \sigma a_t dW_t.$$

Equation (6) is the same equation in terms of $\varepsilon_t dt = \sigma dW_t$.

Proof of Proposition A3: In the proof it is convenient to use slightly different notation than in the statement of the Proposition: for reasons that will become apparent momentarily, we use B_0 in place of the variable $\bar{c}$ used in the statement of the Proposition. As already mentioned we look for solutions of the form (A25). Further, we restrict our attention to solutions such that $v_1'(a) > 0$ for all a which ensures that $v'(a) \approx v_0'(a) + wv_1'(a) > 0$ for all $w > 0$ and therefore consumption $c(a) = (v'(a))^{-1/\gamma}$ is positive. However, we do not make any assumptions about the sign of $v_1(a)$. Substituting (A25) into (A22)

$$\rho(v_0(a) + wv_1(a)) = \max_c \; u(c) + (v_0'(a) + wv_1'(a))(w + ra - c) + (v_0''(a) + wv_1''(a))\frac{\sigma^2}{2}a^2$$

and the first-order condition is $(c_0(a)+wc_1(a))^{-\gamma} = v_0'(a)+wv_1'(a)$. Differentiating both the HJB equation and the first-order condition with respect to w and evaluating at $w=0$:

$$\rho v_1(a) = v_1'(a)(ra - c_0(a)) + v_1''(a)\frac{\sigma^2}{2}a^2 + v_0'(a), \tag{A27}$$

$$-\gamma c_0(a)^{-\gamma-1}c_1(a) = v_1'(a). \tag{A28}$$

Substituting the expressions for v_0 and c_0 from Lemma A1 into (A27)

$$\rho v_1(a) = v_1'(a)\left(\frac{r-\rho}{\gamma} + (\gamma-1)\frac{\sigma^2}{2}\right)a + v_1''(a)\frac{\sigma^2}{2}a^2 + B_0^{-\gamma}a^{-\gamma} \tag{A29}$$

It remains to find a solution $v_1(a)$ that solves the ODE (A29). We solve it using a guess-and-verify strategy. Guess $v_1(a) = B_1 a^{-\gamma}$. Then $v_1'(a) = -\gamma B_1 a^{-\gamma-1}$ and $v_1''(a) = \gamma(1+\gamma)B_1 a^{-\gamma-2}$. Substituting into (A29)

$$\rho B_1 a^{-\gamma} = -\gamma B_1 a^{-\gamma}\left(\frac{r-\rho}{\gamma} + (\gamma-1)\frac{\sigma^2}{2}\right) + \gamma(1+\gamma)B_1 a^{-\gamma}\frac{\sigma^2}{2} + B_0^{-\gamma}a^{-\gamma}$$

Rearranging we find that $B_1 = \frac{1}{r-\gamma\sigma^2}B_0^{-\gamma}$. Therefore

$$v_1(a) = \frac{1}{r-\gamma\sigma^2}B_0^{-\gamma}a^{-\gamma}$$

Similarly, substituting the expressions for $v_1'(a)$ and $c_0(a) = B_0 a$ into (A28) we find

$$c_1(a) = \frac{1}{r-\gamma\sigma^2}B_0$$

Substituting v_1 and c_1 as well as v_0 and c_0 from Lemma A1 into (A25), we obtain (A26) (recall again that the statement of the Proposition uses $\bar{c}$ to denote B_0).■

A.2 Housing (Section 1.2)

Recall that households maximize (10) subject to (11). As explained in the text, the budget constraint can be written in terms of total wealth $a = b + ph$ as

$$\dot{a} + c + Rh = w + ra, \quad \text{where} \quad R := rp - \dot{p}$$

is the user cost of housing. It is easy to see that this problem splits into separate inter- and intratemporal problems and can be solved as a two-stage budget problem. The intertemporal problem is

$$\begin{aligned} \max_{\{C_t\}_{t\geq 0}} \int_0^\infty e^{-\rho t}\frac{C_t^{1-\gamma}}{1-\gamma}dt \quad \text{s.t.} \\ \dot{a}_t = w + ra_t - P_tC_t \end{aligned} \tag{A30}$$

where

$$P_t = \left(\alpha + (1-\alpha)R_t^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{A31}$$

is a CES price index. Given the solution to this intertemporal problem, non-durable consumption and housing can then be found from the intratemporal problem. This problem is just a standard static utility maximization problem with CES utility. The solution is given by

$$c = \frac{\alpha}{R^{1-\eta}(1-\alpha)+\alpha}PC, \qquad h = \frac{R^{-\eta}(1-\alpha)}{R^{1-\eta}(1-\alpha)+\alpha}PC. \tag{A32}$$

A.2.1 Proof of Proposition 1

Problem (A30) is just a consumption-saving problem with a time-varying price level. The solution satisfies

$$C^{-\gamma} = \lambda P, \tag{A33}$$

$$\dot{\lambda} = (\rho - r)\lambda, \tag{A34}$$

$$\dot{a} = w + ra - PC. \tag{A35}$$

The budget constraint can be integrated to yield a lifetime budget constraint

$$\int_0^\infty e^{-rt}P_tC_tdt = \frac{w}{r} + a_0. \tag{A36}$$

Integrating the Euler equation (A34) forward in time, we have

$$\lambda_t = \lambda_0 e^{(\rho-r)t}.$$

Hence from (A33)

$$C_t^{-\gamma}/P_t = C_0^{-\gamma}/P_0 e^{(\rho-r)t}.$$

And in particular

$$P_tC_t/P_0C_0 = (P_t/P_0)^{1-\frac{1}{\gamma}}e^{-\frac{\rho-r}{\gamma}t}.$$

Substituting into the lifetime budget constraint (A36)

$$P_0C_0\int_0^\infty (P_t/P_0)^{-\frac{1}{\gamma}}e^{-\left(\frac{\rho-r}{\gamma}+r\right)t}dt = \frac{w}{r}+a_0,$$

and hence consumption expenditure is

$$P_0C_0 = \left(\int_0^\infty (P_t/P_0)^{1-\frac{1}{\gamma}}e^{-\left(\frac{\rho-r}{\gamma}+r\right)t}dt\right)^{-1}\left(\frac{w}{r}+a_0\right),$$

which is the expression for C_t in (12). Since $\dot{a}_0 = w + ra_0 - P_0C_0$, we also get the saving response, namely

$$\dot{a}_0 = w + ra_0 - \left(\int_0^\infty (P_t/P_0)^{1-\frac{1}{\gamma}}e^{-\left(\frac{\rho-r}{\gamma}+r\right)t}dt\right)^{-1}\left(\frac{w}{r}+a_0\right),$$

which is the expression for $\dot{a}_t$ in (12).■

A.2.2 Transitory Housing Capital Gains

The following corollary to Proposition 1 states our results for a transitory housing capital gain that we summarized informally in the main text.

