Online Appendix for "Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model"

A Investment in a canonical HANK model

In this section, we write down a canonical HANK model with investment.[43] We then show that, in such a model, investment acts a strong amplifier of monetary policy shocks. This confirms that our findings regarding the role of investment are due to the presence of heterogenous agents with high average MPCs, rather than some other feature of our model in the main text.

The model here is a sticky-wage, flexible-price HANK model with capital adjustment costs, as in Auclert and Rognlie (2018) and Auclert, Rognlie and Straub (2018). In Auclert and Rognlie (2017), we explain why this is a more natural starting point for a HANK model than the opposite assumption of flexible wages and sticky prices.[44] The model in the main text adds features that are necessary to obtain a good micro and macro fit: household inattention, sticky prices, government spending and debt, and indexation of both prices and wages; and it replaces capital with investment adjustment costs. But the core complementarity between investment and MPCs that we highlight here is robust to the addition of these additional features.

A.1 Model setup

Individual-level productivity states s follow a Markov process with transition matrix Π. Unions make all households work an equal number of hours N_t. There is no taxation, so take-home pay for a household in state s at time t is $w_t N_t e(s)$, where $e(s)$ is idiosyncratic productivity. Households can trade in one asset, a liquid deposit ℓ issued by financial intermediaries. The household problem is therefore

$$
\begin{aligned}
V_t(\ell, s) \quad &= \quad \max_{c,a'} u(c) + \beta \mathbb{E}_t\left[V_{t+1}(\ell', s')\right] \\
\text{s.t.} \quad & \quad c + \ell' = \left(1 + r_t^{\ell}\right)\ell + w_t N_t e(s) \\
& \quad \ell' \geq 0
\end{aligned}
$$

Nominal wages are set by unions, subject to Calvo wage rigidity. The optimization problem of unions implies a standard Phillips curve for wages[45], which can be written to first order (see

[43] The model and main results in this appendix previously appeared in a June 2018 SED presentation, "Forward Guidance is More Powerful Than You Think," where we pointed out that the MPC-investment interaction, in the absence of other frictions like informational rigidities, could aggravate the forward guidance puzzle. Here we make the same point about amplification for standard AR(1) monetary policy shocks.

[44] This also avoids the (counterfactual) very high countercyclicality of profits in a flexible-wage, sticky-price model.

[45] This is included only for completeness: given the exogenous monetary policy for the real interest rate r_t and the lack of any other nominal rigidity, the slope of this Phillips curve is irrelevant for real equilibrium outcomes.

Auclert, Rognlie and Straub 2018) as

$$\pi_t^w = \kappa^w \int N_t \left(v'(N_t) - \frac{\epsilon - 1}{\epsilon} u'(c_{it}) \right) di + \beta \mathbb{E}_t \left[\pi_{t+1}^w \right]$$

A financial intermediary issues liquid deposits to households and invests them in firm shares. At the beginning of the period, the value of its outstanding deposits must be equal to the liquidation value of firm shares, i.e.

$$\left(1 + r_t^\ell\right) L_{t-1} = (p_t + D_t) v_{t-1}$$

At the end of the period, the value of newly-purchased shares must be equal to the value of newly issued deposits and reserves, i.e.

$$p_t v_t = L_t$$

We also allow the financial intermediary to invest in nominal reserves that pay a promised return of i_t and are in zero net supply. The financial intermediary maximizes the expected return to depositors $\mathbb{E}_t [r_{t+1}]$. The optimal portfolio choice of the financial intermediary results in the pricing equations

$$\mathbb{E}_t \left[1 + r_{t+1}^\ell\right] = \frac{\mathbb{E}_t [p_{t+1} + D_{t+1}]}{p_t} = (1 + i_t) \mathbb{E}_t \left[\frac{P_{t+1}}{P_t}\right] \equiv 1 + r_t$$

where we have defined r_t as the ex-ante real interest rate.

A representative final goods firm produces with technology

$$Y_t = \Theta K_t^\alpha N_t^{1-\alpha}$$

Prices are flexible, and firms have no monopoly power, so the real wage w_t and the rental rate of capital are respectively equal to

$$\begin{aligned} w_t &= \Theta (1 - \alpha) K_t^\alpha N_t^{-\alpha} \\ r_t^K &= \Theta \alpha K_t^{\alpha - 1} N_t^{1-\alpha} \end{aligned}$$

A capital firm owns the capital stock K_t and rents it to the representative final good producer. It faces quadratic adjustment costs to capital. In period t, it enters the period with capital stock K_t, invests I_t to obtain a capital next period of $K_{t+1} = (1 - \delta) K_t + I_t$, and pays the adjustment cost, resulting in a dividend of

$$D_t = r_t^K K_t - I_t - \frac{\Psi}{2} \left(\frac{K_{t+1} - K_t}{K_t}\right)^2 K_t$$

where Ψ indexes the size of adjustment costs. The firm has a unit share outstanding, $v_t = 1$.

The capital firm chooses investment to maximize the sum of its dividend and its end-of period share price, $D_t + p_t$. Defining $Q_t = \frac{\partial p_t}{\partial K_{t+1}}$ as the responsiveness of the share price to the capital chosen by the firm, simple algebra shows that, given the asset pricing equations above, this optimization problem involves the standard equations from Q theory:

$$\frac{I_t}{K_t} - \delta = \frac{1}{\Psi}\left(Q_t - 1\right) \tag{26}$$

and

$$Q_t = \frac{1}{1+r_t}\mathbb{E}_t\left[r^K_{t+1} - \frac{I_{t+1}}{K_{t+1}} - \frac{\Psi}{2}\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)^2 + \frac{K_{t+2}}{K_{t+1}}Q_{t+1}\right]$$

Finally, monetary policy sets the nominal interest rate i_t in order to achieve a target for the ex-ante real interest rate r_t.

In an equilibrium of this model, households, unions, financial intermediaries, final goods firms and capital firms optimize, and markets clear, so that:

$$\begin{aligned} C_t + I_t + \frac{\Psi}{2}\left(\frac{K_{t+1} - K_t}{K_t}\right)^2 K_t &= Y_t \\ L_t &= p_t \quad (= Q_t K_{t+1}) \end{aligned}$$

A.2 Investment as an amplifier of monetary policy

We now demonstrate that investment acts as a amplifier of monetary policy in this model. This force is unique to the presence of heterogeneous agents (HA), in the sense that the amplification we highlight is entirely absent with a representative agent (RA).

To show this, we study the effects of an AR(1) monetary policy shock $r_t = r + \epsilon_0 \rho^t$ under several different sets of assumptions. First, we compare the HA model described above to an RA model that is identical except that the household sector is replaced by a representative agent whose consumption is governed by the Euler equation:

$$u'\left(C_t\right) = \beta\left(1 + r_t\right)u'\left(C_{t+1}\right) \tag{27}$$

Second, we compare a model with "no investment", where the capital adjustment cost in (26) is infinite ($\Psi = \infty$) and investment therefore cannot respond to a monetary shock, to a model where it is finite ($\Psi = \Psi_0$) and the investment response has empirically reasonable magnitude.

Calibration. As in the main text, we assume an elasticity of intertemporal substitution of $\sigma = 1$ and target a steady-state $r = 5\%$, for both the HA and RA models. We depart, however, by choosing zero depreciation $\delta = 0\%$. This makes our point especially stark, since then the HA and RA output responses are exactly identical under the "no investment" case where capital adjustment costs are infinite. (In subsection A.5, we show that this equivalence carries over numerically, though not analytically, to the $\delta = 5.3\%$ case from the main text.)

In the HA model, we use the same Markov process for $e(s)$ as in the main text, but rescale $\log e(s)$ so that the standard deviation of log income is the same as in the main text, despite the absence of permanent type heterogeneity here. Also as in the main text, we target steady-state liquidity holdings L such that the income-weighted average quarterly MPC matches the value in figure 2, of 0.194. Since steady-state liquidity equals capital in our simplified model, this implies a very low capital-output ratio of $\frac{K}{Y} = .304$ in annualized terms. We set $\alpha = (r+\delta)\frac{K}{Y}$ for both models, and calibrate the discount factor β^{HA} for the HA households to be consistent with L. The discount rate β^{RA} for the RA households is, by necessity for a steady state, $1/(1+r)$.

We calibrate the persistence of the monetary shock to be $\rho = 0.9$—close to the persistence of the section 4 monetary shock via inertia in the Taylor rule—so that it lasts an average of 10 quarters. We calibrate the size ϵ_0 of the shock such that the date-0 consumption response in the RA model is 1%. We calibrate the capital adjustment cost with investment $\Psi = \Psi_0$ so that the ratio of the investment to consumption response in the RA model is comparable to the ratio of the peak empirical impulses in section 4.2, at about 2/3.[46]

Main result. Figure A.1 presents our main result. The left part of the graph shows the response in the RA model, the right part shows the response in the HA model. The top row shows the calibration with no investment ($\Psi = \infty$), the bottom row the calibration with investment ($\Psi = \Psi_0$). In the RA model, turning on investment has no effect on the consumption response to a monetary policy shock. This follows from the Euler equation in (27): in equilibrium, the path of consumption is entirely dictated by monetary policy $\{r_t\}$.

Similarly, when investment is turned off, the addition of heterogeneous agents makes no difference to the impulse response to a monetary shock. This result is an instance of Werning (2015)'s neutrality result under log utility, and we prove it formally in section A.4. Intuitively, the general equilibrium effects of monetary policy shocks in this model affect asset prices and labor earnings in proportion to output in every period. (Log utility is needed for the former and Cobb-Douglas production for the latter.) Therefore, agents just scale their decisions, relative to the steady state, by $\frac{Y_t^{RA}}{Y_{ss}^{RA}}$, where Y_t^{RA} is the representative-agent allocation at date t, and the representative-agent allocation obtains in the aggregate.

By contrast, when investment is turned on, the effects on consumption are more than *double* the case without investment. The effects of investment are also a little larger than in the representative agent model, reflecting the fact that output and hence the marginal product of capital is higher at every point. This complementarity is the main result of this section: it is the simultaneous presence of heterogeneous agents *and* investment that generates an amplification that is absent if only one of the two is present. Since MPCs are much higher in the HA model, households consume out of labor income from the investment boom, creating a large multiplier on the investment response to monetary policy.

Interestingly, this mechanism leads to continued amplification even after capital firms start to draw down their investment: for instance, the consumption response at $t = 10$ in the HA-investment model is more than double the others, despite the investment response being slightly

[46] We handle equilibrium selection by assuming that, in all models we consider, the economy returns to its initial steady state in the long-run. This is sufficient to uniquely pin down equilibrium for these cases, and can be implemented (for instance) by monetary policy reverting to a Taylor rule at some far-out date t.

Figure A.1: Complementarity between investment and high MPCs

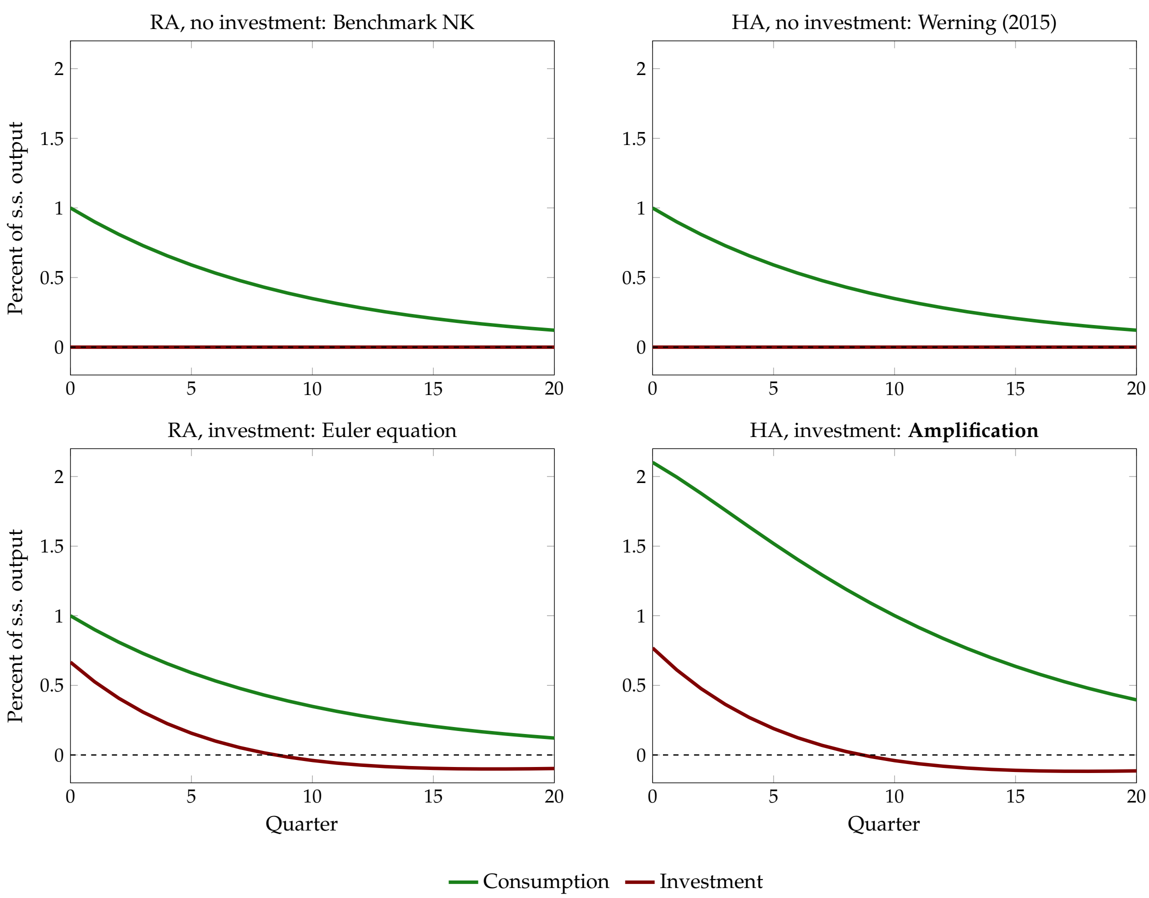
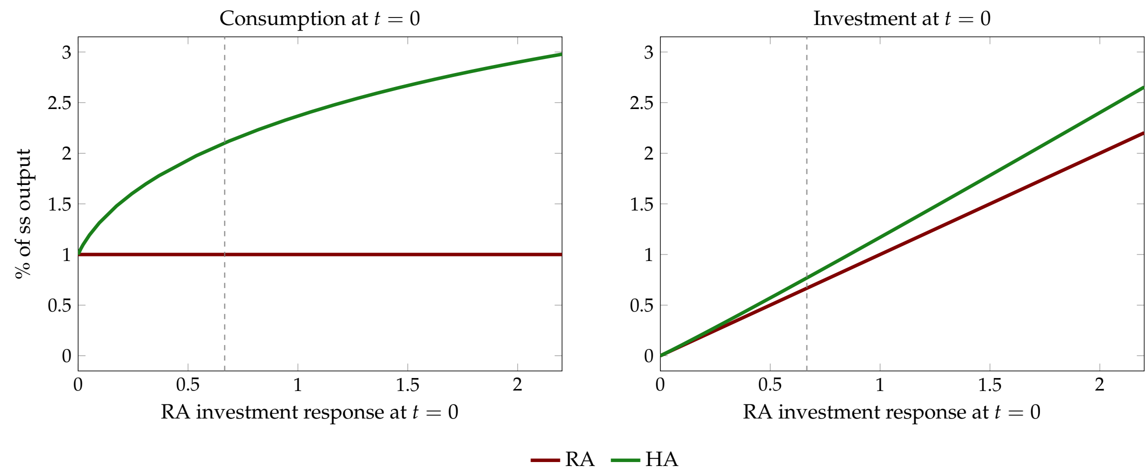


Figure A.2: Investment flexibility and amplification



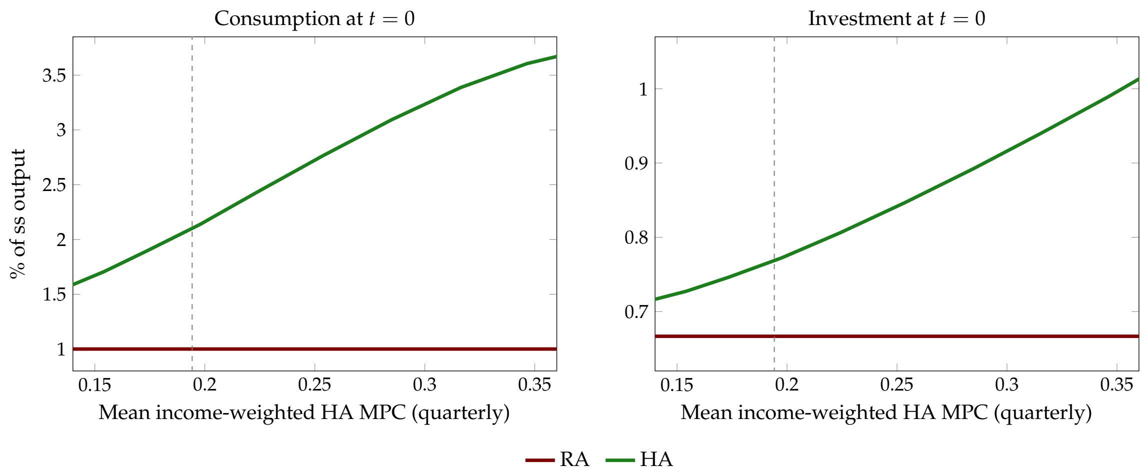
negative at that point. This is thanks to intertemporal demand spillovers, which result from the high iMPCs in the HA model.

