Web Appendix for: The Life-cycle Growth of Plants: The Role of Productivity, Demand and Wedges.

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May 9, 2020

1 Appendix A: price indices

1.1 CUPI price index

Our baseline results use CUPI price indices at the plant level as deflators. Here, we explain the details of their construction. The change in prices from one period to the next is:

$$\frac{P_{ft}}{P_{ft-1}} = \left(\frac{\sum_{\Omega_t^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}}{\sum_{\Omega_t^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \tag{1}$$

Defining as $\Omega_{t,t-1}^f$ the set of goods that is common to both periods, and multiplying both the numerator and the denominator by $\left(\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma} * \sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ we obtain:

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$$\frac{P_{ft}}{P_{ft-1}} = \left(\frac{\sum_{\Omega_t^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}}{\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}} \frac{\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}}{\sum_{\Omega_{t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}} \frac{\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}}{\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \quad (2)$$

$$= \frac{\lambda_{ft-1,t}}{\lambda_{ft,t-1}} \left(\frac{\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}}{\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \quad (3)$$

where $\lambda_{ft-1,\Omega_{t,t-1}^f} = \left(\frac{\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}}{\sum_{\Omega_{t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$ and $\lambda_{ft,\Omega_{t,t-1}^f} = \left(\frac{\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}}{\sum_{\Omega_t^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$. The term $\frac{\lambda_{ft-1,t}}{\lambda_{ft,t-1}}$ is the Feenstra (2004) adjustment for changing varieties between two periods.

Furthermore, since

$$s_{fjt} = \frac{p_{fjt} q_{fjt}}{R_{ft}} = \frac{p_{fjt}^{1-\sigma} \left(d_{fjt}^{\sigma} \right)}{P_{fjt}^{1-\sigma}} \quad (4)$$

we have that:

$$\lambda_{ft-1,\Omega_{t,t-1}^f} = \left(\sum_{\Omega_{t,t-1}^f} \frac{d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}}{\sum_{\Omega_{t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} = \left(\sum_{\Omega_{t,t-1}^f} s_{fjt-1} \right)^{\frac{1}{1-\sigma}}$$

That is, $\left(\lambda_{ft-1,\Omega_{t,t-1}^f} \right)^{1-\sigma}$ is the share of period $t-1$ expenditures devoted to goods that are common to both periods. Similarly, $\left(\lambda_{ft,\Omega_{t,t-1}^f} \right)^{1-\sigma}$ is the share of period t expenditure devoted to goods common to both periods.

With this, the change in prices between the two periods can be written:

$$\frac{P_{ft}}{P_{ft-1}} = \left(\frac{\sum_{\Omega_{t,t-1}^f} s_{fjt}}{\sum_{\Omega_{t,t-1}^f} s_{fjt-1}} \right)^{\frac{1}{\sigma-1}} \frac{P_{ft}^*}{P_{ft-1,\Omega_{t,t-1}^f}^*} \quad (5)$$

where $P_{ft}^* = \left(\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ is a period t price index for the basket of goods common to t and $t-1$ for firm f, and $P_{ft-1,\Omega_{t,t-1}^f}^* = \left(\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$

is a period $t-1$ price index for that same basket. Term $\left(\frac{\sum_{\Omega^f_{t,t-1}} s_{fjt}}{\sum_{\Omega^f_{t,t-1}} s_{fjt-1}}\right)^{\frac{1}{\sigma-1}} = \lambda^{Qfee}_{ft}$ is the Feenstra adjustment for changing varieties, expressed in terms of observables.

The Marshalian demands, given by $q_{fjt} = d^{\sigma}_{ft} d^{\sigma}_{fjt} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma} \left(\frac{p_{fjt}}{P_{ft}}\right)^{-\sigma} \frac{E_t}{P_t}$, imply

$$s^*_{fjt} = \frac{d^{\sigma}_{ft} d^{\sigma}_{fjt} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma} \frac{p^{1-\sigma}_{fjt}}{P^{-\sigma}_{ft}} \frac{E_t}{P_t}}{\sum\limits_{\Omega^f_{t,t-1}} d^{\sigma}_{ft} d^{\sigma}_{fjt} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma} \frac{p^{1-\sigma}_{fjt}}{P^{-\sigma}_{ft}} \frac{E_t}{P_t}} = \frac{d^{\sigma}_{fjt} p^{1-\sigma}_{fjt}}{\left(P^*_{ft}\right)^{1-\sigma}}$$

and

$$s^*_{fjt-1,\Omega^f_{t,t-1}} = \frac{d^{\sigma}_{ft-1} d^{\sigma}_{fjt-1} \left(\frac{P_{ft-1}}{P_{t-1}}\right)^{-\sigma} \frac{p^{1-\sigma}_{fjt-1}}{P^{-\sigma}_{ft-1}} \frac{E_{t-1}}{P_{t-1}}}{\sum\limits_{\Omega^f_{t,t-1}} d^{\sigma}_{ft-1} d^{\sigma}_{fjt-1} \left(\frac{P_{ft-1}}{P_{t-1}}\right)^{-\sigma} \frac{p^{1-\sigma}_{fjt-1}}{P^{-\sigma}_{ft-1}} \frac{E_{t-1}}{P_{t-1}}} = \frac{d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}}{\left(P^*_{ft-1,\Omega^f_{t,t-1}}\right)^{1-\sigma}}$$

so that $\left(\frac{p_{fjt}}{p_{fjt-1}}\right) = \left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right) \left(\frac{s^*_{fjt}}{s^*_{fjt-1,\Omega^f_{t,t-1}}}\right)^{\frac{1}{1-\sigma}} \left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{-\frac{\sigma}{1-\sigma}}$. Given this and $\sum\limits_{\Omega^f_{t,t-1}} s^*_{fjt,\Omega^f_{t,t-1}} = 1$, for plant-product weights $\omega_{ft} = \frac{1}{\left\|\Omega^f_{t,t-1}\right\|}$ such that $\sum\limits_{\Omega^f_{t,t-1}} \omega_{ft,t-1} = 1$,

$$\begin{aligned} & \sum_{\Omega^f_{t,t-1}} \ln \left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} \\ = \; & \ln \left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right) + \frac{1}{(1-\sigma)} \sum_{\Omega^f_{t,t-1}} \ln \left(\frac{s^*_{fjt}}{s^*_{fjt-1,\Omega^f_{t,t-1}}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} \\ & + \frac{\sigma}{\sigma-1} \sum_{\Omega^f_{t,t-1}} \ln \left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} \end{aligned}$$

Shocks d_{fjt} have been defined relative to plant appeal, d_{ft}, such that $\prod_{\Omega^f_{t,t-1}} d_{fjt}^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} = 1$, with the implication that

$\sum_{\Omega^f_{t,t-1}} \ln\left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} = 0$. Notice that this normalization still allows for a distribution of product appeal that varies over time. [1]

The consecutively common good price index growth $\left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right)$ therefore corresponds to

$$\ln\left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right) = \sum_{\Omega^f_{t,t-1}} \ln\left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} - \frac{1}{(1-\sigma)} \sum_{\Omega^f_{t,t-1}} \ln\left(\frac{s^*_{fjt}}{s^*_{fjt-1,\Omega^f_{t,t-1}}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}}$$

The term $\ln \lambda^{QRW}_{ft} = \sum_{\Omega^f_{t,t-1}} \ln\left(\frac{s^*_{fjt}}{s^*_{fjt-1,\Omega^f_{t,t-1}}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}}$ adjusts for changes in appeal for continuing products, addressing the consumer valuation bias.

We similarly obtain a measure of materials by deflating material expenditure by plant-level price indices for materials, pm_{ft}, using information on individual prices and quantities of material inputs. We construct pm_{ft} using an analogous approach to that used to construct output prices. The underlying assumption is that M_{ft}, the index of materials quantities used, is a CES aggregate of individual inputs. As is the case with output prices, until we have an estimate of the elasticity of substitution, we can only build a consecutively-common-basket price index $\overline{pm^*_{ft}}$ for plant f, and carry an adjustment factor $\Lambda^M_{ft} = \Lambda^{MRW}_{ft}\Lambda^{Mfee}_{ft}$ for which we later adjust prices. In particular, we deflate

[1] This is by contrast to empirical price indices that weight across products with variable weights $\omega_{fjt} \neq \omega_{ft}$, such as the commonly used Sato-Vartia approach (Sato, 1974, Vartia, 1974, Feenstra, 2004). Under such variable weights the assumption $\sum_{\Omega^f_{t,t-1}} \ln\left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{\omega_{fjt}} = 0$ does not hold. The fact that traditional approaches using variable weights ignore this term leads to what Redding and Weinstein (2017) have called the "consumer valuation bias" the traditional empirical approaches to economically motivated price indices.

materials expenditures to obtain $M^*_{ft} = \frac{\text{materials expenditure}_{ft}}{pm_{fB}\overline{pm^*_{ft}}} = M_{ft} * \left(\Lambda^M_{ft}\right)^{\frac{1}{\sigma-1}}$. Once we have obtained an estimate of the elasticity of substitution we calculate $pm_{ft} = pm_{fB} * \overline{pm^*_{ft}} * \left(\Lambda^M_{ft}\right)^{\frac{1}{\sigma-1}}$, which is one of the fundamentals on the cost side in our growth decomposition. We use this price index as deflator for materials expenditure to obtain our $TFPQ$ measure. We use for inputs the same elasticity of substitution estimated for outputs. We recognize that using the same elasticity for inputs and outputs is a strong assumption, but find that it does no affect our results in an important way. In particular, we find in Appendix I that using a Divisia price index (with updated input mix each period) generates about the same contribution for materials prices in sales and output volatility as the UPI. The Divisia materials price index does not depend on the elasticity of substitution, suggesting that this assumption is not critical for our results.