Corollary 3 *Assume that $r=\rho$ and $\dot{p}=0$. Then a surprise increase in the house price, i.e. a transitory housing capital gain, has no effect on total consumption and welfare, $\partial C/\partial p = 0$, and the housing capital gain translates one-for-one into higher wealth.*

Proof of Corollary 3 When $\dot{p}=0$ the problem is stationary and we can drop t subscripts from variables. Equation (12) in Proposition 1 becomes

$$PC = \left(\frac{\rho-r}{\gamma}+r\right)\left(a+\frac{w}{r}\right), \qquad \dot{a} = \frac{r-\rho}{\gamma}\left(a+\frac{w}{r}\right) \tag{A37}$$

We are now in a position to derive the marginal effect on total consumption from a surprise and permanent change in the house price. If $r=\rho$, total consumption expenditure in (A37) is simply $PC = rb + rph + w$ where we have used that $a = b + ph$. Totally differentiating this expression with respect to p, we have

$$
\begin{aligned}
&P\frac{\partial C}{\partial p} + \frac{\partial P}{\partial p}C = rh \\
&P\frac{\partial C}{\partial p} + PCr\frac{(rp)^{-\eta}(1-\alpha)}{\alpha + (1-\alpha)(rp)^{1-\eta}} = rh \\
&P\frac{\partial C}{\partial p} + rh = rh \\
&\frac{\partial C}{\partial p} = 0
\end{aligned}
$$

where we have used (A31) and (A32). Hence, a surprise permanent increase in the house price has no effect on total consumption and therefore not on welfare. Further, we have that $\dot{a} = 0$ when $r = \rho$ from (A37). This implies that, wealth moves only on impact, immediately after the initial surprise housing capital gain, but not thereafter. That is, the surprise housing capital gain translates into higher wealth one-for-one.■

A.2.3 Illustration of the Response to Persistent Housing Capital Gains

Corollary 1 implies that the consumption aspect of housing will *not*, by itself, result in households keeping housing and consumption unchanged in the face of rising house prices. Why is this the case? The answer is that this intuition ignores that, if housing were divisible and freely adjustable, households would engage in "intertemporal substitution of housing." Figure A13 explains what we mean by this and shows how, in our model, consumption, wealth, and saving rates respond to a persistent housing capital gain. In the Figure, households

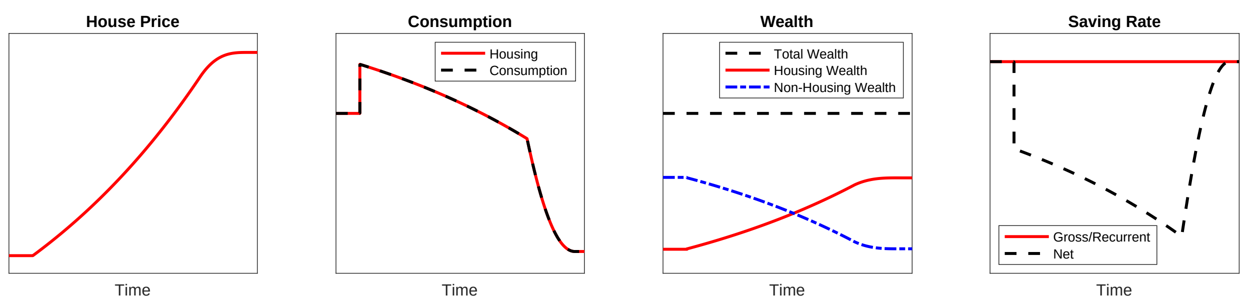


Figure A13: Response of key variables to a change in persistent housing capital gains.
Notes: the figure assumes $\eta = 0$, $\alpha = 0.5$, IES $= 1/\gamma = 1$, and $\rho = r = 0.05$.

suddenly learn that house prices will grow by two percent per year for the next twenty years. The figure assumes that the IES equals one and that the elasticity of substitution between housing and consumption η is zero (Leontief utility).[56] Because consumption and housing are perfect complements, they move together. On impact, they both increase because households purchase more housing in anticipation of the future price increases (which lower the user cost $R = rp - \dot{p}$). Along the house price path, households gradually sell off housing to finance its above-normal consumption. Since the IES equals one, gross saving is always zero and the market value of wealth is constant. Households only re-balance their portfolio between housing and bonds, but never change the market value of wealth. After twenty years of house price growth, households have the same market value of wealth $b + ph$ with a larger share in the form of housing wealth ph, but fewer physical units of housing h.

A.3 Common Extensions (Section 1.3)

This section presents the model in Section 1.3 and the calibration we use to produce Figure 3. A continuum of ex-ante identical and infinitely-lived households maximize the discounted utility flow from consumption,

$$\mathbb{E}_t \int_t^{\infty} e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt,$$

subject to a budget constraint

$$c_t + \dot{a}_t = w_t + r a_t,$$

where w_t is labor income that evolves stochastically according to an Ornstein-Uhlenbeck process (continuous-time analogue of an AR(1)) in logs

$$d \log w_t = -\nu \log w_t + \sigma_w dW_t.$$

We impose a no-borrowing constraint, $a_t \geq 0$. Markets are incomplete and households self-insure by accumulating wealth a. Conditional on their earnings history, households differ in their level of wealth and income. Table A2 presents the calibration we use to produce Figure 3.

[56]The figure also assumes that relative weight on non-durable consumption and housing α equals 0.5.

Table A2: Calibration of Huggett model

	Value	
γ	2	Relative risk aversion / inverse IES
ρ	0.05	Discount rate
r	0.045	Dividend yield
ν	0.03	Persistence of income innovations (annual autocorrelation = 0.97)
σ_w	0.14	Standard deviation of income innovations