Varying investment flexibility and MPCs. To illustrate this interaction further, in figure A.2 we vary the capital adjustment cost Ψ and study the effect on the date-0 responses of consumption C_0 and investment I_0. We quantify adjustment costs on the horizontal axis with the date-0 response of investment I_0 in the RA model (declining in Ψ); the gray dashed line at $2/3$ corresponds to our main calibration. Although the consumption response in the RA model is unaffected by investment flexibility, in the HA model the pattern is monotonic: more flexible investment implies a larger consumption response.

In figure A.3, we vary the level of labor income risk between 50% to 150% of its value in the HA model, while leaving all other aspects of the steady-state calibration, including total liquidity L, unchanged. This matters for our mechanism through the effect on iMPCs (see Auclert, Rognlie and Straub 2018), which here we summarize by plotting the average income-weighted MPC on the horizontal axis, with the gray dashed line at .194 corresponding to our main calibration. In the low-risk, low-MPC calibrations, amplification is small, since the household sector is closer to a representative agent; in high-risk, high-MPC calibrations, amplification becomes much larger. The high-MPC calibrations even have a slightly larger investment response, reflecting the elevated marginal product of capital from high consumption demand.

Together, figures A.2 and A.3 show the robustness of our mechanism: the interaction between investment and MPCs leads to amplification, which becomes stronger when we raise either investment flexibility or MPCs.

Figure A.3: MPCs and amplification



A.3 Understanding the mechanism: capital gains vs labor income

Thus far, we have emphasized transmission via labor income: when investment responds to monetary policy, the higher output demand leads to increased labor income, which raises consumption and output further via high MPCs.

There is another potentially important channel, however, through which the investment sector influences consumption: the return r_0^ℓ on assets between dates -1 and 0. In this model, $r_0^\ell = (p_0 + D_0)/p_{-1} - 1$, which includes the surprise revaluation effect on $p_0 = Q_0 K_0$ from the monetary shock. This effect shrinks when investment is made more flexible, since more capital investment leads to lower future rental rates on capital, offsetting the increase in valuation from lower real interest rates. A smaller return at date 0 makes households poorer, causing them to spend less.

To what extent does this limit amplification in the HA model? In figure A.4 we follow Auclert (2019) and Kaplan, Moll and Violante (2018) and decompose the first-order household consumption response into three sources, which together sum to the aggregate. First, there is the "direct" effect of changing ex-ante real interest rates $r_t = r_{t+1}^\ell$, which is unaffected by the general equilibrium investment response. Second, there is the "indirect" effect from higher labor income, which is the channel we have emphasized so far. Third, there is another "indirect" effect, from unexpected capital gains resulting in a higher date-0 return r_0^ℓ. We perform this decomposition both for the no-investment model $\Psi = \infty$ and the model with investment $\Psi = \Psi_0$.

The direct effect is the same in both models, but relatively muted. Instead, most of the increase in consumption is a response to rising labor income in general equilibrium. This response is much, much larger—by a factor of three—in the model with investment.

As expected, although capital gains contribute positively to consumption in both models, they play less of a role in the model with investment. Their influence is small enough in both cases, however, that this difference is barely visible in figure A.4.

Figure A.4: Decomposing the consumption response

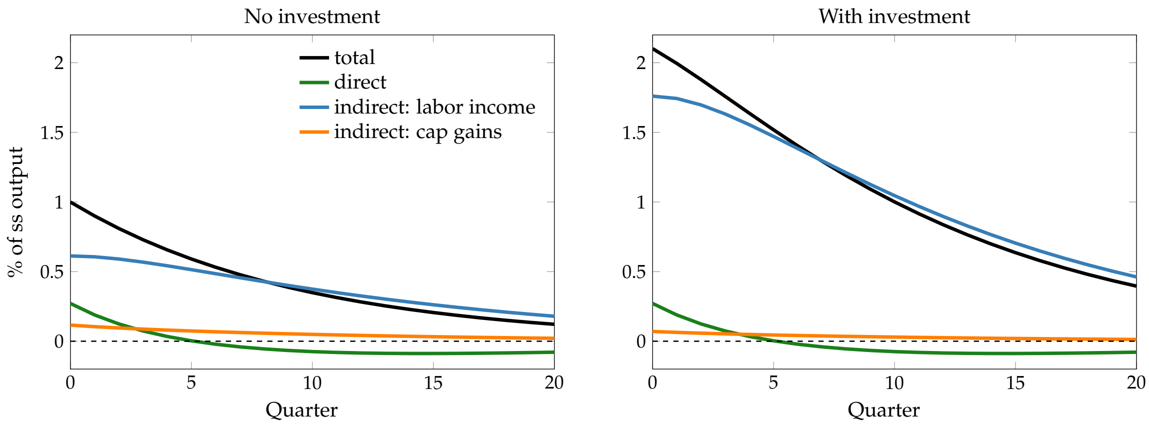
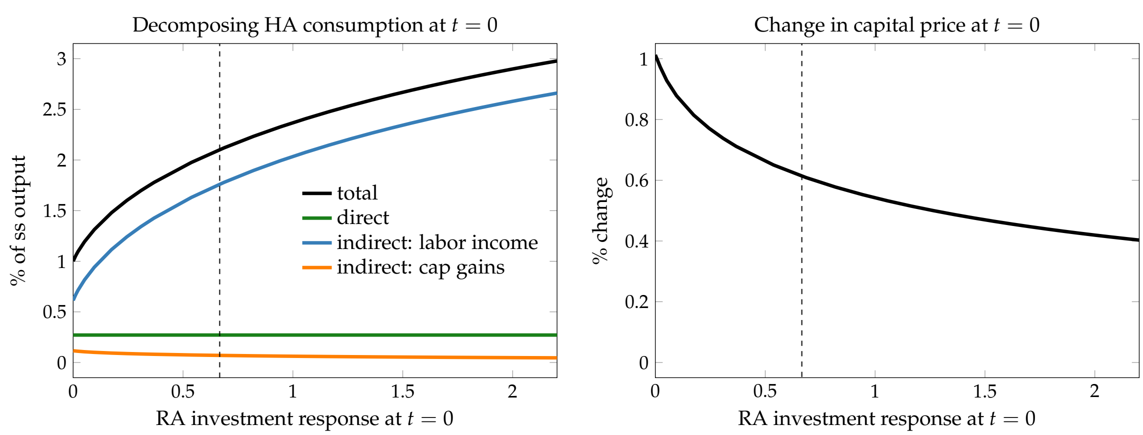


Figure A.5: Decomposition and capital gains by investment flexibility



Why are capital gains so unimportant in comparison to labor income? First, there is an important distributional difference: capital gains are earned by asset-holders, and MPCs are much lower for the asset-rich than for income-earners. For instance, in our calibration, the asset-weighted average quarterly MPC is 0.094, compared to the income-weighted quarterly MPC of 0.194 to which we calibrate. Second, although the decline in rental rates and corresponding rise in real wages causes redistribution from asset owners to income earners, the overall change in income is not zero-sum: indeed, this redistribution is swamped by the rise in aggregate income, most of which goes to labor.

Figure A.5 provides additional detail. The left panel decomposes the date-0 consumption response for the HA model, as a function of investment flexibility, that previously appeared in aggregate terms in figure A.2. We see that as investment flexibility rises, the indirect labor income effect steadily grows, while the indirect capital gains effect shrinks but with far smaller magnitude.

The right panel shows the impulse to date-0 return r_0^ℓ itself. The capital gains effect on consumption at $t = 0$ equals this, times the asset-weighted average MPC of 0.094. Although the change in r_0^ℓ is in relative terms quite dramatic—from a 1.0% to 0.6% increase as we go from the no-investment calibration to our main calibration—this becomes insignificant when multiplied by 0.094 to obtain the effect in the left panel.

Size and liquidity of the capital stock. Since all capital is liquid in this simple model, matching the average MPC from the main text implied a very low capital-output ratio, .304 in annual terms. The calibration in the main text instead has a value that is consistent with the macro data, 2.23.

Importantly, however, this larger capital stock in the main text is entirely held within the illiquid account. Although the larger stock suggests a larger role for capital gains, the illiquidity sharply limits this role: households receive the annuity value of their illiquid accounts into their liquid accounts as a flow, and this flow does not immediately change when the illiquid account gains value. As a result—as figure 5 makes clear—investment makes a strong positive contribution to output, despite its negative effect on the value of the illiquid account.

Our finding of a small contribution from capital gains is likely to be very robust to our calibration choice. If more capital is held in liquid accounts, then capital-holders will have low liquid MPCs. If, instead, more capital is held in illiquid accounts, capital prices' influence on consumption will be limited, since high liquid MPCs will then no longer be directly relevant. Furthermore, holding fixed the magnitude of the investment response, the effect of investment on total capital gains does not grow with the capital stock: if the stock is larger, the price moves proportionately less. Hence, even if a much larger capital stock was held in equally high-MPC liquid accounts, a realistic investment response would not exert any more downward pressure on consumption through the capital gains channel than in figure A.4.

Comparison to other papers. In ongoing and parallel work, Alves, Kaplan, Moll and Violante (2019) argue the opposite: that capital adjustment costs do not matter for aggregate consumption, since the capital gains and labor income effects offset. However, in their framework, another friction shapes capital accumulation: an illiquid account that cannot hold bonds, only capital and monopolists' equity. The properties of this friction may play an important role in their contrasting result.[47]

Bilbiie, Känzig and Surico (2019) argue that investment plays an important role in amplification primarily in conjunction with unequal cyclical incidence of labor income. By contrast, in both the main text and this section, we intentionally abstract away from unequal incidence—which is known in the literature to matter for amplification—to show that a combination of high MPCs and investment, on its own, is enough to deliver major amplification.

A.4 Neutrality proof with inelastic investment ($\Psi = \infty$)

Here we prove a neutrality result for the model with a fixed capital stock and no investment, explaining why the top left and right panels of figure underlying figure A.1 are identical. This is an instance of Werning (2015)'s finding for an EIS of 1.

First we need a lemma.

Lemma 1. *In perfect foresight equilibria of the RA model with $\sigma = 1$ and no investment ($\Psi = \infty$ and $\delta = 0$), $D_t = \alpha Y_t$ and $p_t = \frac{\alpha\beta^{RA}}{1-\beta^{RA}} Y_t$.*

Proof. Since $\Psi = \infty$, capital is always at its steady-state level K^{ss} and investment is always 0, so the dividend at t is $D_t = \alpha \frac{Y_t}{K^{ss}} K^{ss} = \alpha Y_t$, as desired. The asset price is

$$p_t = \frac{1}{1+r_t}\left(\alpha Y_{t+1} + p_{t+1}\right)$$

Iterating forward, we can write[48]

$$p_t = \sum_{s=1}^{\infty}\left(\prod_{u=0}^{s-1}\frac{1}{1+r_{t+u}}\right)\alpha Y_{t+s} \tag{28}$$

Note that iterating forward the Euler equation with $\sigma = 1$, we also have

$$C_t = (\beta^{RA})^{-s}\left(\prod_{u=0}^{s-1}\frac{1}{1+r_{t+u}}\right) C_{t+s}$$

which, using $Y_t = C_t$, we can substitute into (28) to get $p_t = \sum_{s=1}^{\infty}(\beta^{RA})^{-s}\alpha Y_t = \frac{\alpha\beta^{RA}}{1-\beta^{RA}} Y_t$, as desired. □

[47] They also perform a different experiment: they compare the model with *no* adjustment costs to the model *with* adjustment costs, whereas we compare the model *with* adjustment costs to the model with *infinite* adjustment costs (and more generally among different levels of the adjustment cost). We do not perform the first comparison, since the standard New Keynesian model explodes without adjustment costs in response to a real interest rate shock.

[48] This assumes that monetary policy $\{r_t\}$ does not permanently deviate so far from $r^{ss} > 0$ that the product does not converge to zero as $s \to \infty$.

Proposition 1. *Given any monetary policy $\{r_t\}$, a perfect foresight equilibrium allocation $\{Y_t, C_t, N_t, D_t, w_t, p_t\}$ in the representative-agent model with $\sigma = 1$ and no investment ($\Psi = \infty$ and $\delta = 0$) is also an equilibrium allocation in the heterogeneous-agent model with $\sigma = 1$ and no investment.*

In particular, monetary policy shocks will have the same effects on output.

Proof. Consider the HA model. Starting from the ergodic steady-state distribution at $t = 0$, and at first assuming real interest rates remain at r^{ss}, let $c_t^{ss}(\ell_{-1}, s^t)$ and $\ell_t^{ss}(\ell_{-1}, s^t)$ denote the date-t policies of agents as a function of liquidity at ℓ_{-1} and the history $s^t = (s_0, \ldots, s_t)$ of idiosyncratic shocks from date 0 to date t. (This *sequential form* of the problem will be more convenient for the proof.)

Optimal behavior is characterized for all t by

$$\left(c_t^{ss}\left(\ell_{-1}, s^t\right)\right)^{-1} \geq \beta\left(1+r^{ss}\right)\mathbb{E}_t\left[\left(c_{t+1}^{ss}\left(\ell_t^{ss}\left(\ell_{-1}, s^t\right), s^{t+1}\right)\right)^{-1}|s^t\right] \tag{29}$$

$$c_t^{ss}\left(\ell_{-1}, s^t\right) + \ell_t^{ss}\left(\ell_{-1}, s^t\right) = \left(1+r^{ss}\right)\ell_{t-1}^{ss}\left(\ell_{-1}, s^{t-1}\right) + w^{ss}N^{ss}e\left(s_t\right) \tag{30}$$

$$\ell_t^{ss}\left(\ell_{-1}, s^t\right) \geq 0 \tag{31}$$

where the Euler equation and the borrowing constraint hold with complementary slackness.

Now take an arbitrary monetary policy path $\{r_t\}$ and corresponding RA equilibrium sequences $\{Y_t, C_t, N_t, D_t, w_t, p_t\}$, and consider optimal household behavior subject to these equilibrium sequences . Denote policies by $c_t(\ell_{-1}, s^t)$ and $\ell_t(\ell_{-1}, s^t)$. Optimal behavior is characterized by

$$\left(c_t\left(\ell_{-1}, s^t\right)\right)^{-1} \geq \beta\left(1+r_{t+1}^{\ell}\right)\mathbb{E}_t\left[\left(c_{t+1}\left(\ell_{-1}, s^{t+1}\right)\right)^{-1}|s^t\right] \tag{32}$$

$$c_t\left(\ell_{-1}, s^t\right) + \ell_t\left(\ell_{-1}, s^t\right) = \left(1+r_t^{\ell}\right)\ell_{t-1}\left(\ell_{-1}, s^{t-1}\right) + w_tN_te\left(s_t\right) \tag{33}$$

$$\ell_t\left(\ell_{-1}, s^t\right) \geq 0 \tag{34}$$

We now guess and verify that if $c_t^{ss}(\ell_{-1}, s^t)$ and $\ell_t^{ss}(\ell_{-1}, s^t)$ satisfy (29)-(31), then $c_t(\ell_{-1}, s^t) = \frac{Y_t}{Y^{ss}}c_t^{ss}(\ell_{-1}, s^t)$ and $\ell_t(\ell_{-1}, s^t) = \frac{Y_t}{Y^{ss}}\ell_t^{ss}(\ell_{-1}, s^t)$ satisfy (32)-(34). We will do so by explicitly showing the two sets of equations are equivalent.

This is trivially true for (34) and (31). For (32), plug in the candidate policy to obtain

$$\left(\frac{C_t}{C^{ss}}\right)^{-1}\left(c_t^{ss}\left(\ell_{-1}, s^t\right)\right)^{-1} \geq \beta\left(1+r_{t+1}^{\ell}\right)\left(\frac{C_{t+1}}{C^{ss}}\right)^{-1}\mathbb{E}_t\left[\left(c_{t+1}^{ss}\left(\ell_{-1}, s^{t+1}\right)\right)^{-\sigma}|s^t\right]$$

Here, dividing both sides by the RA Euler equation $C_t^{-1} = \beta(1+r_t)C_{t+1}^{-1}$, applying perfect foresight $r_t = r_{t+1}^{\ell}$, and dividing by C^{ss} gives us (29).