1.2 Initializing a plant's CUPI price index

A plant's price index is constructed as

$$P_{ft} = P_{fB} * \overline{P^*_{ft}} * \left(\Lambda^Q_{ft}\right)^{\frac{1}{\sigma-1}}$$

The initial level P_{fB}, where B is the base year for plant f, is constructed as: $P_{fB} = P_{base,B} \prod_{\Omega^f_B} \left(\frac{p_{fjB}}{\overline{p}_{jB}}\right)^{s_{fjB}}$, where $\overline{p_{jB}}$ is the geometric average of the price of product j in year B across plants, year B is the first year in which plant f is present in the survey, and $P_{base,B}$ is an overall base. We use 1982 as the base year, so $P_{base,1982} = 1$. For plants with $B \neq 1982$, $P_{base,B}$ is set equal to the geometric mean of the price index across plants that we observe prior to year B. Notice that our approach takes advantage of cross sectional variability across plants for any given product or input j. In the plant's base year B, $\left(\frac{P_{fjB}}{\overline{P_{jB}}}\right) = 1$ for the average producer of product j. For other plants, it will capture dispersion in price levels around that average.[2]

[2]We deal with excessive noise from partial year reporting and other sources by eliminating outliers. In particular, in any given year we consider only products that represent at least 2% of sales of the respective plant. Shares are re-calculated accordingly for this restricted basket. We also winsorize the 2% tails at each step of the process of build-

1.3 Sato-Vartia indices

The Sato-Vartia approach (used in the results in Appendix C) is an alternative way of computing $\ln\left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right)$, using weights $\omega^{SV}_{fjt,t-1} = \frac{\frac{(s^*_{fjt}-s^*_{fjt-1,t})}{\ln s^*_{fjt}-\ln s^*_{fjt-1,t}}}{\sum\limits_{\Omega^f_{t,t-1}}\left(\frac{(s^*_{fjt}-s^*_{fjt-1,t})}{\ln s^*_{fjt}-\ln s^*_{fjt-1,t}}\right)}$ and imposing $-\frac{\sigma}{\sigma-1}\sum\limits_{\Omega^f_{t,t-1}}\ln\left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{\omega^{SV}_{fjt}} = 0$. That is, $\ln\left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right)^{SV} = \sum\limits_{\Omega^f_{t,t-1}}\ln\left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\omega^{SV}_{fjt}}$.

Notice, in the derivation above, that when using variable weights $\omega_{fjt} \neq \omega_{ft}$, the assumption $\sum\limits_{\Omega^f_{t,t-1}}\ln\left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{\omega_{fjt}} = 0$ would not hold. In the Sato-Vartia case, since product demand shocks $\frac{d_{fjt}}{d_{fjt-1}}$ are positively correlated with the weights $\omega^{SV}_{fjt,t-1}$ (Redding-Weinstein, 2020), $\sum\limits_{\Omega^f_{t,t-1}}\ln\left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{\omega^{SV}_{fjt}} > 1$ and the consumer valuation bias would be positive. That is, the Sato Vartia approach likely overstates price inflation for the common goods produced by plant f in both $t-1$ and t. Such overstatement of price inflation implies understatement of quantity growth and therefore $TFPQ$.

1.4 Törnqvist index

Appendix C also presents results using Törnqvist indices, first imposing a basket of goods that is fixed over the life cycle and constant weights for

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ing price indices. In particular, we winsorize $\frac{\sum_{\Omega^f_{l,l-1}} s_{fjl}}{\sum_{\Omega^f_{l,l-1}} s_{fjl-1}}$; $\prod\limits_{\Omega^f_{t,t-1}}\left(\frac{s^*_{fjt}}{s^*_{fjt-1,\Omega^f_{t,t-1}}}\right)^{\frac{1}{\|\Omega_{t,t-1}\|}}$; $\frac{p_{fjt}}{p_{fjt-1}}$; $\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}$; $\frac{P_{ft}}{P_{ft-1}}$.

We also winsorize adjustment factors at the 5% level. Extreme changes in the baskets of goods, where common $(t,t-1)$ products represent a negligible share of revenue in either t or $t-1$ imply extreme values for $\ln\Lambda^Q_{ft}$. These extreme changes may partly reflect measurement error in an enviroment where baskets of goods are auto-reported into relatively wide product components.

them, and then imposing constant baskets only over consecutive periods (the "divisia" case). Törnqvist indices for the growth of prices of plant f at time t are constructed as $\frac{P_{ft}}{P_{ft-1}} = \prod_{\Omega^f_{t,t-1}} \left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\overline{s}_{fjt}}$.

In the constant baskets of goods version of the Törnqvist index, $\Omega^f_{t,t-1} = \Omega^f$ is a basket of all products ever produced (or materials ever used) by plant f, and $\overline{s}_{fj}$ is the average share of j in that basket of products (or materials) plant f produces over the whole period. In this approach, the plant level index is initialized at $\ln P_{fA} = \sum_{\Omega^f} s_{fj} \left(\ln p_{fjA} - \ln \overline{p_{jA}}\right)$. If product j is not produced (or used as input) in years t or $t-1$ (or both), $\Delta \ln(P_{fjt})$ is inputed at the average growth of the price of that product (or input) for other plants within the sector. If no plant in the sector produces that good in t, then the average over all plants is used, independent of sector.

The divisia version of the Törnqvist index is similar, but $\Omega^f_{t,t-1}$ is the basket of goods produced by f in either t or $t-1$ and $\overline{s}_{fjt}$ is the average share of product j in plant $f's$ sales over t and $t-1$.

Törnqvist prices with a constant basket of products do not quality-adjust prices in any way, for either product turnover of changing quality of surviving products. Compared to this version, all versions allowing for evolving baskets of goods (including divisia) have the advantage of capturing evolving expenditure shares over time and therefore quality-adjusting prices, but the disadvantage of being more biased by errors from product coding and coarse aggregation, which are more likely in our context than in that of prices from scan bar codes (Hottman et al 2016). Compared to our baseline estimation with UPI prices, even versions with changing baskets quality-adjust in a less precise (i.e. not exact) manner, by imposing restrictions to the extent to which appeal many vary.

2 Appendix B: firm problem with Cobb Douglas production function

Firm chooses X_t to solve:

$$\underset{\{X_{ft}\}}{Max} \quad \pi_{ft} = (1-\tau_{ft})\, R_{ft} - C_{ft}X_{ft} = D_{ft}A_{ft}^{1-\frac{1}{\sigma}} X_{ft}^{\gamma\left(1-\frac{1}{\sigma}\right)} - C_{ft}X_{ft}$$

where $R_{ft} = P_{ft}Q_{ft}$ and $P_{ft} = D_{ft}Q_{ft}^{-\frac{1}{\sigma}}$.Optimal input demand is

$$X_{ft} = \left(\frac{(1-\tau_{ft})\, D_{ft} A_{ft}^{1-\frac{1}{\sigma}} \gamma}{\mu_{ft} C_{ft}} \right)^{\frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}} \tag{6}$$

Proof. The firm's problem can be written

$$\underset{X_{it}}{Max} \;\; (1-\tau_{ft})\, D_{ft} Q_{ft}^{1-\frac{1}{\sigma}} - C_{ft} X_{ft}$$

where $D_{ft} = d_{ft} \frac{P_t^{1-\frac{1}{\sigma}}}{E_t^{\frac{1}{\sigma}}}$.

If the firm has market power, then $\frac{\partial P_t}{\partial X_{ft}} \neq 0$. The first order condition for the firm is then given by

$$\begin{aligned}
(1-\tau_{ft}) \left(1 - \frac{1}{\sigma}\right) \frac{d_{ft}}{E_t^{\frac{1}{\sigma}}} (P_t Q_{ft})^{-\frac{1}{\sigma}} \left(P_t + Q_{ft} \frac{\partial P_t}{\partial Q_{ft}} \right) \frac{\partial Q_{ft}}{\partial X_{ft}} &= C_{ft} \\
(1-\tau_{ft}) \left(\frac{\sigma - 1}{\sigma} \right) D_{ft} Q_{ft}^{-\frac{1}{\sigma}} (1 - s_{ft}) \frac{\partial Q_{ft}}{\partial X_{ft}} &= C_{ft} \\
\frac{(1-\tau_{ft})}{\mu_{ft}} D_{ft} Q_{ft}^{-\frac{1}{\sigma}} \left(\gamma A_{ft} X_{ft}^{\gamma - 1} \right) &= C_{ft} \qquad (7) \\
\frac{(1-\tau_{ft})\, D_{ft} A_{ft}^{1-\frac{1}{\sigma}} \gamma}{\mu_{ft} C_{ft}} &= X_{ft}^{1-\gamma\left(1-\frac{1}{\sigma}\right)} \qquad (8)
\end{aligned}$$

Where the second line uses Sheppard's lemma $\left(-\frac{\partial P_t}{\partial Q_{ft}} \frac{Q_{ft}}{P_t} = s_{ft} \right)$, and the third line uses (see Appendix D) $\mu^{-1} = 1 - \left(\frac{1}{\sigma} + \left(\frac{\sigma - 1}{\sigma} \right) s_{ft} \right) = \frac{\sigma - 1 - (\sigma - 1) s_{ft}}{\sigma} = \frac{(\sigma-1)\left(1 - s_{ft}\right)}{\sigma}$. ■