B Appendix for Section 2

B.1 Data Sources and Variables

Source:	Variables:
Income and wealth from tax returns	Labor income
Annual, 1993 -	Business income
	Capital income
	Transfers received
	Taxes paid
	Asset holdings (e.g deposits, mutual funds, bonds, real estate)
	Debt (total debt)
	Pensionable income (since 1967)
https://www.ssb.no/en/omssb/tjenester-og-verktoy/data-til-forskning/inntekt	
Housing wealth database	Value of housing (including cabins and secondary homes)
Annual, 1993 -	(as in Fagereng, Holm and Torstensen, 2019)
Norwegian educational database	Highest completed education (length and type)
Annual, 1964 -	
https://www.ssb.no/en/omssb/tjenester-og-verktoy/data-til-forskning/utdanning	
Stockholder registry	ISIN / firm ID
Annual, 2004 -	Owner ID
	Quantity owned of stock
https://www.ssb.no/383782/utlan-av-data-om-aksjonaerer-aksjeselskaper-og-allmennaksjeselskaper	
Firm balance sheet and tax return data data	Balance sheet information (e.g. book value of equity, retained earnings)
Annual, 1995 -	Assessed value of private companies
https://www.ssb.no/en/omssb/tjenester-og-verktoy/data-til-forskning/regnskap	
The central population register	Region of residence at the end of the year
Annual, 1964 -	Date (month) of birth
	Gender, indicator variable for gender
	Marital status indicator variable
	Spousal id (unique identifier of spouse)
https://www.ssb.no/en/omssb/tjenester-og-verktoy/data-til-forskning/befolkning	
Ambita / Norwegian mapping authority	Buyer/seller ID
Event data, 1993 -	Price
https://www.ambita.com/tjenester/eiendomsinformasjon/	
Other public data sources:	
Consumer price index, Statistics Norway	
https://www.ssb.no/en/priser-og-prisindekser/statistikker/kpi	
Flow of funds, Statistics Norway	
https://www.ssb.no/en/nasjonalregnskap-og-konjunkturer/statistikker/finsekv	
Stock price index and general stock prices, Oslo Børs	
https://www.oslobors.no/ob_eng/markedsaktivitet/#/details/OBX.OSE	
House price indices, Eitrheim and Erlandsen (2004)	
https://www.norges-bank.no/en/topics/Statistics/Historical-monetary-statistics/	
Exchange rate data, Norges Bank	
https://www.norges-bank.no/en/topics/Statistics/exchange_rates/	
MSCI stock index	
https://finance.yahoo.com/quote/MSCI/history/	

B.2 Net, Gross and Recurrent Saving with Multiple Assets

Section 1.1 defined net, gross and recurrent saving with one asset. The data feature multiple assets and in this section, we generalize the saving definitions.

A household receives annual income w_t (labor income and transfers) and pays taxes τ_t. There are J assets indexed by $j = 1, ..., J$. Let $k_{j,t-1}$ denote the household's holdings of asset j at the end of period $t-1$. To simplify notation, assume that the household holds each asset $k_{j,t-1}$ throughout the year and only makes transactions at the end of the year. Throughout the year, asset holdings $k_{j,t-1}$ earn capital income $\theta_{j,t}p_{j,t}k_{j,t-1}$. The general versions of (7) and (8) are then

$$c_t + \underbrace{\sum_{j=1}^{J} p_{j,t}(k_{j,t} - k_{j,t-1})}_{\text{net saving}} = \underbrace{w_t - \tau_t + \sum_{j=1}^{J} \theta_{j,t}p_{j,t}k_{j,t-1}}_{\text{disposable income}} \tag{A38}$$

$$c_t + \underbrace{\sum_{j=1}^{J}(p_{j,t}k_{j,t} - p_{j,t-1}k_{j,t-1})}_{\text{gross saving}} = \underbrace{w_t - \tau_t + \sum_{j=1}^{J} \left(\theta_{j,t} + \frac{p_{j,t} - p_{j,t-1}}{p_{j,t-1}}\right) p_{j,t-1}k_{j,t-1}}_{\text{Haig-Simons income}}. \tag{A39}$$

Similarly, the analogue of (9) defining recurrent saving and income is (A39) but with only the persistent component of capital gains added to both sides.

B.3 Separating Gross Saving into Net Saving and Capital Gains

B.3.1 Housing: Using Transaction Data

To explain our approach, it is helpful to introduce some notation. Time is continuous and we consider a household that makes housing transactions at discrete time intervals. We denote by $h(t), p(t)$ and $a_h(t) = p(t)h(t)$ the household's physical number of housing units, the price of housing and the value of the house at the beginning of year t. Throughout a year, i.e. between dates t and $t + 1$, the household makes $N \geq 0$ transactions at ordered dates τ_n: $t \leq \tau_1 < \tau_2 < ... < \tau_N < t+1$. We decompose gross saving, i.e. the change over the year in housing wealth $a_h(t) = p(t)h(t)$, into net saving and capital gains using the following decomposition

$$\underbrace{a_h(t+1) - a_h(t)}_{\text{gross saving}} = \underbrace{\sum_{n=1}^{N+1} (p(\tau_n) - p(\tau_{n-1}))h(\tau_{n-1})}_{\text{capital gains}} + \underbrace{\sum_{n=1}^{N} p(\tau_n)(h(\tau_n) - h(\tau_{n-1}))}_{\text{net saving}} . \tag{A40}$$

For our purpose, the main implication is that net saving can only be non-zero for households with housing transactions. Hence, for households without housing transactions, net saving in housing is zero and all changes in housing wealth are due to capital gains. In the case with transactions, on the other hand, (A40) implies that net saving is equal to net transactions at market value during the year. Capital gains in housing is then the change in housing wealth minus net transactions at market value.

B.3.2 Stocks: Using Ownership Data

For most asset classes, we do not know the individual transactions within the year. We therefore approximate capital gains and net saving based only on available information at time t and $t+1$. For example, for stocks we know the number of shares q in each stock and its price p at the beginning and end of the year. We make three simplifying assumptions to compute capital gains and net saving:

1. All transactions are of the same size and direction: $dq_{\tau_n} = \frac{q_{t+1}-q_t}{N}$.
2. All prices move monotonically and with same step size within a year: $p_{\tau_n} = (\tau_n - t)(p_{t+1} - p_t) + p_t = \frac{n}{N+1}p_{t+1} + \frac{N+1-n}{N+1}p_t$.
3. All transactions are distributed uniformly across the year: $\tau_n - \tau_{n-1} = d\tau \ \forall n$.

Under these assumptions, we derive an expression for net saving from observables

$$\begin{aligned}
\sum_{n=1}^{N} p_{\tau_n} dq_{\tau_n} &= \sum_{n=1}^{N} \left(\frac{n}{N+1} p_{t+1} + \frac{N+1-n}{N+1} p_t \right) \frac{q_{t+1} - q_t}{N} \\
&= \frac{q_{t+1} - q_t}{N} \sum_{n=1}^{N} \left(\frac{n p_{t+1} + (N+1-n) p_t}{N+1} \right) \\
&= \frac{q_{t+1} - q_t}{N} \sum_{n=1}^{N} \left(\frac{n(p_{t+1} - p_t)}{N+1} + p_t \right) \\
&= \frac{q_{t+1} - q_t}{N} (N p_t + \frac{1}{2} N (p_{t+1} - p_t)) \\
&= \frac{1}{2} (p_t + p_{t+1})(q_{t+1} - q_t)
\end{aligned}$$

Capital gains is next defined as the change in total value of an asset not accounted for by net saving.