Similarly, for (33), plug in the candidate policy to obtain for the $t > 0$ case

$$\frac{Y_t}{Y^{ss}}c_t^{ss}\left(\ell_{-1}, s^t\right) + \frac{Y_t}{Y^{ss}}\ell_t^{ss}(\ell_{-1}, s^t) = \left(1+r_t^{\ell}\right)\frac{Y_t}{Y^{ss}}\ell_{t-1}^{ss}\left(\ell_{-1}, s^{t-1}\right) + w_tN_te\left(s_t\right)$$

Here, noting that $w_t N_t = (1-\alpha) Y_t = \frac{Y_t}{Y^{ss}} w^{ss} N^{ss}$, we see that the equation is just (30) with $\frac{Y_t}{Y^{ss}}$ multiplying every term.

For $t = 0$, we have instead

$$\frac{Y_0}{Y^{ss}} c_0^{ss}\left(\ell_{-1}, s^t\right) + \frac{Y_0}{Y^{ss}} \ell_0^{ss}(\ell_{-1}, s) = \left(1 + r_0^{\ell}\right) \ell_{-1} + w_0 N_0 e\left(s_t\right)$$

which seems problematic since there is no longer a $\frac{Y_0}{Y^{ss}}$ multiplying the first term on the right. However, applying lemma 1,

$$1 + r_0^{\ell} = \frac{p_0 + D_0}{p^{ss}} = \frac{\frac{\alpha\beta^{RA}}{1-\beta^{RA}} Y_0 + \alpha Y_0}{\frac{\alpha\beta^{RA}}{1-\beta^{RA}} Y^{ss}} = (\beta^{RA})^{-1} \frac{Y_0}{Y^{ss}} = (1 + r^{ss}) \frac{Y_0}{Y^{ss}}$$

so we again have equation (30) multiplied by $\frac{Y_0}{Y^{ss}}$. We conclude that $c_t(\ell_{-1}, s^t) = \frac{Y_t}{Y^{ss}} c_t^{ss}(\ell_{-1}, s^t)$ and $\ell_t(\ell_{-1}, s^t) = \frac{Y_t}{Y^{ss}} \ell_t^{ss}(\ell_{-1}, s^t)$ are an optimal plan for each household faced with RA equilibrium sequences $\{Y_t, C_t, N_t, D_t, w_t, p_t, r_t\}$.

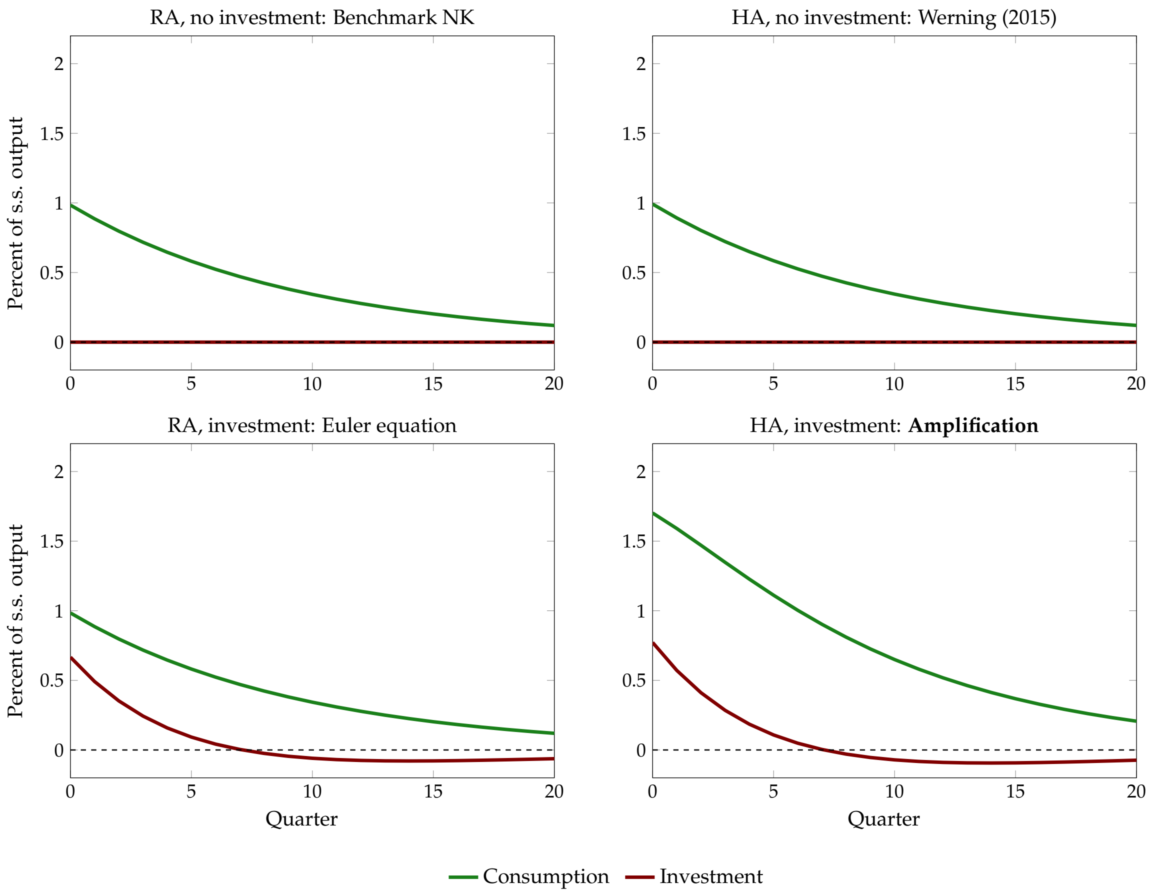
Since each household's consumption is scaled up by the same factor $\frac{Y_t}{Y^{ss}}$, aggregate consumption is also scaled up by that factor. Hence consumption $C_t = Y_t$ is the same as its RA equilibrium value, and goods market clearing holds. Asset market clearing follows from Walras' law, and all other equilibrium conditions are the same as in the RA model. We conclude that $\{Y_t, C_t, N_t, D_t, w_t, p_t, r_t\}$ is also an equilibrium for the HA model. □

A.5 Robustness to positive depreciation

In figure A.6, we recalculate figure A.1 in a model that is calibrated in exactly the same way, except that depreciation is set at its value $\delta = .054$ (annualized) from the main text, rather than at zero. The "no investment" case still features $\Psi = \infty$, in which case $K_t = K^{ss}$ and $I_t = \delta K^{ss}$ in all periods.

Although the analytical proof of proposition 1 no longer goes through when $\delta > 0$, we see numerically in the top panel of figure A.6 that the RA and HA models still deliver nearly identical results when $\Psi = \infty$. Similarly, in the bottom right panel, the interaction of the HA model with positive investment delivers substantial amplification. This is slightly smaller than in figure A.1 because a shock to r_t has less proportional effect on the user cost $r_t + \delta$, and therefore the incentive to invest, when δ is positive. (Although Ψ is recalibrated to match the same investment response at $t = 0$ in the RA model, the cumulative investment response adding subsequent periods is smaller.)

Figure A.6: Complementarity between investment and high MPCs: positive depreciation $\delta > 0$



B Appendix to section 2

B.1 RA model with additive internal habits

In partial equilibrium, given a process for income and interest rates $\{y_t, r_t\}$ and initial assets a_{-1}, a representative agent with external additive habit formation solves the following problem:

$$\begin{aligned} \max \quad & \mathbb{E}\left[\sum \beta^t u\left(c_t - \gamma c_{t-1}\right)\right] \\ & c_t + a_t = y_t + \left(1 + r_{t-1}\right) a_{t-1} \end{aligned}$$

where $0 \leq \gamma < 1$. The associated Euler equation is:

$$u'\left(c_t - \gamma c_{t-1}\right) - \mathbb{E}_t\left[\beta\gamma u'\left(c_{t+1} - \gamma c_t\right)\right] = \beta\left(1 + r_t\right)\mathbb{E}_t\left[u'\left(c_{t+1} - \gamma c_t\right) - \beta\gamma u'\left(c_{t+2} - \gamma c_{t+1}\right)\right] \tag{35}$$

Linearized solution. Linearizing (35) around a steady state with constant consumption c and $\beta^{-1} = (1 + r)$, we obtain

$$-\gamma dc_{t-1} + \left(1 + \gamma + \beta\gamma^2\right) dc_t - \left(1 + \beta\gamma + \beta\gamma^2\right)\mathbb{E}_t\left[dc_{t+1}\right] + \beta\gamma\mathbb{E}_t\left[dc_{t+2}\right] = -\frac{1}{\sigma}\left(1 - \beta\gamma\right)\left(1 - \gamma\right) c\frac{dr_t}{1 + r} \tag{36}$$

where $\frac{1}{\sigma} = -\frac{u''(c)c}{u'(c)}$ is the inverse curvature of u. We can rewrite (36) as

$$\mathbb{E}_t\left[P\left(L\right)\left(1 - L\right) dc_t\right] = -\kappa dr_t \tag{37}$$

where $\kappa \equiv \frac{1}{\sigma}\left(1 - \beta\gamma\right)\left(1 - \gamma\right)\frac{c}{1+r}$, and

$$P\left(X\right) \equiv \frac{b\left(X - \beta\gamma\right)\left(X - \frac{1}{\gamma}\right)}{X^2}$$

has two roots, one greater and one smaller than 1. The linear solution to the habits problem therefore jointly solves the Euler equation and the linearized version of the budget constraint,

$$\mathbb{E}_t\left[dc_{t+1} - dc_t\right] = \gamma\left(dc_t - dc_{t-1}\right) + \kappa\mathbb{E}_t\left[\sum_{k\geq 0}\left(\beta\gamma\right)^k dr_{t+k}\right] \tag{38}$$

$$dc_t = dy_t + \frac{1}{\beta} da_{t-1} - da_t + a dr_{t-1} \tag{39}$$

Intertemporal marginal propensities to consume. If $\mathbb{E}_t\left[dr_t\right] = 0$ for all t, then we can solve (38) to obtain

$$\mathbb{E}_0\left[dc_t\right] = \frac{1 - \gamma^{t+1}}{1 - \gamma} dc_0 \tag{40}$$

Integrating (39) and plugging in (40) , we then find

$$\frac{1}{1-\gamma}\left(\sum_{t=0}^{\infty}\beta^{t}\left(1-\gamma^{t+1}\right)\right)dc_{0}=dy_{0}$$

Hence, the expected path of consumption after an initial increase in income is:

$$\frac{\mathbb{E}_{0}\left[dc_{t}\right]}{dy_{0}}=\left(1-\beta\right)\left(1-\beta\gamma\right)\frac{1-\gamma^{t+1}}{1-\gamma}\tag{41}$$

which is plotted on the red line of figure 2, for an illustrative calibration with $\beta=0.95$ and $\gamma=0.6$. The initial MPC is depressed relative to $(1-\beta)$—that of the representative-agent model—by a factor $(1-\beta\gamma)$, reflecting the desire of the agent with additive habits to limit the initial increase in his habit stock.

B.2 HA model with additive internal habits

The heterogeneous-agent habit problem can be formulated as follows:

$$\begin{aligned}V\left(\ell,c_{-},s\right) &= \max_{c,\ell'} u\left(c-\gamma c_{-}\right)+\beta\mathbb{E}\left[V\left(\ell',c,s'\right)|s\right]\\ & \quad c+\ell'=\left(1+r\right)\ell+ye\left(s\right)\\ & \quad \ell'\geq 0\end{aligned}$$

The first-order conditions for c and ℓ' are

$$\lambda=u'\left(c-\gamma c_{-}\right)+\beta\mathbb{E}\left[V_{c}\left(\ell',c,s'\right)|s\right]\tag{42}$$

$$\lambda+\mu=\beta\mathbb{E}\left[V_{\ell}\left(\ell',c,s'\right)|s\right]\tag{43}$$

where $\mu\geq 0$ is the multiplier on the borrowing constraint $\ell'\geq 0$. Moreover, the envelope conditions for ℓ and c_{-} imply

$$V_{\ell}\left(\ell,c_{-},s\right)=\lambda\left(1+r\right)\tag{44}$$

$$V_{c_{-}}\left(\ell,c_{-},s\right)=-\gamma u'\left(c-\gamma c_{-}\right)\tag{45}$$

We calibrate the model to $\gamma=0.6$, $r=0.05$, $u=\log$, and the same annual $\beta=0.8422$ as that found in calibrating our no-habit HA model to match a first-year MPC of 0.55. We solve the model using standard methods, on a grid for (ℓ,c_{-}).

C Appendix to section 3

C.1 Euler equation for inattentive households

The optimal policy functions $c_{g,t}(\ell, a, a_{-k}, s, k)$ and $\ell'_{g,t}(\ell, a, a_{-k}, s, k)$ for the household problem (5), when the household is not constrained at the liquid asset lower bound $\ell'_{g,t}(\ell, a, a_{-k}, s, k) \geq 0$, satisfy the intertemporal Euler equation

$$u'(c_{g,t}(\ell, a, a_{-k}, s, k)) = \beta_g \mathbb{E}_{t-k}\left[(1 + r^{\ell}_{t+1})\left(\theta u'(c_{g,t+1}(\ell'_{g,t}(\ell, a, a_{-k}, s, k), a', a_{-k}, s', k+1)\right.\right.$$
$$\left.\left. +(1-\theta)u'(c_{g,t+1}(\ell'_{g,t}(\ell, a, a_{-k}, s, k), a', a', s', 0))\right) |s\right] \quad (46)$$

which follows immediately from combining the first-order condition and envelope condition for (5). When the household is constrained, (46) is an inequality $\geq$.

C.2 Extended financial intermediary problem and monetary policy implementation

Here we extend the model of the financial intermediary in section 3.2 to allow it to allow nominal reserves M_t at the central bank, that pay a pre-determined interest rate of i_t. Since all assets are real, the flow-of-funds constraint (7) in date-t nominal units is modified to

$$(1 + r^a_t) P_t A_{t-1} + \left(1 + r^{\ell}_t\right) P_t L_{t-1} = (1 + \delta q_t) P_t B_{t-1} + \int \left(p_{jt} + D_{jt}\right) P_t v_{jt-1} dj - \xi P_t L_{t-1} + (1 + i_{t-1}) M_{t-1} \quad (47)$$

and portfolio-investment constraint (8) now reads

$$P_t \int p_{jt} v_{jt} dj + P_t q_t B_t + M_t = P_t A_t + P_t L_t \quad (48)$$

The financial intermediary's problem is now to choose v_{jt}, B_t, L_t and M_t so as to maximize the expected return on illiquid liabilities, $\mathbb{E}_t\left[r^a_{t+1}\right]$, subject to (48) and (47). Since (47) implies that

$$\mathbb{E}_t\left[1 + r^a_{t+1}\right] = \mathbb{E}_t\left[\frac{(1 + \delta q_{t+1}) B_t + \int \left(p_{jt+1} + D_{jt+1}\right) v_{jt} dj + (1 + i_t) \frac{M_t}{P_{t+1}} - \left(1 + r^{\ell}_{t+1} + \xi\right) L_t}{\int p_{jt} v_{jt} dj + q_t B_t + \frac{M_t}{P_t} - L_t}\right]$$

the first order conditions lead to equalization of all expected returns

$$\mathbb{E}_t\left[\frac{1 + \delta q_{t+1}}{q_t}\right] = \mathbb{E}_t\left[\frac{p_{jt+1} + D_{jt+1}}{p_{jt}}\right] = (1 + i_t)\, \mathbb{E}_t\left[\frac{P_t}{P_{t+1}}\right] = 1 + r^{\ell}_{t+1} + \xi$$

which are equations (9) and (10) in the main text, where we also define these to all be equal to the ex-ante real interest rate $1 + r_t$.

The central bank implements monetary policy by setting the nominal interest rate on reserves i_t, using open-market operations. We consider the limit where it does so using a net supply of reserves that is at all times equal to $M_t = 0$.