Suppose $X_{ft} = K_{ft}^{\frac{\alpha}{\gamma}} L_{ft}^{\frac{\beta}{\gamma}} M_{ft}^{\frac{\phi}{\gamma}}$ where K, L and M are, respectively, capital, labor and material inputs, and $\gamma = \alpha + \beta + \phi$. Consequently, C_{ft} is itself a Cobb Douglas aggregate of factor prices: $C_{ft} = r_{ft}^{\frac{\alpha}{\gamma}} w_{ft}^{\frac{\beta}{\gamma}} pm_{ft}^{\frac{\phi}{\gamma}}$. Consequently and

$$\frac{X_{ft}}{X_{f0}} = \left(\frac{d_{ft}}{d_{f0}}\right)^{\kappa_1}\left(\frac{a_{ft}}{a_{f0}}\right)^{\kappa_2}\left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\frac{\phi}{\gamma}\kappa_1}\left(\frac{w_{ft}}{w_{f0}}\right)^{-\frac{\beta}{\gamma}\kappa_1}\kappa_t\widehat{\kappa}_{ft} \tag{9}$$

$$\frac{L_{ft}}{L_{f0}} = \left(\frac{d_{ft}}{d_{f0}}\right)^{\kappa_1}\left(\frac{a_{ft}}{a_{f0}}\right)^{\kappa_2}\left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_2}\left(\frac{w_{ft}}{w_{f0}}\right)_t^{-\kappa_1+(\alpha+\phi)\kappa_2}\vartheta_t\vartheta_{ft} \tag{10}$$

$$\frac{Q_{ft}}{Q_{f0}} = \left(\frac{d_{ft}}{d_{f0}}\right)^{\gamma\kappa_1}\left(\frac{a_{ft}}{a_{f0}}\right)^{\kappa_1}\left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_1}\left(\frac{w_{ft}}{w_{f0}}\right)^{-\beta\kappa_1}\chi_t\chi_{ft} \tag{11}$$

where $\kappa_1 = \frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}$; $\kappa_2 = \left(1-\frac{1}{\sigma}\right)\kappa_1$; $\kappa_t = \left(\frac{D_t}{D_0}\right)^{\kappa_1}\left(\frac{A_t}{A_0}\right)^{\kappa_2}\left(\frac{C_t}{C_0}\right)^{-\kappa_1}$ captures aggregate growth between birth and age t, and ;$\widehat{\kappa}_{ft} = \frac{\left(1-\tau_{ft}\right)^{\kappa_1} r_{ft}^{\frac{-\alpha\kappa_1}{\gamma}}}{\left(1-\tau_{f0}\right)^{\kappa_1} r_{f0}^{\frac{-\alpha\kappa_1}{\gamma}}}$ captures residual variation from wedges, and the unobserved user cost of capital. ϑ_t, ϑ_{ft}, χ_t and χ_{ft} are analogous residuals for the specific cases of employment and output. In particular: $\chi_t == \kappa_t^{\gamma}\left(\frac{A_t}{A_0}\right)$ and $\chi_{ft} = \widehat{\kappa}_{ft}^{\gamma}\frac{\alpha_t}{\alpha_0}$. We have used the fact that $1+\gamma\kappa_1\left(1-\frac{1}{\sigma}\right) = \kappa_1$.

Moreover, since $R_{ft} = D_{ft}Q_{ft}^{1-\frac{1}{\sigma}}$ and $1+\gamma\kappa_1\left(1-\frac{1}{\sigma}\right) = \kappa_1$ then

$$\frac{R_{ft}}{R_{f0}} = \left(\frac{d_{ft}}{d_{f0}}\right)^{\kappa_1}\left(\frac{a_{ft}}{a_{f0}}\right)^{\kappa_2}\left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_2}\left(\frac{w_{ft}}{w_{f0}}\right)^{-\beta\kappa_2}\left(\frac{\mu_{ft}}{\mu_{f0}}\right)^{-\gamma\kappa_2}\left(\widehat{\chi}_t\chi_{ft}\right)^{1-\frac{1}{\sigma}}$$

3 Appendix C: Sensitivity to Revenue Curvature

To assess the contribution of $TFPQ\_HK_{ft}$ and *composite* wedges to sales growth, we first calculate $TFPQ\_HK_{ft} = R_{ft}^{1/(1-\frac{1}{\sigma})}/X_{ft}^{\gamma}$ using our estimates of σ, ϕ, β, α, and the implied $X = M_{ft}^{\frac{\phi}{\gamma}}L_{ft}^{\frac{\beta}{\gamma}}K_{ft}^{\frac{\alpha}{\gamma}}$. We call this calculation $TFPQ\_HK_{ft}$ "unconstrained", since we use detailed parameter estimates that would be hard to obtain if one were constrained by the lack of plant-level data on prices. We also build an estimate of $TFPQ\_HK_{ft}$ "constrained" where, following usual practice, we impose monopolistic competition, $\gamma = 1$, ϕ, β, α equal to the corresponding cost shares, and a constant number for σ

Table C1. Decomposition of sales under baseline and constrained fundamentals

	Structural				
	(1)	(2)	(3)	(4)	(5)
TFPQ_HK unconstrained	1,280				
TFPQ_HK constrained			1,116	1,161	2,250
TFPQ		0,154			
Demand shock		1,128			
ln Input prices		-0,078			
ln Average wage		-0,081			
ln Markup		-0,006			
Sales Wedge	-0,280	-0,117	-0,116	-0,161	-1,250
Avg Rev Curvature	**0,641**	**0,641**	**0,641**	**0,666**	**0,877**
Max Rev Curvature	**0,877**	**0,877**	**0,641**	**0,666**	**0,877**
	Reduced				
	(1)	(2)	(3)	(4)	(5)
TFPQ_HK unconstrained	0,574				
TFPQ_HK constrained			0,703	0,684	0,412
TFPQ		0,050			
Demand shock		0,476			
ln Input prices		-0,007			
ln Average wage		0,036			
ln Markup		0,107			
Sales Wedge	0,426	0,322	0,297	0,316	0,588
Avg Rev Curvature	**0,641**	**0,641**	**0,641**	**0,666**	**0,877**
Max Rev Curvature	**0,877**	**0,877**	**0,641**	**0,666**	**0,877**

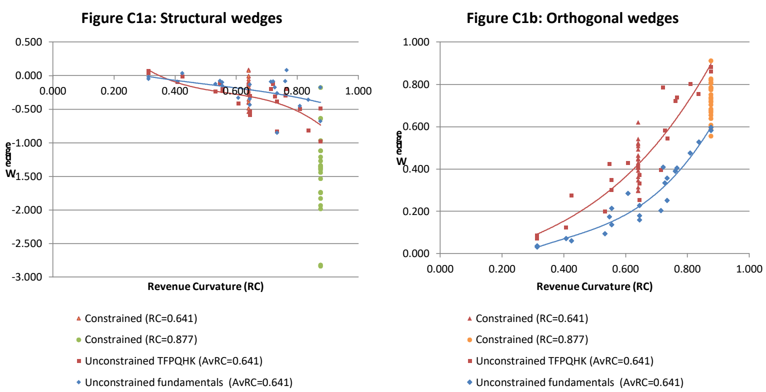
TFPQ_HK is a function of TFPQ, demand shocks, and the elasticity of substitution. The unconstrained version uses the factor and substitution elasticities estimated using P and Q data, reported in Table 1. The constrained version uses cost shares as factor elasticities consistent with CRS in production and a demand elasticity consistent with the curvature of the revenue function in the reported column.

(as in the macro misallocation literature).[3] While in the unconstrained case M is the materials quantitities index built deflating with our UPI plant-level deflators for materials, in the constrained one it is materials expenditure deflated with the PPI.

Figure C1 depicts the contribution of wedges vs. returns to scale in revenue by sector, for different columns of Table C1. It shows that the estimated contribution of wedges to sales is higher when the revenue curvature parameter $\gamma\left(1-\frac{1}{\sigma}\right)$ is high (i.e. curvature is low), and that the increase is nonlinear: in sectors when $\gamma\left(1-\frac{1}{\sigma}\right)$ is close to 1, wedges tend to dominate the contribution of fundamentals.

[3] Under these assumptions, $\mu_{it}=\mu=\frac{\sigma}{\sigma-1}$. Since cost minimization implies $\beta=\frac{w_{ft}L_{ft}}{Cost}\gamma=\frac{w_{ft}L_{ft}}{R_{ft}}\mu$ (Hall, 1994), we impose for each sector $\beta=\frac{\sum_f w_{ft}L_{ft}}{\sum_f R_{ft}}\frac{\sigma}{\sigma-1}$, calculating β first for each year and then averaging over years. We proceed similarly for ϕ, and then obtain $\alpha=1-\beta-\phi$.