B.4 Private Businesses

The portfolio shares in Figure 4 show that many of the wealthiest households hold a substantial share of their wealth in private businesses. Since these firms are not publicly traded, there is no available market price. In this appendix, we describe how we account for private businesses.

A private business is a company that is not listed on a stock exchange and owned by a small number of shareholders. Control of the firm is therefore limited to a few persons. These firms are typically small to medium sized businesses or holding companies. In 2006, Norway introduced a dividend tax at the individual level. One response to this tax reform was that the number of holding companies grew such that individuals could retain earnings in firms to avoid paying the dividend tax. These holding companies are therefore common, especially at the top of the wealth distribution.

Our aim is to find the ultimate owners of private businesses to be able to allocate retained earnings, public stock ownership, debt, and capital gains onto the ultimate owner's balance sheet. The approach is similar to other papers using Norwegian data (Alstadsæter et al., 2016; Fagereng et al., 2019).

Ultimate owners of private businesses. We use the stock holder registry to find the ultimate owners of private businesses. The stock holder registry contains information of individuals' and firms' ownership of stocks in all companies in Norway. Some companies are held directly. In this case, the ownership share is the fraction of total shares owned by the individual. However, many companies are owned by other firms. To fix ideas, assume an individual owns shares in company A and company A owns shares in company B. In this case, the individual holds an ownership share in company B equal to that individual's ownership share in company A multiplied with company A's ownership share in company B. We compute indirect ownership through up to 7 layers.

Retained earnings. Retained earnings is the profit of the firm that is withheld in the firm by not paying dividends. These are profits that accrue to the company but will not be accounted for on the income statement of individuals. Alstadsæter et al. (2016) show that retained earnings in private businesses have grown sharply after the dividend tax reform in 2006. By not accounting for retained earnings properly, we underestimate earnings of (wealthy) individuals.

To compute retained earnings, we follow the method in Alstadsæter et al. (2016) and exploit the Norwegian accounting concept of earned equity, defined as accumulated retained earnings. Retained earnings in a private business in year t is therefore the difference between earned equity at the end of year t and the beginning of year t. After obtaining retained earnings at the level of the private business, we allocate it to the ultimate owners' income using the ownership register.

Balance sheets. To more precisely measure individuals' portfolio shares and exposure to risky assets and debt, we allocate all publicly-traded stocks and debt onto the ultimate owners' balance sheets. At the end of each year, the private business reports its balance sheet to the tax authorities. Both publicly-traded stocks and debt are directly observed on these firm balance sheets and we allocate these to the ultimate owners' balance sheet using the ownership register.

Capital gains. Private businesses hold publicly traded stocks that accumulate capital gains. From the stockholder registry, we can see which stocks a private business hold. We are therefore able to compute capital gains in a private business in the same way as for publicly traded stocks held by individuals in Appendix B.5. Once we obtain a measure of capital gains at the level of the private business, we allocate these capital gains to ultimate owners' capital gains using the ownership register.

B.5 Stock Ownership in Norway

The portfolio shares in Figure 4 show that ownership of publicly-traded stocks, held either directly by the individual or indirectly though stock funds or private businesses, is relatively low in Norway compared with other OECD countries. For example, the mean portfolio share in publicly-traded stocks is about 1.5% for all individuals and less than 5% for the top 1% of the wealth distribution. In contrast, the top 1% in the US hold more than 40% of their assets in public equity (Campbell, 2006). There are two main reasons why the portfolio share of publicly-traded stocks is lower in Norway than in the US. First, Norway has a public pension system that holds a substantial position of Oslo Stock Exchange (OSE) on behalf of the Norwegian population. This indirect ownership of publicly-traded stocks does not enter individuals' balance sheets in the way for example 401k accounts enter the balance sheets of US citizens. Second, Oslo Stock Exchange is smaller as a share of GDP than in other similar countries. For example, the market capitalization of listed domestic companies relative to GDP is about twice as large in Sweden as in Norway.[57] In this appendix we document the ownership structure of Oslo Stock Exchange and to what extent we are able to account for aggregate stock ownership at the individual level.

Ownership structure of Oslo Stock Exchange. Table A3 presents the ownership structure in aggregate data from the Oslo Stock Exchange and in the ownership registry in 2015. In 2015, 33.6% of the market capitalization of Oslo Stock Exchange was held by the government sector. There are two main reasons why the government sector has such a large ownership share. First, the government sector includes pension funds both at the state and municipality level.[58] Norway has a public pension system where all citizens are enrolled. A part of the pension funds is invested in public stocks in Norway. Second, it includes the government's direct ownership of firms. The Norwegian government owns substantial fractions of many publicly-traded companies in Norway, both for historic and strategic reasons. For example, the Norwegian government still holds large positions in many Norwegian banks after the re-capitalization of the banking system in the early 1990s.

In 2015, foreign investors held 36.8% of Oslo Stock Exchange. This ownership share has also been stable between 33% and 41% in our sample period. Next, 8.8% of the stock exchange is held by the financial sector. This is mainly stock funds (6.8%), while the rest

[57] See https://data.worldbank.org/indicator/CM.MKT.LCAP.GD.ZS?locations=NO-SE.

[58] Note that the Norwegian Sovereign Wealth Fund does not hold stocks in Norway and is therefore not included in this ownership share.

Table A3: Ownership structure of Oslo Stock Exchange (OSE), 2015

Owner sector	OSE Annual report	Ownership registry	Potentially controlled by individuals	Allocated to individuals
Government[1]	33.6%	33.0%		
Foreign investors	36.8%	32.4%		
Financial sector[2]	8.8%	8.4%		
Other companies	16.7%	11.2%	9.7%	6.0%
Private investors	3.9%	3.7%	3.7%	3.5%
Others	0.1%	0.0%		
Sum	100.0%	88.6%	13.4%	9.4%

Notes: "OSE Annual report" refers to the annual report of Oslo Stock Exchange and "Ownership registry" refers to the ownership registry that is available with individual owners. "Potentially held by individuals" refers to the share of Oslo Stock Exchange that is potentially controlled directly by individuals. The difference between "ownership registry" and "potentially controlled by individuals" is the stocks that are held by companies that are listed on the stock exchange. "Allocated to individuals" is the share of stocks at Oslo Stock Exchange that we ultimately allocate to individuals, either via indirect ownership through private businesses or directly held stocks.
[1] Government includes the categories "government and municipalities" and "companies with government ownership."
[2] Financial sector includes the categories "banks and mortgage companies," "private pension funds/life insurance," "stock funds," and "general insurance."