C.3 Intermediate goods firm price-setting

We first derive final demand. Individual consumers minimize $\int P_{jt} Y_{jt} dj$ subject to (11), which results in the first order condition

$$\frac{P_{jt}}{P_t} = G_p'\left(\frac{Y_{jt}}{Y_t}\right) = \frac{1 - \left(\frac{Y_{jt}}{Y_t}\right)^{\frac{\nu_p}{\epsilon_p}}}{\nu_p}$$

hence, intermediate goods firms face the static demand curve

$$\frac{Y_{jt}}{Y_t} = \mathcal{Y}_p\left(\frac{P_{jt}}{P_t}\right)$$

where we have defined $\mathcal{Y}_p$ as

$$\mathcal{Y}_p(x) \equiv \left(1 - \nu_p \log x\right)^{\frac{\epsilon_p}{\nu_p}}$$

Define the static profit function of an intermediate goods firm with current price p, when the price index is P, real marginal costs are s and aggregate demand is Y as

$$D(p; P, Y, s) \equiv \left(\frac{p}{P} - s\right) \mathcal{Y}_p\left(\frac{p}{P}\right) Y$$

and note that the derivative of D with respect to own price p is

$$\frac{\partial D}{\partial p} = \left(\frac{1}{P} + \left(\frac{s}{p} - \frac{1}{P}\right) \epsilon_p\left(\frac{p}{P}\right)\right) \mathcal{Y}_p\left(\frac{p}{P}\right) Y \tag{49}$$

where $\epsilon_p(x)$ is the elasticity of demand,

$$\epsilon_p(x) \equiv -\frac{\mathcal{Y}_p'(x)\, x}{\mathcal{Y}_p(x)} = \frac{\epsilon_p}{1 - \nu_p \log x} \tag{50}$$

We next work out the optimal reset price for a firm. Upon receiving an opportunity to reset its price, a firm chooses P_t^* to maximize the sum of its dividend and its stock price,

$$D\left(P_t^*; P_t, Y_t, s_t\right) + p_t\left(P_t^*\right)$$

where by the no-arbitrage condition in (9) and the price indexation formula (12), we have

$$\begin{aligned} p_t\left(P_t^*\right) \quad &= \quad \frac{1}{1+r_t}\mathbb{E}_t\left[\zeta_p\left(D\left(P_t^*\frac{P_t}{P_{t-1}}; P_{t+1}, Y_{t+1}, s_{t+1}\right) + p_{t+1}\left(P_t^*\frac{P_t}{P_{t-1}}\right)\right)\right. \\ &\qquad \left. + \left(1-\zeta_p\right)\max_{\widehat{p}} \theta\left(D\left(\widehat{p}; P_{t+1}, Y_{t+1}, s_{t+1}\right) + p_{t+1}\left(\widehat{p}\right)\right)\right] \end{aligned}$$

Hence, defining $M_{t,t+k} \equiv \prod_{s=t}^{t+k-1} \frac{1}{1+r_s}$, P_t^* also solves

$$P_t^* = \arg\max_x \mathbb{E}_t \left[\sum_{k \geq 0} \zeta_p^k M_{t,t+k} D\left(x \frac{P_{t+k-1}}{P_{t-1}}; P_{t+k}, Y_{t+k}, s_{t+k} \right) \right]$$

Taking the first-order condition and using (49), we find that P_t^* solves

$$\begin{aligned}
&\mathbb{E}_t \left[\sum_{k \geq 0} \zeta_p^k \cdot M_{t,t+k} \cdot Y_{t+k} \cdot \mathcal{Y}_p \left(\frac{P_t^*}{P_{t-1}} \frac{P_{t+k-1}}{P_{t+k}} \right) \left(\frac{P_t^*}{P_{t-1}} \frac{P_{t+k-1}}{P_{t+k}} + \epsilon_p \left(\frac{P_t^*}{P_{t-1}} \frac{P_{t+k-1}}{P_{t+k}} \right) \left(s_{t+k} - \frac{P_t^*}{P_{t-1}} \frac{P_{t+k-1}}{P_{t+k}} \right) \right) \right] \\
&= \mathbb{E}_t \left[\sum_{k \geq 0} \zeta_p^k \cdot M_{t,t+k} \cdot Y_{t+k} \cdot f_p \left(\frac{P_t^*}{P_{t-1}} \frac{P_{t+k-1}}{P_{t+k}}, s_{t+k} \right) \right] \qquad (51) \\
&= 0
\end{aligned}$$

where we have defined the function $f_p(x, s)$ as

$$f_p(x, s) \equiv \mathcal{Y}_p(x) \left(x + \epsilon_p(x)(s - x) \right)$$

To derive a first-order approximation to the solution to (51), observe that

$$\begin{aligned}
\frac{\partial f_p}{\partial x} &= \mathcal{Y}_p'(x) \left(x + \epsilon_p(x)(s - x) \right) + \mathcal{Y}_p(x) \left(1 + \epsilon_p'(x)(s - x) - \epsilon_p(x) \right) \\
&= \mathcal{Y}_p(x) \left(-\frac{\epsilon_p(x)}{x} \left(x + \epsilon_p(x)(s - x) \right) + 1 + \epsilon_p(x) \frac{\epsilon_p'(x) x}{\epsilon_p(x)} \left(\frac{s - x}{x} \right) - \epsilon_p(x) \right) \\
&= \mathcal{Y}_p(x) \epsilon_p(x) \left(\frac{1}{\epsilon_p(x)} + \left[\frac{\nu_p}{\epsilon_p} - 1 \right] \epsilon_p(x) \left(\frac{s - x}{x} \right) - 2 \right)
\end{aligned}$$

where we have made use of the fact that $\frac{\epsilon_p'(x) x}{\epsilon_p(x)} = \frac{\nu_p}{\epsilon_p} \epsilon_p(x)$ from (50). Hence, around the steady state where $x^{ss} = 1$ and $s^{ss} = \frac{\epsilon_p - 1}{\epsilon_p}$, we have

$$\frac{\partial f_p}{\partial x}(x^{ss}, s^{ss}) = 1 \cdot \epsilon_p \cdot \left(\frac{1}{\epsilon_p} - \frac{\nu_p}{\epsilon_p} + 1 - 2 \right) = \epsilon_p \left(\frac{1 - \nu_p}{\epsilon_p} - 1 \right) = 1 - \nu_p - \epsilon_p$$

and similarly,

$$\frac{\partial f_p}{\partial s}(x^{ss}, s^{ss}) = \mathcal{Y}_p(x^{ss}) \epsilon(x^{ss}) = \epsilon_p$$

Totally differentiating (51) around the steady state where $M_{t,t+k}^{ss} Y_{t+k}^{ss} = \left(\frac{1}{1+r} \right)^k Y^{ss}$, we next find

$$\begin{aligned}
\mathbb{E}_t \Bigg[&\sum_{k \geq 0} \zeta_p^k d\left(M_{t,t+k} \cdot Y_{t+k} \right) \cdot f_p(x^{ss}, s^{ss}) + \sum_{k \geq 0} \left(\frac{\zeta_p}{1+r} \right)^k \cdot Y^{ss} \frac{\partial f_p}{\partial x}(x^{ss}, s^{ss}) \, d\left(\frac{P_t^*}{P_{t-1}} \frac{P_{t+k-1}}{P_{t+k}} \right) \\
&+ \sum_{k \geq 0} \left(\frac{\zeta_p}{1+r} \right)^k Y^{ss} \frac{\partial f_p}{\partial s}(x^{ss}, s^{ss}) \, ds_{t+k} \Bigg] = 0
\end{aligned}$$

and since $f_p(x^{ss}, s^{ss}) = 0$, this gives

$$\left(\epsilon_p + \nu_p - 1\right) \mathbb{E}_t \left[\sum_{k \geq 0} \left(\frac{\zeta_p}{1+r} \right)^k d \left(\frac{P_t^*}{P_{t-1}} \frac{P_{t+k-1}}{P_{t+k}} \right) \right] = \epsilon_p \mathbb{E}_t \left[\sum_{k \geq 0} \left(\frac{\zeta_p}{1+r} \right)^k ds_{t+k} \right] \tag{52}$$

Now write $p_t^* \equiv \log P_t^*$, $p_t \equiv \log P_t$, and $\pi_t \equiv \log \left(\frac{P_t}{P_{t-1}} \right)$. Since we are linearizing around a zero inflation steady state, we have

$$d \left(\frac{P_t^*}{P_{t-1}} \frac{P_{t+k-1}}{P_{t+k}} \right) = d \log \left(\frac{P_t^*}{P_{t-1}} \frac{P_{t+k-1}}{P_{t+k}} \right) = p_t^* - p_{t-1} + \pi_{t+k}$$

hence

$$\left(\epsilon_p + \nu_p - 1\right) \mathbb{E}_t \left[\sum_{k \geq 0} \left(\frac{\zeta_p}{1+r} \right)^k \left(p_t^* - p_{t-1} + \pi_{t+k} \right) \right] = \epsilon_p \mathbb{E}_t \left[\sum_{k \geq 0} \left(\frac{\zeta_p}{1+r} \right)^k ds_{t+k} \right]$$

which can be rewritten as

$$p_t^* - p_{t-1} \quad = \quad \left(1 - \frac{\zeta_p}{1+r} \right) \mathbb{E}_t \left[\sum_{k \geq 0} \left(\frac{\zeta_p}{1+r} \right)^k \cdot \left(\pi_{t+k} + \frac{\epsilon_p}{\epsilon_p + \nu_p - 1} ds_{t+k} \right) \right]$$

or recursively as

$$p_t^* - p_{t-1} = \left(1 - \frac{\zeta_p}{1+r} \right) \left(\pi_t + \frac{\epsilon_p}{\epsilon_p + \nu_p - 1} ds_t \right) + \frac{\zeta_p}{1+r} \mathbb{E}_t \left[p_{t+1}^* - p_t \right] \tag{53}$$

Moreover, the price index satisfies

$$\zeta_p \left(\frac{P_{t-1}\Pi_{t-1}}{P_t} \right) \cdot \mathcal{Y} \left(\frac{P_{t-1}\Pi_{t-1}}{P_t} \right) + \left(1 - \zeta_p\right) \frac{P_t^*}{P_t} \cdot \mathcal{Y} \left(\frac{P_t^*}{P_t} \right) = 1 \tag{54}$$

Differentiating (54) around the zero inflation steady state, we obtain

$$\zeta_p \left(\pi_{t-1} - \pi_t \right) + \left(1 - \zeta_p\right) \left(p_t^* - p_t \right) = 0$$

or

$$\pi_t = \zeta_p \pi_{t-1} + \left(1 - \zeta_p\right) \left(p_t^* - p_{t-1} \right)$$

Combining with (53) and rearranging delivers

$$\pi_t - \zeta_p \pi_{t-1} = \left(1 - \zeta_p\right) \left(1 - \frac{\zeta_p}{1+r} \right) \left(\frac{\epsilon_p}{\epsilon_p + \nu_p - 1} ds_t + \pi_t \right) + \frac{\zeta_p}{1+r} \mathbb{E}_t \left[\pi_{t+1} - \zeta_p \pi_t \right]$$

which can be rearranged as

$$\pi_t = \frac{1}{1+\frac{1}{1+r}}\pi_{t-1} + \frac{\left(1-\zeta_p\right)\left(1-\frac{\zeta_p}{1+r}\right)}{\zeta_p\left(1+\frac{1}{1+r}\right)}\frac{\epsilon_p}{\epsilon_p+\nu_p-1}ds_t + \frac{\frac{1}{1+r}}{1+\frac{1}{1+r}}\mathbb{E}_t\left[\pi_{t+1}\right]$$

or alternatively as

$$\pi_t - \pi_{t-1} = \frac{\left(1-\zeta_p\right)\left(1-\frac{\zeta_p}{1+r}\right)}{\zeta_p}\frac{\epsilon_p}{\epsilon_p+\nu_p-1}\mathbb{E}_t\left[\sum_{k\geq 0}\left(\frac{1}{1+r}\right)^k ds_t\right]$$

which is the expression used in the main text, equation (13).

C.4 Capital firms

The capital firm comes in with planned investment I_t and capital stock K_t, conducts this investment and pays dividend of

$$D_t^K\left(K_t, I_{t-1}, I_t\right) = r_t^K K_t - I_t\left(1 + S\left(\frac{I_t}{I_{t-1}}\right)\right)$$

leaving it with $K_{t+1} = (1-\delta)K_t + I_t$ for next period. It also chooses I_{t+1} for next period in order to maximize its stock price of

$$p_t^K\left(K_{t+1}, I_t, I_{t+1}\right) = \frac{1}{1+r_t}\mathbb{E}_t\left[D_{t+1}^K\left(K_{t+1}, I_t, I_{t+1}\right) + p_{t+1}^K\left(K_{t+2}, I_{t+1}, I_{t+2}\right)\right]$$

The problem can therefore be written as

$$\max_{I_{t+1}}\left\{\mathbb{E}_t\left[D_{t+1}^K\left(K_{t+1}, I_t, I_{t+1}\right) + \max_{I_{t+2}} p_{t+1}^K\left(I_{t+1} + (1-\delta)K_{t+1}, I_{t+1}, I_{t+2}\right)\right]\right\}$$

The first order condition for I_{t+1} is

$$\mathbb{E}_t\left[1 + S\left(\frac{I_{t+1}}{I_t}\right) + \frac{I_{t+1}}{I_t}S'\left(\frac{I_{t+1}}{I_t}\right)\right] = Q_t + Q_t^I \tag{55}$$

where we have defined Q_t and Q_t^I as, respectively,

$$\begin{aligned} Q_t &\equiv \mathbb{E}_t\left[\frac{\partial p_{t+1}^K}{\partial K_{t+2}}\right] \\ Q_t^I &\equiv \mathbb{E}_t\left[\frac{\partial p_{t+1}^K}{\partial I_{t+1}}\right] \end{aligned}$$

The envelope conditions for capital and previous investment are

$$\begin{aligned}\frac{\partial p_t^K}{\partial K_{t+1}} &= \frac{1}{1+r_t}\mathbb{E}_t\left[r_{t+1}^K + (1-\delta)\frac{\partial p_{t+1}^K}{\partial K_{t+2}}\right] \\ &= \frac{1}{1+r_t}\mathbb{E}_t\left[r_{t+1}^K + (1-\delta)\,Q_t\right]\end{aligned}$$

hence

$$Q_t = \mathbb{E}_t\left[\frac{\partial p_{t+1}^K}{\partial K_{t+2}}\right] = \mathbb{E}_t\left[\frac{1}{1+r_{t+1}}\left(r_{t+2}^K + (1-\delta)\,Q_{t+1}\right)\right]$$

which is equation (15) in the main text.

The envelope condition for previous investment is

$$\frac{\partial p_t^K}{\partial I_t} = \frac{1}{1+r_t}\mathbb{E}_t\left[\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)\right]$$

hence

$$Q_t^I = \mathbb{E}_t\left[\frac{1}{1+r_{t+1}}\left(\frac{I_{t+2}}{I_{t+1}}\right)^2 S'\left(\frac{I_{t+2}}{I_{t+1}}\right)\right]$$

plugging this expression back in (55) implies

$$1 + S\left(\frac{I_{t+1}}{I_t}\right) + \frac{I_{t+1}}{I_t}S'\left(\frac{I_{t+1}}{I_t}\right) = Q_t + \mathbb{E}_t\left[\frac{1}{1+r_{t+1}}\left(\frac{I_{t+2}}{I_{t+1}}\right)^2 S'\left(\frac{I_{t+2}}{I_{t+1}}\right)\right]$$

which is equation (14) in the main text. (15) and (14) jointly characterize investment dynamics.