Figure C1: Wedges vs. Revenue Curvature (by 3-digit sector)



4 Appendix D: markups

The firm's (potentially variable) markup after the distortion, $\mu_{ft} = \frac{P_{ft}}{mc_{ft}\left(1-\tau_{ft}\right)^{-1}}$, is given by:

$$\mu_{ft} = \frac{1}{1-\left(\frac{1}{\sigma}-\left(\frac{\sigma-1}{\sigma}\right)s_{ft}\right)} = \frac{\sigma}{\left(\sigma-1\right)\left(1-s_{ft}\right)} \tag{12}$$

Proof that:

$$\mu_{ft} = \frac{1}{1-\left(\frac{1}{\sigma}-\left(\frac{\sigma-1}{\sigma}\right)s_{ft}\right)} = \frac{\sigma}{\left(\sigma-1\right)\left(1-s_{ft}\right)} \tag{13}$$

Proof. $\underset{Q_{ft}}{Max}\left(1-\tau_{ft}\right)P_{ft}Q_{ft} - CT$ leads to first orden condition $\left(P_{ft}+Q_{ft}\frac{dP_{ft}}{dQ_{ft}}\right) = \frac{mc_{ft}}{\left(1-\tau_{ft}\right)}$. Dividing by P_{ft} we obtain $\frac{1}{\mu_{ft}} = 1+\frac{Q_{ft}}{P_{ft}}\frac{dP_{ft}}{dQ_{ft}} = 1-\epsilon^{-1}$ (where we have denoted $\epsilon_{ft} \equiv -\frac{Q_{ft}}{P_{ft}}\frac{dP_{ft}}{dQ_{ft}}$), so that

$$\mu_{ft} = \left(\frac{\epsilon_{ft}}{\epsilon_{ft}-1}\right) \tag{14}$$

In turn, under $Q_{ft} = d_{ft}^{\sigma} P_{ft}^{-\sigma} \frac{E_t}{P_t^{1-\sigma}}$ and its implication that $P_{ft} = d_{ft} Q_{ft}^{-\frac{1}{\sigma}} \left(\frac{E_t}{P_t^{1-\sigma}}\right)^{\frac{1}{\sigma}} = d_{ft} Q_{ft}^{-\frac{1}{\sigma}} \left(\frac{Q_t}{P_t^{-\sigma}}\right)^{\frac{1}{\sigma}}$ and allowing for market power so that $\frac{dP_t}{dQ_{ft}} \neq 0$, the inverse of the demand elasticity as perceived by the firm ($\epsilon_{ft}^{-1} \equiv -\frac{dP_{ft}}{dQ_{ft}} \frac{Q_{ft}}{P_{ft}}$) is:

$$\begin{aligned}
\epsilon_{ft}^{-1} &= -\left(\frac{\partial P_{ft}}{\partial Q_{ft}} + \frac{\partial P_{ft}}{\partial P_t} \frac{\partial P_t}{\partial Q_{ft}}\right) \frac{Q_{ft}}{P_{ft}} &(15)\\
&= -\left(-\frac{1}{\sigma} \frac{P_{ft}}{Q_{ft}} + \left(\frac{\sigma-1}{\sigma}\right) \frac{P_{ft}}{P_t} \frac{\partial P_t}{\partial Q_{ft}}\right) \frac{Q_{ft}}{P_{ft}} \\
&= \left(\frac{1}{\sigma} - \left(\frac{\sigma-1}{\sigma}\right) \frac{\partial P_t}{\partial Q_{ft}} \frac{Q_{ft}}{P_t}\right) \\
&= \left(\frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right) s_{ft}\right) &(16)
\end{aligned}$$

where the last line uses Sheppard's lemma: $-\frac{\partial P_t}{\partial Q_{ft}} \frac{Q_{ft}}{P_t} = s_{ft}$.

Equations (14) and (16) together imply $\mu_{ft}^{-1} = 1 - \epsilon_{ft}^{-1} = 1 - \left(\frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right) s_{ft}\right) = \left(\frac{\sigma-1}{\sigma} - \left(\frac{\sigma-1}{\sigma}\right) s_{ft}\right)$, so that

$$\begin{aligned}
\mu_{ft} &= \frac{1}{1 - \left(\frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right) s_{ft}\right)} \\
\mu &= \frac{\sigma}{\sigma - 1} \text{ if } s_{ft} = 0
\end{aligned}$$

■

The markup $\mu_{ft} = \frac{\sigma}{(\sigma-1)\left(1-s_{ft}\right)}$ is increasing in the firm's market share. Thus, the markup is itself a fuction of fundamentals:

$$\begin{aligned}
s_{ft} &= \frac{P_{ft} Q_{ft}}{E_t} = \frac{D_{ft} Q_{ft}^{1-\frac{1}{\sigma}}}{E_t} = \frac{D_{ft} A_{ft}^{1-\frac{1}{\sigma}} X_{ft}^{\gamma\left(1-\frac{1}{\sigma}\right)}}{E_t} &(17)\\
&= \frac{D_{ft} A_{ft}^{1-\frac{1}{\sigma}}}{E_t} \left(\frac{\gamma\left(1-\tau_{ft}\right)\left(1-\frac{1}{\sigma}\right) D_{ft} A_{ft}^{1-\frac{1}{\sigma}}}{C_{ft} \mu_{ft} \left(\frac{\sigma-1}{\sigma}\right)}\right)^{\frac{\gamma\left(1-\frac{1}{\sigma}\right)}{1-\gamma\left(1-\frac{1}{\sigma}\right)}} &(18)
\end{aligned}$$

so that

$$s_{ft}\left(\frac{\sigma-(\sigma-1)s_{ft}}{\sigma-(\sigma-1)s_{ft}-1}\right)^{\frac{\gamma\left(1-\frac{1}{\sigma}\right)}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}=\frac{D_{ft}^{\frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}A_{ft}^{\frac{1-\frac{1}{\sigma}}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}}{E_t}\left(\frac{\gamma\left(1-\tau_{ft}\right)\left(1-\frac{1}{\sigma}\right)}{C_{ft}\left(\frac{\sigma-1}{\sigma}\right)}\right)^{\frac{\gamma\left(1-\frac{1}{\sigma}\right)}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}$$

The LHS is increasing in s and the RHS is increasing in D and A, and decreasing in τ and C. Thus, s_{ft} and the markup are increasing in D and A, and decreasing in τ and C.

5 Appendix E: Persistence in Fundamentals and Endogenous Innovation

We have taken fundamentals as given, but noted that our results should help guide future work, both theoretical and empirical, about the specific drivers of measured productivity. To further understand the nature of $TFPQ$ vs. demand shock, and potential mechanisms through which businesses accumulate each of them, we have studied the relationship between these fundamentals and reported innovation efforts. The Colombian Manufacturing Survey can be merged with the Innovation Survey at the level of the firm (tax ID). Since 2006 firms report number of innovations by type, defined by categories named "product", "process", and "organizational" innovation. They also report innovation expenditures, unfortunately not broken down in the same categories.

Results from our structural decomposition of growth show that, given fundamentals, high-fundamentals plants are being implicitly taxed while low-fundamentals plants are implicitly subsidized (by the environment, not necessarily by the government). Causality in the opposite direction is also likely: technical efficiency and product-plant appeal, while partly determined by exogenous stochastic dynamics (as in, e.g., Hopenhayn (1992) and Hopenhayn and Rogerson (1993)), partly also result from endogenous investments to improve performance (as in Acemoglu et. al., 2017, or Aw, Roberts and Xu, 2011). In the latter class of models, firms invest in future fundamentals (e.g. via R&D expenditure) to the extent that they expect high returns from such investments. High fundamentals plants should, therefore, invest more in a context with persistence in fundamentals. Since wedges make future profitability less dependent in fundamentals, they should reduce the incentive to invest given by high fundamentals, especially if wedges are negatively correlated with fundamentals (e.g. HK, 2014). Wedges may also have a direct effect on investment if, for instance, the presence of fixed costs of production implies that a subsidy directly increases the chances of surviving to enjoy the returns from R&D.

Table E1 presents an OLS analysis of the persistence in wedges, and the role of lagged wedges for the evolution of sales, output, $TFPQ$ and demand.[4] Wedges are standardized to facilitate interpretation. Both structural wedges (upper panel) and reduced form wedges exhibit considerable positive persistence, though less so in the case of structural wedges. This is consistent with structural wedges in part reflecting non-convex adjustment costs. Such cost generate a wedge that is correlated with fundamentals and that only persists up to the moment in which the benefit of adjusting overcomes its fixed cost.

As in models of endogenous fundamentals, contemporaneous fundamentals and wedges correlate with higher *lagged* wedges (higher implicit lagged subsidies), even after controlling for persistence in fundamentals. The correlation with lagged structural wedges is larger than that with reduced form wedges but neither accounts for much variation in outcomes and fundamentals. For example, a one standard deviation increase in lagged structural wedges yields a 0.06 increase in $TFPQ$ and a 0.01 increase in demand. These are small effects relative to the standard deviations of $TFPQ$ and demand reported in Table 2 (0.84 and 0.67, respectively). [5] In turn, as hypothesized, the interaction effect between the lagged dependent variable and lagged structural wedges (negatively correlated with lagged fundamentals, as seen above) is negative. That is, while higher lagged structural wedges boost outcomes and fundamentals, they correlate with reduced persistence in outcomes and fundamentals. But, the interacted effects are also small.

Even though we find modest effects of lagged wedges on current fundamentals, this should not be interpreted as evidence against the endogenous evolution of fundamentals via endogenous investment in innovation. For a limited sample of the firms with positive R&D expenditures, Table E2 presents evidence of the determinants of R&D expenditures.[6] Lagged funda-

[4]As background, the standard deviation of reduced-form (uncorrelated) wedges lies in the same ball-park as the standard deviation of $TFPQ$ and demand (Table 2), while that of structural wedges doubles that (all in log points).

[5]Lagged wedges also have modest impact on current output and sales.