(2.0%) is held by banks and mortgage companies, private pension funds or life insurance companies, and general insurance companies.

The next two ownership categories, "other companies" and "private investors," are the most interesting for our purpose because a large share of these categories are controlled by Norwegian individuals. Other companies includes all stocks that are held by Norwegian companies. Many individuals hold stocks through private businesses and these are included in this sector. The private investors sector includes all stocks that are directly held by Norwegian individuals. The sum of other companies and private investors is therefore an upper bound the share of the stock exchange that is controlled by Norwegian individuals and that we may be able to find using the available data.[59]

Although the sum of "other companies" and "private investors" is the upper bound for the share of stocks that are directly controlled by Norwegian individuals in the annual report, it is not the upper bound that we can unravel from the registry data for two reasons. First, there is a discrepancy between the ownership registry in the micro data and the official data from Oslo Stock Exchange. This discrepancy brings the ownership share of other companies

[59] Note that the sum of "other company" and "private investors" is not the strict upper bound of ownership that is held by Norwegian individuals since they can hold stocks on Oslo Stock Exchange indirectly through foreign companies. For example, Alstadsæter, Johannesen and Zucman (2019) document that almost 30% of taxes among the top 0.01% are evaded in Norway.

down from 16.7% to 11.2%. Second, a share of the sector other companies are stocks that are either held by the company itself, held by other publicly traded companies or held by foreign companies. By excluding the share that are owned by other publicly-traded companies, the share of the sector other companies declines further from 11.2% to 9.7%. 9.7% is therefore the upper bound on what share of stocks on Oslo Stock Exchange that we potentially can allocate to individuals. At the end of the day, we are able to allocate 6.0% of the stock exchange to the ultimate owners held through private businesses.

B.6 Public Pension Wealth

This appendix describes how we compute public pension wealth. We define public pension wealth as the net present value of future pension income, discounted at the risk free interest rate and accounting for the probability of living to the retirement age. The main complication is that there are currently four different pension systems depending on birth cohorts. We describe each system in detail before we define public pension wealth, savings, and income. A common feature of all public pension systems is that an individual accumulates claims pen_t^c in units of the basis-amount in the social security system G_t.

B.6.1 The Four Pension Systems

Cohorts born prior to 1944. For these cohorts, we observe pension transfers in our data. Pension wealth is therefore computed as the net present value of these pension earnings.

Cohorts born between 1944 and 1953. For these cohorts, there are two parts of the pension system: the social security and the service pension.

1. **Social Security.** The social security system is based on a point system. Each year, individuals accumulate pension points based on the following formula

$$point_t = \begin{cases} \frac{y_t}{G_t} - 1 & \text{if } G_t \leq y_t \leq 6G_t \\ 5 + \frac{y_t - 6G_t}{3G_t} & \text{if } 6G_t \leq y_t \leq 12G_t \\ 7 & \text{if } y_t \geq 12G_t \end{cases}$$

   where y_t is gross earnings in year t and G_t is the basis-amount in the social security system.

An individual's pension number P is then defined as the average of the 20 years with the highest pension points, or the average of the n years that person has earned pension points. The payouts from the social security pension is approximately

$$pen^{SC} = \alpha + \max\left\{\kappa P \frac{min\{n, 40\}}{40}, 1\right\}$$

where $\alpha = 1$ for singles and 0.85 for couples, and κ is a proportionality factor equal to 0.45 if income was accumulated prior to 1992 and 0.42 after 1992.

2. **Service Pension.** All public sector employees and about 50 percent of private sector employees have an additional service pension on top of their social security pension. This service pension guarantees an individual a fraction ψ of their final income. This fraction depends on how long the individual has worked for the company/government, but the maximum is $\psi = 0.66$.

   The service pension pays the difference between the sum implied by the fraction rule and the level received from the social security pension. We can therefore approximate the pensions in units of G as

$$pen_t^c = \max\left\{\alpha + \max\left\{\kappa P \frac{min\{n, 40\}}{40}, 1\right\}, \frac{0.66 y^{final}}{G^{final}}\right\}$$

   where n is the number of working years, and y^{final} and G^{final} are income and basis-unit in social security in your final working year, respectively.

Cohorts born between 1954 and 1962. Pensions in units of G is a linear combination of the system for those born prior to 1954 outlined above and those born after 1963 outlined below:

$$pen_t^c = \frac{63 - c}{10} pen_t^{44 \leq c \leq 53} + \frac{c - 53}{10} pen_t^{c \geq 63}.$$

Cohorts born after 1963. In 2010, the government simplified the pension system for all earners born after 1964. The new system implies that every year, 18.1 % of your gross income below 7.1 G is added to your pension holdings ("pensjonsbeholdning," P_t).

$$P_t^c = \left(\sum_{\tau=c+13}^{t} \max\left\{0.181 \frac{y_\tau}{G_\tau}, 0.181 \cdot 7.1\right\}\right)$$

P_t^c is, as before, defined in units of G, the basis-amount in the Norwegian pension system.

One complication is that you can start taking out pensions from age 62. However, to simplify the exposition, we assume that all households value their pension as if they would start taking out pensions from age 67. From age 67, your income from pensions is the value of your pension holdings P_t^c in units of G divided by your expected remaining years (e.g. 16.02 for the cohort born in 1964).

$$pen_t^c = \frac{P_t^c}{d_c}$$

B.6.2 Pension Wealth, Saving, and Income

We define pension wealth as the net present value of pension income from age 67. That income is $pen_t^c G_t$ in each year t when t is greater than 67, where pen_t^c is defined by one of the four systems above depending on your birth cohort. In order to calculate the net present value, we discount the pension contributions net of taxes by the risk free real interest rate and the survival probability[60]

$$V_t^c = (1-\tau) pen_t^c G_t \mathcal{M}_{t,c+67} \left\{ \sum_{\tau=\max\{c+67,t\}}^{\max\{c+67+d_c,t\}} \frac{\prod_{s=t}^{\tau}(1+\pi_{w,s})}{\prod_{s=t}^{\tau}(1+r_s)} \right\} \tag{A41}$$

where τ is the median tax rate on pensions (17 %), $\mathcal{M}_{t,c+67}$ is the probability of surviving from year t to year $c+67$ when the household is born in year c, and $\pi_{w,s}$ is the real growth rate in G in year s and r_s is the real interest rate in year s. The max operator is there because an individual start withdrawing from the pension account after age 67. We have to make two assumptions to calculate pension wealth in the data. First, we assume perfect foresight in the years where we observe r_s and $\pi_{w,s}$. Second, we assume that after 2015, the expected real interest rate and growth rate of G are the observed geometric mean in the years from 1993 to 2015. For example, in order to calculate pension wealth in 2006, we discount by the observed real interest rate from 2006 to 2015, and with the mean real interest rate after 2015.