C.5 Unions

Demand for labor services from union j is given by:

$$\frac{N_{jt}}{N_t} = \mathcal{Y}_w\left(\frac{W_{jt}}{W_t}\right) \tag{56}$$

where $\mathcal{Y}_w$ is Kimball demand,

$$\mathcal{Y}_w(x) \equiv (1 - \nu_w \log x)^{\frac{\epsilon_w}{\nu_w}}$$

The maximization problem of union j at time t is then

$$\mathbb{E}\left[\sum_{k\geq 0}\beta^k \zeta_w^k\left(\int \left\{u\left(c_{it+k}\right) - v\left(n_{it+k}\right)\right\} d\mathcal{D}_{it+k}\right)\right]$$

taking as given (56), the wage indexation rule, and households budget constraints. Since each union is infinitesimal, it only takes into account its marginal effect on every household's consumption and labor supply. Note that the total real earnings of household i are

$$\begin{aligned} z_{it} &= (1-\tau_t)e_{it}\frac{\int_0^1 W_{jt}n_{ijt}dj}{P_t} \\ &= (1-\tau_t)e_{it}\frac{\int_0^1 W_{jt}\mathcal{Y}_w\left(\frac{W_{jt}}{W_t}\right)dj}{P_t}N_t \end{aligned}$$

The envelope theorem implies that we can evaluate indirect utility as if all income from the union wage change is consumed. Hence $\frac{\partial c_{it}}{\partial W_{jt}} = \frac{\partial z_{it}}{\partial W_{jt}}$, where

$$\begin{aligned} \frac{\partial z_{it}}{\partial W_{jt}} &= (1-\tau_t)e_{it}\frac{N_t}{P_t}\left(\mathcal{Y}_w\left(\frac{W_{jt}}{W_t}\right)+\frac{W_{jt}}{W_t}\mathcal{Y}'_w\left(\frac{W_{jt}}{W_t}\right)\right) \\ &= (1-\tau_t)e_{it}\frac{N_t}{P_t}\mathcal{Y}_w\left(\frac{W_{jt}}{W_t}\right)\left(1-\epsilon_w\left(\frac{W_{jt}}{W_t}\right)\right) \end{aligned}$$

where $\epsilon_w(x) \equiv -\frac{x\mathcal{Y}'_w(x)}{\mathcal{Y}_w(x)}$. On the other hand, household i's total hours worked are

$$n_{it} \equiv \int_0^1 \mathcal{Y}_w\left(\frac{W_{jt}}{W_t}\right)N_t dj$$

so that

$$\begin{aligned} \frac{\partial n_{it}}{\partial W_{jt}} &= \mathcal{Y}'_w\left(\frac{W_{jt}}{W_t}\right)\frac{N_t}{W_t} \\ &= -\epsilon\left(\frac{W_{jt}}{W_t}\right)\mathcal{Y}_w\left(\frac{W_{jt}}{W_t}\right)\frac{N_t}{W_{jt}} \end{aligned}$$

hence, denoting by

$$U_s \equiv \int \{u(c_{is}) - v(n_{is})\}\, d\mathcal{D}_{is}$$

as average utility in period s, we have that the marginal change in aggregate utility induced by a change in a wage w is

$$\begin{aligned} \frac{\partial U_s}{\partial w} &= N_s\mathcal{Y}_w\left(\frac{w}{W_s}\right)\int\left\{(1-\tau_s)\,u'(c_{is})\,e_{is}\frac{1}{P_s}\left(1-\epsilon_w\left(\frac{w}{W_s}\right)\right)+v'(n_{is})\,\epsilon_w\left(\frac{w}{W_s}\right)\frac{1}{w}\right\}d\mathcal{D}_{is} \\ &= \left(\frac{MU_s}{P_s}+\left(\frac{MV_s}{w}-\frac{MU_s}{P_s}\right)\epsilon_w\left(\frac{w}{W_s}\right)\right)\mathcal{Y}_w\left(\frac{w}{W_s}\right)N_s \end{aligned}$$

where we have defined $MU_s \equiv (1-\tau_s)\int u'(c_{is})\,e_{is}d\mathcal{D}_{is}$ and $MV_s \equiv \int v'(n_{is})\,d\mathcal{D}_{is}$.

The union resets the wage knowing that, if it chooses wage w today, then its wage at any future time before it can reindex will be $w\frac{P_{t+k-1}}{P_{t-1}}$. The reset wage then solves

$$W_t^* = \arg\max \mathbb{E}\left[\sum_{k\geq 0}\beta^k\zeta_w^k U_{t+k}\left(w\frac{P_{t+k-1}}{P_{t-1}}\right)\right]$$

whose first order condition is

$$\mathbb{E}_t \left[\sum_{k \geq 0} \beta^k \zeta_w^k N_{t+k} \mathcal{Y}_w \left(\frac{W_t^*}{W_{t+k}} \frac{P_{t+k-1}}{P_{t-1}} \right) \right.$$
$$\left. \cdot \left(\left(1 - \epsilon_w \left(\frac{W_t^*}{W_{t+k}} \frac{P_{t+k-1}}{P_{t-1}} \right) \right) \frac{W_t^*}{W_{t+k}} \frac{P_{t+k-1}}{P_{t-1}} \frac{MU_{t+k} W_{t+k}}{P_{t+k}} + \epsilon_w \left(\frac{W_t^*}{W_{t+k}} \frac{P_{t+k-1}}{P_{t-1}} \right) MV_{t+k} \right) \right] = 0$$

We can rewrite this condition as

$$\mathbb{E} \left[\sum_{k \geq 0} \beta^k \zeta_w^k N_{t+k} \frac{MU_{t+k} W_{t+k}}{P_{t+k}} f_w \left(\frac{W_t^*}{W_{t+k}} \frac{P_{t+k-1}}{P_{t-1}}, s_{w,t+k} \right) \right] = 0$$

with f_w defined symmetrically to f_p in section C.3, and the inverse wage markup is defined as

$$s_{w,t} \equiv \frac{MV_t}{\frac{MU_t W_t}{P_t}} = \frac{\int v'(n_{it})\, di}{(1-\tau_t)\, w_t \int e_{it} u'(c_{it})\, di} = \frac{v'(N_t)}{(1-\tau_t)\, w_t u'(C_t^*)}$$

with C_t^* satisfying $u'(C_t^*) \equiv \int e_{it} u'(c_{it})\, di$, as in Auclert, Rognlie and Straub (2018).

The derivation follows the same steps as in section C.3. Linearizing around the steady state, where $s_w = \frac{\epsilon_w - 1}{\epsilon_w}$, we obtain

$$(\epsilon_w + \nu_w - 1)\, \mathbb{E}_t \left[\sum_{k \geq 0} (\beta \zeta_w)^k \frac{d\left(\frac{W_t^*}{W_{t+k}} \frac{P_{t+k-1}}{P_{t-1}} \right)}{\frac{W_t^*}{W_{t+k}} \frac{P_{t+k-1}}{P_{t-1}}} \right] = \epsilon_w \mathbb{E}_t \left[\sum_{k \geq 0} (\beta \zeta_w)^k d(s_{w,t+k}) \right]$$

rearranging, and defining $\omega_t = \log W_t$, this is also

$$(\epsilon_w + \nu_w - 1)\, \mathbb{E}_t \left[\sum_{k \geq 0} (\beta \zeta_w)^k (\omega_t^* - p_{t-1} - (\omega_{t+k} - p_{t+k-1})) \right] = \epsilon_w \mathbb{E}_t \left[\sum_{k \geq 0} (\beta \zeta_w)^k d(s_{w,t+k}) \right]$$

or

$$\omega_t^* - p_{t-1} \quad = \quad (1 - \beta \zeta_w) \left(\omega_t - p_{t-1} + \frac{\epsilon_w}{\epsilon_w + \nu_w - 1} ds_t^w \right) + \beta \zeta_w \mathbb{E}_t \left[w_{t+1}^* - p_t \right] \tag{57}$$

Moreover, the wage index satisfies

$$\zeta_w \frac{W_{t-1}}{W_t} \Pi_{t-1} \mathcal{Y}_w \left(\frac{W_{t-1}}{W_t} \Pi_{t-1} \right) + (1 - \zeta_w) \frac{W_t^*}{W_t} \mathcal{Y}_w \left(\frac{W_t^*}{W_t} \right)$$

which, in linear form, reads

$$(1 - \zeta_w)(\omega_t^* - p_{t-1}) = \omega_t - p_{t-1} - \zeta_w (\omega_{t-1} - p_{t-2})$$

Then, plugging in (57)

$$\begin{aligned}(1-\zeta_w)(\omega_t^*-p_{t-1}) &= \omega_t - p_{t-1} - \zeta_w(\omega_{t-1}-p_{t-2})\\ &= (1-\zeta_w)(1-\beta\zeta_w)\left(\omega_t - p_{t-1} + \frac{\epsilon_w}{\epsilon_w+\nu_w-1}ds_{w,t}\right) + \beta\zeta_w(1-\zeta_w)\mathbb{E}_t\left[\omega_{t+1}^*-p_t\right]\\ &= (1-\zeta_w)(1-\beta\zeta_w)\left(\omega_t - p_{t-1} + \frac{\epsilon_w}{\epsilon_w+\nu_w-1}ds_{w,t}\right)\\ &\quad +\beta\zeta_w\mathbb{E}_t\left[\omega_{t+1}-p_t-\zeta_w(\omega_t-p_{t-1})\right]\end{aligned}$$

and using $1-(1-\zeta_w)(1-\beta\zeta_w)+\beta\zeta_w^2=\zeta_w(1+\beta)$, we obtain

$$\omega_t - p_{t-1} = \frac{1}{1+\beta}(\omega_{t-1}-p_{t-2}) + \frac{\beta}{1+\beta}\mathbb{E}_t\left[\omega_{t+1}-p_t\right] + \frac{(1-\zeta_w)(1-\beta\zeta_w)}{1+\beta}\frac{\epsilon_w}{\epsilon_w+\nu_w-1}ds_{w,t}$$

In present value form, defining $\pi_{w,t}\equiv\omega_t-\omega_{t-1}=\log\left(\frac{W_t}{W_{t-1}}\right)$, this reads

$$\pi_{w,t}-\pi_{t-1} = \frac{(1-\beta\zeta_w)(1-\zeta_w)}{\zeta_w}\frac{\epsilon_w}{\epsilon_w+\nu_w-1}\mathbb{E}_t\left[\sum_k\beta^k\left(s_{w,t+k}-\frac{\epsilon_w-1}{\epsilon_w}\right)\right]$$

which is expression (18) in the main text.

C.6 Walras's law

Aggregate across all households,

$$\begin{aligned}C_t+L_t &= (1+r_{t-1}-\xi)L_{t-1}+Z_t+d_t\\ A_t &= (1+r_t^a)A_{t-1}-d_t\end{aligned}$$

where d_t are aggregate distributions from liquid to illiquid account. Consolidating, and using the definition of Z_t, we find

$$C_t+L_t+A_t=\left(1+r_{t-1}^{\ell}\right)L_{t-1}+(1+r_t^a)A_{t-1}+(1-\tau_t)w_tN_t$$

Using the government budget constraint (19), we next have

$$C_t+G_t+L_t+A_t+(1+\delta q_t)B_{t-1}=\left(1+r_{t-1}^{\ell}\right)L_{t-1}+(1+r_t^a)A_{t-1}+w_tN_t+q_tB_t$$

Finally, using the incoming flow of funds constraint for the financial intermediary (7),

$$C_t+G_t+L_t+A_t=(p_t+D_t)v_{t-1}-\xi L_{t-1}+w_tN_t+q_tB_t$$

then using the outgoing flow of funds constraint (8),

$$C_t + G_t + p_t v_t = (p_t + D_t)\, v_{t-1} - \xi L_{t-1} + w_t N_t$$

using market clearing condition for shares $v_t = 1$,

$$C_t + G_t + \xi L_{t-1} = w_t N_t + D_t$$

and finally, using the expression for dividends in (16), we obtain

$$C_t + G_t + I_t + I_t S\left(\frac{I_t}{I_{t-1}}\right) + \xi L_{t-1} = Y_t$$

which is the goods market clearing condition.

D Appendix to section 4

D.1 Calibration of the fiscal rule

Our calibration of the fiscal rule parameter ψ in (20) is informed by existing estimates from the fiscal rule literature, following Leeper (1991)'s seminal paper. We calibrate rather than estimate this parameter, because our model outcomes are insensitive to the value of ψ within a wide range, as we show below.

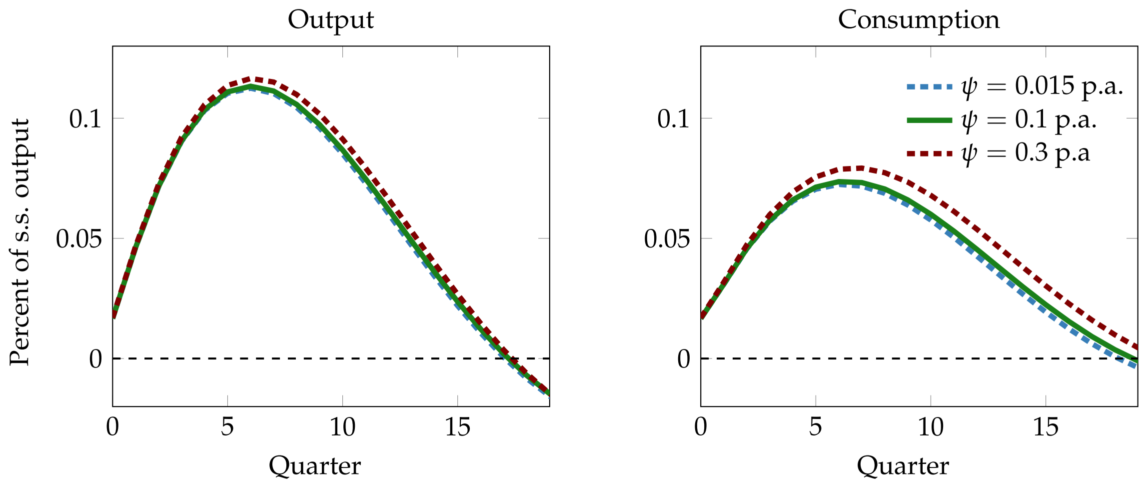
There exists a wide range of estimates for ψ, all of which tend to imply that the fiscal adjustment to shocks is delayed. Two representative examples from the literature are Davig and Leeper (2011) and Auclert and Rognlie (2018).

Davig and Leeper (2011) regress the ratio of federal receipts net of federal transfers to GDP on the debt-to-GDP ratio $\frac{q^{ss} B_{t-1}}{Y_t}$. Their estimate corresponds to an annualized value of $\psi = 0.28$. However, this is only an estimate for the active fiscal regime, which they estimate to be in place for half of their 1949:Q1 to 2008:Q4 sample (the estimate for the passive fiscal regime is $\psi = -0.1$ annually). Moreover, their numbers do not directly correspond to our specification in (20), which divides the face value of debt $q^{ss} B_{t-1}$ by steady-state rather than current GDP.

Auclert and Rognlie (2018)'s specification is closer to ours, since their regressor is the face value of debt divided by potential GDP, $\frac{q^{ss} B_{t-1}}{Y_t^{pot}}$. Combining their estimates for government spending and deficits, we obtain $\psi = -0.015 + 0.0288 \approx 0.015$ at an annual level. The implied estimates for ψ from Fernández-Villaverde, Guerrón-Quintana, Kuester and Rubio-Ramírez (2015) and Bianchi and Melosi (2017) lie somewhere between $\psi = 0.015$ and $\psi = 0.3$.

Altogether, we take $\psi = 0.015$ and $\psi = 0.30$ to be extreme points from the literature, and therefore pick $\psi = 0.1$ as our baseline calibration value.

Figure D.1: Impulse responses of output and consumption for various calibrated values of ψ



Note. This figure shows the impulse responses of output and consumption for different calibrated values of ψ. Our central model estimates are for $\psi = 0.1$ (solid green line). We then hold all other parameters fixed and recompute impulse responses for our extreme values of $\psi = 0.015$ and $\psi = 0.3$.

Robustness to alternative calibrations of ψ. Figure D.1 illustrates that our model outcomes are very insensitive to the value of ψ, given the existing range from the literature. Starting from our central estimates with our calibrated value of $\psi = 0.1$, we recompute impulse responses for the extreme values of $\psi = 0.015$ and $\psi = 0.3$ discussed above. We find that the impulse responses are almost identical, irrespective of ψ.

Estimating ψ. The results above suggest that there is little information in our impulse responses that can help identify ψ. However, in a further robustness exercise, we add nominal federal government current tax receipts divided by nominal GDP to our list of observables, estimate the impulse response of tax revenue to a monetary policy shock, and use this together with the other impulse responses in the main text to produce an estimate of ψ. This exercise yields $\psi = 0.13 \pm 0.18$. The point estimate suggests that our calibrated value of ψ is reasonable. The confidence bands are very wide, however, because with long-term debt, the fiscal impact of a monetary shock is in practice not large enough to provide sufficient identifying variation.

D.2 Model DAG and sequence-space Jacobian solution method

Figure D.2 displays the blocks for our model as a directed acyclic graph (DAG).