[6]R&D expenditures are asked to manufacturing firms in a survey paralell to the Annual Manufacturing Survey, that can be linked to the AMS via firm identifiers internal to the

Table E1. Wedge and Fundamental persistence

VARIABLES	(1) Sales Wedge (subsidy)	(2) Output	(3) Sales	(4) TFPQ	(5) Demand shock
	Structural wedge (both orthogonal and correlated sources)				
Lagged Dependent Variable		0.981***	0.984***	0.928***	0.966***
		(0.0008)	(0.0008)	(0.0014)	(0.0010)
Lagged Wedge (subsidy, standarized)	0.754***	0.0264***	0.0289***	0.0603***	0.0081***
	(0.0020)	(0.0015)	(0.0013)	(0.0014)	(0.0007)
Lagged Wedge (subsidy, standarized)*Lagged DV		-0.0043***	-0.0052***	-0.00185**	-0.0112***
		(0.0008)	(-0.0007)	(-0.0008)	(0.0011)
Constant	0.0250***	-0.0074***	-0.0163***	-0.0103***	-0.0094***
	(0.0017)	(0.0013)	(0.0011)	(0.0011)	(0.0006)
Observations	114,231	114,231	114,231	114,231	114,231
R-squared	0.570	0.928	0.932	0.799	0.899
Sector*Time FE	Yes	Yes	Yes	Yes	Yes
	(1) Sales Wedge (subsidy)	(2) Output	(3) Sales	(4) TFPQ	(5) Demand shock
	Reduced-form wedge (orthogonal to fundamentals)				
Lagged Dependent Variable		0.971***	0.973***	0.904***	0.964***
		(0.0010)	(0.0010)	(0.0014)	(0.0010)
Lagged Wedge (subsidy, standarized)	0.933***	0.0219***	0.0237***	0.0202***	0.0111***
	(0.0012)	(0.0018)	(0.0015)	(0.0011)	(0.0007)
Lagged Wedge (subsidy, standarized)*Lagged DV		0.00202***	0.000925	-0.0094***	-0.0111***
		(0.0006)	(0.0006)	(0.0011)	(0.0009)
Constant	0.0032***	-0.0075***	-0.0148***	-0.0081***	-0.0088***
	(0.0011)	(0.0014)	(0.0012)	(0.0011)	(0.0006)
Observations	114,231	114,231	114,231	114,231	114,231
R-squared	0.848	0.928	0.932	0.796	0.899
Sector*Time FE	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table E2. Innovation vs. Fundamentals

	(1)	(2)	(3)
VARIABLES	R&D spendings (logs, standarized)		
Lagged TFPQ (demeaned)	0.254***	0.314***	0.226***
	(0.00838)	(0.00892)	(0.00772)
Lagged Demand shock (demeaned)	0.682***	0.738***	0.666***
	(0.0104)	(0.0107)	(0.00946)
Lagged Wedge (subsidy, structural, standarized)		0.139***	
		(0.00894)	
Lagged Wedge (subsidy, structural, standarized)* Lagged TFPQ		-0.00115	
		(0.00434)	
Lagged Wedge (subsidy, structural, standarized)* Lagged Demand shock		0.0504***	
		(0.0113)	
Lagged Wedge (subsidy, reduced, standarized)			0.331***
			(0.00629)
Lagged Wedge (subsidy, reduced, standarized)* Lagged TFPQ			-0.00673
			(0.00561)
Lagged Wedge (subsidy, reduced, standarized)* Lagged Demand shock			-0.00460
			(0.00801)
Constant	-0.199***	-0.210***	-0.289***
	(0.00698)	(0.00712)	(0.00652)
Observations	16.547	16.539	16.539
R-squared	0.268	0.284	0.399
Sector*Time FE	Yes	Yes	Yes

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

mentals and lagged wedges (subsidies) increase current R&D expenditures. Lagged fundamentals account for much more of the variation in observed R&D expenditures than lagged wedges. These endogenous innovation findings are broadly consistent with the literature (see, e.g., Aw, Roberts and Xu (2011)) and help provide further perspective on the strong persistence in fundamentals.

6 Appendix F: details for the joint estimation of production and demand functions

As in proxy methods for the estimation of the production function, the joint estimation of production and demand is preceded by a first stage that ensures that $TFPQ$ can be proxied by an observable factor, in this case materials,

two databases. Most firms report zero R&D expenditures. As is well known in the R&D literature, this may reflect measurement limitations of expenditures in R&D. Thus, Table E1 is only suggestive. In unreported results, we have estimated Table E1 using a Tobit approach treating the zeros as reflecting true zero R&D expenditures. We obtain similar qualitative results.

which is conditionally monotonic in $TFPQ$. The free input M_{ft} is a function of $TFPQ_{ft}$, conditional on quasi-fixed inputs. The FOC for materials is

$$\begin{aligned} M_{ft} &= \frac{\phi(1-\tau_{ft})R_{ft}}{pm_{ft}}(1-1/\sigma) \\ &= \frac{\phi(1-\tau_{ft})P_{ft}Q_{ft}}{pm_{ft}}(1-1/\sigma) \\ M_{ft}^{1-\phi} &= \frac{P_{ft}A_{ft}K_{ft}^{\alpha}L_{ft}^{\beta}(1-\tau_{ft})(\phi\frac{\sigma-1}{\sigma})}{pm_{ft}} \end{aligned}$$

Within a sector, ϕ and σ display no variability. We have measures for all the variable terms in this FOC, except for τ. Since all firm choices (L_{ft}, K_{ft}, P_{ft}, and Q_{ft}) are themselves functions of τ, we condition on a flexible polinomial on s_{ft} rather than τ_{ft}. Furthermore, $P_{ft} = \overline{P_{ft}^{*}}P_{fB}\left(\Lambda_{ft}^{Q}\right)^{\frac{1}{\sigma-1}}$ and similarly $pm_{ft} = \overline{PM_{ft}^{*}}PM_{fB}\left(\Lambda_{ft}^{M}\right)^{\frac{1}{\sigma-1}}$We thus re-write

$$\ln M_{ft} = h\left(\ln A_{ft}, \ln K_{ft}, \ln L_{ft}, \ln\frac{\overline{P_{ft}^{*}}P_{fB}}{\overline{PM_{ft}^{*}}PM_{fB}}, \ln\Lambda_{ft}^{Q}, \ln\Lambda_{ft}^{m},\ \ln s_{ft}\right)$$

so that

$\ln A_{ft} \ = \ h^{-1}\left(\ln M_{ft}, \ln K_{ft}, \ln L_{ft}, \ln\frac{\overline{P_{ft}^{*}}P_{fB}}{\overline{PM_{ft}^{*}}PM_{fB}}, \ln\Lambda_{ft}^{Q}, \ln\Lambda_{ft}^{m}, \ln s_{ft}\right) \equiv h^{-1}\left(\overrightarrow{W}\right).$

Incorporating this expression, recognizing that Q_{ft} is subject to measurement error and other shocks not observed by either the econometrician or the firm at the time of making input choices, and denoting by $\widehat{Q_{ft}} = Q_{ft}\varepsilon_{ft}$ measured Q_{ft}, we write:

$$\begin{aligned} \widehat{Q_{ft}} &= \alpha\ln K_{ft} + \beta\ln L_{ft} + \phi\ln M_{ft} + h^{-1}\left(\overrightarrow{W}\right) + \varepsilon_{ft} \\ &\quad \text{so that} \\ \widehat{Q_{ft}^{*}} &= \alpha\ln K_{ft} + \beta\ln L_{ft} + \phi\ln M_{ft} - \frac{1}{\sigma-1}\ln\Lambda_{ft}^{Q} + \frac{\phi}{\sigma-1}\ln\Lambda_{ft}^{M} \quad (19) \\ &\quad + h^{-1}\left(\overrightarrow{W}\right) + \varepsilon_{ft} \end{aligned}$$

where ε_{ft} is measurement error, and the "$*$" refers to the fact that we are estimating the transformed $Q^*_{ft} = \frac{R_{ft}}{P^*_{ft}}$ rather than $Q_{ft} = \frac{R_{ft}}{P_{ft}}$.

In the first stage we proxy productivity and eliminate measurement error by estimating 19 through a flexible third-degree polinomial $\varphi^*\left(\overrightarrow{W}\right)$estimated via OLS and obtaining the predicted $\widehat{\varphi}^*\left(\overrightarrow{W}\right)$.

We then estimate the system of demand and production functions replacing $\ln Q^*_{ft}$ with $\varphi^*\left(\overrightarrow{W}\right)$ in the production function. We use GMM methods and rely on the moment conditions presented in the main text for identification. For sectors where the estimated returns to scale in revenue exceed 0.9, we re-estimate imposing this bound for the curvature of revenue, in the context of a positive elasticity of substitution.

Our estimates of production coefficients are initialized at the respective OLS estimates of the production function augmented with Λ^Q_{ft} and Λ^M_{ft} regressors (coefficients for Λ^Q_{ft} and Λ^M_{ft} also freely estimated by OLS). Our σ estimate is initialized through an IV estimation of demand function, where the instrument for Q is the residual from the OLS production function. The IV procedure follows the spirit of Foster et al (2008), though only for initialization.

7 Appendix G: Variance decomposition

This appendix explains the structural and reduced form variance decompositions presented in Figures 5 and 6. We follow a two stage procedure, similar to that in Hottman et al. (2016).

7.1 Structural decomposition

The structural decomposition for sales is guided by:

$$\frac{R_{ft}}{R_{f0}} = \left(\frac{d_{ft}}{d_{f0}}\right)^{\kappa_1} \left(\frac{a_{ft}}{a_{f0}}\right)^{\kappa_2} \left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_2} \left(\frac{w_{ft}}{w_{f0}}\right)^{-\beta\kappa_2} \left(\frac{\mu_{ft}}{\mu_{f0}}\right)^{-\gamma\kappa_2} \left(\widehat{\chi}_t \chi_{ft}\right)^{1-\frac{1}{\sigma}}$$

Results for this decomposition are reported in Figure 3 of the main text and reproduced in the top left panel of Figure G1. We conduct an analogous decomposition for output, following the corresponding equation in the main text, and report its results in the bottom left panel of Figure G1.