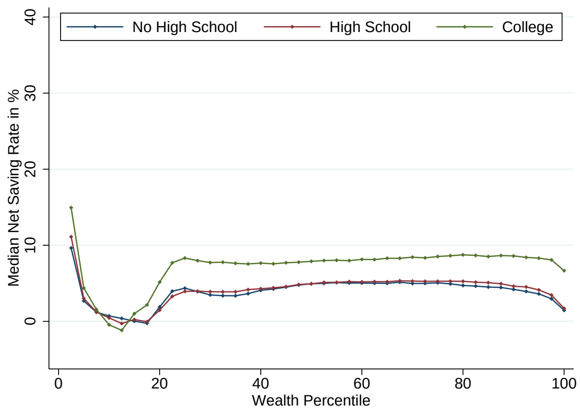
We define pension saving as the change in pension wealth. Arguably, changes in pension wealth may be due one of the following three reasons: (i) net withdrawals or contributions, (ii) revaluation due to discounting by the real interest rate, or (iii) revaluation because the probability of surviving to age 67 increases. We count all changes in pension wealth as net saving. Furthermore, we define pension income in such a way to ensure that the budget

[60] We calculate the survival probability of living from age t to $c+67$ from the Norwegian mortality tables. It is about 90 % for a 20 year old in our sample and increases toward 1 as the individual ages.

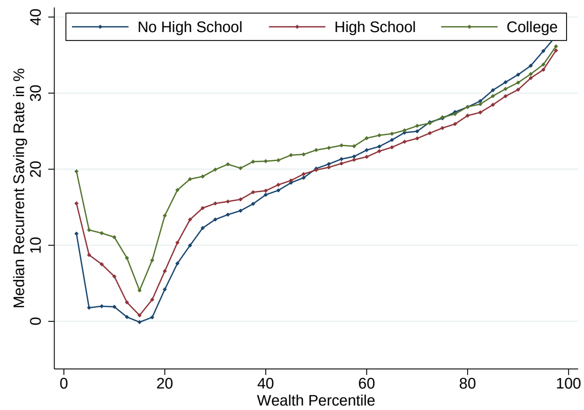
constraint adds up. This implies that pension income always equals pension saving.

C Appendix for Section 3: Additional Exercises

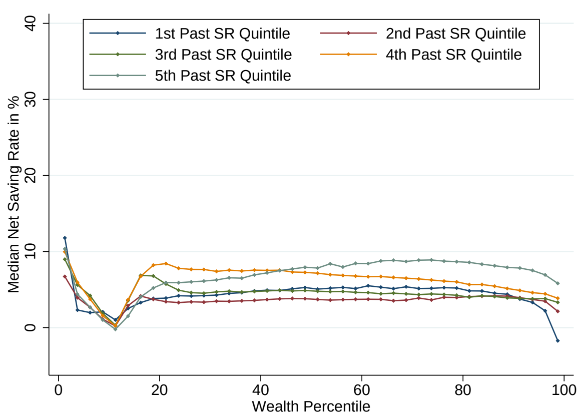
Saving Rates by Education and within Deciles of Historical Saving Rates. These results and the approach behind them are discussed in the main text. Figure A14 plots the results.



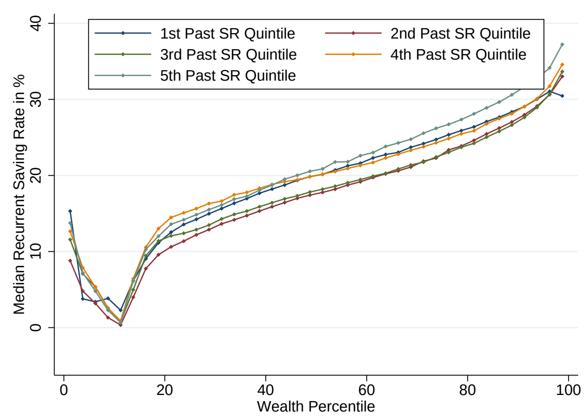
(a) Net saving rate by education group



(b) Recurrent saving rate by education group



(c) Net saving rate by past saving rate group



(d) Recurrent saving rate by past saving rate group

Figure A14: Saving rates across the wealth distribution by education and within deciles of past saving rates

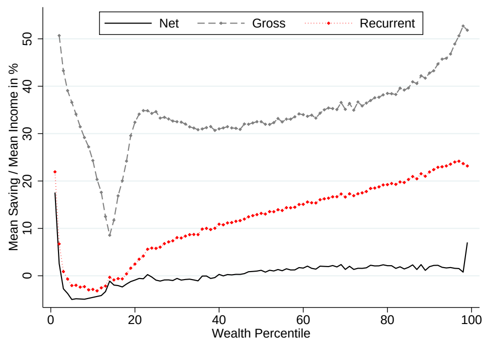
Notes: The figures display the median net saving rates (left) and median recurrent saving rates (right) within percentiles of past saving rates. Conditional upon observing a household for at least 4 prior years, we compute each household's past recurrent saving rate for every year, and thereafter stratify each household-year observation by average past recurrent saving rate. All variables are computed as the median within wealth percentile and year, averaged across all years (2005-2015).

Saving Rate as Ratio of Means. An alternative approach to what we do in the main text is to compute the saving rate in a percentile as the ratio of average saving and average income. For example, Krueger, Mitman and Perri (2016, Table 2) show that expenditures as a share of disposable income in the U.S. is declining in wealth. The comparable prediction in our data is to say that the net saving rate, as measured by the ratio of average saving and average income within wealth percentiles, should be increasing in wealth. In Figure A15(a), we do find that the net saving rate measured as ratios of means is approximately flat across the wealth distribution, while the recurrent saving rate is increasing in wealth, similar to our main results.

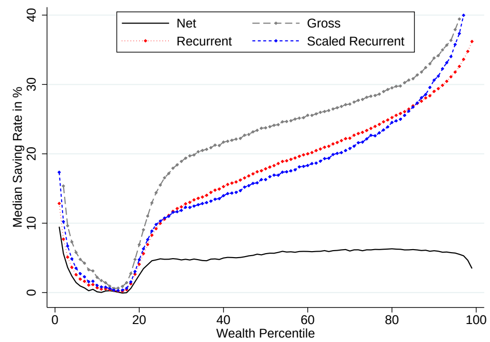
Saving Rates Adjusted for Scale-Effects in Returns. In the paper, we assume that all households have the same recurrent capital gain. However, Fagereng et al. (2019) show that wealthy individuals tend to have higher returns to wealth, indicating that our assumption of the same recurrent capital gains for all households may be misleading. In this appendix, we compute an alternative measure of recurrent capital gains where we take into account the scale-effects of wealth on returns. In particular, we compute percentile-specific estimates of μ for each asset class using the realized capital gains in our sample. For example, for housing, we first compute the median capital gains rate on housing in the percentile. Next, we adjust the μ by adding this median percentile-specific capital gains rate and subtracting the economy-wide median capital gains rate.

