HPC for Structural Estimation

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2018 NBER SI
Outline

1. Introduction
2. HPC for Estimation
3. Examples
4. Finite Sample Performance
5. Conclusion
I am going to talk about what I know.

- Structural estimation is a broad field that uses many methods for many kinds of models.

- I work only in one corner of this very large field.

- Full solution estimation of dynamic models of the firm.
High performance computing is extremely useful for full solution methods

- One simple estimation can (now) be done on a workstation.

- An entire paper is hard to do on a single workstation.

- High performance computing has allowed me to do things with papers that I could never have done otherwise.
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The models I solve look like this

\[ V(w, z) = \max_{w'} \pi(w, w', z) + \beta \int V(w', z') dq(z', z) \]

- \( V(\cdot) \) is the value of the firm.
- A prime means tomorrow. No prime means today.
- \( w \) is a vector of endogenous state variables.
- \( z \) is a vector of exogenous state variables that follow a Markov process.
- \( q(z' \mid z) \) is the Markov transition function.
- \( \beta \) is a discount factor.
The solution is easy to parallelize

- Discretize \((w, z)\) into a finite number of feasible points.

\[
\{\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_N\}, \quad \{\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_M\}
\]

- Use
  - value function iteration (slow, reliable)
  - policy function iteration (faster, less reliable)
  - polynomial approximations to \(V\) (faster, squirrely)

  to solve the model

- Farm out the solution for each \((\tilde{w}_i, \tilde{z}_j)\) tuple out to a different thread on a workstation.

- Use OpenMP (intuitive) or MPI (unintuitive and usually faster)
Shared memory and unshared memory parallelization

- OpenMP is a set of compiler directives that make loops operate in parallel.
  - All of the instances of the loop can share variables in memory.

- MPI is a method of parallelization that does not require shared memory.
  - An entire section of code runs as many identical copies that are utterly independent of each other.
  - You can send info back and forth as necessary.
  - Hence, the name: Message Passing Interface.
Simulated minimum distance is the tool I use for estimation

- Compute statistics in actual data.
  \[ n^{-1} \sum_{i=1}^{n} h(x_i) \]

- Solve a model and simulate data from the model.

- Compute the exact same statistics in the simulated data.
  \[ S^{-1} \sum_{s=1}^{S} h(y_{is}(b)) \]

- Simulated data are a function of the model parameters, \( b \).

- Try to get the two sets of statistics as close as possible by choosing the model parameters.
More formally, . . .

Define

\[ g_n(b) = n^{-1} \sum_{i=1}^{n} \left[ h(x_i) - S^{-1} \sum_{s=1}^{S} h(y_{is}(b)) \right] . \]

The simulated moments estimator of \( b \) is then defined as the solution to the minimization of

\[ \hat{b} = \arg \min_b Q(b, n) \equiv g_n(b)' \hat{W}_n g_n(b) , \]

\( \hat{W}_n \) is a positive definite matrix.
I have tried many different minimization algorithms

- Cannot use gradient based methods.

- Need to rely on heuristic methods
  - Multistart Nelder Meade (can be parallelized, unreliable)
  - Simulated Annealing (cannot be parallelized, slow, reliable)
  - Differential Evolution (can be parallelized)
  - Particle Swarm (can be parallelized)

- The last two algorithms are useful on a many node cluster.
  - Very useful for models that take longer to solve: Michaels, Page, and Whited (2018), Gao, Whited, and Zhang (2018)
Hennessy and Whited (2005, 2007)

- I have been using distributed computing (though not high performance) since 2003.

- I used a multi-start Nelder Meade algorithm on a bunch unused PCs in the UW-Madison plasma physics lab.

- That was inefficient but the only thing available.
DeAngelo, DeAngelo, and Whited (2011)

- A kind PhD student gave me access to his account on his local HPC cluster.

- I produced this.
Cross-sectional heterogeneity

▶ The models I use are models of a single economic agent or of an industry with limited heterogeneity.

▶ It makes no sense to estimate these models on data generated by many extremely heterogeneous firms.

▶ I have found access to HPC clusters to be invaluable for examining cross-sectional heterogeneity.
  ▶ Nikolov and Whited (2014)
  ▶ Warusawitharna and Whited (2016)
  ▶ Li, Whited, and Wu (2016)
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All of the econometric estimators I use are based on the analogy principle

- GMM, M-estimators, Minimum distance estimators are all examples

- While they are asymptotically efficient and consistent

- They can have terrible finite sample properties
  - Arellano and Bond (1991), Altonji and Segal (1996), Hansen, Heaton, and Yaron (1996)

- Very few evaluations of the finite sample properties of the simulation counterparts of these estimators — SMM, SMD
Bazdresch, Kahn, and Whited (2018)

▶ And nothing for the class of simulated minimum distance estimators used in corporate finance.

▶ We evaluated the finite sample performance of these estimators on data generated from models of the firm.

▶ How do you do a Monte Carlo study of an estimator that can take days to converge?
  ▶ Use a relatively simple model.
  ▶ And ...
The finite sample properties of these estimators are surprisingly good!

▶ Estimators of the parameters have low RMSE and bias.

▶ Optimal weight matrices help with this a lot.

▶ Specification tests have excellent power to detect model misspecification.
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HPC is becoming more and more widespread

- Basic operating knowledge of Linux
- Knowledge of a fast language: C++, Fortran, Julia
- Learn parallelization paradigms
- Not as intimidating as it looks