1. Guided by the above equation, we obtain $\ln \chi_{ft}$ as a residual from the following equation:

$$\begin{aligned} \ln \frac{R_{ft}}{R_{f0}} &= \beta_D \ln\left(\frac{d_{ft}}{d_{f0}}\right) + \beta_A \ln\left(\frac{a_{ft}}{a_{f0}}\right) + \beta_\mu \ln \frac{\mu_{ft}}{\mu_{f0}} \\ &+\beta_M \ln\left(\frac{pm_{ft}}{pm_{f0}}\right) + \beta_w \ln\left(\frac{w_{ft}}{w_{f0}}\right) + \ln\left(\chi_{ft}\right)^{\left(1-\frac{1}{\sigma}\right)} \end{aligned} \tag{20}$$

where $\beta_D = \kappa_1$; $\beta_A = \kappa_2$; $\beta_\mu = -\gamma\kappa_2$; $\beta_M = -\phi\kappa_2$; $\beta_w = -\beta\kappa_2$; $\kappa_1 = \frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}$; $\kappa_2 = \left(1-\frac{1}{\sigma}\right)\kappa_1$. We calculate these parameters using our estimates of factor elasticities in technology and the elasticity of substitution. Because we use these parameters that stem from the structure of the model, we label the residual as a "structural" wedge. The fundamentals d_{ft}, a_{ft}, pm_{ft} and w_{ft} correspond to the idiosyncratic components of demand, technology and input price shocks, estimated as already described ($D_{ft} = D_t d_{ft}$ and so on).

2. We then estimate the following equations:

$$\begin{aligned} \beta_D \ln\left(\frac{d_{ft}}{d_{f0}}\right) &= \rho_{0,D} + \rho_D \ln \frac{R_{ft}}{R_{f0}} + \nu_{ft,D} \\ \beta_A \ln\left(\frac{a_{ft}}{a_{f0}}\right) &= \rho_{0,A} + \rho_A \ln \frac{R_{ft}}{R_{f0}} + \nu_{ft,A} \\ \beta_\mu \ln\left(\frac{g\left(s_{ft}\right)}{g\left(s_{f0}\right)}\right) &= \rho_{0,\mu} + \rho_\mu \ln \frac{R_{ft}}{R_{f0}} + \nu_{ft,A} \\ \beta_M \ln\left(\frac{pm_{ft}}{pm_{f0}}\right) &= \rho_{0,M} + \rho_M \ln \frac{R_{ft}}{R_{f0}} + \nu_{ft,M} \\ \beta_w \ln\left(\frac{w_{ft}}{w_{f0}}\right) &= \rho_{0,w} + \rho_w \ln \frac{R_{ft}}{R_{f0}} + \nu_{ft,w} \\ \ln \widehat{\chi_{ft}} &= \rho_{0,\upsilon} + \rho_\upsilon \ln \frac{R_{ft}}{R_{f0}} + \nu_{ft,\upsilon} \end{aligned} \tag{21}$$

We now prove that the contribution of each fundamental to the variance of sales equals the ratio of its covariance with sales to the variance of sales multiplited by its structural parameter in equation 20. Also that, by the properties of OLS, the contributionof the different factors considered add up to 1. We conduct the proof for the two-covariance case for simplicity

For any given log-linear equation (such as 20):

$$Y_f = \beta_1 X_{1f} + \beta_2 X_{2f} + \varepsilon_i \tag{22}$$

If one estimates by OLS The set of equations

$$\beta_1 X_{1f} = \gamma_{1,0} + \gamma_1 Y_f + \nu_{1i} \tag{23}$$

$$\beta_2 X_{2f} = \gamma_{1,0} + \gamma_2 Y_f + \nu_{2i} \tag{24}$$

and

$$\varepsilon_f = \gamma_{\varepsilon,0} + \gamma_\varepsilon Y_f + \nu_{\varepsilon f} \tag{25}$$

The estimated parameters for $j = \{1, 2\}$ are:

$$\begin{aligned}\hat{\gamma}_j &= \frac{Cov(\beta_j X_{jf}, Y_f)}{Var(Y_f)} = \beta_j \frac{Cov(X_{jf}, Y_f)}{Var\left(Y_f\right)} \\ &= \beta_j Corr(X_{ji}, Y_f) \left(\frac{Var(X_{jf})}{Var\left(Y_f\right)}\right)^{\frac{1}{2}}\end{aligned}$$

Since $\varepsilon_f = Y_f - (\beta_1 X_{1f} + \beta_2 X_{2f})$, $\hat{\gamma}_\varepsilon$ can be re-written as:

$$\begin{aligned}\hat{\gamma}_\varepsilon &= \frac{Cov(Y_f - (\beta_1 X_{1f} + \beta_2 X_{2f}), Y_f)}{\sigma_Y^2} \\ &= \frac{Var\left(Y_f\right) - \beta_1 Cov(X_{1f}, Y_f) - \beta_2 Cov(X_{2f}, Y_f)}{Var\left(Y_f\right)} = 1 - \hat{\gamma}_1 - \hat{\gamma}_2\end{aligned}$$

7.2 Reduced form decomposition

The reduced form decomposition follows a procedure analogous to the one just described, but where the first stage estimates an OLS coefficient for each fundamental rather than imposing the coefficients imposed by our structure. In particular, the first stage estimates by OLS

$$\ln\frac{R_{ft}}{R_{f0}} = \beta_D^r \ln\left(\frac{d_{ft}}{d_{f0}}\right) + \beta_A^r \ln\left(\frac{a_{ft}}{a_{f0}}\right) + \beta_\mu^r \ln\frac{\mu_{ft}}{\mu_{f0}} + \beta_M^r \ln\left(\frac{pm_{ft}}{pm_{f0}}\right) + \beta_w^r \ln\left(\frac{w_{ft}}{w_{f0}}\right) + \ln\left(\chi_{ft}\right)^{\left(1-\frac{1}{\sigma}\right)} \tag{26}$$

where the "r" index in each coefficient stands for "reduced form". Once OLS estimates of each of these coefficients are obtained, the second stage is implemented as in the structural decomposition, replacing each β_x with β_x^r, where x stands for any fundamental. Results of this decomposition are reported in Figure G1, right panels.

7.3 Decomposition by ages

To conduct the decomposition by ages, we expand equations 20 and 21 to include interactions with the different age groups. Suppose there are two mutualy exclusive groups: B and C. We redefine the equation 20 as:

$$Y_f = \beta_1 X_{1f} + \beta_2 X_{2f} + \varepsilon_i \tag{27}$$

$$\ln\frac{Q_{ft}}{Q_{f0}} = \beta_{1,C} X_{1f} d_{Cf} + \beta_{1,B} X_{1f} d_{Bf} \tag{28}$$

$$+\beta_{2,C} X_{2f} d_{Cf} + \beta_{2,B} X_{2f} d_{Bf} + \varepsilon_i \tag{29}$$

where $d_{Cf} = 1$ if f belongs to group C (say, an age), and everything else as defined previously.

The new decomposition equation for, say, X_1 will be given by:

$$\beta_{1,C} X_{1f} d_{Cf} + \beta_{1,B} X_{1f} d_{Bf} = \gamma_{C1} Y_f d_{Cf} + \gamma_{B1} Y_f d_{Bf} + \nu_{1f} \tag{30}$$

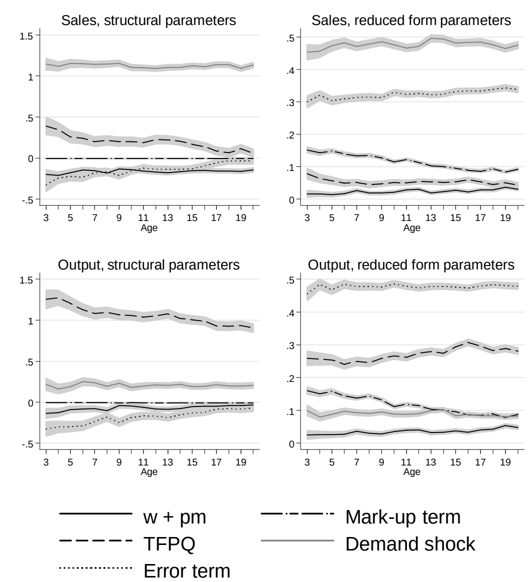
$$\varepsilon_f = \gamma_{C\varepsilon} Y_f d_{Cf} + \gamma_{B\varepsilon} Y_f d_{Bf} + \nu_{\varepsilon f} \tag{31}$$

Just as before $\hat{\gamma_{C1}} + \hat{\gamma_{C\varepsilon}} = \hat{\gamma_{B1}} + \hat{\gamma_{B\varepsilon}} = 1$.