As discussed in Auclert et al. (2019), DAGs are useful devices to summarize how the model is computed and obtain impulse responses by chaining Jacobians. Endogenous sequences that are not the output of any block are at the left of the DAG, labeled "unknowns". Stacking these sequences in a vector $\mathbf{U}$, and stacking any exogenous sequences in $\mathbf{Z}$, we can evaluate each

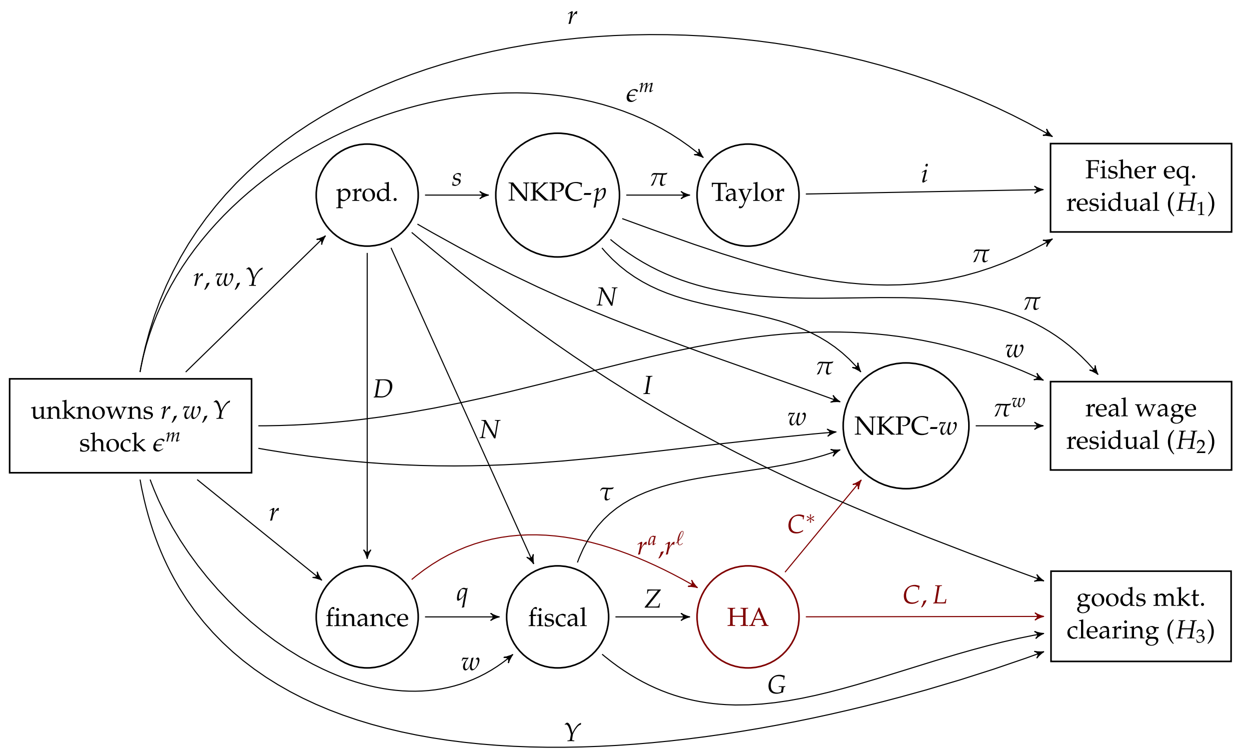


Figure D.2: Directed Acyclic Graph of the model

block in suitable order along the DAG to obtain every other endogenous sequence, including several—labeled H_1, H_2, and H_3 in the figure—that must be zero in equilibrium, and which we call "targets". Overall, then, the DAG represents a mapping

$$\mathbf{H}(\mathbf{U}, \mathbf{Z}) = 0 \tag{58}$$

As Figure D.2 shows, we set up our model so that the unknowns are $U_t \equiv (r_t, w_t, Y_t)$, the sequences of real interest rates, wages, and output. Corresponding to these are our three targets: first, the Fisher equation residual under perfect foresight,

$$H_{1t} = 1 + r_t - (1 + i_t)\,\frac{P_{t+1}}{P_t}$$

which in equilibrium, when $H_{1t} = 0$, corresponds to equation (10). Second, the real wage residual,

$$H_{2t} = \log\left(\frac{w_t}{w_{t-1}}\right) - (\pi^w_t - \pi_t)$$

which in equilibrium, when $H_{2t} = 0$, imposes consistency between the definition of the real wage

$w_t = \frac{W_t}{P_t}$, wage inflation $\pi_t^w = \log\left(\frac{W_t}{W_{t-1}}\right)$, and price inflation $\pi_t = \log\left(\frac{P_t}{P_{t-1}}\right)$. Finally, we have the goods market clearing residual

$$H_{3t} = C_t + G_t + I_t + I_t S\left(\frac{I_t}{I_{t-1}}\right) + \xi L_{t-1} - Y_t$$

which in equilibrium ensures goods market clearing at all times.

Our procedure solves equation (58) for $\mathbf{U}$ to first order around the steady state, as follows. Each block in the DAG is characterized to first order by the Jacobian matrices $\mathcal{J}^{o,i}$ for input sequences i and output sequences o. We combine these $\mathcal{J}$ using the chain rule to obtain the Jacobians $\mathbf{H_U}$ and $\mathbf{H_Z}$ of (58). This then provides a linear map from exogenous shocks to unknowns,

$$d\mathbf{U} = -\mathbf{H}_\mathbf{U}^{-1}\mathbf{H_Z}d\mathbf{Z}$$

Finally, we obtain all other sequences to first order given $d\mathbf{U}$ and $d\mathbf{Z}$, by applying $\mathcal{J}$'s along the DAG an extra time.

D.3 Details on solution method with informational rigidities

Deriving the recursion for sticky expectations. As discussed in section 4.3, if $\tau \leq s$, the impulse response of a household learning at date τ about a date-s change in input i is the same as the impulse response of a household who learns at date 0 about a date-$(s-\tau)$ change in i, shifted by τ periods. Both are the impulse response to a news shock about the value of i, $(s-\tau)$ periods in the future. This can be written as

$$\mathcal{J}_{t,s}^{o,i,\tau} = \mathcal{J}_{t-1,s-1}^{o,i,\tau-1} = \cdots = \mathcal{J}_{t-\tau,s-\tau}^{o,i,0} \tag{59}$$

If $\tau > s$, on the other hand, then we have $\mathcal{J}_{t,s}^{o,i,\tau} = \mathcal{J}_{t,s}^{o,i,s}$ for all t: we assume that the household is aware at date s of all inputs i to its problem at date s, so if not prior to s, τ is irrelevant.

These two observations allow us to simplify (23) for a given s, writing

$$\mathcal{J}_{t,s}^{o,i} = \theta^s \mathcal{J}_{t,s}^{o,i,s} + (1-\theta)\sum_{\tau=0}^{s-1}\theta^\tau \mathcal{J}_{t,s}^{o,i,\tau} \tag{60}$$

Applying (59) to each term of (60) except where $\tau = 0$, we can write for any $t, s > 0$

$$\begin{aligned}
\mathcal{J}_{t,s}^{o,i} &= \theta^s \mathcal{J}_{t-1,s-1}^{o,i,s-1} + (1-\theta)\sum_{\tau=0}^{s-2}\theta^{\tau+1}\mathcal{J}_{t-1,s-1}^{o,i,\tau} + (1-\theta)\mathcal{J}_{t,s}^{o,i,0} \\
&= \theta\mathcal{J}_{t-1,s-1}^{o,i} + (1-\theta)\mathcal{J}_{t,s}^{o,i,0}
\end{aligned} \tag{61}$$

where the second step consolidates the first two terms in the previous line using (60).

For $s = 0$, (60) simplifies to just $\mathcal{J}_{t,0}^{o,i} = \mathcal{J}_{t,0}^{o,i,0}$. For $t = 0$ and $s > 0$, there is no response unless

$\tau = 0$, so $\mathcal{J}_{0,s} = (1-\theta)\mathcal{J}^0_{0,s}$. Combining all results, we obtain

$$\mathcal{J}^{o,i}_{t,s} = \begin{cases} \theta \mathcal{J}^{o,i}_{t-1,s-1} + (1-\theta)\mathcal{J}^{o,i,0}_{t,s} & t > 0,\ s > 0 \\ \mathcal{J}^{o,i,0}_{t,s} & s = 0 \\ (1-\theta)\mathcal{J}^{o,i,0}_{t,s} & t = 0,\ s > 0 \end{cases}$$

But $\mathcal{J}^{o,i,0}_{t,s}$, the Jacobian for households that learn at date $\tau = 0$ about shocks, is also, by definition, the full-information Jacobian $\mathcal{J}^{o,i,FI}_{t,s}$, so (24) follows.

Implementation for other behavioral or informational frictions. Here, to illustrate the method's generality, we use an analogous approach to derive the transformation of the full-information Jacobian associated with some other frictions.

Cognitive discounting. Under Gabaix (2016)'s "cognitive discounting" friction, at the micro level agents' expectations of disturbances k periods in the future shrink by a factor of $\bar{m}^k$ relative to rational expectations, where $\bar{m} \in [0,1]$ is a parameter capturing cognitive discounting.

If at date 0 there is a news shock about some change to input i at date s, agents subject to cognitive discounting perceive this instead as a series of news shocks: they learn about a fraction $\bar{m}^s$ of the change at date 0, a fraction $\bar{m}^{s-1} - \bar{m}^s$ of the change at date 1, and so on, up until they learn about the final fraction $1-\bar{m}$ when the change actually happens at date s.

Using the same notation, the analog of (60) here is then

$$\mathcal{J}^{o,i}_{t,s} = \bar{m}^s \mathcal{J}^{o,i,0}_{t,s} + (\bar{m}^{s-1} - \bar{m}^s)\mathcal{J}^{o,i,1}_{t,s} + (\bar{m}^{s-2} - \bar{m}^{s-1})\mathcal{J}^{o,i,2}_{t,s} \ldots + (1-\bar{m})\mathcal{J}^{o,i,s}_{t,s} \tag{62}$$

Applying (59) to each term of (62) except the first, we can write for any $t, s > 0$

$$\begin{aligned} \mathcal{J}^{o,i}_{t,s} &= \bar{m}^s \mathcal{J}^{o,i,0}_{t,s} + (\bar{m}^{s-1} - \bar{m}^s)\mathcal{J}^{o,i,0}_{t-1,s-1} + (\bar{m}^{s-2} - \bar{m}^{s-1})\mathcal{J}^{o,i,1}_{t-1,s-1} \ldots + (1-\bar{m})\mathcal{J}^{o,i,s-1}_{t-1,s-1} \\ &= \bar{m}^s (\mathcal{J}^{o,i,0}_{t,s} - \mathcal{J}^{o,i,0}_{t-1,s-1}) + \bar{m}^{s-1}\mathcal{J}^{o,i,0}_{t-1,s-1} + (\bar{m}^{s-2} - \bar{m}^{s-1})\mathcal{J}^{o,i,1}_{t-2,s-2} + \ldots + (1-\bar{m})\mathcal{J}^{o,i,s-1}_{t-1,s-1} \\ &= \bar{m}^s (\mathcal{J}^{o,i,0}_{t,s} - \mathcal{J}^{o,i,0}_{t-1,s-1}) + \mathcal{J}^{o,i}_{t-1,s-1} \end{aligned} \tag{63}$$

For $s = 0$, (62) simplifies to just $\mathcal{J}^{o,i}_{t,0} = \mathcal{J}^{o,i,0}_{t,0}$, and for $t = 0$ and $s > 0$, $\mathcal{J}^{o,i,\tau}_{t,s} = 0$ for all $\tau > 0$, so that $\mathcal{J}^{o,i}_{0,s} = \bar{m}^s \mathcal{J}^{o,i,0}_{0,s}$. Combining all results and writing $\mathcal{J}^{o,i,FI}_{t,0} = \mathcal{J}^{o,i,0}_{t,0}$ for the full-information Jacobian, we have the recursion

$$\mathcal{J}^{o,i}_{t,s} = \begin{cases} \bar{m}^s (\mathcal{J}^{o,i,FI}_{t,s} - \mathcal{J}^{o,i,FI}_{t-1,s-1}) + \mathcal{J}^{o,i}_{t-1,s-1} & t > 0,\ s > 0 \\ \mathcal{J}^{o,i,FI}_{t,s} & s = 0 \\ \bar{m}^s \mathcal{J}^{o,i,FI}_{t,s} & t = 0,\ s > 0 \end{cases} \tag{64}$$

which can transform the full-information Jacobian $\mathcal{J}^{o,i,FI}_{t,s}$ into the Jacobian $\mathcal{J}^{o,i}_{t,s}$ with cognitive discounting with only a single evaluation for each Jacobian entry.

Noisy information about shocks. Following a date-0 shock $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$ that causes expected future inputs i to change, suppose that at each date $t \geq 0$, all agents receive independent private signals $\epsilon + \nu_t$ about the shock, where $\nu_t \sim \mathcal{N}(0, \sigma_\nu^2)$. Agents receive no other information about the shock.[49] Then, applying standard Bayesian updating, the average belief about ϵ at date j is

$$\bar{\mathbb{E}}_j \epsilon = \underbrace{\frac{(j+1)\tau_\nu}{\tau_\epsilon + (j+1)\tau_\nu}}_{\equiv a_j} \epsilon \tag{65}$$

where $\tau_\epsilon \equiv 1/\sigma_\epsilon^2$ and $\tau_\nu \equiv 1/\sigma_\nu^2$ are the precisions of the shock and signal, respectively. This is because by date j, each agent has combined the prior on ϵ with $j+1$ noisy signals $\epsilon + \nu_0, \ldots, \epsilon + \nu_j$.

On average, then, agents receive a news shock about ϵ of $a_0 \epsilon$ at date 0, $(a_1 - a_0)\epsilon$ at date 1, and so on. They receive proportional news shocks about the changes in inputs i, until they learn fully about the change in input i at date s once date s actually arrives.

The analog of (60) is then

$$\mathcal{J}_{t,s}^{o,i} = (1 - a_{s-1})\mathcal{J}_{t,s}^{o,i,s} + \sum_{\tau=0}^{s-1} (a_\tau - a_{\tau-1})\mathcal{J}_{t,s}^{o,i,\tau} \tag{66}$$

where we take $a_{-1} = 0$. Applying (59) to this to reduce all superscripts τ to 0, and also using $\mathcal{J}_{t,s}^{o,i,\tau} = 0$ for $t < s, \tau$, along with $\mathcal{J}_{t,0}^{o,i,FI} = \mathcal{J}_{t,0}^{o,i,0}$, we get

$$\mathcal{J}_{t,s}^{o,i} = \begin{cases} (1 - a_{s-1})\mathcal{J}_{t-s,0}^{o,i,FI} + \sum_{\tau=0}^{s-1} (a_\tau - a_{\tau-1})\mathcal{J}_{t-\tau,s-\tau}^{o,i,FI} & t \geq s \\ \sum_{\tau=0}^{t} (a_\tau - a_{\tau-1})\mathcal{J}_{t-\tau,s-\tau}^{o,i,FI} & t < s \end{cases} \tag{67}$$

Since the a_τ do not decay exponentially, it is impossible to simplify this further into a recursive form as in the prior examples. Still, directly applying (67) to calculate $\mathcal{J}^{o,i}$ from $\mathcal{J}^{o,i,FI}$, when both are $T \times T$ matrices, only takes $\mathcal{O}(T^3)$ operations, the same as matrix multiplication and inversion—which are already done many times as part of the Auclert, Bardóczy, Rognlie and Straub (2019) solution method. Even here, therefore, the additional computational burden from converting the full-information Jacobian to the frictional Jacobian is slight.[50]

[49] In particular, agents do not extract information about the shock ϵ from changes in variables like i, once they are actually observed. If agents did, then without noise from additional individual-level shocks, they would be able to back out ϵ perfectly. Though we conjecture that our methods should still apply in a model augmented with such additional shocks—with somewhat greater complexity due to the endogeneity of the observed variables—this is beyond the scope of the current paper.

[50] Since the sums in (67) are convolutions of the sequence $\{a_\tau - a_{\tau-1}\}$ with the diagonals of the $\mathcal{J}^{o,i,FI}$ matrix, it is possible to use the Fast Fourier Transform to speed up computation to $\mathcal{O}(T^2 \log T)$, but since it is already not a bottleneck, in practice this seems unnecessary.