Figure A15(b) shows our new measure of recurrent saving together with the three measures of saving rates from the main text. The scale-adjustment neither affects the net nor the gross saving rate since it is an adjustment only for the recurrent capital gains rate. But it does affect the recurrent saving rate. Since there is a scale-effect in our data, the adjustment increases the recurrent saving rate for the wealthiest while it decreases the recurrent saving rate for the wealth-poor. However, we still observe a net saving rate that is approximately flat across the wealth distribution while the saving rates including capital gains are increasing in wealth.

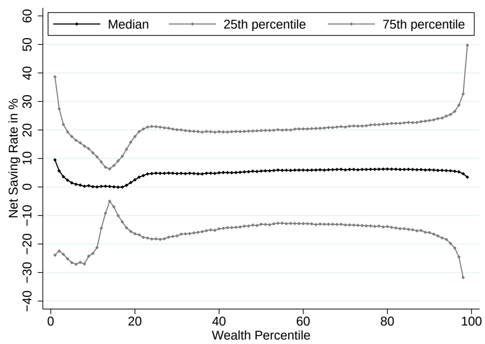
Dispersion in Saving Rates. In our main exercise, we compute medians within wealth percentiles. While interesting and informative for models, our approach ignores the dispersion in saving rates that exists within wealth percentiles. Figure A15(c) and A15(d) present the net saving rate and the recurrent saving rate together with their respective 25th and 75th percentile within the wealth percentile. The additional lines are computed in the same



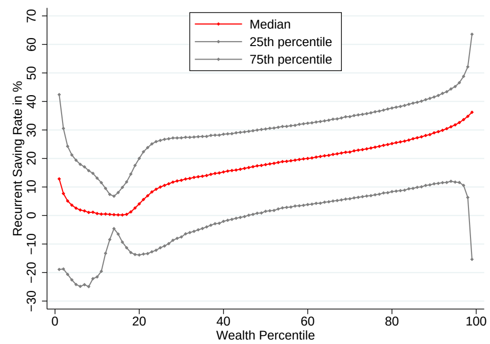
(a) Mean(saving)/mean(income)



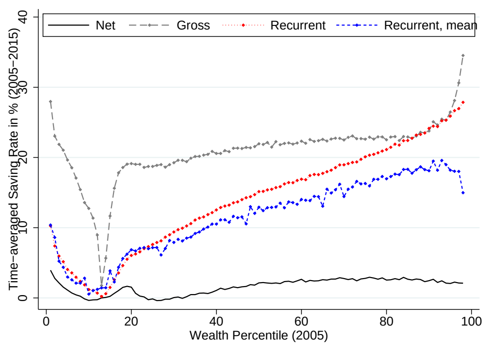
(b) Saving rates adjusted for scale effects



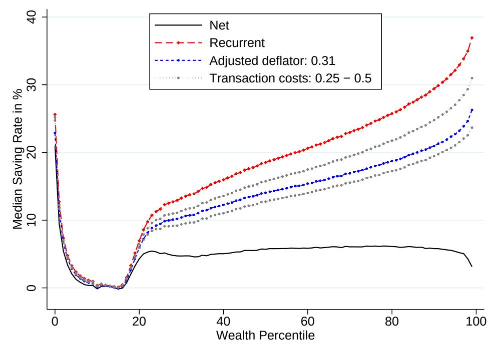
(c) Dispersion in net saving rate



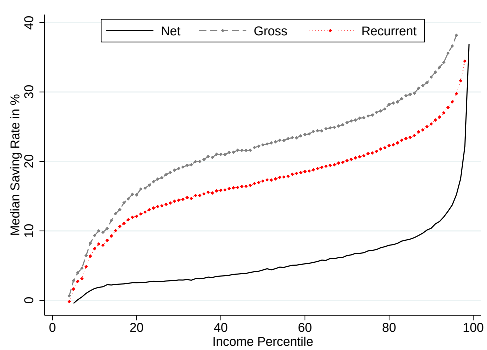
(d) Dispersion in recurrent saving rate



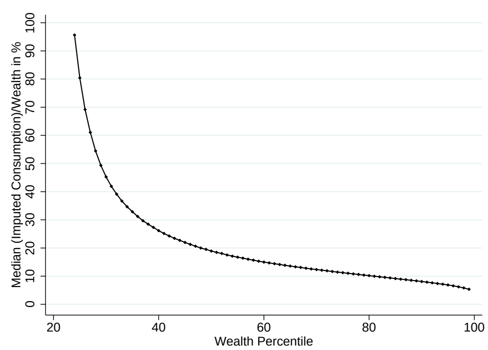
(e) Alternative Time-averaging



(f) Consumption-equivalent saving rates



(g) Saving rates across income distribution



(h) Imputed consumption/wealth

Figure A15: Additional Exercises

way as the main graph. For example, the 25th percentile line is computed by first computing the 25th percentile within the wealth percentile in each year, and then averaging across all years (2005-2015). The most striking feature of the dispersion graphs is that, dispersion is relatively stable across the wealth distribution except for the tails. For the net saving rate, also the 25th and 75th percentiles are flat across the wealth distribution for households with positive net worth. Similarly, for the recurrent saving rate, the 25th and 75th percentiles are increasing in wealth for households with positive net worth.

Alternative Time-Averaging. Our main graph was constructed by first computing median saving rates within each percentile for every year, and then plotting the average of these yearly median saving rates. A concern with this approach is that if households transact irregularly, for example if they buy homes every fifth year, the median households might always be non-transactors who therefore save a relatively large fraction of capital gains. To address this concern, we here present an alternative way of time-averaging saving rates. We first compute the average saving rate for each household over the entire period (2005-2015). Thereafter we stratify households by their percentile in the 2005 wealth distribution. Then we compute the within-percentile median and mean of households' time-averaged saving rate. This way, the saving rate we compute also includes years in which households transact. Figure A15(e) presents the median net, gross, and recurrent saving rates, in addition to the mean recurrent saving rate, plotted against the 2005 wealth distribution on the horizontal axis. We see that also with this definition, our main qualitative result withstands: the net saving rate is approximately flat across the wealth distribution, while the recurrent saving rate increases with wealth.