8 Appendix H: Selection

By construction we focus on survivor growth: growth from birth to age a of plants that have survived to age a. However, because we are able to follow

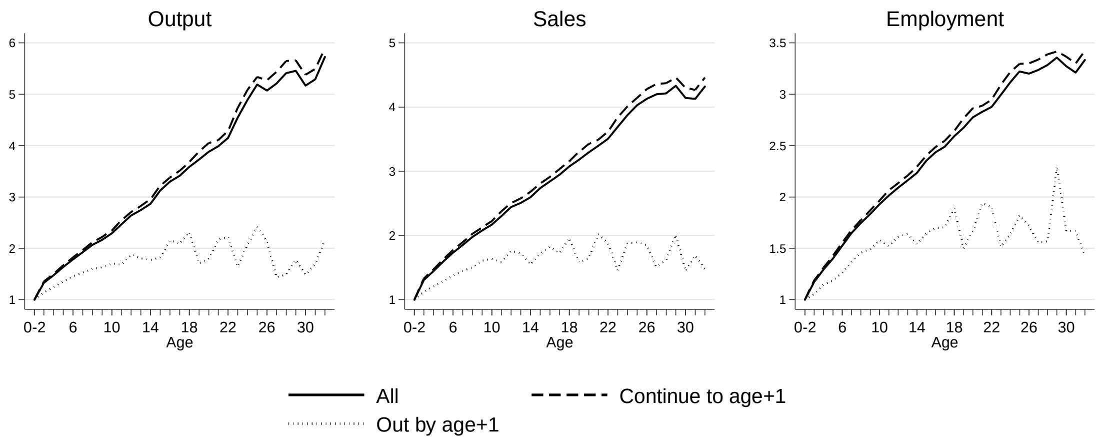
Figure G1: Life-cycle growth variance decomposition by age



Within sector*year.

Figure H1: Life Cycle Growth

Current to initial

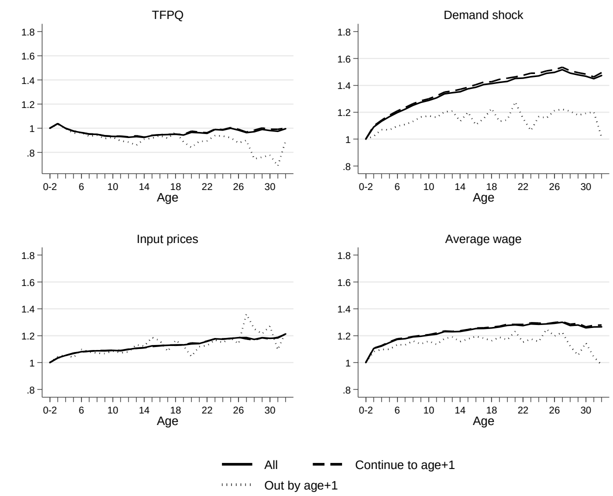


Within sector*year.

life cycle growth directly at the plant level–by contrast to cross sectional comparisons of cohorts–the usual concern that selection drives average growth because size at the initial age is biased downwards by exits-to-be does not apply. We compare plant i's size at age a to i's own birth size. It is the case, still, that plants that eventually exit may grow slower than others before they exit and, in that sense, even true life-cycle average growth is affected by selection: if the exiting plant had instead continued to the following age, average growth would have been lower. Figure H1 illustrates that this is indeed the case, since the life-cycle growth of plants that exit in the next period does depart significantly, downwards, from that of continuers. But, this growth of plants that exit only affects marginally the overall average. That is, the average patterns described in the previous paragraph are mainly driven by continuous plants (plants of age t that continue on to age $t+1$). Still, in this section, we also explore how fundamentals relate to selection vs. continuer growth.

Figure H2 illustrates average growth of fundamentals separately for plants that continue for at least one additional year and those that exit the following year. The most noteworthy difference is much poorer growth in demand shocks for plants about to exit compared to those that will continue, suggestive of demand side fundamentals being particularly important determinants of exit.

Figure H2: Life cycle growth of fundamentals: exiters vs. continuers



Within sector*year.

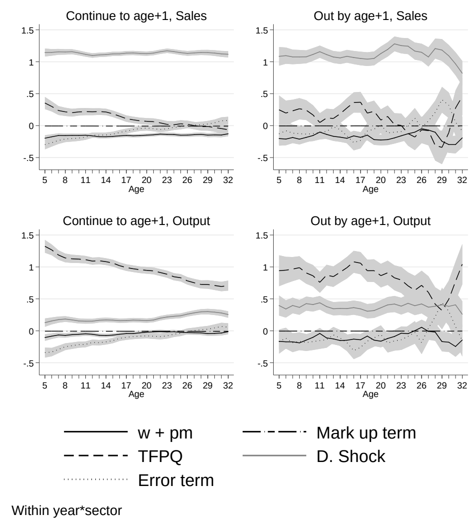
Figure H3 further carries our decomposition of drivers of output growth for these two groups of plants. Because exits are more likely at earlier ages and we know the contribution of fundamentals varies across ages, we compute the decomposition for each group at specific ages. We present three-year moving averages because the patterns for plants about to exit are noisy.[7] Fundamentals still play an important role for exiters in explaining their growth from birth to the moment in which they are about to exit. Despite demand shocks being the dimension where most marked differences are observed between exits-to-be and continuers, especially for young ages (Figure H2), $TFPQ$ tends to play a more significant (at least more sustained) role in explaining growth up to age t for plants about to exit compared to continuers, simply capturing the extremely poor $TFPQ$ behavior of exits-to-be.

8.1 Appendix I: The value of Quality Adjustment

UPI plant price indices adjust real output for intra-firm quality/appeal differences. Moreover, in the context of UPI prices, sales measure output that is additionally adjusted for cross-plant quality differences. We now compare re-

[7]Since each point (age) in a figure for plants about to exit contains the plants that will exit at age+1, the plants included in a given line are different for each age. This explains the noisy patterns.

Figure H3: Life cycle growth decomposition by age
Continuers vs. Exiters, 3 years moving average - Structural parameters

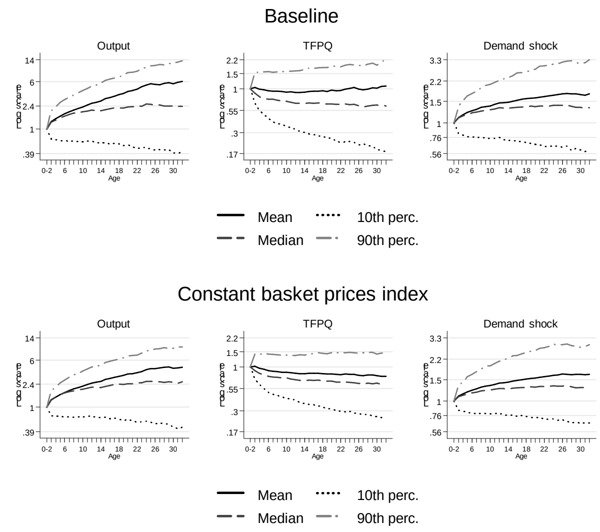


sults to what would be obtained under two alternatives to price measurement. First, we implement a “statistical” approach based on Törnqvist indices for a constant basket of goods within the plant or, alternatively, on the divisia price index that allows that basket to change and uses average t, $t-1$ expenditure shares. We implement a second alternative approach, using prices based on the insights offered by Sato (1976), Vartia (1976) and Feenstra (1994). The Sato-Vartia approach is economically motivated but keeps appeal shifters and baskets of goods constant over two consecutive periods, implying a much slower quality adjustment for both continuing products and those that enter and exit. The Feenstra adjustment for changing varieties incorporated into our UPI approach can also be added to the Sato-Vartia index to adjust for changing baskets of goods over consecutive periods (it was, in fact, originally implemented by Feenstra, 2004, within the Sato-Vartia approach). The UPI, meanwhile, allows for both changing baskets of goods and varying appeal shifters, dimensions of flexibility which respectively deal with the "consumer valuation bias" and the "variety bias" (Redding and Weinstein, 2020). (For a detailed discussion of each of these alternatives, contrasted with the UPI, see Appendix A, and Redding and Weinstein, 2020).

In each approach, the aggregation from the plant to the sector level is analogous to the aggregation from the product to the plant level, using weights and shares that correspond to the basket of plants in aggregate expen-

Figure I1: Distribution of output and fundamentals life-cycle growth

Alternative price indices



Within sector*year

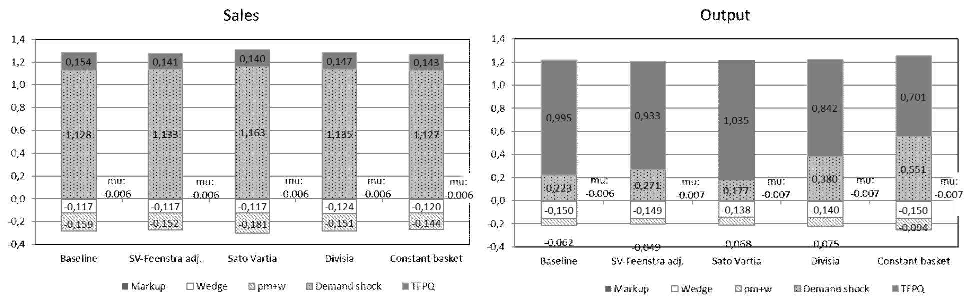
diture by contrast to the basket of products in plants' sales. For theory-based indices this is directly implied by theory. For statistical indices we impose it for consistency.

If the quality mix within the plant improves over time, plant-level quality adjusted price indices will grow less than unadjusted ones, as a result yielding less deflated output growth and less $TFPQ$ growth. This composes with overall plant quality growth to imply economically motivated aggregate prices that grow less than unadjusted ones. Not properly adjusting plant-level prices for quality changes biases estimated idiosyncratic output and technical efficiency growth downwards because such estimates will ignore the part of price increases that reflects increasing valuation of goods and the services of plants to their costumers, and thus mistakenly translate those price increases into welfare decreases for given expenditure. Figure I1 shows that output and $TFPQ$ growth is dampened when revenue is deflated with price indices that do not adjust for quality.