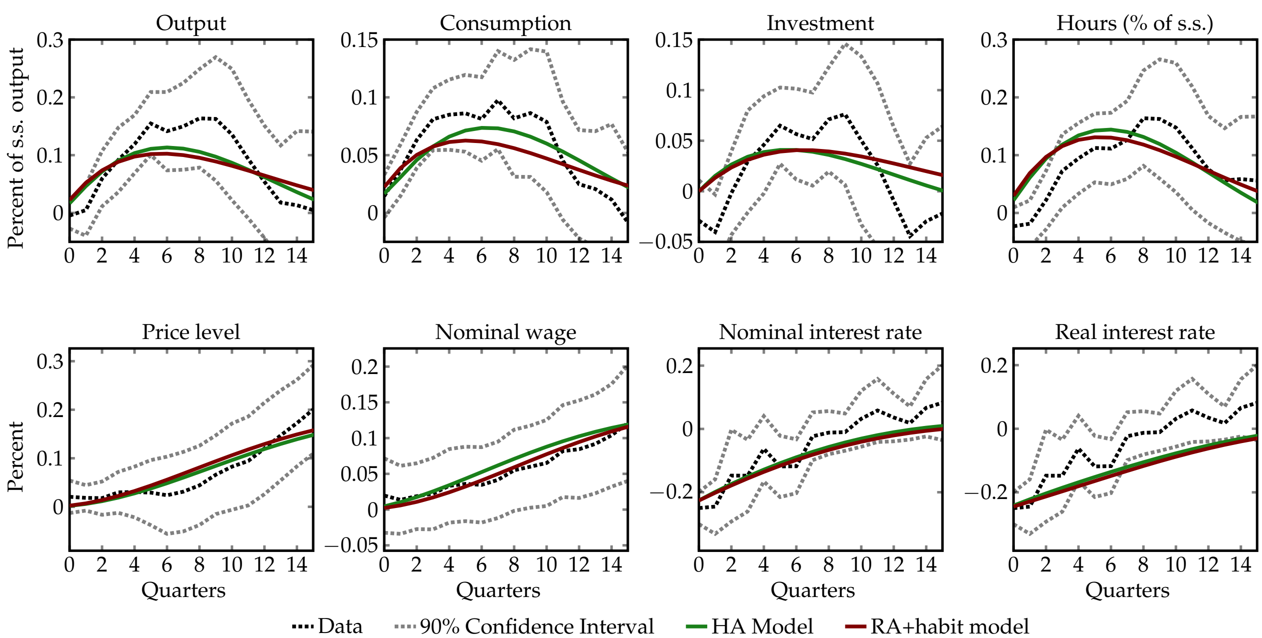
D.4 Estimated RA-habit model

Here we estimate the RA-habit model, using the procedure described in section 4.3 on the set of impulse responses described in section 4.2. Table D.1 displays the estimated parameters, figure D.3 shows the fit compared to that of our estimated HA model.

Table D.1: Estimated parameters for RA model.

Parameter		Value	std. dev.
γ	Household habit parameter	0.878	(0.012)
ϕ	Investment adjustment cost parameter	14.851	(4.016)
ζ_p	Calvo price stickiness	0.880	(0.042)
ζ_w	Calvo wage stickiness	0.946	(0.025)
ρ^m	Taylor rule inertia	0.904	(0.008)
σ^m	Std. dev. of monetary shock	0.057	(0.005)

Figure D.3: Impulse response to a monetary policy shock vs. model fit of HA and RA



Note. This figure shows our estimated set of impulse responses to an identified Romer and Romer (2004) monetary policy shock (dashed black, with gray confidence intervals). The solid lines are the impulse responses implied by our estimated inattentive heterogeneous-agent model (green) and a representative-agent model (red).

E Appendix to section 5

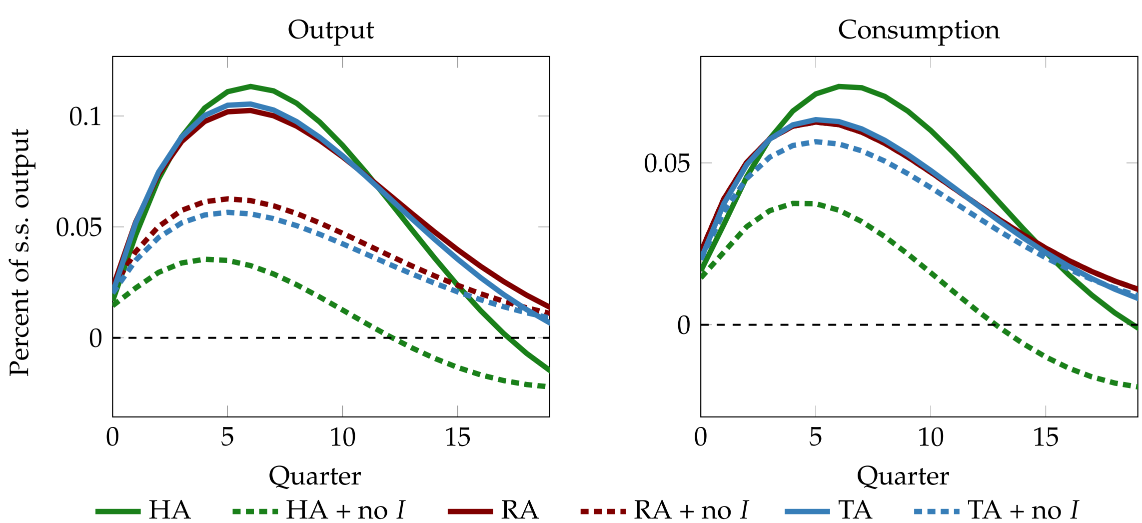
E.1 Investment counterfactual in TA-habit

Here we set up a two-agent version of our RA-habit model. The model is identical to the RA-habit model described in the main text, except that it features a share μ of hand-to-mouth households who consume their net-of tax income, $C_t^{HTM} = Z_t$. We choose $\mu = 0.20$ in line with the average MPC in figure 2. Table E.1 shows the estimated parameter values. Figure E.1 repeats the investment counterfactual of section 5, but using the estimated TA model as baseline.

Table E.1: Estimated parameters for TA model.

Parameter		Value	std. dev.
γ	Household habit parameter	0.884	(0.012)
ϕ	Investment adjustment cost parameter	13.150	(3.385)
ζ_p	Calvo price stickiness	0.898	(0.031)
ζ_w	Calvo wage stickiness	0.931	(0.024)
ρ^m	Taylor rule inertia	0.902	(0.008)
σ^m	Std. dev. of monetary shock	0.057	(0.005)

Figure E.1: Role of investment with estimated TA habit model

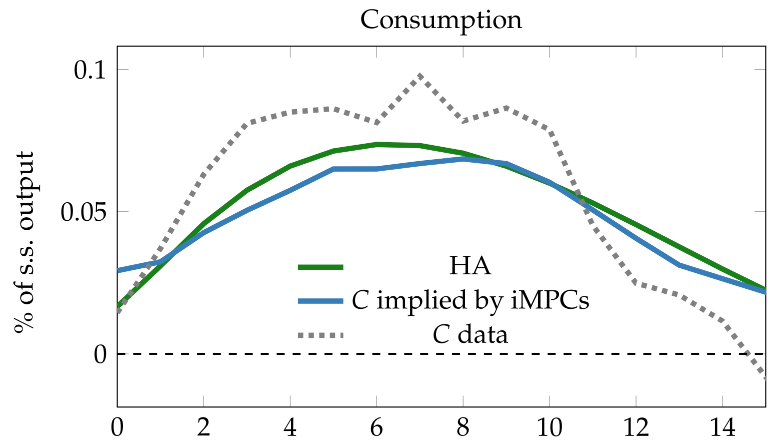


Note. This figure shows the general equilibrium paths of output and consumption in: our estimated HA model (green), an RA model with habits (red), and a TA model with habits (blue). Dashed lines correspond to an investment adjustment cost parameter $\phi = \infty$.

E.2 iMPCs and the path of income

In figure E.2 we perform a simple experiment, which is independent of our supply-side calibration and depends only on the pattern of intertemporal MPCs. In this experiment, we suppose

Figure E.2: Consumption implied by iMPCs and the output response to the monetary policy shock



Note. This figure shows the estimated model (green) and data (gray dashed) responses to the monetary policy, as well as the implied consumption response (blue) if agents were only to receive the income stream $(1-\alpha)Y_t^{data}$ where Y_t^{data} is the empirical impulse response to the monetary policy shock. This consumption response only depends on intertemporal MPCs.

that households' aggregate before-tax labor income is given by $(1-\alpha)Y_t^{data}$, where Y_t^{data} is the empirical impulse response of output to the monetary shock, and then feed in this labor income shock—*and no other shocks*—to the full-attention household sector.

The blue line shows the resulting consumption impulse, which is already quite large, both relative to the estimated model consumption response (green) as well as the empirical consumption response (gray, dashed). There is no room for intertemporal substitution to add to consumption in the first few quarters: the entire impulse is explained by the consumption response to labor income alone. Some friction, therefore, must be dampening the overall consumption response, especially on impact. In our model, this friction is inattention.

F Appendix to section 6

F.1 Estimated Habit-RA model

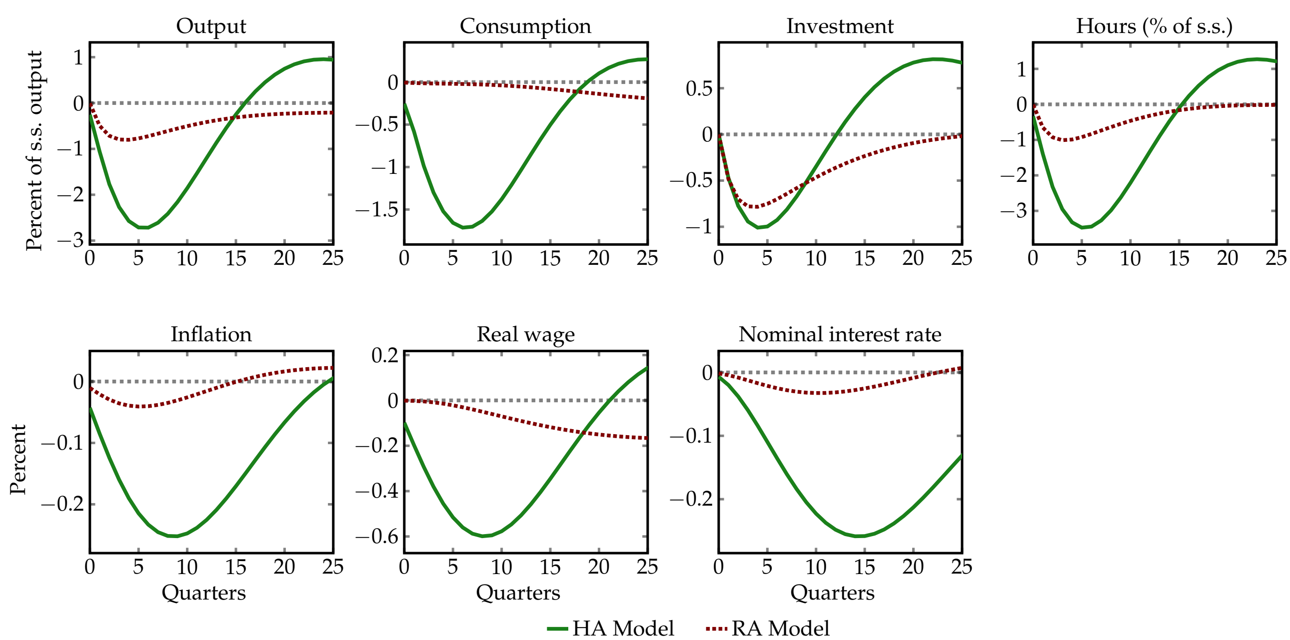
Table F.1: Priors and posteriors for the representative-agent model

Supply shock		Prior distribution	Posterior Mode	Posterior std. dev	Demand shock		Prior distribution	Posterior Mode	Posterior std. dev
TFP Θ_t	s.d.	Invgamma(0.1, 2)	0.330	(0.016)	Mon. policy ϵ_t^m	s.d.	Invgamma(0.1, 2)	0.215	(0.010)
	AR	Beta(0.5, 0.2)	0.970	(0.015)		AR	Beta(0.5, 0.2)	0.139	(0.051)
w markup $\epsilon_{w,t}$	s.d.	Invgamma(0.1, 2)	0.415	(0.028)	G shock G_t	s.d.	Invgamma(0.1, 2)	0.313	(0.015)
	AR	Beta(0.5, 0.2)	0.690	(0.132)		AR	Beta(0.5, 0.2)	0.884	(0.031)
	MA	Beta(0.5, 0.2)	0.647	(0.155)	C shock ϵ_t^C	s.d.	Invgamma(0.1, 2)	4.253	(1.067)
p markup $\epsilon_{p,t}$	s.d.	Invgamma(0.1, 2)	0.246	(0.016)		AR	Beta(0.5, 0.2)	0.759	(0.040)
	AR	Beta(0.5, 0.2)	0.266	(0.129)	I shock ϵ_t^I	s.d.	Invgamma(0.1, 2)	31.746	(7.832)
	MA	Beta(0.5, 0.2)	0.393	(0.095)		AR	Beta(0.5, 0.2)	0.528	(0.044)

Note. For an ARMA(1,1) process of the form $x_{t+1} - \rho x_t = \epsilon_{t+1} - \theta \epsilon_t$, "AR" refers to ρ, "MA" refers to θ. To be comparable with Smets and Wouters (2007) we scale the markup shocks such that $\epsilon_{w,t}, \epsilon_{p,t}$ appear with a coefficient of 1 in the Phillips curves (18) and (13).

F.2 Impulse response functions for HA and RA

Figure F.1: Impulse responses to 1-sd investment shock



Note. This is a positive shock to risk premia ϵ_t^I.