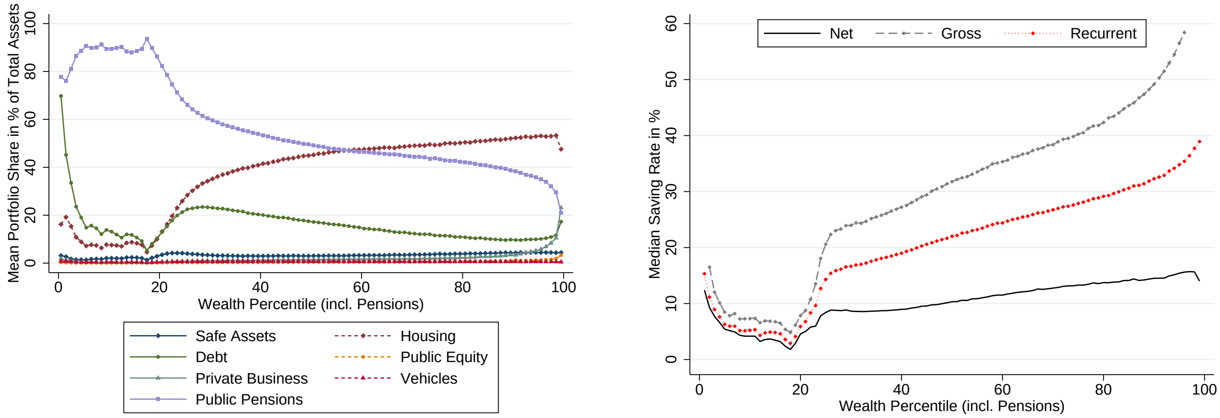
Consumption-equivalent Saving Rates. Our last exercise entails computing something we term the consumption-equivalent saving rate. One issue in our data is that capital gains are mainly due to house price appreciations and households might not want to consume capital gains in housing for at least two reasons: (1) since the price of housing services also increases with capital gains in housing such that the household is not necessarily wealthier with capital gains, and (2) because consuming out of capital gains in housing often requires transaction costs such that one should expect households to save a large share of the capital gains in housing. We compute two additional saving rate concepts to check whether these are quantitatively relevant reasons why we observe a gap between recurrent and net saving rates that is increasing with wealth. First, we compute recurrent saving rates, but this time adjusted with a price index that includes house price appreciations. The consumer price index (CPI) we use to normalize our data contains house prices, but house prices in the CPI grow at a significantly lower rate than in our data. The blue line in Figure A15(f) presents the recurrent saving rate when we replace housing in the CPI (31%) with a market-based house price index. Second, we compute recurrent saving rates taking into account that only a fraction of the capital gains can be converted into consumption due to transaction costs. The two grey lines in Figure A15(f) presents this case when we assume transaction costs of 25% and 50%, respectively. The take-away here is that any such adjustment will not change our main story, i.e. that we observe a flat net saving rate and an increasing recurrent saving rate, only decrease the gap between recurrent and net saving rates.

Saving Rates across the Wealth Distribution Including Pensions. Figure A16 presents the portfolio shares and the saving rates when we include pensions.[61] The first thing to note is that the public pension system has complete coverage of the Norwegian population. This means that including public pensions adds wealth to every person with a Norwegian passport. Furthermore, the public pension system is relatively generous. In particular, everyone is entitled to a minimum pension equal to approximately $20,000 every year after retirement. Since the discounting approximately cancels out the real wage growth, the value of this pension claim is approximately equal to $20,000 per year multiplied by 20 years, net of taxes on pension benefits (approximately 17%) and the probability of living until age 67. This implies that every 20 year old has a pension wealth approximately equal to $300,000. Figure A16(a) reveals that almost no Norwegian has negative net wealth if we count their claims to public pensions. Furthermore, public pensions are a substantial share of net worth across the wealth distribution, ranging from almost 100% to about 20% in the top 1% wealth group.[62]

Figure A16(b) shows the saving rates across the wealth distribution when we include pensions. The first thing to note is that all saving rates shift up when we include pension saving. For almost all Norwegians, pension wealth increases from one year to the other because (i) they work that year and add to their pension wealth or (ii) they live another year such that the probability of living until age 67 increases. Hence, including pensions adds saving for almost all individuals.

[61] We exclude retirees from sample when we look on portfolios and saving rates including pensions. This is because retirees typically have approximately zero income when we include pension income (their income = pension benefits - reduction in pension wealth ≈ 0), making saving rates explosive.

[62] The 20% share at the top may seem large. However, most individuals in the top 1% wealth group has the maximum public pension, equal to about $55,000 per year and worth about $900,000. Taking into account that the threshold for entering the top 1% wealth is a little less than $2,000,000 (excluding pensions), 20% is a reasonable average.



(a) Portfolio Shares

(b) Saving Rates

Figure A16: Portfolio shares and saving rates, including pension wealth, and income and savings from pensions. The sample excludes retirees.

The second notable change when comparing figure A16(b) with figure 5 is that by including pension saving, the saving rate becomes increasing in wealth percentiles, even for net saving rates. The minimum pension is the main reason this happens. Young individuals, typically at the lower part of the wealth distribution, have not yet worked long enough to accumulate pension wealth above the minimum pension. Hence, from year to year, their pension saving is small. Older individuals, on the other hand, have long enough earnings history to have accumulated pension wealth above the minimum pension. Hence, for every working year, old individuals add a part of their labor earnings to their pension wealth. Since old individuals typically are wealthier, this saving channel makes saving rates more increasing in wealth percentiles.

D Calibration of the Two-Asset Model in Section 5

Table A4 presents the calibration of the two-asset model in Section 5. There are two set of parameters that matter for our results. First, the wedge between borrowing and lending rates κ is sizable, ensuring that the borrowing rate is greater than the sum of the discount rate and the mortality rate. The wedge ensures that saving rates are high for households with debt. Second, the sum of dividends θ and capital gains μ on the investment asset is lower than the cost of debts. This ensures that households repay debt before they accumulate the investment asset. At the same time, the return on the investment asset (the sum of dividends and capital gains) is higher than both the return on the consumption asset and the sum of the discount rate and the mortality rate, making the investment asset attractive.

Table A4: Calibration of two-asset model

	Value	
Preferences		
γ	2	Relative risk aversion / inverse IES
ρ	0.04	Discount rate
η	0.01	Mortality rate
Capital markets		
θ	0.10	Dividend yield
μ	0.015	Capital gains
r^b	0.02	Interest rate on consumption asset
κ	0.10	Wedge between borrowing and lending rates
Adjustment costs		
χ_0	0.50	
χ_1	6.50	
Income		
w	5	Wage
ν	0.03	Persistence of income innovations (annual autocorrelation = 0.97)
σ_z	0.14	Standard deviation of income innovations