Figure I2 displays growth decomposition using alternative price indices.

Figure I2: Life-cycle growth variance decomposition

Structural parameters, alternative Price indices



Adjusting output for quality changes assigns a much larger weight to technical efficiency, $TFPQ$, and a lesser role to demand, in explaining output life cycle growth (see Appendix I for detailed results). While with constant-weights-Törnqvist-indices $TFPQ$ and demand are estimated to contribute roughly equally to output growth, $TFPQ$ is assigned progressively more relative importance as one moves to the Sato-Vartia and then to the UPI approaches. But quality adjusting prices matters much more in decomposing output than for sales because, beyond the more precise measurement of fundamentals when quality is adjusted for, the measure of output itself is affected by price indices. In addition, quality adjusting materials input prices plays more of a modest role than quality adjusting output prices.

Table J1: Decomposition of sales and output under baseline and adjusted wages (2000 - 2012)

	Structural			
	Sales		Output	
	Wages as in baseline	Quality adjusted wages	Wages as in baseline	Quality adjusted wages
	(1)	(2)	(3)	(4)
TFPQ	0.287	0.244	1.129	1.080
Demand shock	1.141	1.141	0.273	0.273
ln pm	-0.051	-0.051	0.018	0.018
ln wage	-0.065	-0.026	-0.070	-0.025
ln markup	-0.005	-0.005	-0.005	-0.005
Wedge	-0.307	-0.303	-0.345	-0.341
Average R curvature parameter	**0.641**	**0.641**	**0.641**	**0.641**
Max R curvature parameter	**0.877**	**0.877**	**0.877**	**0.877**

Restricting to 2000 onwards due to the availability of information necessary to quality-adjust wages.

9 Appendix J: Decomposition with quality adjusted wages

We take advantage of data on broad skill categories available for 2000-2012 to construct quality-adjusted wages and a quality-adjusted labor input. Quality adjusted wages are built using a procedure analogous to the construction of our price indices for outputs and material inputs. Quality adjusted labor corresponds to the payroll deflated using quality-adjusted wages. The available skill categories are production workers without tertiary education, production workers with tertiary education and administrative workers.

Results of our sales growth decomposition using quality adjusted wages are reported in table J1, compared to results with unadjusted wages for the comparable subperiod. In turn, Table J2 presents results of the welfare analysis with quality adjusted wages.

Table J2: Counterfactual welfare relative to HK efficient welfare, without and with quality adjusted wages (2000-2012). Sector level parameters.

		Unadjusted wage		Quality adjusted wages	
		Average sector (1)	Average Sector - Revenue Weighted (2)	Average sector (3)	Average Sector - Revenue Weighted (4)
Panel A: Actual to HK Efficient Welfare					
		0.295	0.126	0.286	0.123
Panel B: Counterfactual to HK Efficient Welfare					
Plant attribute set to conterfact. Level	TFPQ HK	0.252	0.097	0.260	0.096
	Demand Shock	0.249	0.227	0.235	0.213
	Input prices + Markup	0.563	0.304	0.519	0.283
	Input prices	0.470	0.283	0.445	0.265
	Markup	0.329	0.134	0.321	0.131
	Wedge	0.488	0.288	0.496	0.299

10 Appendix K: Hottman, Redding and Weinstein framework accounting explicitly for wedges

Our framework closely follows the modeling of the demand side in Hottman, Redding and Weinstein (2016). On the cost side, however, they model total costs rather than efficiency and input prices individually, and do so at the product level rathe than the firm level. They also abstract from wedges. Expanding HRW's framework to include wedges explicitly, and focusing on the case of uniproduct firms where their approach and ours are equivalent, the firm solves:

$$\underset{Q_{ft}}{Max}\,(1-\tau_{ft})\,P_{ft}Q_{ft} - CT_{ft}\,(Q_{ft})$$

where $CT_{ft}\,(Q_{ft})$ is total cost as a function of output. Profit maximization leads to first orden condition $\left(P_{ft} + Q_{ft}\frac{dP_{ft}}{dQ_{ft}}\right) = \frac{\frac{\partial CT_{ft}}{\partial Q_{ft}}}{\left(1-\tau_{ft}\right)}$, so that at the optimum

$$\mu_{ft} = \frac{P_{ft}}{\frac{\partial CT_{ft}}{\partial Q_{ft}} (1-\tau_{ft})^{-1}} \tag{32}$$

. The associated optimal markup is given by (see appendix D):

$$\mu_{ft} = \frac{1}{1 - \left(\frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right) s_{ft}\right)} \tag{33}$$

Moreover, our demand structure is the same as in HRW. The implied demand function in the case of a uniproduct firm is:

$$Q_{ft} = d_{ft}^{\sigma} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma} \frac{E_t}{P_t} \tag{34}$$

$$or$$

$$R_{ft} = d_{ft}^{\sigma} \left(\frac{P_{ft}}{P_t}\right)^{1-\sigma} E_t \tag{35}$$

$$\frac{P_{ft}}{P_t} = d_{ft}^{\frac{\sigma}{\sigma-1}} s_{ft}^{\frac{1}{1-\sigma}} \tag{36}$$

where $R_{ft} = P_{ft} Q_{ft}$ is firm sales and $s_{ft} = \frac{R_{ft}}{E_{ft}}$ is the firm's share in aggregate (sector) sales. Equation 34 is HRW's equation (5) for the uniproduct case (where $d_{ft} = \varphi_{ft}^{\frac{\sigma-1}{\sigma}}$ and φ_{ft} is the notation used in HRW. Equation 36 is obtained by direct manipulation of 35.

Replacing the optimal markup rule 32 into 35 HRW decompose firm sales into:

$$R_{ft} = d_{ft}^{\sigma} \frac{E_t}{P_t^{1-\sigma}} \left(\mu_{ft} \frac{\frac{\partial CT_{ft}}{\partial Q_{ft}}}{1-\tau_{ft}}\right)^{1-\sigma} \tag{37}$$

which is equivalent to HRW's equation (16). To see the equivalence, notice that in the uniproduct case $\frac{\partial CT_{fjt}}{\partial Q_{fjt}} = \frac{\partial CT_{ft}}{\partial Q_{ft}}$ (where j is a product and HRW have denoted by $\widetilde{\gamma}_{ft}$ the average marginal cost across products of a firm), and that $d_{ft} = \varphi_{ft}^{\frac{\sigma-1}{\sigma}}$. Firm sales variability can thus be decomposed into variation attributable to : 1) an aggregate component; 2) firm idiosyncratic demand d_{ft}; 3) firm markup; 4) a distortion-adjusted marginal cost $\frac{mc_{ft}}{(1-\tau_{ft})}$.

HRW's empirical procedure is as follows:

1) Estimate the demand function 34, in differences with respect to aggregates and over time, to obtain σ and decompose price (observable) into d_{ft} (not observable) and s_{ft} (observable).

2) Estimate the markup μ_{ft} based on observables, using 33.

3) With these components decompose the idiosyncratic variation of sales from equation 37 into the contributions of d_{ft}, μ_{ft} and the residual component: $\frac{\frac{\partial CT_{ft}}{\partial Q_{ft}}}{(1-\tau_{ft})}$. This is a distortion-adjusted marginal cost component, which HRW do not further decompose into its $\frac{\partial CT_{ft}}{\partial Q_{ft}}$ and $(1-\tau_{ft})$ components.

11 Appendix L: Supplementary results

Production function coefficients by sector are shown in Table L1.

Table L1. Factor and demand elasticities by sector

Sector	β	α	ϕ	σ	γ	γ(1-1/σ)
311-313	0,14	0,06	0,81	7,61	1,01	0,88
321	0,16	0,09	0,70	4,34	0,94	0,69
322	0,10	0,08	0,77	5,12	0,95	0,76
323	0,12	0,06	0,75	5,63	0,93	0,77
324	0,25	0,13	0,57	2,26	0,96	0,53
331-332	0,48	0,07	0,36	1,53	0,91	0,31
341-342	0,37	0,12	0,55	2,15	1,04	0,55
351	0,32	0,29	0,43	2,40	1,04	0,61
352	0,18	0,07	0,81	4,29	1,06	0,81
355-356	0,25	0,18	0,62	3,34	1,05	0,71
362,369,371	0,50	0,21	0,36	2,50	1,08	0,65
381	0,26	0,09	0,66	3,39	1,01	0,60
382	0,49	0,08	0,44	1,68	1,01	0,41
383	0,27	0,06	0,66	6,26	1,00	0,84
384	0,10	0,07	0,79	4,00	0,96	0,72
385	0,18	0,15	0,76	1,64	1,08	0,42
390	0,05	0,14	0,85	2,12	1,04	0,55
Average	0,28	0,12	0,61	3,47	1,01	0,63

Counterfactual analysis of the impact of life cycle of wedges and fundamentals on welfare.

Table L2: Counterfactual welfare - relative to HK efficient welfare. Average sector, sector-level parameters

		Specific plant attributes set to constant mean value	Specific attributes of high life cycle growth plants (>P75) set to average life cycle growth for the rest	Specific attributes of low life cycle growth plants (<P25) set to average life cycle growth for the rest
		(1)	(2)	(3)
Benchmark: Actual to HK Efficient Welfare		0,278	0,278	0,278
Plant attribute set to counterfactual level	Demand Shock	0,126	0,122	0,315
	D+TFPQ	0,123	0,108	0,354
	Input prices + Markup	0,551	0,334	0,402
	Wedge	0,490	0,335	0,285