Figure F.2: Impulse responses to 1-sd consumption shock

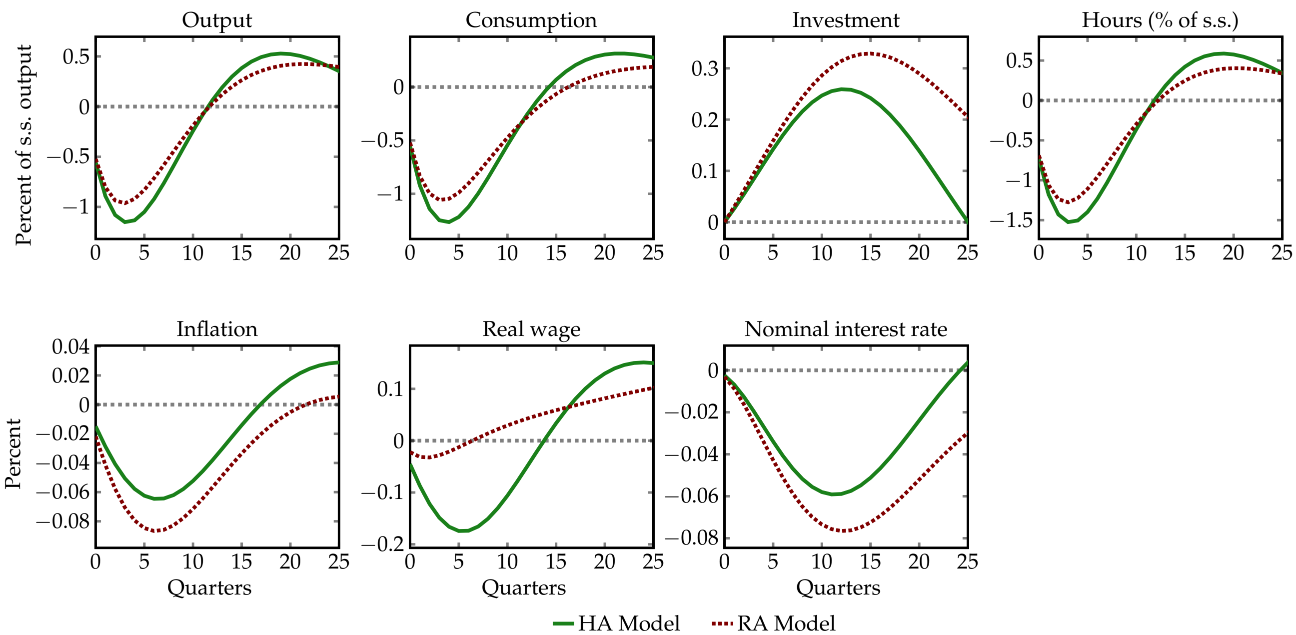


Figure F.3: Impulse responses to 1-sd government spending shock

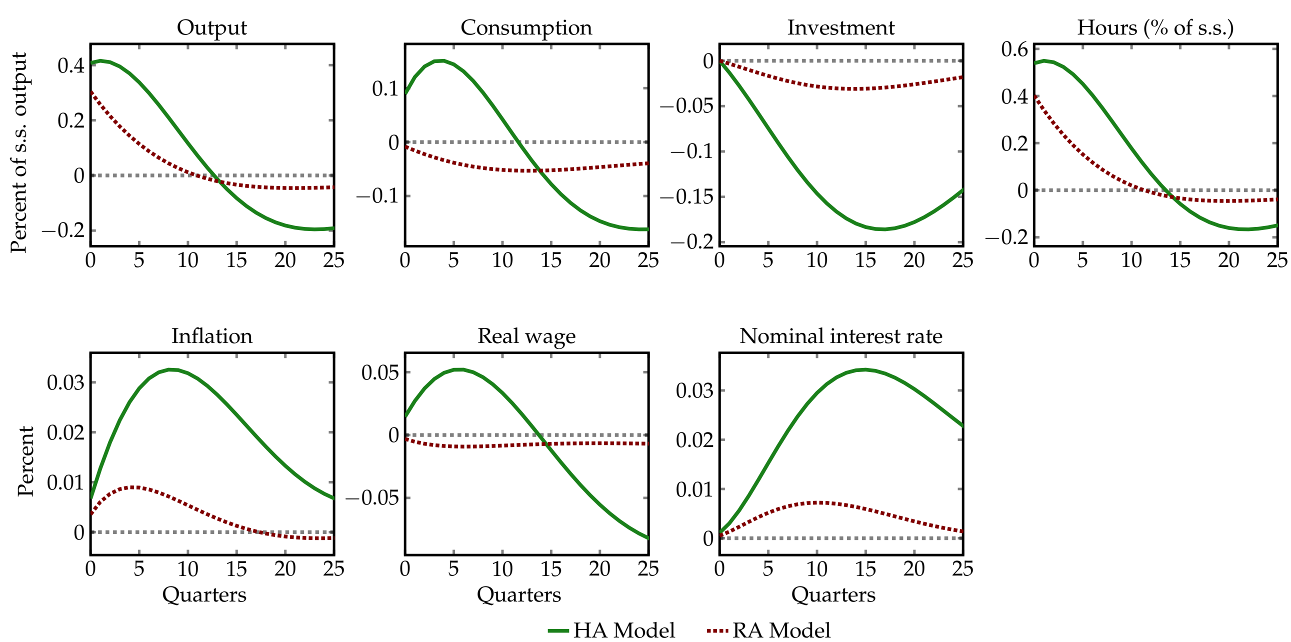
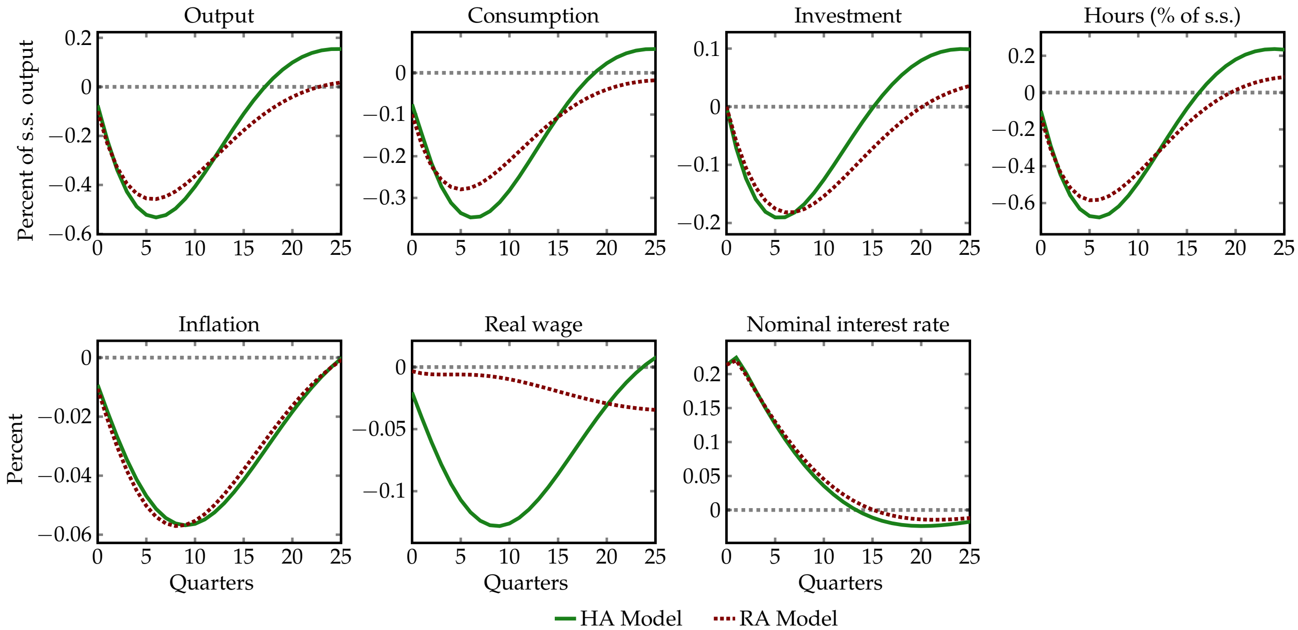


Figure F.4: Impulse responses to 1-sd monetary policy shock



Note. This is a positive shock to nominal rates ϵ_t^m.

Figure F.5: Impulse responses to 1-sd wage markup shock

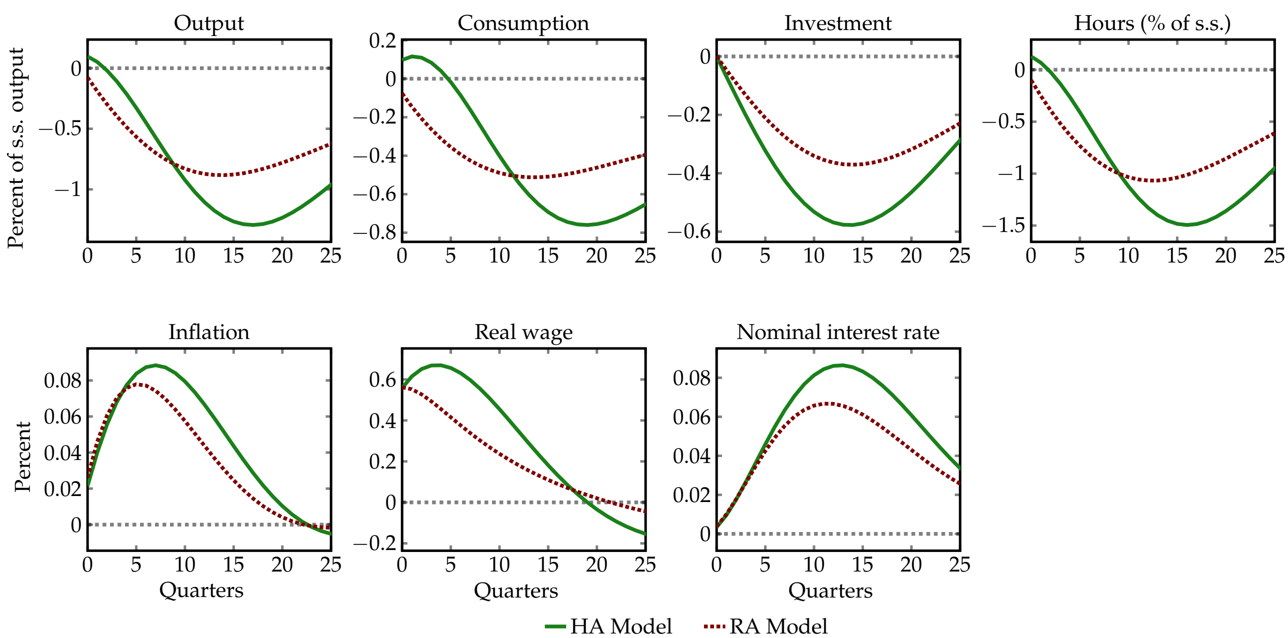


Figure F.6: Impulse responses to 1-sd productivity shock

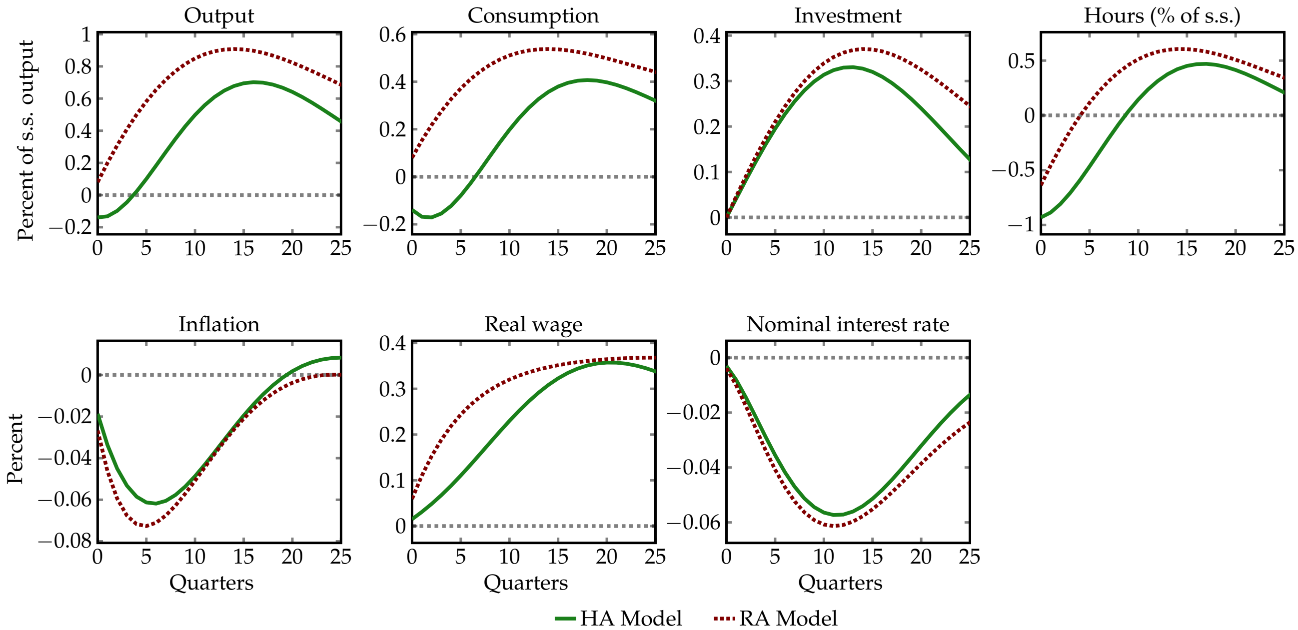
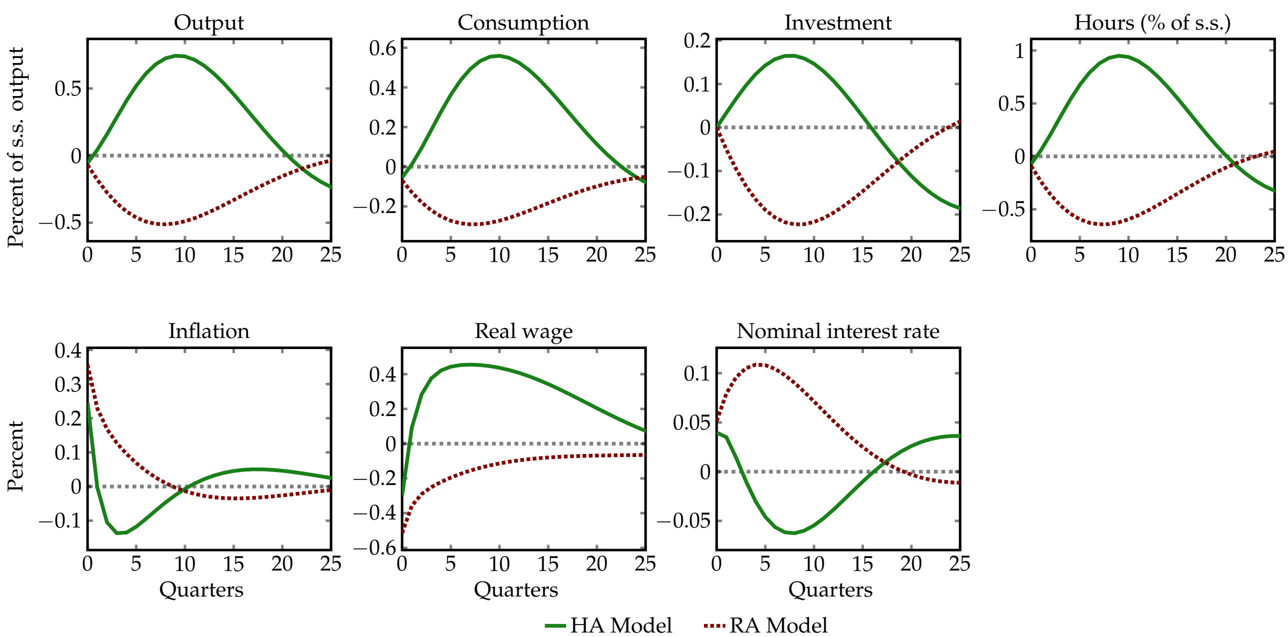
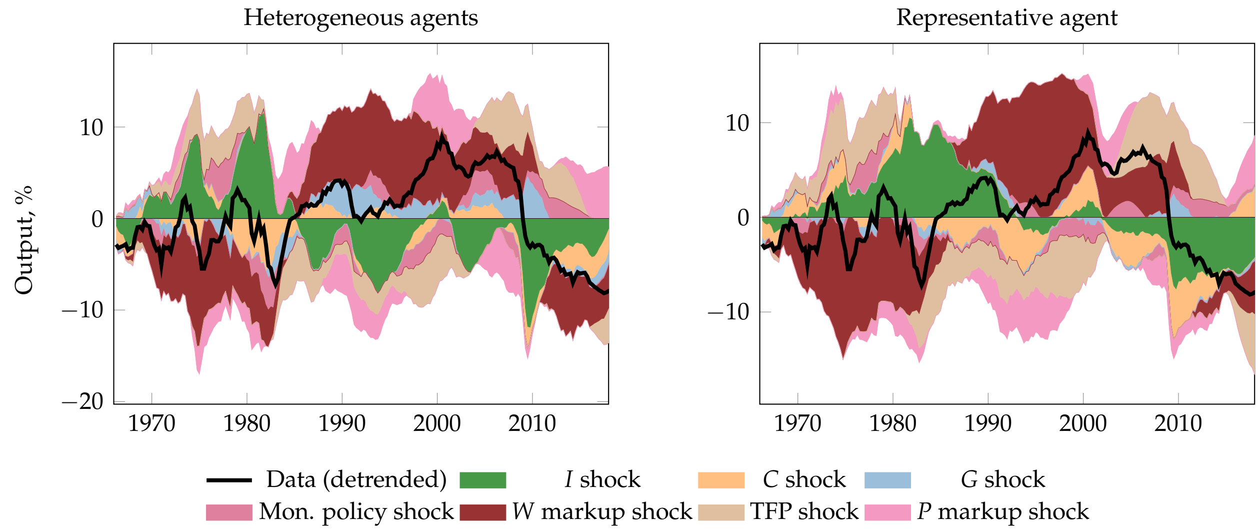


Figure F.7: Impulse responses to 1-sd price markup shock



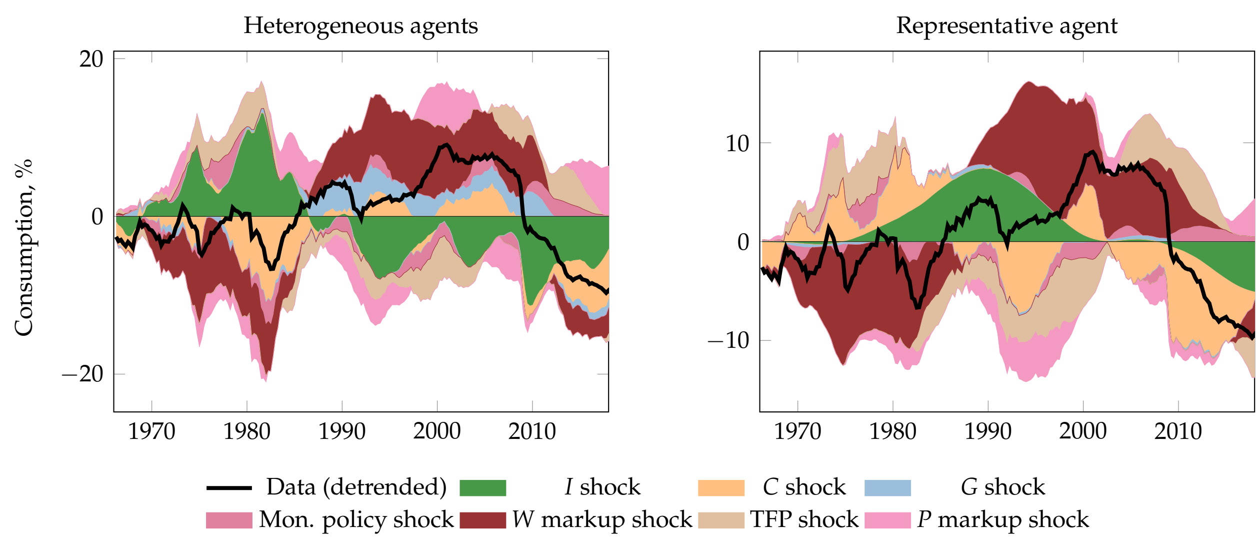
F.3 Historical shock decompositions of output and consumption

Figure F.8: Shock decompositions for output



Note. This figure decomposes the observed (linearly detrended real) output path Y_t into components driven by the seven shocks in the RA and HA models.

Figure F.9: Shock decompositions for consumption



Note. This figure decomposes the observed (linearly detrended real) consumption path C_t into components driven by the seven shocks in the RA and HA models.